Modeling, theory and numerics for PDEs (kinetic and hyperbolic systems)

Rapport sur les contributions

https://indico.math.cnrs.fr/e/12641

ID de Contribution: **28** Type: **Non spécifié**

Transport and semi-Lagrangian particles

jeudi 10 octobre 2024 09:00 (45 minutes)

Orateur: M. COTTET, Georges-Henri

ID de Contribution: **29** Type: **Non spécifié**

Long-time behavior for Vlasov-Fokker-Planck with general potentials

mardi 8 octobre 2024 09:50 (45 minutes)

In this presentation, I will talk about the well-posedness, steady states and long time behavior of solutions to Vlasov-Fokker-Planck equation with external confinement potential and self-consistent interactions. Compared to previous works on this topic, our results allow for large, singular and non-symmetric interactions. As a corollary of our main results, we show exponential decay of solutions to the Vlasov-Poisson-Fokker-Planck equation in dimension 3, for low regularity initial data, and arbitrarily small Debye length. This is a work in collaboration with Pierre Gervais.

Orateur: M. HERDA, Maxime

ID de Contribution: **31** Type: **Non spécifié**

Linear and Non Linear Stability for the kinetic plasma sheath on a bounded interval

mardi 8 octobre 2024 09:00 (45 minutes)

Plasma sheaths are inhomogeneous equilibrium that form when a plasma is in contact with an absorbing wall. We prove linear and non linear stability of a kinetic sheath equilibrium for a Vlasov-Poisson type system in a bounded interval. Notably, in the linear setting, we obtain exponential decay of the fluctuation provided the rate of injection of particles at equilibrium is smaller that the rate of absorption at the wall. In the non linear setting, we prove a similar result for small enough equilibrium and small localized perturbation of the equilibrium.

Orateur: M. BADSI, Mehdi

ID de Contribution: **32** Type: **Non spécifié**

Hyperbolic approximation of parabolic systems : hypocoercivity and harmonic analysis

mercredi 9 octobre 2024 09:50 (45 minutes)

Orateur: M. CRIN-BARRAT, Timothée

ID de Contribution: **33** Type: **Non spécifié**

Hydrodynamic limit of elastic kinetic equations by a spectral approach

jeudi 10 octobre 2024 14:35 (45 minutes)

[FRANÇAIS] Limite hydrodynamique d'équations cinétiques conservatives par une approche spectrale Parmi les 23 problèmes listés par D. Hilbert durant le Congrès International des Mathématiciens en 1900, le 6ème concerne la dérivation de descriptions macroscopiques des fluides à partir de leurs descriptions microscopique. Une des stratégies possibles consiste à passer par un niveau de description intermédiaire qualifié de mésoscopique, ou cinétique, tels que les modèles de Boltzmann ou Landau. On parle de problème de **limites hydrodynamiques**. Au début des années 1990, C. Bardos, F. Golse et D. Levermore prouvèrent que l'on pouvait dériver formellement les équations de Navier-Stokes à partir d'équations cinétiques conservant la masse, vitesse et énergie, et dissipant l'entropie, et les cas spécifiques des équations de Boltzmann et Landau furent progressivement et indépendamment traités durant les trois décennies suivantes, malgré leur **structure commune**. Les travaux de limites hydrodynamiques sont en partie limités par des outils remontant aux débuts de la théorie de Boltzmann dans les années 1960, permettant seulement de considérer des solutions satisfaisant une hypothèse très contraignante d'intégrabilité, mais aussi par des résultats établis à l'aide d'arguments **non-constructifs**. Dans le cas des théories de Cauchy d'équations cinétiques, ces restrictions ont été levées grâce aux outils modernes de "théorie d'élargissement" et d'hypocoercivité développés à partir des années 2000, notamment par C. Mouhot, S. Mischler et M. Gualdani. Dans cet exposé, je présente un travail en collaboration avec Bertrand Lods dans lequel nous avons, d'une part, considéré la question de limite hydrodynamique pour une équation cinétique **sous des hypothèses génériques** proches de celles de Bardos-Golse-Levermore, unifiant ainsi les résultats antérieurs, d'autre part, modernisé l'étude spectrale nécessaire grâce aux nouvelles théories d'élargissement et d'hypocoercivité, fournissant ainsi les premiers résultats de limites hydrodynamiques **entièrement quantitatif**. [ENGLISH] Hydrodynamic limit of elastic kinetic equations by a spectral approach Among the 23 problems listed by D. Hilbert during the International Congress of Mathematicians in 1900, the 6th one concerns the derivation of macroscopic descriptions of fluids from their microscopic descriptions. One possible strategy involves going through an intermediate level of description called mesoscopic, or kinetic, such as the Boltzmann or Landau models. This is referred to as the problem of **hydrodynamic limits**. In the early 1990s, C. Bardos, F. Golse, and D. Levermore proved that one could formally derive the Navier-Stokes equations from kinetic equations conserving mass, velocity, and energy, and dissipating entropy, and the specific cases of the Boltzmann and Landau equations were gradually and independently addressed over the following three decades, despite their **common structure**. The work on hydrodynamic limits is partly constrained by tools dating back to the early days of Boltzmann theory in the 1960s, allowing only for solutions satisfying a very restrictive integrability assumption, but also by results established using **non-constructive** arguments. In the case of Cauchy theories of kinetic equations, these restrictions have been lifted thanks to modern tools of "enlargement theory" and hypocoercivity methods developed from the 2000s onwards, notably by C. Mouhot, S. Mischler, and M. Gualdani. In this talk, I present a collaboration with Bertrand Lods in which we have, on the one hand, considered the question of hydrodynamic limit for a kinetic equation **under generic assumptions** close to those of Bardos-Golse-Levermore, thus unifying previous results, and, on the other hand, modernized the necessary spectral study using the new theories of enlargement and hypocoercivity, thus providing the first fully quantitative results of

