JHH80

JUNE 12

Caroline Davis

freelance mathematician for June ☺

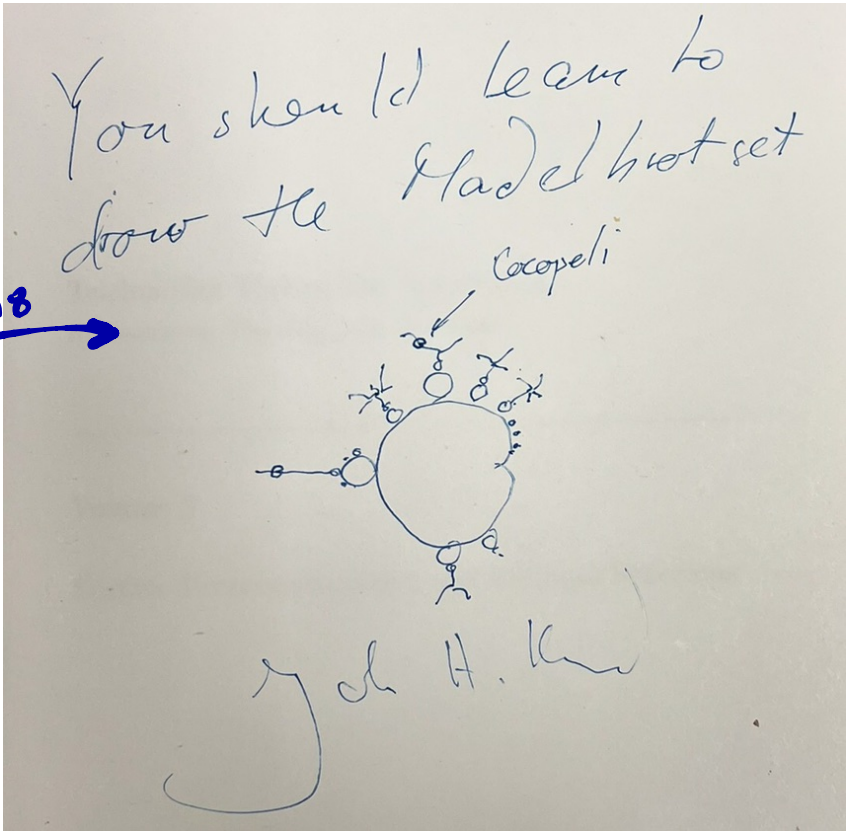
Math 636!

2018

end May 31, '25

Stony Brook University

start July 1, '25

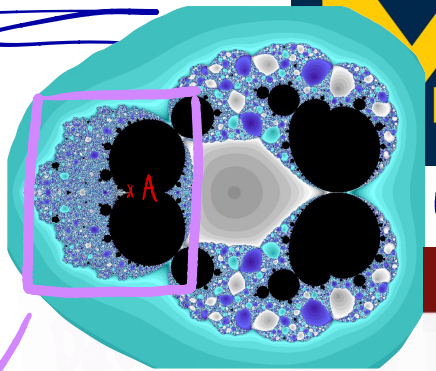


Locus of Matings + Captures in $Per_n(0)$

or: 2-sided DH theory

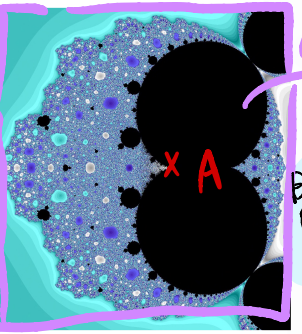
JHH80

Per$_3$(0)



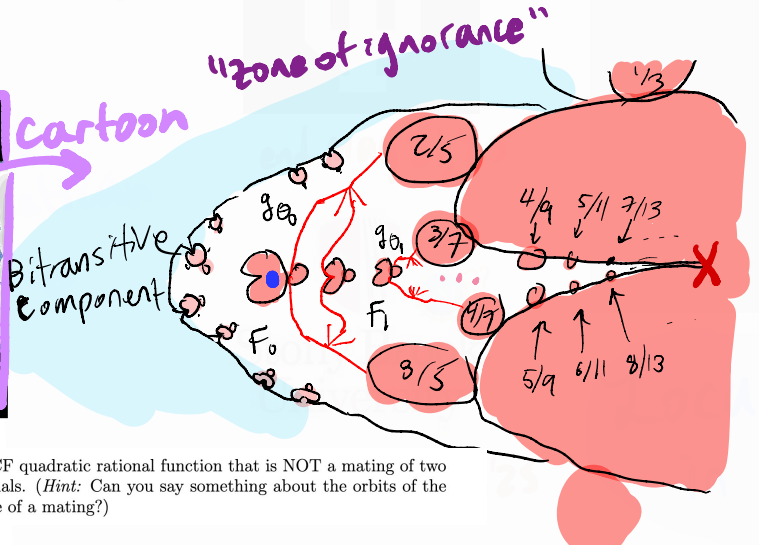
636!

Zoom



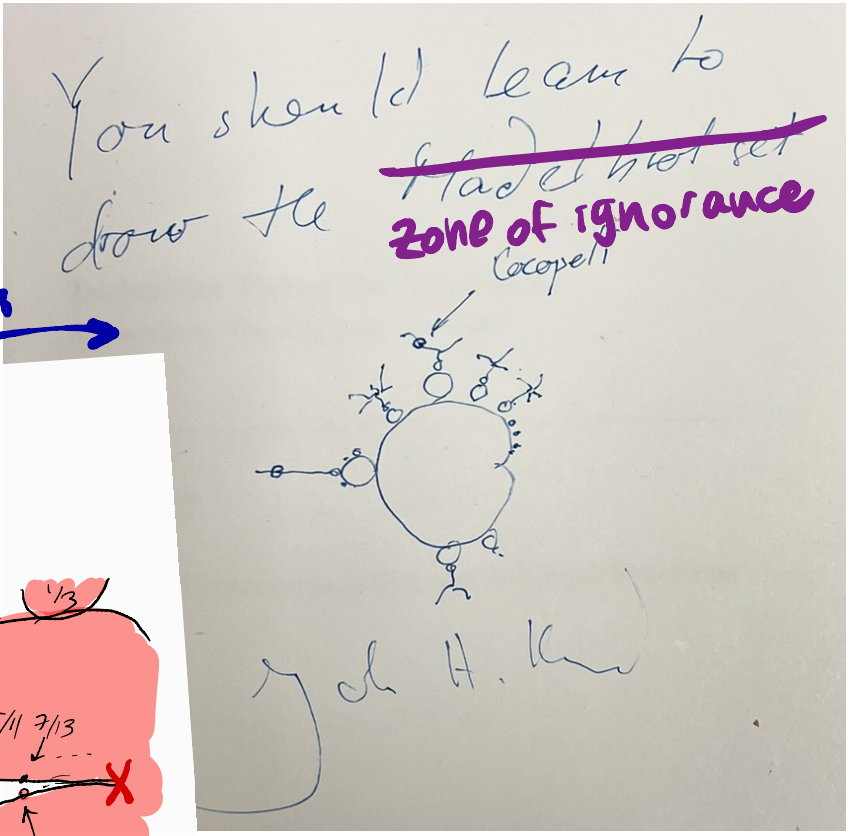
cartoon

"zone of ignorance"



(127) Find an example of a PCF quadratic rational function that is NOT a mating of two PCF quadratic polynomials. (*Hint:* Can you say something about the orbits of the critical points in the case of a mating?)

2018

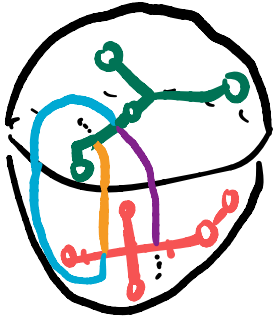


of Matings + Captures

Per$_n$(0) or: 2-sided DH theory

Matings:

- all the ways to combine K_f with K_g

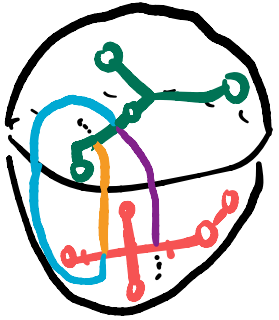


← "Ray equivalence classes" can be long:

- mating is disjoint type, verified by equators

Matings:

- all the ways to combine K_f with K_g

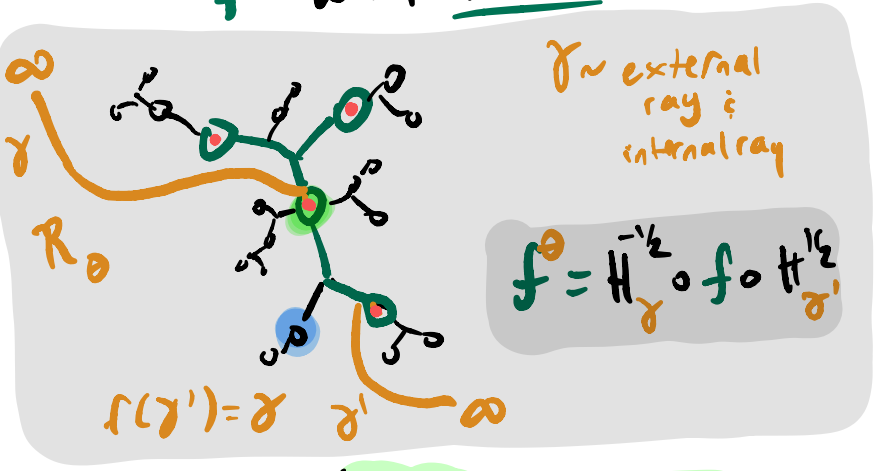


← "Ray equivalence classes" can be long:

- mating is **disjoint type**, verified by equators

Captures:

- all the ways to combine K_f with **itself**.



- Becomes

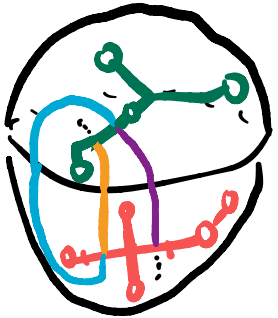
type B ⇝ periodic component

or

type C ⇝ preperiodic component

Matings:

- all the ways to combine K_f with K_g



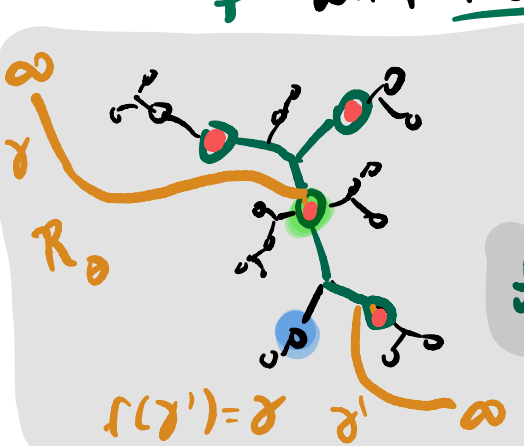
← "Ray equivalence classes" can be long:

- mating is disjoint type, verified by equators

Captures:

side matters

- all the ways to combine K_f with itself.



