Large-time Dynamics of Classical and Quantum Systems

Toulouse, 12-14 November 2024

Programme

Time slot	Speaker	Title
November 12, Tuesday		
9:30 — 10:15	M. Porta	Dynamics of high density Fermi gases on a local scale
10:15 — 10:45	Coffee break	
10:45 — 11:30	M. Wrochna	Modular theory and entropy in QFT on curved spacetimes
11:35 — 12:05	C. Cedzich	Exponential tail estimates for quantum lattice dynamics
12:05 — 14:00	Lunch	
14:00 — 14:45	V. Jaksic	Some remarks on entropy production in quantum statistical mechanics
14:50 — 15:20	C. Tauber	Approach to equilibrium in quantum systems
15:20 — 15:45	Coffee break	
15:45 — 16:30	L. Bruneau	Some recent developments about quantum entropic fluctuations

November 13, Wednesday		
9:30 — 10:15	A. Joye	Quantum Walks Dynamics for use by the ANR
10:15 — 10:45		Coffee break
10:45 — 11:30	H. Cornean	A mathematical approach to N(on)E(quilibrium)G(reen)F(unctions)
11:35 — 12:05	N. Cuneo	Multifractal analysis of decoupled measures: what we know so far
12:05 — 14:00		Lunch
14:00 — 14:45	M. Correggi	Quasi-classical Limit of a Spin Coupled to a Reservoir
14:50 — 15:20	L. Daures	A large deviation principle for discrete, possibly reducible, Markov chains
15:20 — 15:45		Coffee break
15:45 — 16:30	A. Shirikyan	10 challenging problems in classical dynamics
November 14, Thursday		
9:30 — 10:15	T. Benoist	Repeated quantum measurements, recent results and outlook
10:15 — 10:45		Coffee break
10:45 — 11:30	C.-A. Pillet	
11:35 — 12:05	A. Szczepanek	Real-analyticity of the pressure function for products of matrices under irreducibility
12:05 — 14:00		Lunch

Title and abstracts

Tristan Benoist (Thursday, 9:30 — 10:15)

Title: *Repeated quantum measurements, recent results and outlook*

Abstract: I will give a short overview of recent results concerning ergodicity and limit theorems for repeated quantum measurements, both at the level of the measurement outcomes dynamical system and the underlying quantum trajectory Markov chain. I will then present a few conjectures, open questions and interesting extensions to be developed.

Laurent Bruneau (Tuesday, 15:45 — 16:30)

Title: *Some recent developments about quantum entropic fluctuations*

Abstract: Since the seminal works of Evans, Searles, Gallavotti and Cohen in the early 90's the study of entropic fluctuations has encountered a fast growing interest in the last decades, and many developments at least in classical systems. Its quantum counterpart however turned out to be very challenging. In this talk I will present some recent developments about possible extensions of these entropic fluctuations to the quantum domain, as well as some open problems.

The talk is based on the following recent series of papers (in collaboration with T. Benoist, V. Jaksic, A. Panati and C.-A. Pillet):

1. A note on two-times measurement entropy production and modular theory. Lett. Math. Phys. (2024).
2. On the thermodynamic limit of two-times measurement entropy production. To appear in Rev. Math. Phys.
3. Entropic fluctuations in statistical mechanics II. Quantum dynamical systems. Preprint arXiv 2409.15485
4. Entropic fluctuation theorems for Spin-Fermion model. In preparation

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Christopher Cedzich (Tuesday, 11:35 — 12:05)

Title: *Exponential tail estimates for quantum lattice dynamics*

Abstract: We consider the quantum dynamics of a particle on a lattice for large times. Assuming translation invariance, and either discrete or continuous time parameter, the distribution of the ballistically scaled position $Q(t)/t$ converges weakly to a distribution that is compactly supported in velocity space, essentially the distribution of group velocity in the initial state. We show that the total probability of velocities strictly outside the support of the asymptotic measure goes to zero exponentially with t, and we provide a simple method to estimate the exponential rate uniformly in the initial state.

Horia Cornean (Wednesday, 10:45 — 11:30)

Title: *A mathematical approach to N(on)E(quilibrium)G(reen)F(unctions)*

Abstract: Quantities like local particle densities and charge currents in mesoscopic systems are computed by a large community of solid-state physicists via the so-called NEGF formalism. We will rigorously define these Green functions and focus on the Dyson equation for the advanced/retarded interacting Green's function, by constructing its (irreducible) self-energy using the theory of Volterra operators. This is joint work with C.-A. Pillet and V. Moldoveanu.

Michele Correggi (Wednesday, 14:00 — 14:45)

Title: *Quasi-classical Limit of a Spin Coupled to a Reservoir*

Abstract: We consider a spin in contact with a bosonic reservoir, whose state contains a parameter interpolating between quantum and classical reservoir features. For energy conserving interactions we study decoherence and markovianity properties of the effective dynamics of the spin for all values of the parameter. Our main finding is that the spin decoherence

is enhanced (full decoherence) when the spin is coupled to quantum reservoir states while it is dampened (partial decoherence) when coupled to classical reservoir states. The markovianity properties depend in a subtle way on the classicality parameter and on the finer details of the spin-reservoir interaction. We further examine scattering and periodicity properties for energy exchange interactions.

Joint work with M. Falconi, M. Fantechi, M. Merkli.

Noé Cuneo (Wednesday, 11:35 — 12:05)

Title: *Multifractal analysis of decoupled measures: what we know so far*

Abstract: I will present some ongoing work with Vojkan Jakšić and Renaud Raquépas about multifractal analysis of decoupled measures. Our focus is on determining the fractal dimension of level sets for certain observables on shift spaces. I will introduce the necessary definitions and describe how Large Deviation results can be leveraged to compute the dimension of these level sets. If time permits, I will outline some challenges that remain in our approach.