hydrodynamic limits.

Orateur: M. GERVAIS, Pierre

ID de Contribution: **34** Type: **Non spécifié**

TBA

mercredi 9 octobre 2024 09:00 (45 minutes)

Orateur: M. HÉRAU, Frédéric

ID de Contribution: **36** Type: **Non spécifié**

On the quadratic stability of asymmetric Hermite basis and application to plasma physics

mardi 8 octobre 2024 11:10 (45 minutes)

We analyze why the discretization of linear transport with asymmetric Hermite basis functions can be instable in quadratic norm. The main reason is that the finite truncation of the infinite moment linear system looses the skew-symmetry property with respect to the Gram matrix. Then we propose an original closed formula for the scalar product of any pair of asymmetric basis functions. It makes possible the construction of two simple modifications of the linear systems which recover the skew-symmetry property. By construction the new methods are quadratically stable with respect to the natural L2 norm. We explain how to generalize to other transport equations encountered in numerical plasma physics. Basic numerical tests illustrate the unconditional stability properties of our algorithms.

Orateur: M. DAI, Ruiyang

ID de Contribution: **37** Type: **Non spécifié**

Conservative cascade schemes for Vlasov type equations

jeudi 10 octobre 2024 09:50 (45 minutes)

The conservative cascade method permits to replace a complex multi-dimensionnal conservative remapping with a sequence (”cascade”) of 1D conservative remappings. It shares some features with splitting schemes (in particular the 1D feature permits to ease the parallelization), but it has also some differences, as it remains based on multi-dimensionnal characteristics, which can be more convenient in some situations. We will here present and apply such a cascade method in the context of Vlasov type equations.

Orateur: M. MEHRENBERGER, Michel

ID de Contribution: **38** Type: **Non spécifié**

Optimal control in ferromagnetism: minimal time of magnetization switching

vendredi 11 octobre 2024 09:00 (45 minutes)

In this talk, we consider an ellipsoidal ferromagnetic material exposed to an external magnetic field. The magnetization of the material is modeled by the Landau-Lifshitz equation. We are interested in the following question: can we reverse the magnetization of the material in minimal time by using the external magnetic field as our control variable? We prove that, depending on the material's ellipsoidal geometry, there is a threshold value for the magnetic field that allows reversal or not. This is a joint work with Raphaël Côte, Guillaume Ferrière and Yannick Privat.

Orateur: Mme COURTES, Clémentine

ID de Contribution: **39** Type: **Non spécifié**

Spectral scheme for the linear Boltzmann BGK equation on the real line

mercredi 9 octobre 2024 11:10 (45 minutes)

We present a fully spectral scheme in both space and velocity for an inhomogeneous kinetic equation on the real axis. The collision operator admits several conservation laws, and their number depends on the harmonicity of the potential phi. Our scheme is based upon a projection on Hermite polynomials in velocity and on orthonormal polynomials with respect to the weight exp(-phi) in space. For a polynomial potential phi, the scheme preserves every conservation laws and hypocercivity. If there is time, we will talk about approximation by orthonormal polynomials associated with an exponential weight.

Orateur: M. GROSSE, Bastien

ID de Contribution: **41** Type: **Non spécifié**

TBA

Orateur: BOUIN, Emeric

ID de Contribution: **43** Type: **Non spécifié**

Discontinuous Galerkin methods and basis enrichment using neural networks

vendredi 11 octobre 2024 09:50 (45 minutes)

The Discontinuous Galerkin method is a high-order accurate numerical scheme to solve hyperbolic systems. In this talk, we will present a strategy to take advantage of the recent development of neural networks to enhance the method's precision around a parametric family of equilibria. The method is based on introducing a prior solution inside the basis, which can be estimated using Physic Informed Neural Networks (PINNs).