γ ~ external ray & internal ray

$$f^\theta = H_\gamma^{-1/2} \circ f \circ H_{\gamma'}^{1/2}$$

- Becomes

type B ⇝ periodic component

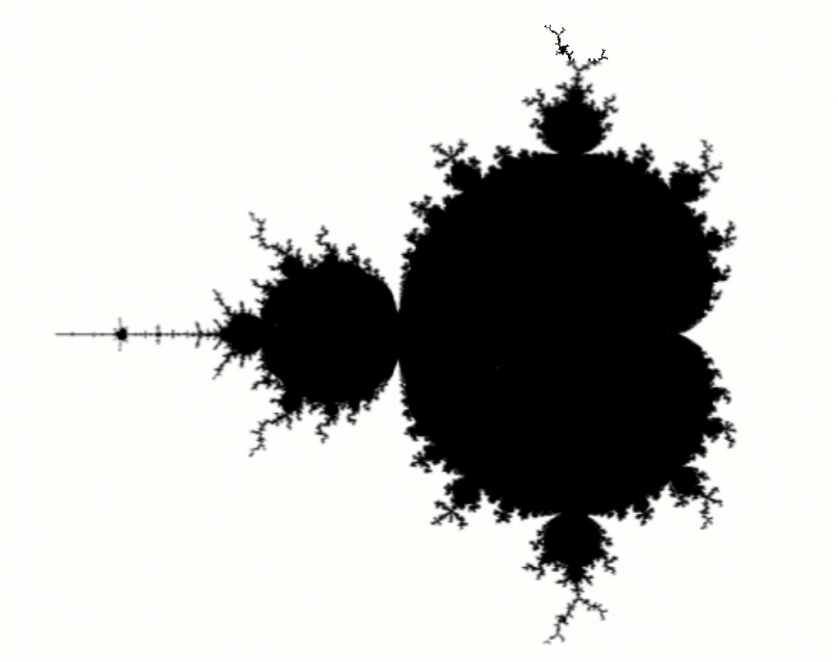
or

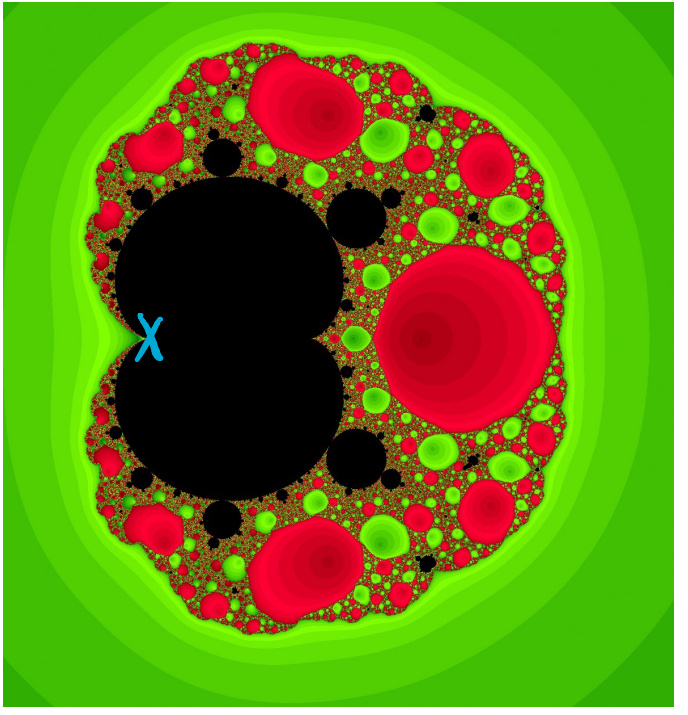
type C ⇝ preperiodic component

wolf video!

$\mathrm{Per}_1(0) = \{$ QRMs w/ superattracting fixed pt $\}$

$\mathrm{Per}_2(0) = \{$ QRMs w/ superattracting 2-cycle $\}$



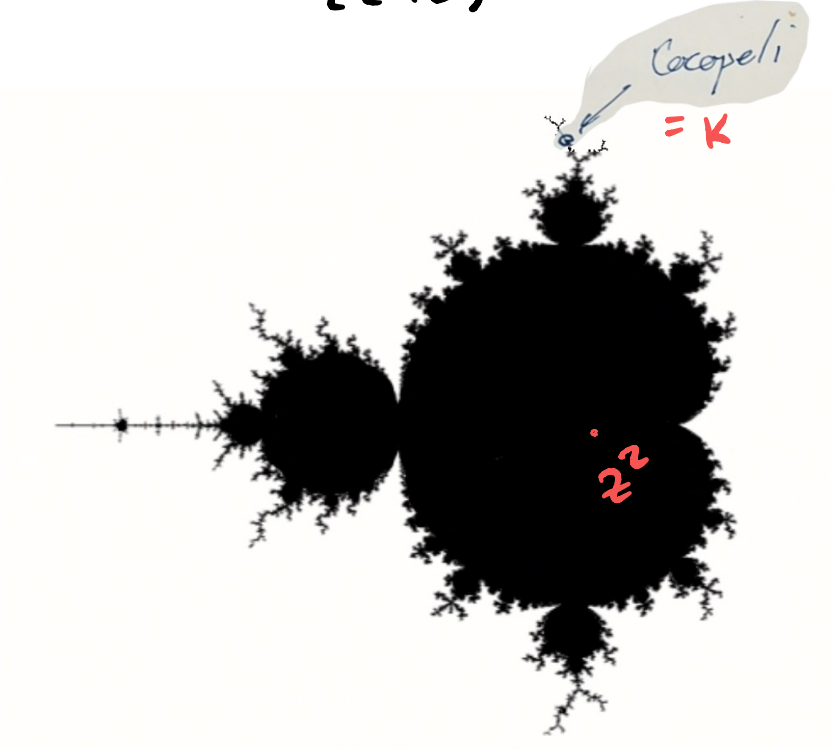


$$\mathrm{Per}_1(0) = \{\text{QRMs w/ superattracting fixed pt } 0\}$$

$$\simeq \{z^2+c\}$$

a) $M \subset \mathrm{Per}_1(0)$

$= \{z^2+c \mid \text{orbit of } 0 \not\to \infty\}$



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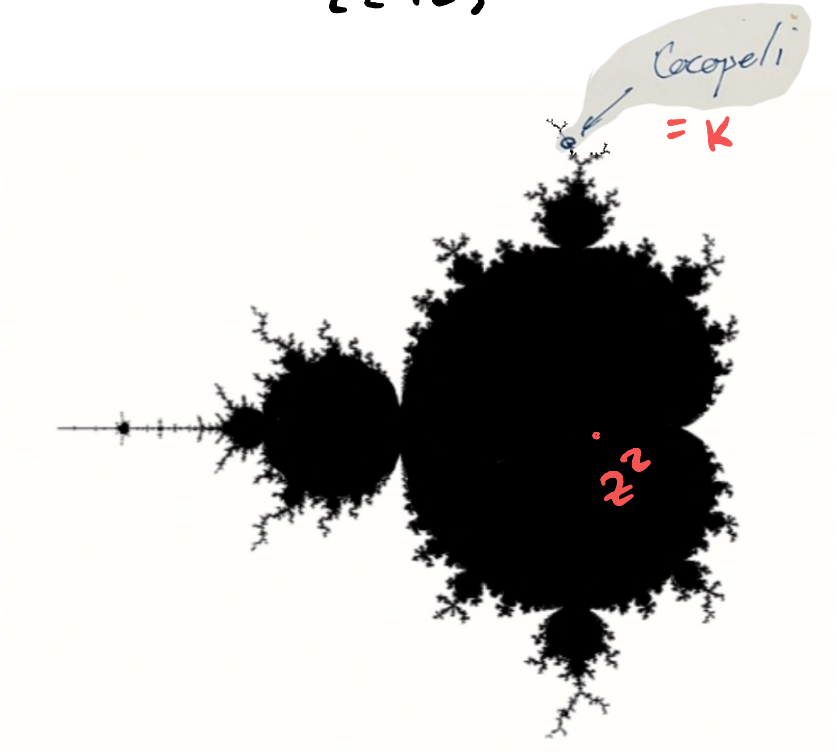

a) $M \subset Per_1(0)$

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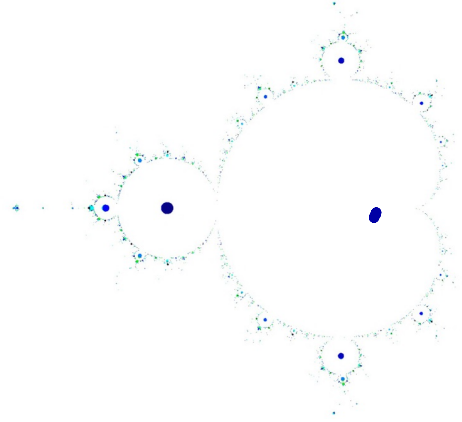
b) $G(n) \subset M$

$= \{z^2+c \mid 0 \text{ in (exact) } n\text{-cycle}\}$



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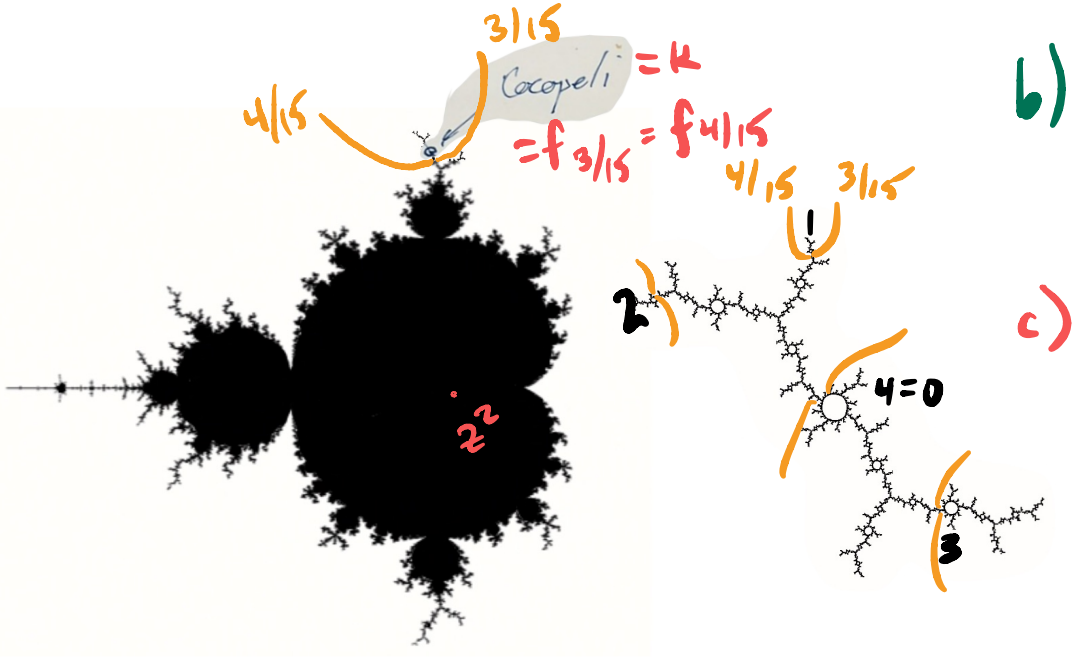

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c) M connected:

$\exists \Phi : \mathbb{C}-\overline{\mathbb{D}} \to \mathbb{C}-M$

$\rightsquigarrow$ external rays

characteristic $f = f_{\theta_-} = f_{\theta_+}$

$Per_1(0) = \{$ QRMs w/ superattracting fixed pt $0\}$

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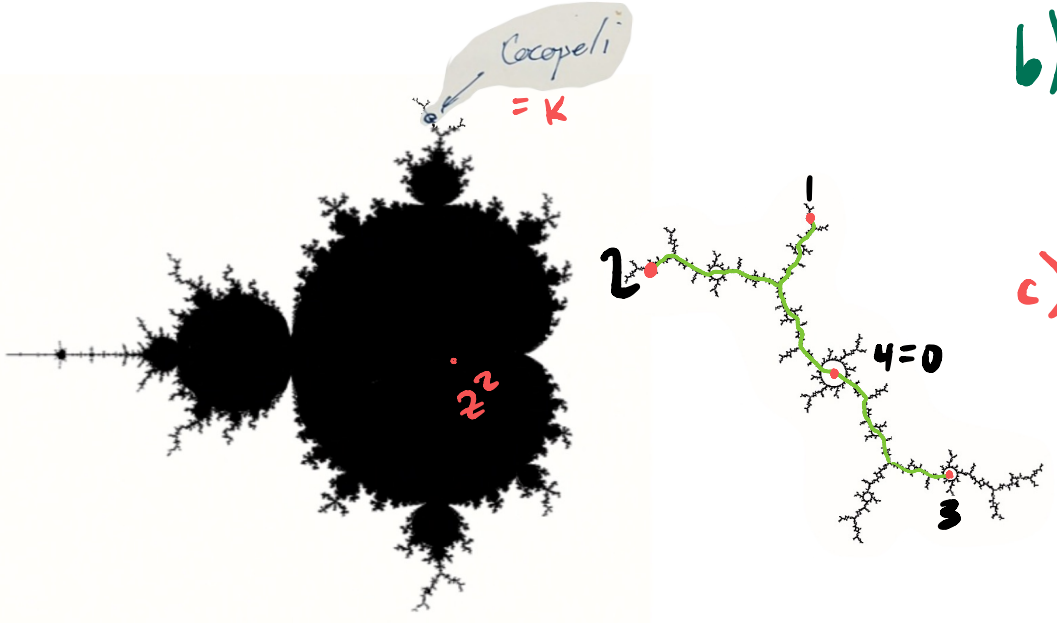