Léo Daures (Wednesday, 14:50 — 15:20)

Title: *A large deviation principle for discrete, possibly reducible, Markov chains.*

Abstract: This talk is part of my ongoing PhD research. I will begin by presenting the subadditive method, an elegant approach for deriving large deviation principles (LDPs). This method is particularly effective for systems with strong decoupling assumptions, such as irreducible Markov chains, where it has become standard under strong recurrence conditions. However, when these assumptions are relaxed—specifically when the Markov chain is no longer irreducible—further adaptations of the method are required. I will highlight the additional challenges posed by this setting and discuss a construction that addresses them, enabling the use of the subadditive method. This construction ultimately leads to the

derivation of a weak LDP for any Markov chain on a discrete state space.

Vojkan Jaksic (Tuesday, 14:00 — 14:45)

Title: *Some remarks on entropy production in quantum statistical mechanics*

Abstract: I will introduce quantum mechanical entropic notions that play a role in the recent series of papers (joint work with T. Benoist, L. Bruneau, A. Panati, C-A. Pillet).

1. A note on two-times measurement entropy production and modular theory
2. On the thermodynamic limit of two-times measurement entropy production
3. Entropic Fluctuations in Statistical Mechanics II. Quantum Dynamical Systems
4. Entropic fluctuation theorems for spin-fermion model

This talk is an introduction to the follow up talk by L. Bruneau.

Alain Joye (Wednesday, 9:30 — 10:15)

Title: *Quantum Walks Dynamics for use by the ANR*

Abstract: We will begin by presenting a construction of unitary and open quantum walks on arbitrary graphs, called scattering quantum walks, and describe some of their key properties. Next, we will discuss the large times dynamics of ensembles of quantum walkers interacting with reservoirs. Finally, we will address several open problems that remain to be tackled within the framework of the ANR Dynacqus project.

Claude-Alain Pillet (Thursday, 10:45 — 11:30)

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Marcello Porta (Tuesday, 9:30 — 10:15)

Title: *Dynamics of high density Fermi gases on a local scale*

Abstract: In this talk I will discuss the dynamics of extended, relativistic Fermi gases, at high density. I will focus on the derivation of the Hartree equation, at zero temperature, as an effective description of many-body quantum dynamics. I will present a result that establishes convergence to the Hartree equation in the sense of expectation values of local observables, supported in regions with volume of order 1, independent of the density. The result applies to initial data describing systems at equilibrium confined in arbitrarily large domains, under the assumption that a suitable local Weyl-type estimate holds true. The proof is based on the approximation of the initial data through low temperature quasi-free states, for which a strong form of local semiclassical structure can be established, that also allows to capture decay of correlations in space. This plays a crucial role in the control of the growth of fluctuations created by the many-body dynamics on a local scale. Joint work with Luca Fresta (Bonn) and Benjamin Schlein (Zürich).

Armen Shirikyan (Wednesday, 15:45 — 16:30)

Title: *10 challenging problems in classical dynamics*

Abstract: We discuss four groups of problems arising in the theory of randomly forced PDE. They are related to mixing of a random flow, Donsker-Varadhan type large deviations, estimation of a parameter, and the vanishing noise limit. For each problem, we give a rather precise formulation, summarise some known results, and outline the main difficulties.

Anna Szczepanek (Thursday, 11:35 — 12:05)

Title: *Real-analyticity of the pressure function for products of matrices under irreducibility*

Abstract: The regularity of the pressure function (also known as the generalized Lyapunov exponent) plays an important role in various fields, e.g. in statistical mechanics or multifractal analysis. Pressure function $s \to k(s)$ is known to be differentiable on $(0,\infty)$ under (weak) irreducibility assumption, and analytic under an additional assumption of contractivity (a.k.a. purification). We claim the latter assumption can be lifted, i.e. irreducibility alone is sufficient for the analyticity of pressure.
The proof consists in constructing an operator $G(s)$ for which k(s) is a simple eigenvalue and spectral radius. The key step then is to show that a related Markov operator $Q(s)$ is quasi-compact: this guarantees that k(s) is an isolated eigenvalue of $G(s)$ and the analyticity of $s \to k(s)$ follows by holomorphic functional calculus. I will discuss how to prove the quasi-compactness of $Q(s)$ via Doeblin-Fortet inequality, using methods recently developed in the context of quantum trajectories. Joint work with T. Benoist, A. Hautecoeur, C. Pellegrini.

Clément Tauber (Tuesday, 14:50 — 15:20)

Title: *Approach to equilibrium in quantum systems*

Abstract: This talk is about the problem of approach to equilibrium in quantum systems. I will formulate the problem for translation invariant quantum systems (spins or fermions) on a lattice within the approach of algebraic quantum statistical

mechanics. Then, I will discuss some recent results relating the zeroth law of thermodynamics (approach to equilibrium) and the second law (increase of entropy). The main result is that approach to equilibrium is necessarily accompanied by a strict increase of the specific (mean) entropy. This talk is based on joint work with Vojkan Jaksic and Claude-Alain Pillet.

Michal Wrochna (Tuesday, 10:45 — 11:30)

Title: *Modular theory and entropy in QFT on curved spacetimes*

Abstract: In this talk I will review how modular theory enters the formalism of Quantum Field Theory on curved spacetimes, stressing the similarities with Quantum Statistical Mechanics. In particular I will discuss the Araki-Woods representation and the problem of defining and computing entanglement entropy, then outline possible directions of research for the ANR project (joint project with T. Benoist, D. Djordjevic, V. Jaksic).