2025 IHES Summer School - Statistical Aspects of Nonlinear Physics

Monday, June 23, 2025 - Friday, July 4, 2025

Le Bois-Marie

Program

Disorder Landscapes, out of equilibrium dynamics and AI

Gérard Ben Arous - Exploring the high-dimensional random landscapes of Data Science

Machine learning and Data science algorithms involve in their last stage the need for optimization of complex random functions in very high dimensions. I will briefly survey how these landscapes can be topologically complex, whether this is important or not, and how the usual simple optimization algorithms like Stochastic Gradient Descent (SGD) perform in these difficult contexts. I will mostly rely on joint works with Reza Gheissari (Northwestern), Aukosh Jagganath (Waterloo), Jiaoyang Huang (Wharton).

I will first introduce the whole framework for non-experts, from the structure of the typical tasks to the natural structures of neural nets used in standard contexts. I will then cover briefly the classical and usual context of SGD in finite dimensions.
We will then concentrate first on a simple example, which happens to be close to statistical physics, i.e. the so-called Tensor PCA model, and show how it is related to spherical spin glasses. We will survey the topological properties of this model, and how simple algorithms perform in the basic estimation task of a single spike. We will then move on to the so-called single index models, and then to the notion of "effective dynamics" and "summary statistics". These effective dynamics, when they exist, run in much smaller dimension and rule the performance of the algorithm.
The next step will be to understand how the system finds these "summary statistics". We will show how this is based on a dynamical spectral transition: along the trajectory of the optimization path, the Gram matrix or the Hessian matrix develop outliers which carry these effective dynamics. I will naturally first come back to the Random Matrix tools needed here (the behavior of the edge of the spectrum and the BBP transition) in a much broader context. We will illustrate this with a few examples from ML: multilayer neural nets for classification (of Gaussian mixtures), and the XOR examples, for instance, or from Statistics like the multi-spike Tensor PCA (from a recent joint work with Cedric Gerbelot (Courant) and Vanessa Piccolo (ENS Lyon)).

Giulio Biroli - Generative AI and Diffusion Models: a Statistical Physics Analysis

We will first present « diffusion models » which are nowadays the state of the art methods used to generate images, videos and sounds. They are very much related to ideas developed in stochastic thermodynamics, and based on time-reversing stochastic processes. The outcome of these methods is a Langevin process that generates from white noise (the initial condition) new images, videos and sounds. We will show that tools of statistical physics allow to characterise two main phenomena emerging during the Langevin generative diffusion process. The first one, that we call 'speciation' transition, is where the gross structure of data is unraveled, through a mechanism similar to symmetry breaking in phase transitions. The second phenomenon is the generalisation-memorisation transition, which turns out to be related to the glass transition of Derrida's random energy model. We will present analytical solutions for simple models and show numerical experiments on real datasets which validate the theoretical analysis.

Long-range interactions

Satya N. Majumdar - Nonequilibrium point processes with long-range correlations generated by stochastic resetting

In this course, I'll first discuss briefly the equilibrium systems with long-range correlations (such as the log-gas or the Riesz gas in coordination with the course by S. Serfaty). Then the main emphasis of my course would be on nonequilibrium stationary states of many-body systems with attractive long-range correlations. I will provide examples of how such states can be generated by stochastic resetting of N diffusing particles in one or higher dimensions. The joint distribution of the positions of the particles, though not factorizable due to strong correlations, has a certain conditionally independent structure that allows them to be exactly solvable for various physical obsevables. For example, in one dimension and in the large N limit, one can compute exactly, the average density, the extreme value statistics, the spacing distribution between particles as well as the full counting statistics. The presence of long-range correlations make these observables drastically different from the equilibrium ideal gas where the joint distribution in the stationary state factorizes.

Sylvia Serfaty - Systems with Coulomb and Riesz Interactions

We will focus on the statistical mechanics of large systems of particles with Coulomb or Riesz long-range repulsion, analyzed via an electrostatic formulation. Topics covered include: equilibrium measure, expansion of the free energy, limit point processes, fluctuations.

Random interfaces

Kazumasa Takeuchi - Introduction to the physics of the KPZ universality class

The Kardar-Parisi-Zhang (KPZ) class is a prominent universality class known to describe a surprising variety of models and phenomena, including growing interfaces, directed polymers in random media, stochastic particle transport, and most recently even quantum systems such as the exciton-polariton condensate and integrable spin chains. Remarkably, the 1D KPZ class is exactly solvable in many aspects, opening a window to detailed statistical properties through its links to various mathematical problems. The lecture will include a pedagogical introduction to basic properties of the KPZ equation, an illustrative outline of characteristic distribution and correlation properties revealed by exact solutions, and a survey of experimental studies on growing interfaces.
1. Introduction: why should we care this?
2. Scaling exponents and universality classes
3. Basic properties of the KPZ equation
4. Distribution and correlation properties: stationary & non-stationary cases
5. Experimental test of predictions from integrable models
6. Distribution properties for general cases and variational formula

Main reference: K. A. Takeuchi, "An appetizer to modern developments on the Kardar-Parisi-Zhang universality class", Physica A 504, 77-105 (2018).

Ivan Corwin

Lecture 1: Extreme diffusion; or was Einstein wrong about diffusion?
In a system of many particles diffusing in a common environment, the first few particles often have outsized importance. How do they behave and what does that behavior tell us about the environment in which they have evolved? We will approach these problems by studying random walks in random environments and utilizing a connection with the Kardar-Parisi-Zhang stochastic PDE and universality class. I will describe recent work which uncovers new power-laws beyond

those of Einstein's theory of diffusion and introduces the Extreme Diffusion Coefficient that captures new microscopic information about the environment.

Lecture 2: How do boundary conditions influence random interface growth?
In modeling real interface growth, there are several types of boundary conditions that can be imposed. In this lecture we will probe some of the predictions and results about how these boundary conditions influence the long-time behavior of interface growth. In particular, we will focus on the stationary measure for such growth models, as well as the time to approach stationarity and the current fluctuations in stationarity.

Lecture 3: What is the universal scaling limit of random interface growth, and what does it tell us?
KPZ scaling leads to a natural renormalization of the entire space-time trajectory of random interface growth models. What is the universal limit of this process? I will first introduce the KPZ fixed point and directed landscape which characterize this limit. Then, I will describe some implications of this full scaling limit, in particular to the theory of fluctuating hydrodynamics and multispecies interacting particle systems. Finally, I will sketch how the Yang-Baxter equation -- the fundamental relation from quantum integrable systems -- gives the key to unlocking this scaling limit.

Collective behavior

Julien Tailleur - Active matter

Bertrand Maury - Crowd movements