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d) Hubbard trees

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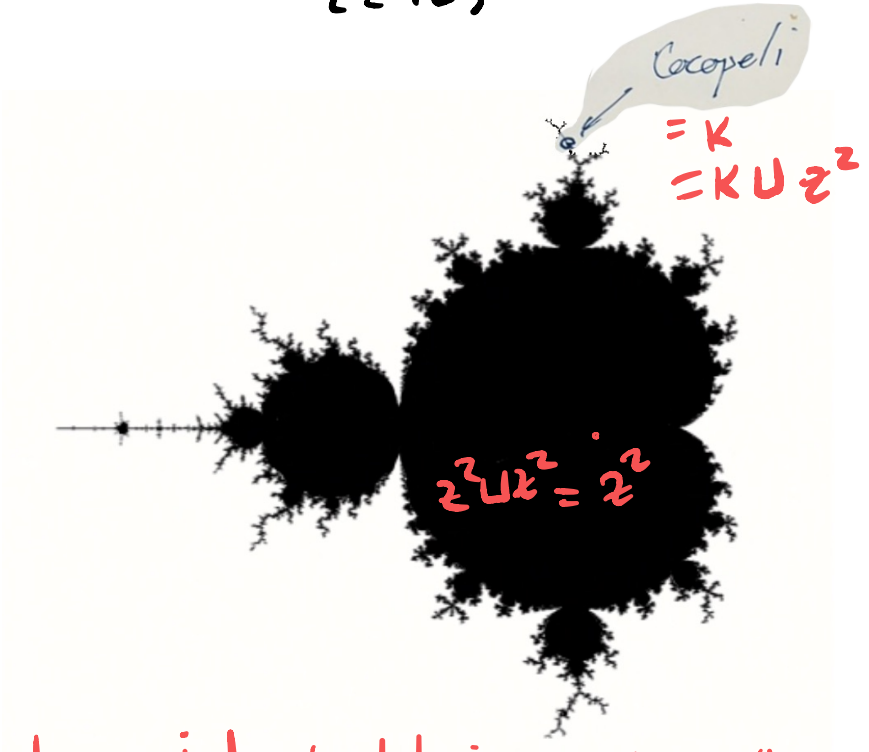

"polynomials tautological matings"

a) $M \subset \mathrm{Per}_1(0) = \{z^2+c \mid \text{orbit of } 0 \not\to \infty\}$

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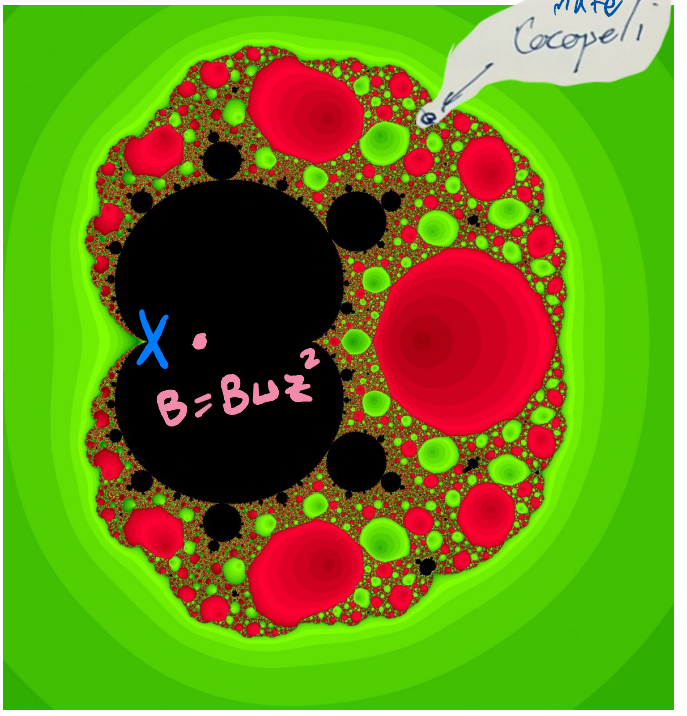
$$M_2 := \{ f \mid f^k(0) \not\to \infty \}$$

$$Per_2(0) = \{ \text{QRMs w/ superattracting 2-cycle} \}$$

$$\simeq \left\{ \frac{z^2+c}{z^2-1} : c \neq -1 \right\}$$



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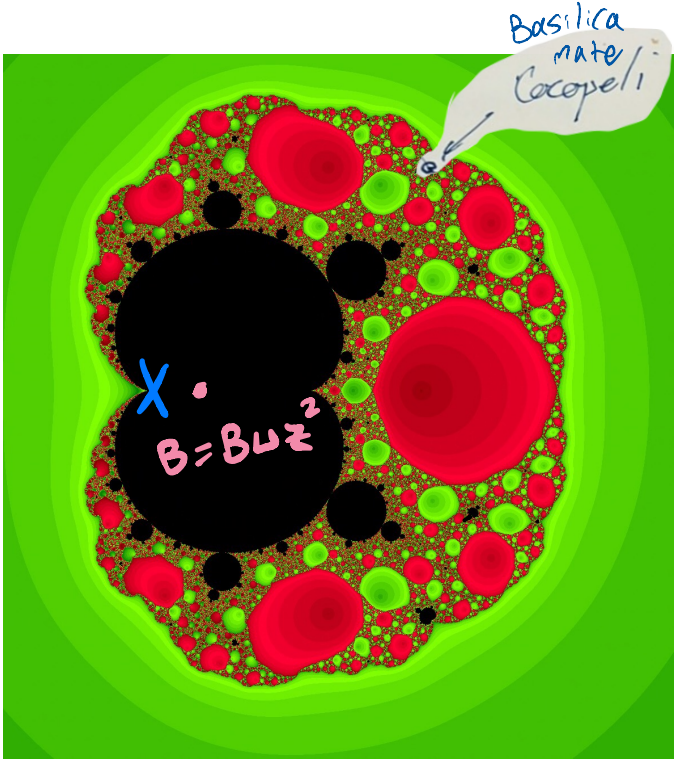

$$\mathcal{M}_2 := \{ f \mid f^k(0) \not\to \infty \}$$

$$\mathcal{M}_n = \{ f \mid f^k(0) \not\to \infty \}$$

$$\mathcal{M}_B = \{ \text{all matings with Basilica} \}$$

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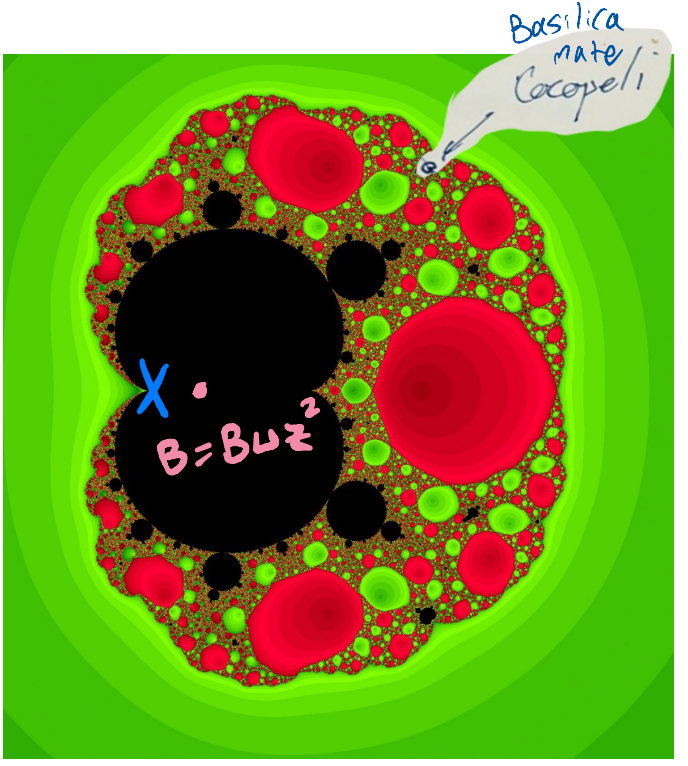

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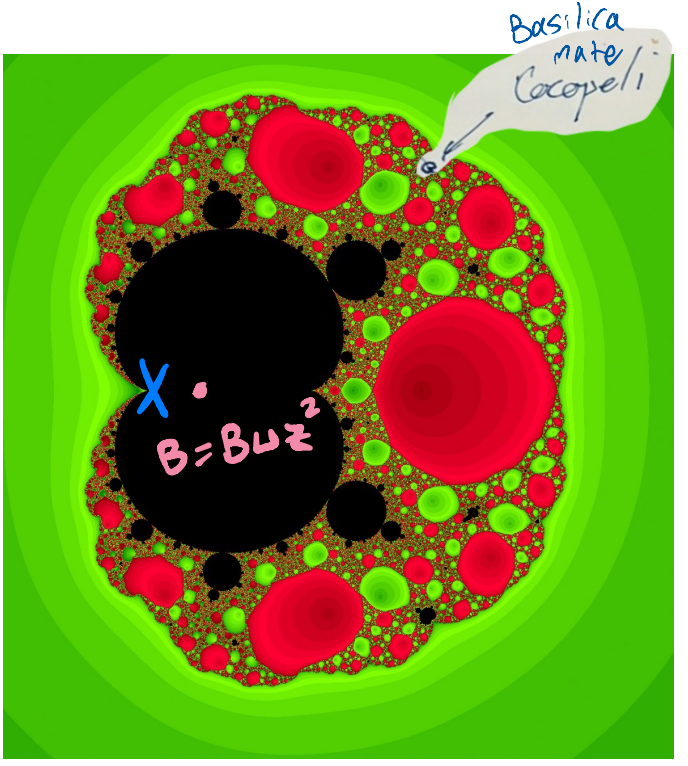

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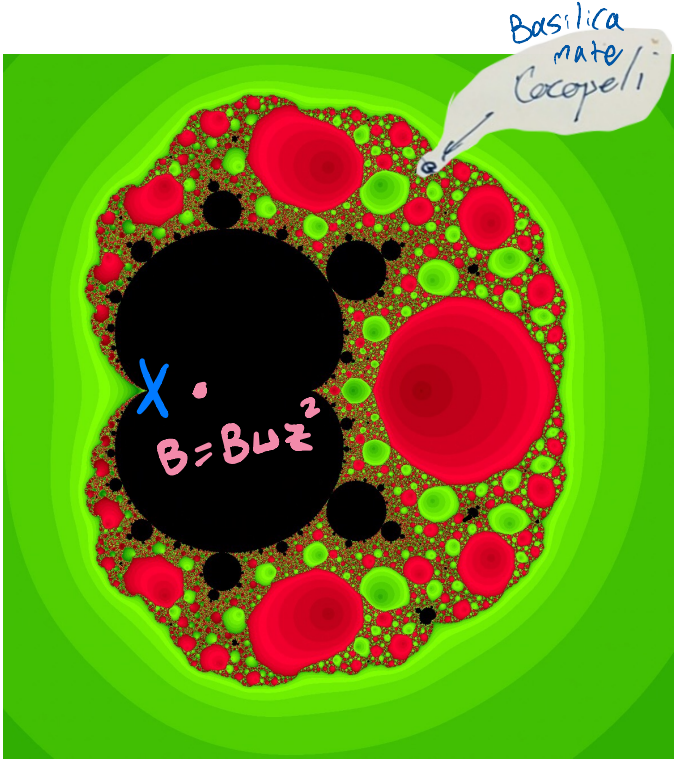