Orateur: M. NAVORET, Laurent

ID de Contribution: **44** Type: **Non spécifié**

Particle-Free-Lagrange methods for transport equations

jeudi 10 octobre 2024 13:45 (45 minutes)

Orateur: M. DESPRÈS, Bruno

ID de Contribution: **45** Type: **Non spécifié**

Dynamic metastability in the self-attention model

vendredi 11 octobre 2024 11:10 (45 minutes)

The pure self-attention model is a simplification of the celebrated Transformer architecture, which neglects multi-layer perceptron layers and includes only a single inverse temperature parameter. Despite its apparent simplicity, the model exhibits a remarkably similar qualitative behavior across layers to that observed empirically in a pre-trained Transformer. Viewing layers as a time variable, the self-attention model can be interpreted as an interacting particle system on the unit sphere. We show that when the temperature is sufficiently high, all particles collapse into a single cluster exponentially fast. On the other hand, when the temperature falls below a certain threshold, we show that although the particles eventually collapse into a single cluster, the required time is at least exponentially long. This is a manifestation of dynamic metastability: particles remain trapped in a "slow manifold" consisting of several clusters for exponentially long periods of time. Our proofs make use of the fact that the self-attention model can be written as the gradient flow of a specific interaction energy functional previously found in combinatorics.

Orateur: M. GESHKOVSKI, Borjan

ID de Contribution: **47** Type: **Non spécifié**

Sub-exponential tails in biased run and tumble equations with unbounded velocities

mardi 8 octobre 2024 14:35 (45 minutes)

In this talk we are going to present some recent results about the Run and Tumble equations, a kinetic model for the movement of bacteria subjected to the presence of a chemotactic substance. Unlike many previous articles, in this work in collaboation with Emeric Bouin and Josephine Evans, we consider the Run and Tumble equation when the set velocities is the whole space. The distribution M of post-reorientation velocities is considered to be sub-exponentially or super-exponentially decaying. We also present an ongoing numerical study of the model, carried out with Francis Filbet.

Orateur: M. ZIVIANI, Luca

ID de Contribution: **48** Type: **Non spécifié**

Linear / nonlinear approaches for the approximation of convection-diffusion equations

mardi 8 octobre 2024 13:45 (45 minutes)

Orateur: Mme CHAINAIS-HILLAIRET, Claire

ID de Contribution: **49** Type: **Non spécifié**

Numerical simulation of Vlasov-Poisson system with strong external magnetic field

jeudi 10 octobre 2024 11:10 (45 minutes)

Orateur: M. TRINH, Kim Han

ID de Contribution: **50** Type: **Non spécifié**

On the stability of numerical schemes for linear transport with boundary

mardi 8 octobre 2024 15:55 (45 minutes)

The goal is the study the stability of explicit finite difference schemes for the one-dimensional advection equation with an inflow boundary condition, the outflow case being rather well understood. We reformulate the so-called strong stability by introducing the intrinsic Kreiss-Lopatinskii determinant, which possesses the same regularity as the vector bundle of discrete stable solutions. In practice, we are able to link this analysis with a (robust and cheap) computation of some winding number.

Orateur: M. SEGUIN, Nicolas

ID de Contribution: **52** Type: **Non spécifié**

A multi-dimensional staggered scheme for the diffusive limit in the radiative transfer equation

mardi 8 octobre 2024 16:45 (45 minutes)

The radiative transfer equation is a kinetic PDE modelling the specific radiation intensity carried by a population of photons described by a statistical description, i.e. a transport equation on the fraction of photons travelling in a given direction. It is well known that as the Knudsen number (which is the ratio of the mean free path length to a representative physical length scale) goes to zero, the radiation intensity tends to a solution of a diffusion problem.

In this talk, we present a numerical scheme for the radiative transfer equation that has the asymptotic preserving property: when the Knudsen number is fixed, we prove that the numerical solution converges to a solution of the radiative transport equation for vanishing discretisation parameters. And for a fixed discretisation, the numerical solution converges to the solution of a stable and consistent numerical scheme for the limit diffusion equation. The numerical scheme considered is an extension to the multidimensional setting of the 1D scheme developed by Lemou and Mieussens in their 2008 SIAM paper. It is based on a micro-macro decomposition of the main unknown and on a staggered discretisation: the macroscopic variable is cell-centred while the microscopic variable is face-centred. We show that special consistency problems arise in the multi-dimensional setting due to the fact that composing a consistent discrete divergence with a (weakly) consistent discret gradient does not generally yield a consistent Laplacian operator, even on admissible grids.

This is a joint work with Mohamed Ghattassi and Nader Masmoudi.

Orateur: M. SALEH, Khaled