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Local linear forms on pseudodifferential operators and index theory

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This talk discusses **local** linear forms $\Lambda : A \mapsto \Lambda(A)$ on classical pseudo-differential operators on a closed manifold, namely linear forms of the type $\Lambda(A) = \int_M \lambda_A(x)\, dx$ given by a density $\lambda_A(x)\, dx$ on the manifold M and their relevance in index theory.

Local linear forms are spanned by the well-known Wodzicki residue $A \longmapsto \mathrm{Res}(A)$ on integer order operators and the somewhat lesser known canonical trace $A \longmapsto \mathrm{TR}(A)$ on non-integer order operators (joint work with S. Azzali).

For a **holomorphic perturbation** $A(z)$ of a differential operator $A(0) = A$, these two linear forms relate by (joint work with S. Scott)
\[
(1)\,\,\,\,\, \,\,lim_{z\to 0}TR(A(z))= -\frac{1}{2}\, Res(A^\prime(0)),
\]
where the residue has been extended to logarithms. Inspired by Gilkey's approach using invariance theory, for a family $A(z)$ of geometric operators, we showed (joint work with J. Mickelsson) that the density $\mathrm{res}_x(A'(0))$ arising in the r.h.s. of (1) is an invariant polynomial which can be expressed in terms of Pontryagin forms on the tangent bundle and Chern forms on the auxillary bundle.

A $\mathbb{Z}_2$-graded generalisation of (1) applied to an appropriate holomorphic perturbation of the identity built from a Dirac operator $D = D_+ \oplus D_-$ acting on a $\mathbb{Z}_2$-graded vector bundle, expresses the **index** of D_+ in terms of a Wodzicki residue. As a result of their locality, the canonical trace and the Wodzicki residue are preserved under lifting to the universal covering of a closed manifold; consequently formula (1) lifts to coverings. This lifted analoque of (1) yields an expression of the L^2-index of a lifted Dirac operator in terms of the Wodzicki residue of the logarithm of its square (joint work with S. Azzali).

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