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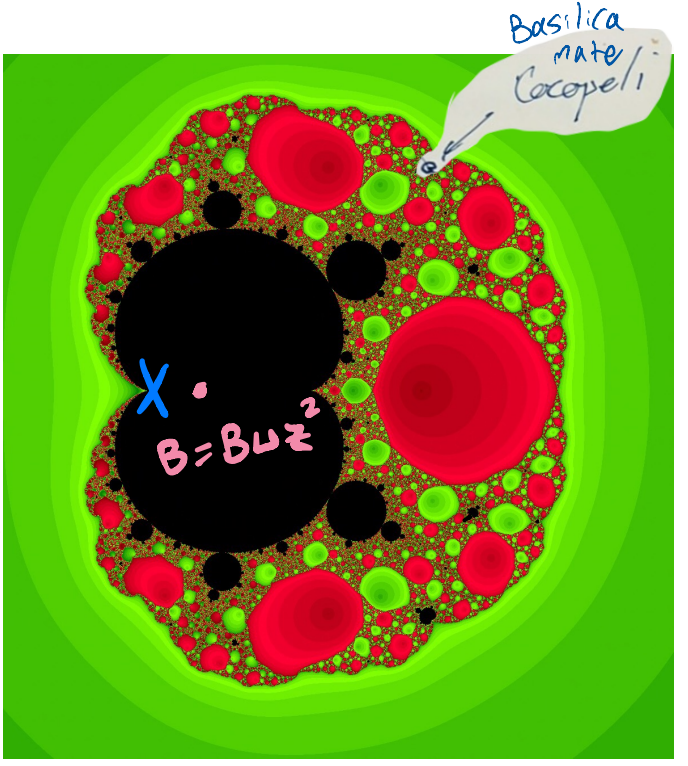

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Sanity Check $\hookrightarrow$ $\mathcal{M}_f \subset \mathcal{M}_n$, $\hat{\mathcal{M}}_f \subseteq \mathcal{M}_n$

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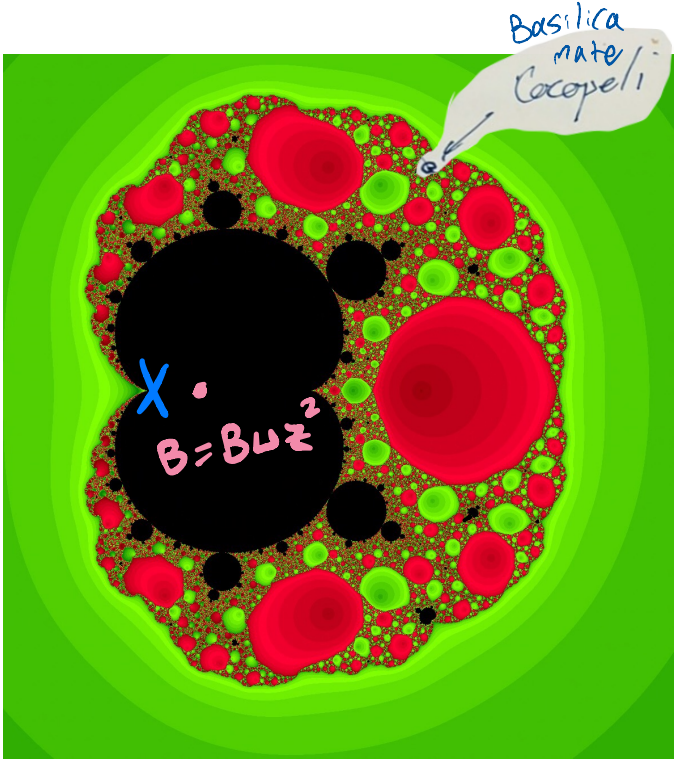
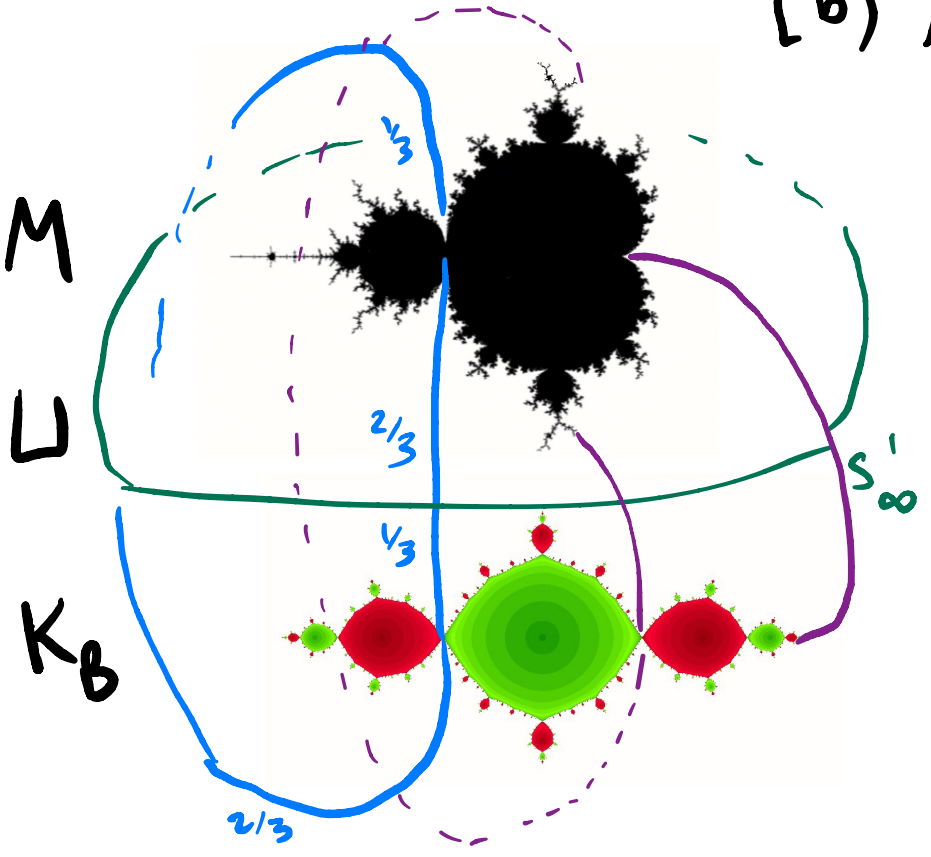
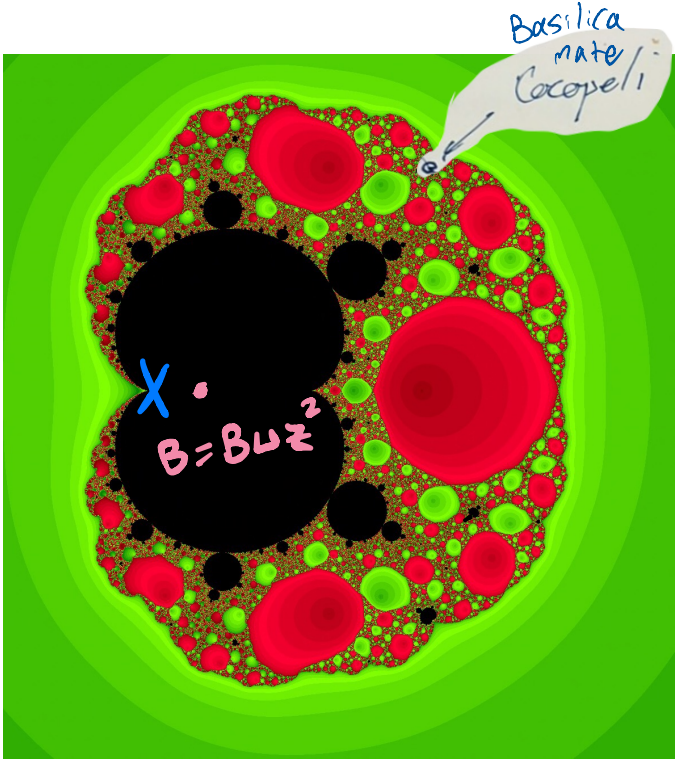


Wittner, Luo, Dudko: [a) $Per_2(0) = \hat{M}_B$
b) $\hat{M}_B = M^* \amalg K_B^*$





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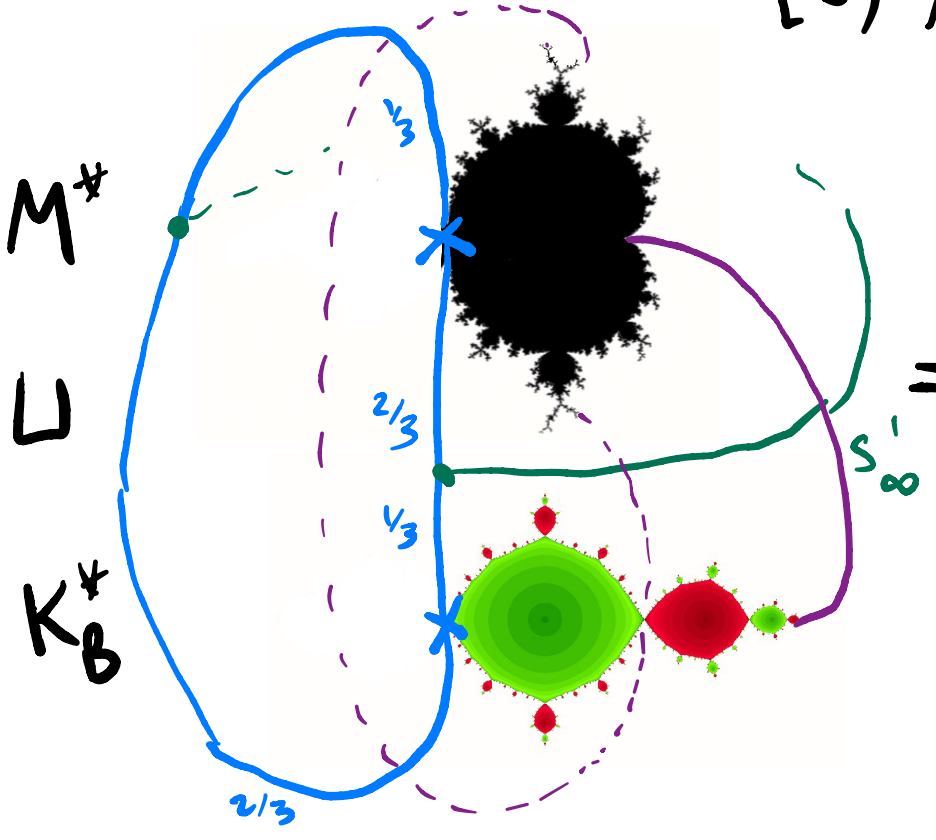

