

$SL(2, \mathbb{Z})$ -action on unipotent characters for finite reductive groups

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The space of class functions on a reductive group over a finite field (such as $GL(n, q)$, $Sp(2n, q)$, etc.) admits two particularly interesting bases:

- an algebraic basis, given by the characters of irreducible representations,
- a geometric basis, given by the characteristic functions of character sheaves.

These two bases are related through a transformation that generalizes the classical Fourier transform on finite abelian groups.

In ongoing joint work with Bonnafé, Broué, Malle, Michel, and Rouquier, we aim to provide a geometric interpretation of this transformation in the case of unipotent class functions, using traces of braid group operators acting on Deligne–Lusztig varieties. This transformation is part of a broader $SL(2, \mathbb{Z})$ -action that encapsulates the Fourier transform, the Frobenius eigenvalues on the cohomology of Deligne–Lusztig varieties, and Shintani’s twisting operator (which interchanges the Frobenius with its inverse).

One of the key advantages of this approach is that it extends naturally to settings where the underlying root datum is governed by complex reflection groups rather than just Weyl groups. This provides natural candidates for Fourier matrices and unipotent character sheaves in the context of “Spetses”.

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