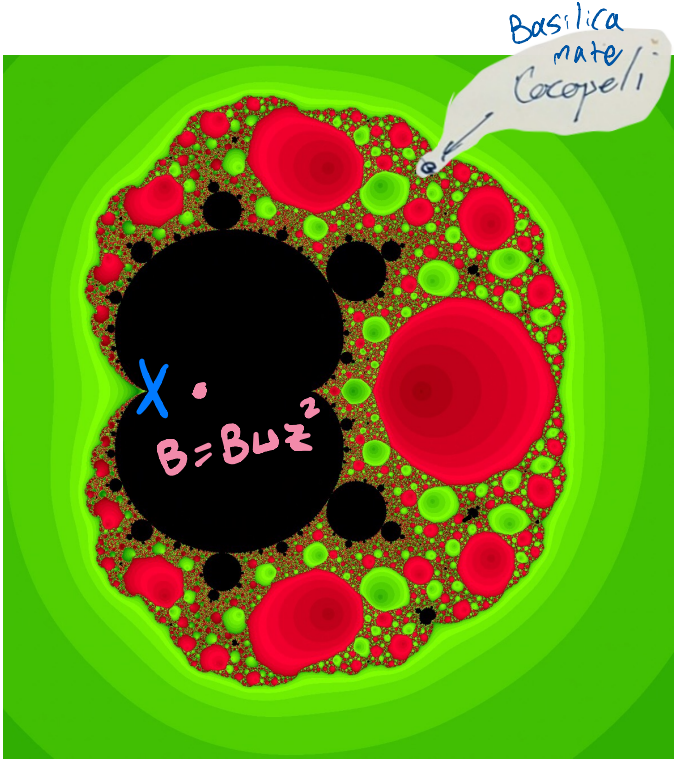

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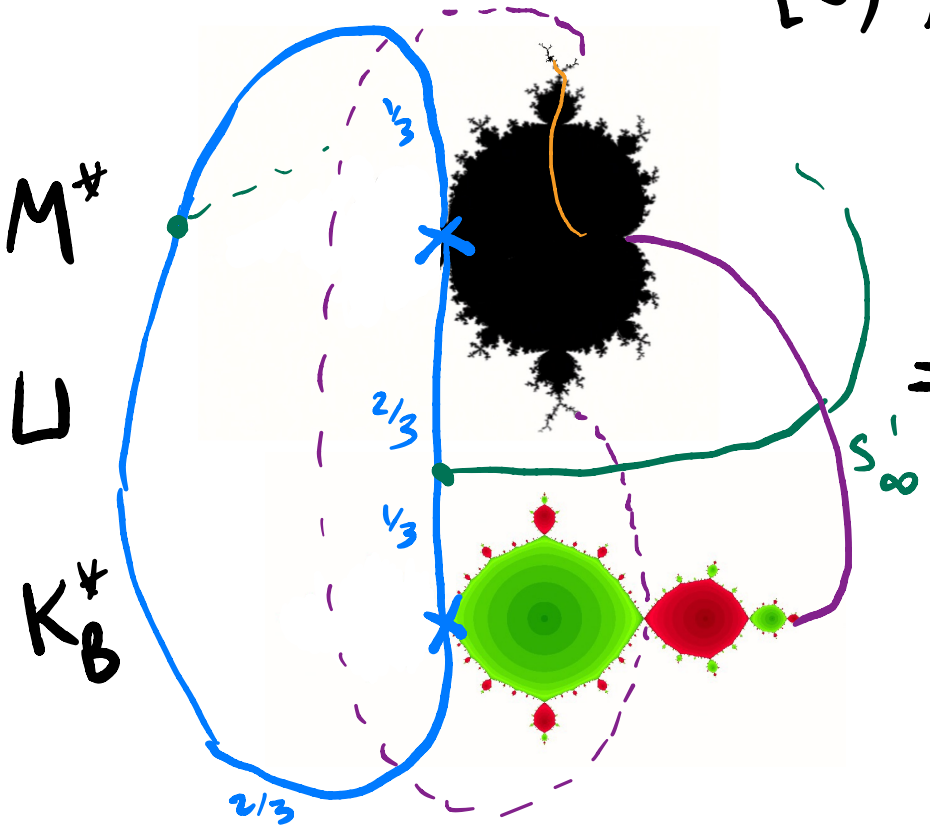

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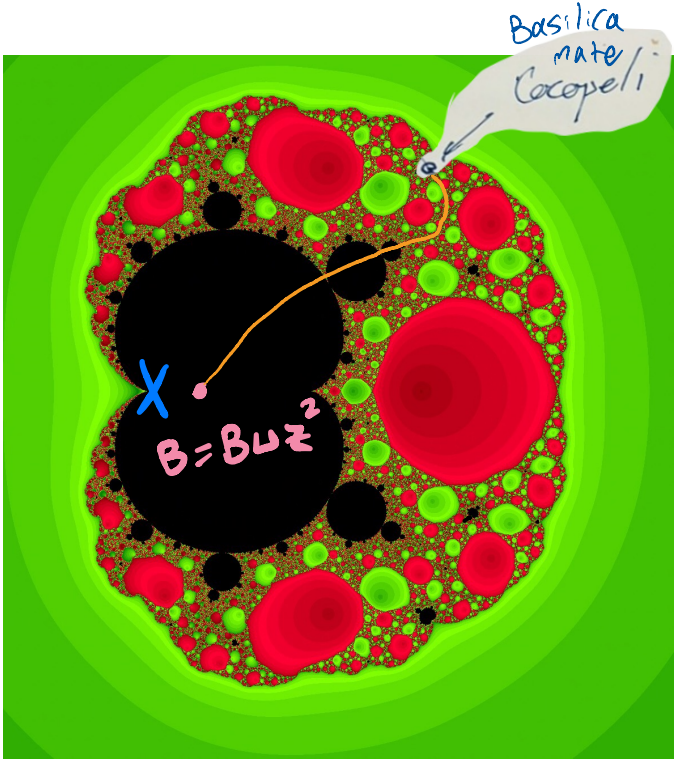
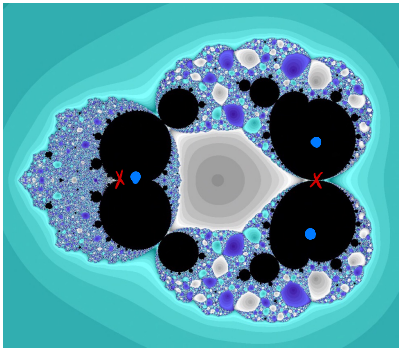


Higher $\mathrm{Per}_n(c)$?

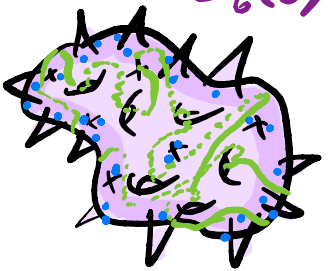
$\#(\mathrm{Gln}) \nearrow \infty$, all smooth

open Q: connected?

$\mathrm{Per}_3(c) \simeq \mathbb{C}^*$



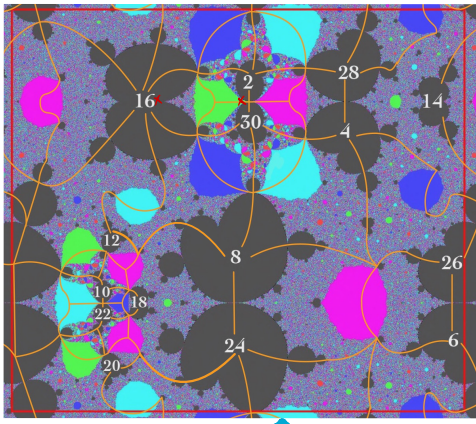
$\mathrm{Per}_6(c)$



Qs

- Who & where are PCF parameters in $\mathrm{Per}_n(c)$?
  - Are they all? matings or captures?
  - How do different $\hat{M}_f$ fit together?
  - When are 2 PCF parameters closeby?
  - M is connected... is M_n? ($\Leftrightarrow$ irreducible)

$\mathrm{Per}_5(c) \simeq \Sigma_{1,1,0}$



15 diff. $\hat{M}_f$

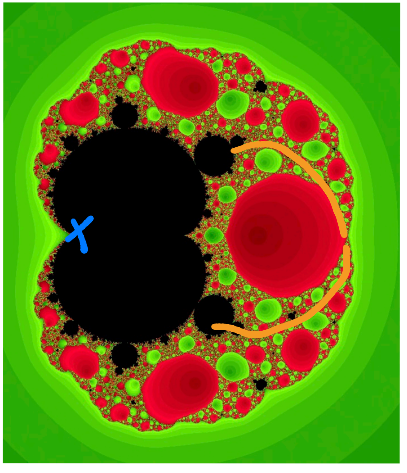
"Best" case scenario re: $\mathrm{Per}_n(0)$

a) every map in $\mathrm{Per}_n(0)$ a mating or capture

$$\begin{cases} \mathrm{Per}_n(0) = \bigcup \hat{\mathcal{M}}_f \\ \mathcal{M}_n = \bigcup \mathcal{M}_f \end{cases}$$

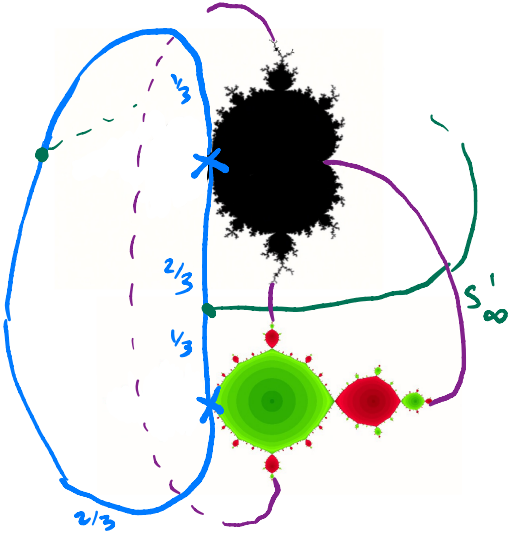
b) each $\hat{\mathcal{M}}_f$ has "nice" organization

$$\hat{\mathcal{M}}_f = \mathcal{M}^* \sqcup K_f^*$$



$=$

Wittner, J. Luo, Dudko



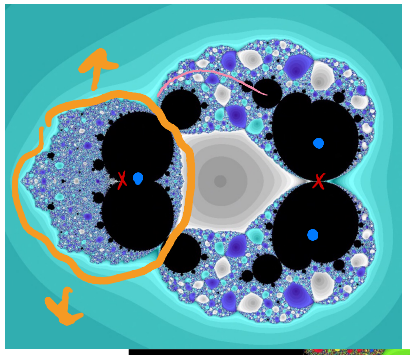
"Best" case scenario re: $\mathrm{Per}_n(0)$

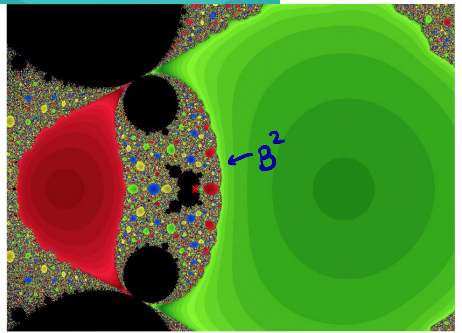
b) $\hat{\mathcal{M}}_f = \mathcal{M}^* \sqcup K_f^*$

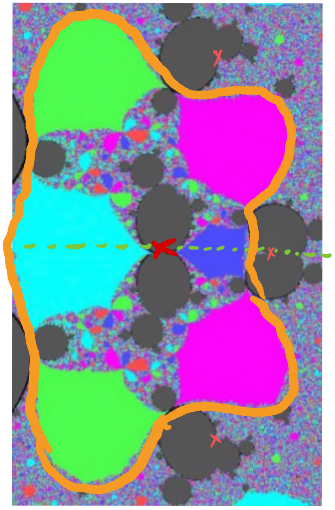
a) $\begin{cases} \mathrm{Per}_n(0) = \bigcup \hat{\mathcal{M}}_f \\ \mathcal{M}_n = \bigcup \mathcal{M}_f \end{cases}$

Zero entropy regions

a) + b): true for rabbits $R_{p/q}$ & folklorely expected, accessible now for main molecule f







$R_{\pm p/q}$

$= \hat{\mathcal{M}}_{R_{\pm p/q}}$

$\simeq \overline{\mathbb{D}}^*$

$\exists \Gamma \ni \{\text{marked } n\text{-cycle}\}$ s.t. $h_\Gamma = 0$

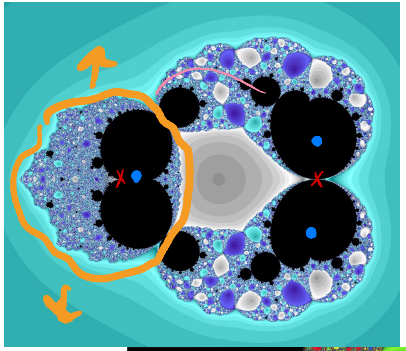
"Best" case scenario re: $\mathrm{Per}_n(0)$

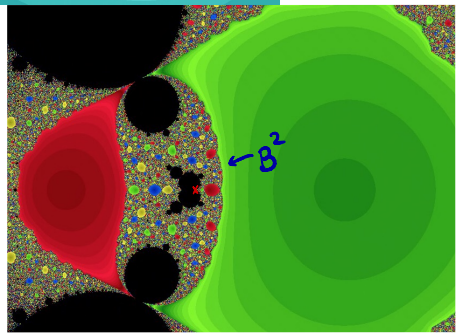
b) $\hat{M}_f = M^* \sqcup K_f^*$

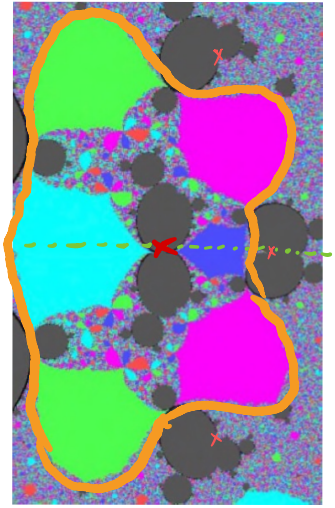
a) $\begin{cases} \mathrm{Per}_n(0) = \bigcup \hat{M}_f \\ M_n = \bigcup M_f \end{cases}$

Zero entropy regions

a) + b): true for rabbits $R_{p/q}$ & folklorely expected, accessible now for main molecule f







$R_{\pm p/q} = \hat{M}_{R_{\pm p/q}} \simeq \overline{\mathbb{D}}^*$

a) $\begin{cases} \mathrm{Per}_n(0) \overset{?}{=} \bigcup \hat{M}_f \\ M_n \overset{?}{=} \bigcup M_f \end{cases}$

"Best" Case Scenario?

b) $\hat{M}_f \overset{?}{=} M^* \sqcup K_f^*$

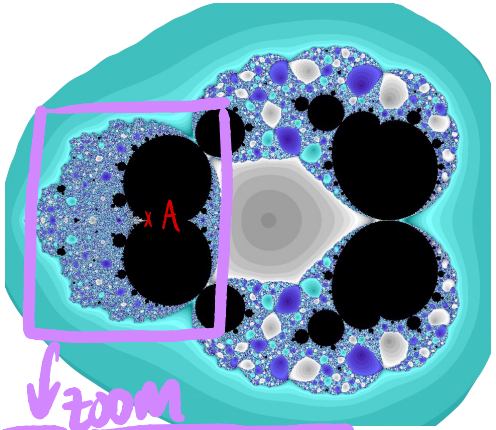
Positive Entropy

a) Definitely false

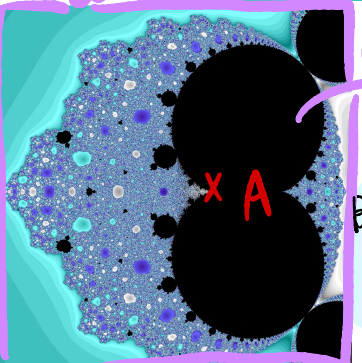
b) suitable modification

Cf. 30m from now

→ no zero entropy graph containing marked n-cycle

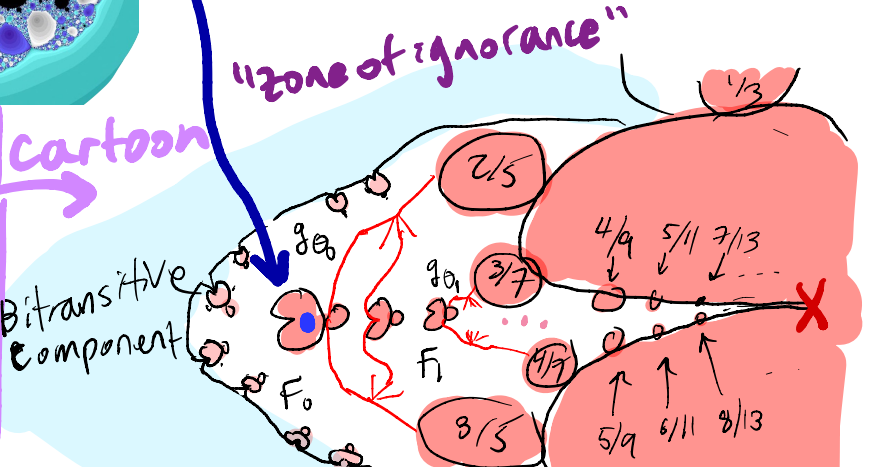


zoom



cartoon

"zone of ignorance"



Non Matings = measure of M_n complexity

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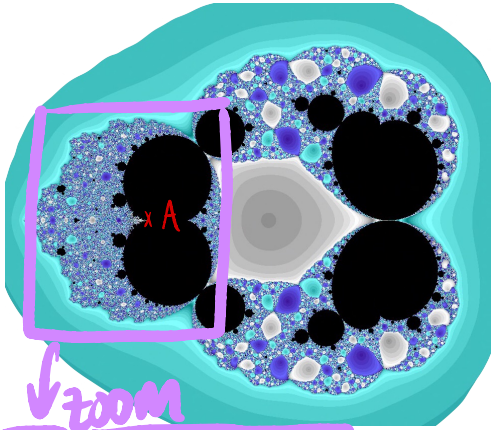

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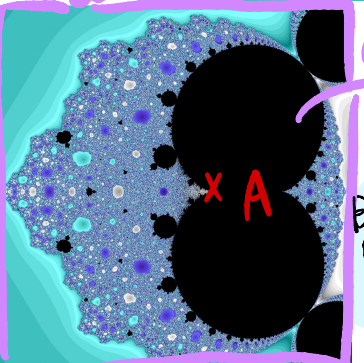

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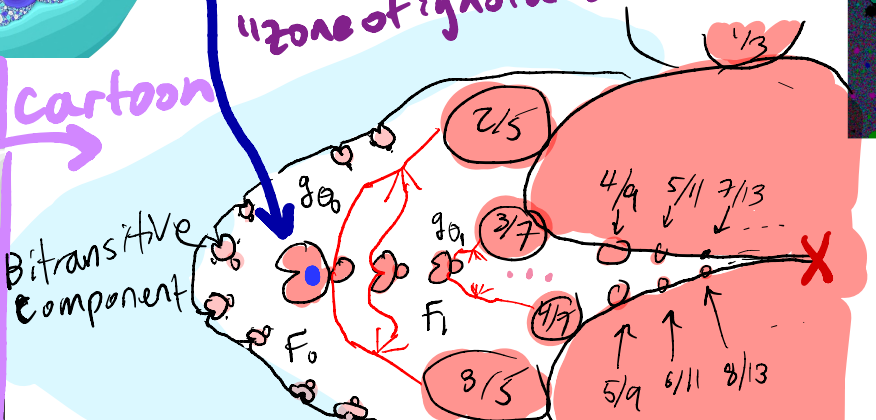
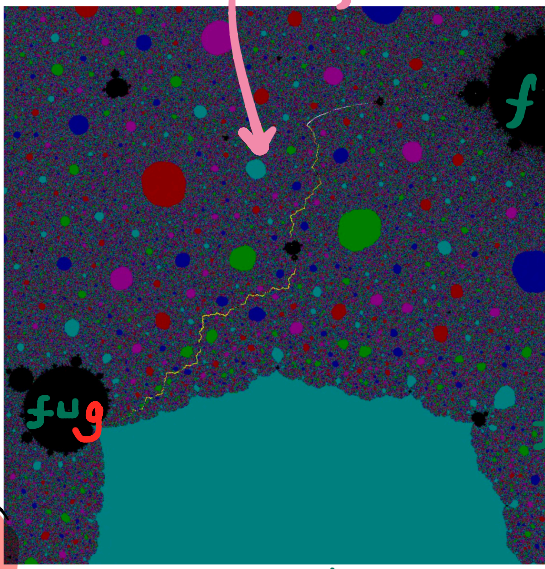


vein in M_f



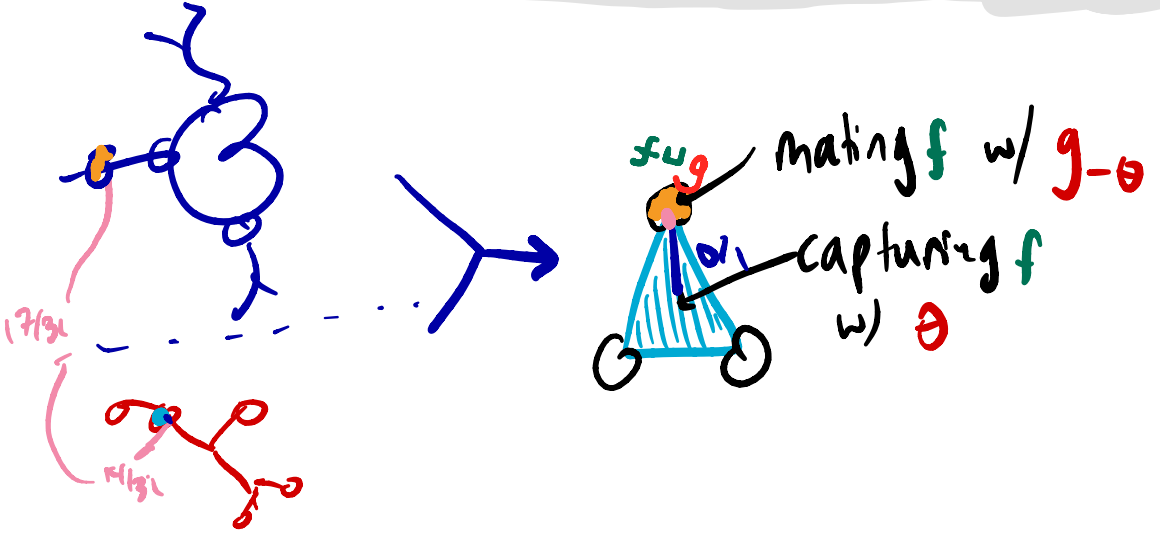
nice fig by Wolfjung

cf Nuria!

What do we know about $\hat{M}_f$ for generic f?

Interplay theorem (D., Timorin)

Suppose Θ lands on a Fatou component in K_f.
Then the mating component $\mathcal{H}_{f \amalg g_{-\Theta}}$ is adjacent to the capture component $\mathcal{H}_{f_\Theta}$



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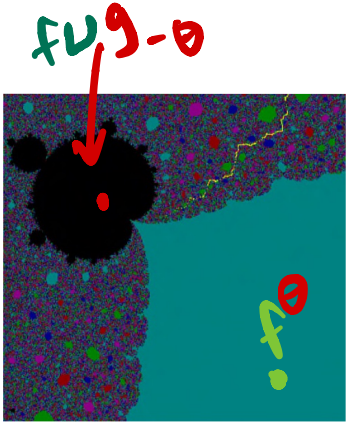

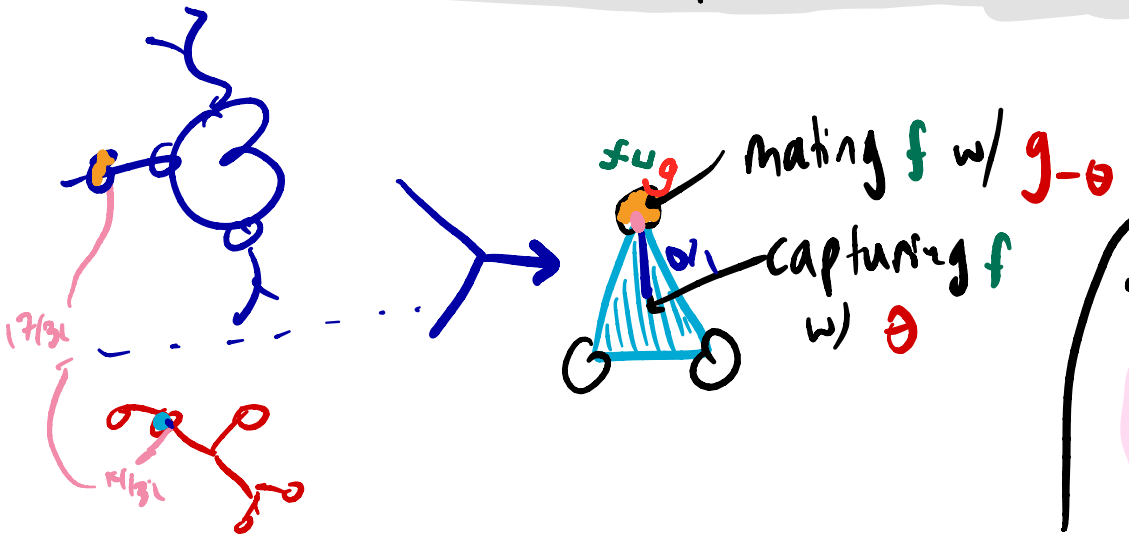

What's left?
- what happens at veins/arcs?

Conjecturally:
Mating cts, capturing cts along veins, along arcs

Overlaps of $\{\hat{M}_f\}$

(128) Find an example of a *shared mating*; that is, find an example of a PCF rational map that is a mating of two polynomials in more than one way.

Thm (D.) The $\hat{M}_f$ have transitive overlaps in $\mathrm{Per}_n(0)$

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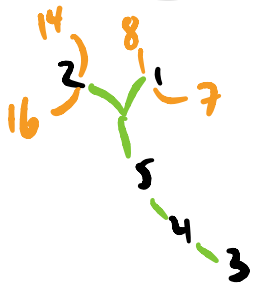
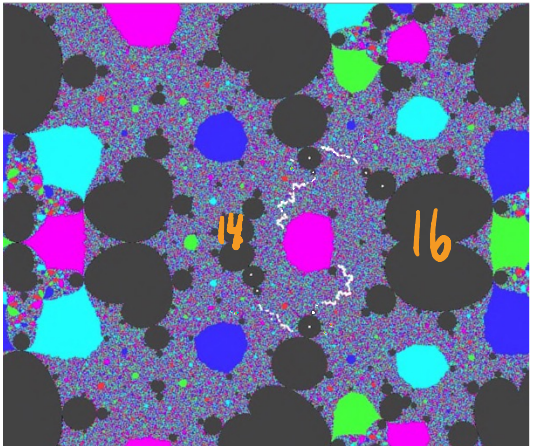
pf: uses shared captures generated by tips in Hubbard trees

eg $f_{8/31}$





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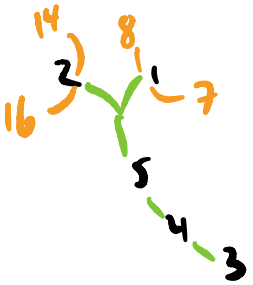
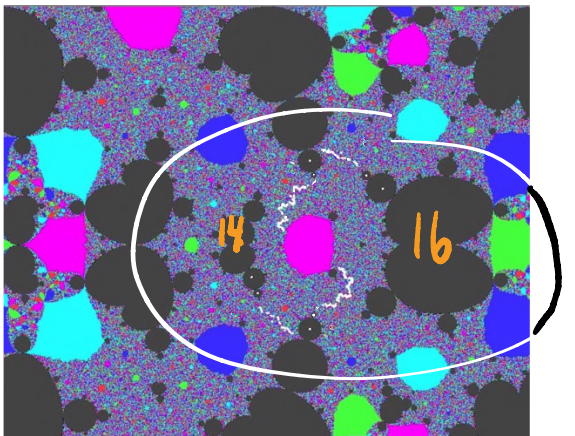
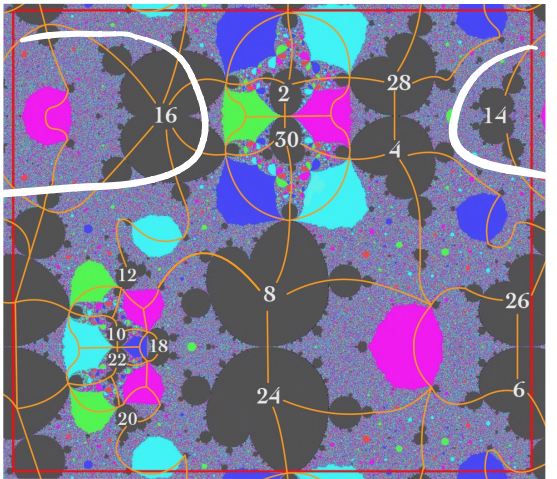
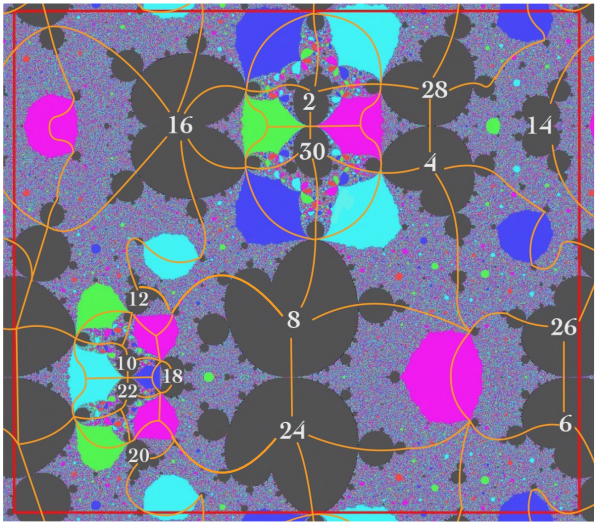






Where could non-matings be?



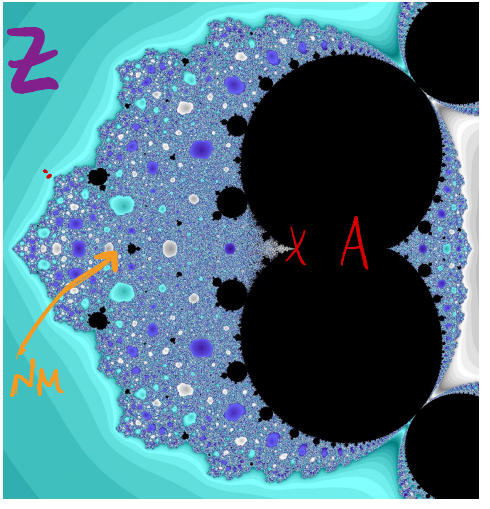
Claim: Can't be "too far" away from polynomials

1) **Topologically** (Milnor):

every connected component of $\mathrm{Per}_n(0)$ contains a polynomial

2) **Dynamically** (Rees) Polynomial + Path Thm

Any $F \in \mathrm{Per}_n(0)$ can be conjugated by half-twists to a polynomial [maybe not $H^{1/2}$ along external rays]

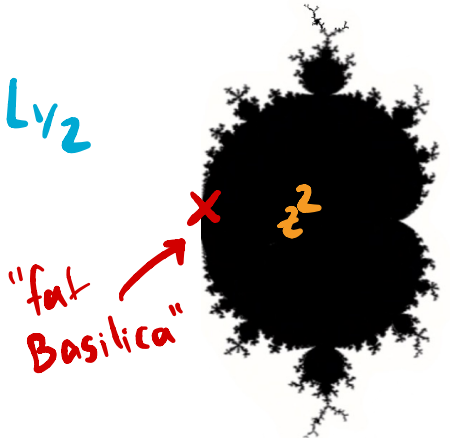


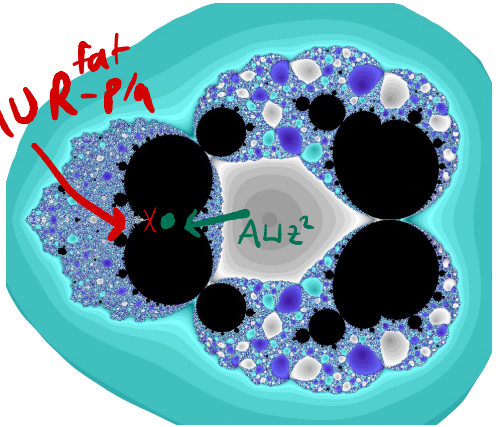
Punctures in $Per_1(0)$

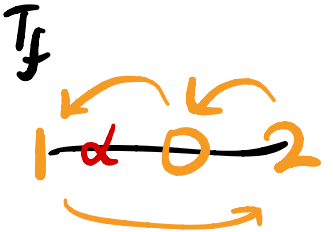
(125) Find an obstructing multicurve for the map f, where f is the formal mating of the basilica with itself.

Tan Lei: $f \in L_{p/q}$, $f \amalg g$ mateable $\iff g \notin L_{-p/q}$ → "begins" at fat Rabbit $R^{fat}_{-p/q}$

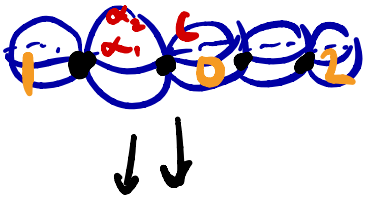
eg $f = A \in L_{1/2}$







Rees-Stimson: Can fill in punctures with $f \amalg R^{fat}_{-p/q} : S^2/\Gamma_2$ — α-rays!



Buff-D.-Kapiamba: Holomorphic compactification exhibiting "parabolic implosion"

Kiwi [cf. 30 min ago :)]

Theorem 1. *Let $\{f_t\}$ be a holomorphic family of degree $d \geq 2$ rational maps. Then there are at most $2d-2$ pairwise dynamically independent rescalings for $\{f_t\}$ such that the corresponding rescaling limits are not postcritically finite.*

Moreover, if $d = 2$, then there are at most two dynamically independent rescalings. Furthermore, in the case that a rescaling of period at least 2 exists, then exactly one of the following holds:

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(1) 2 RSLs:
- a) $q \rightsquigarrow f \in \mathrm{Per}_1(1)$ w/ prefixed critical point
- b) $q' \rightsquigarrow$ polynomial $\in \mathrm{Per}_1(0)$

(2) ∃! RSL:
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In $\mathrm{Per}_n(0)$: All punctures on $\partial \mathcal{H}_f$ for $f \in G(n) \subset M$

Q: How can we see RSLs via M-info?

In $\mathrm{Per}_n(0)$: All punctures on $\partial \mathcal{H}_f$ for $f \in \mathcal{C}(n) \subset \mathcal{M}$

Express any $f \neq z^2$ as $\boxed{f = f' \boxplus g}$ for f' exactly once renormalizable

Decorations of $\mathcal{M}$: $\mathcal{M} = \bigsqcup L_{p/q}$

& each $L_{p/q} = \{ R_{p/q} \boxplus g \mid g \in \mathcal{M} \} \sqcup \bigsqcup D^{p/q}_{t \in \mathbb{R}/2\mathbb{Z}}$

dyadic tip of $\mathcal{M}$

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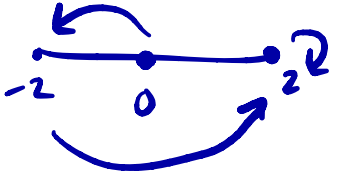
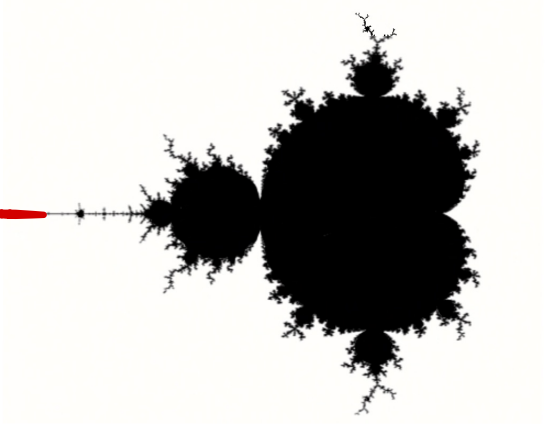


$t_{1/2}$

"chebychev"
$z^2 - 2$



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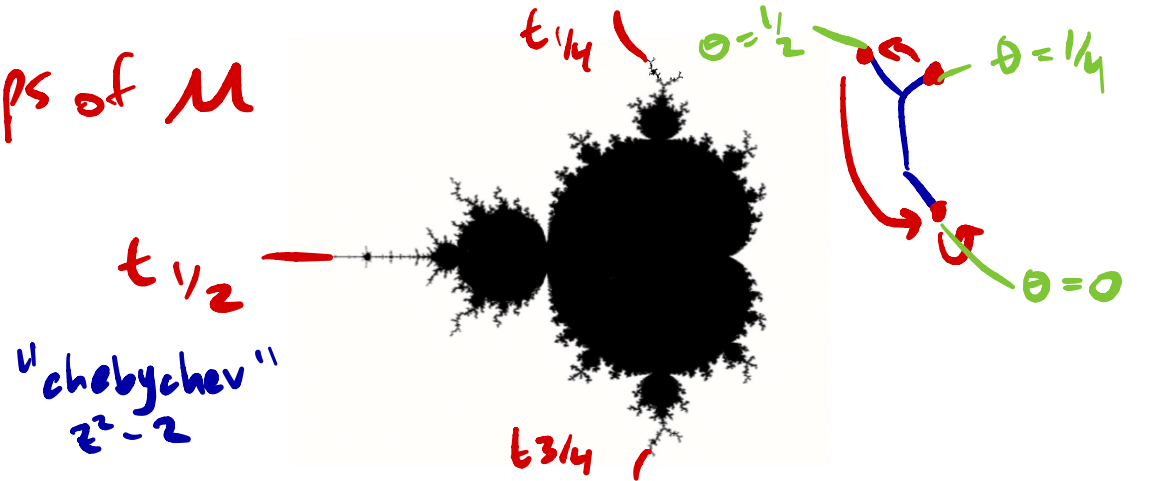

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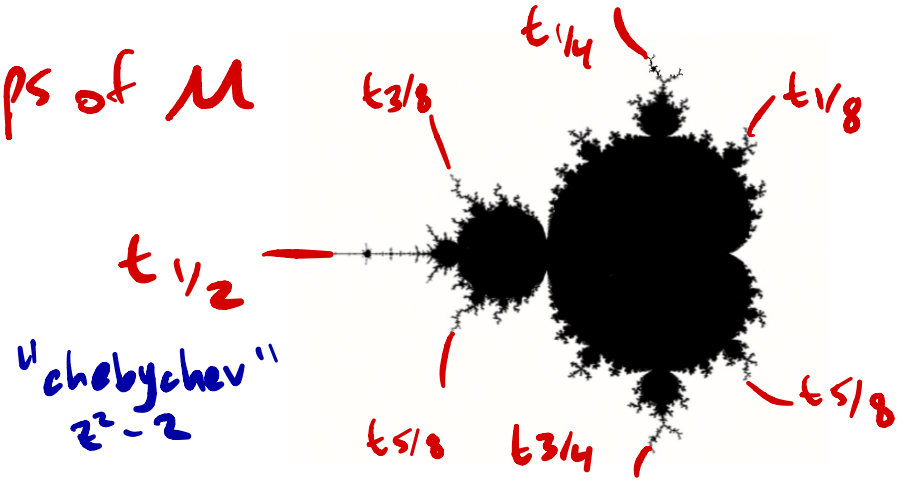

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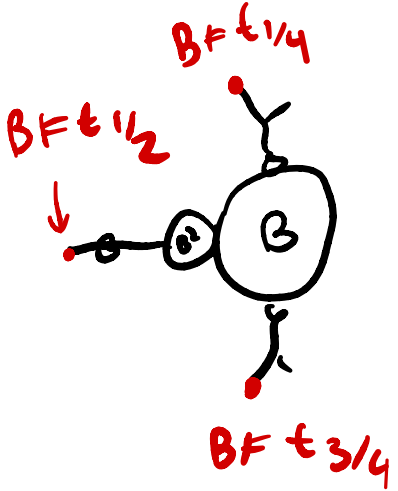

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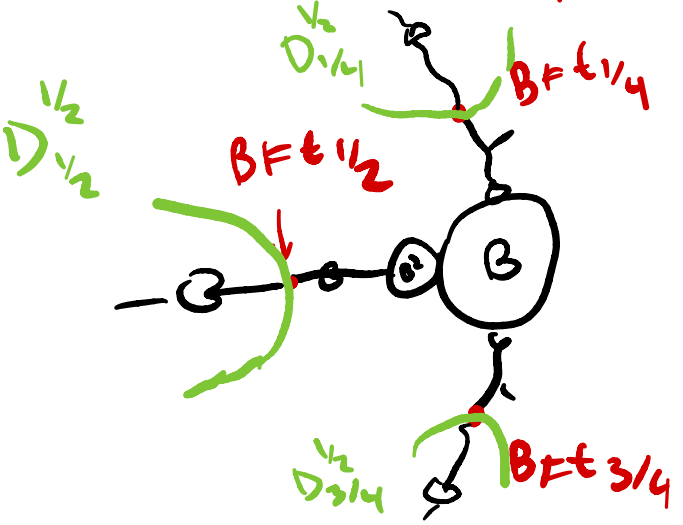
$= \text{wake of } R_{p/q} \circ t_{\frac{l}{2^k}}$



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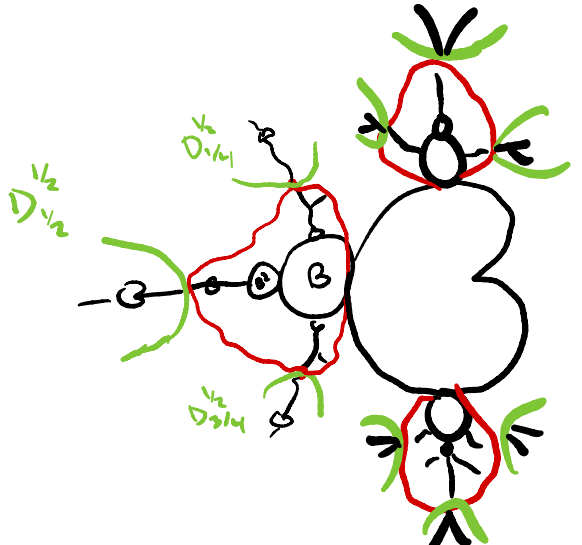

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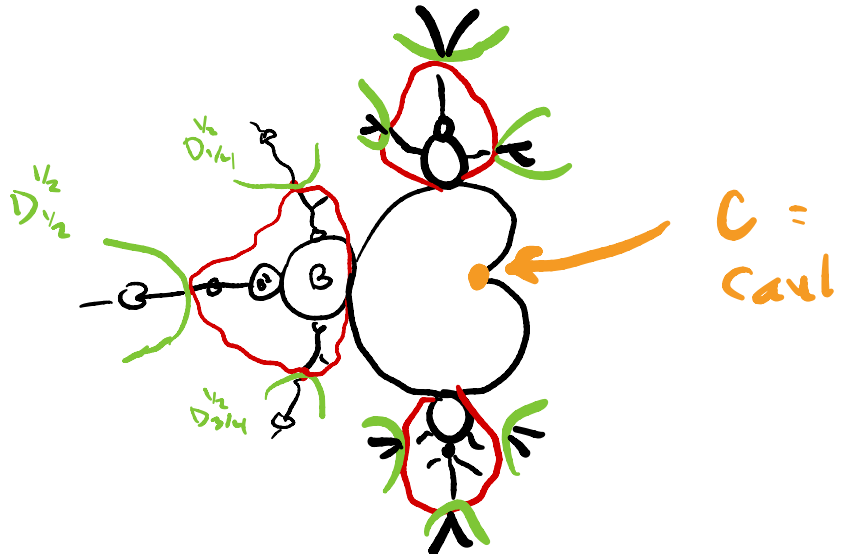

Riwi

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Express f as $\boxed{f = f' \boxplus g}$ for f' exactly once renormalizable

Case(1): f' is primitive so ∃ some Decoration $f \in D^{p/q}_{t_{1/2^k}}$

then 2 RSLs:
- (a) period $q \rightsquigarrow t_{1/2^k} \sqcup \mathrm{LC} \in \mathrm{Per}_1(1)$
- (b) period $q' = \mathrm{period}(f') > q \rightsquigarrow g$, polynomial

Case(2): $f' = R_{p/q}$, some $R_{p/q}$

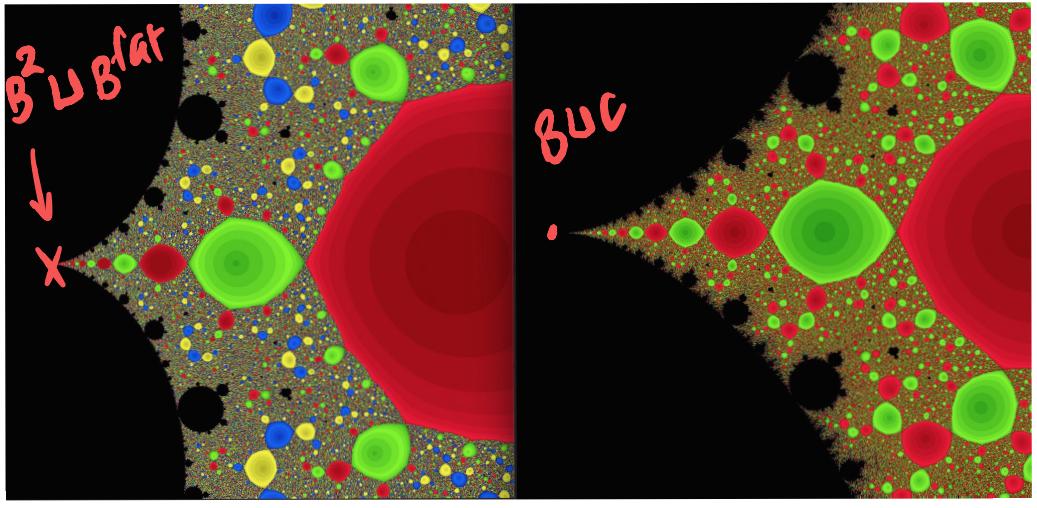
then ∃! RSL of period q: $g \sqcup \mathrm{LC}$

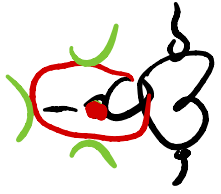
"Echoes Thm" (Bnff-D.-Kapamba)

Asymptotic similarity between neighborhood of puncture $\in \partial \mathcal{U}_f$ and a neighborhood of a given simpler Per_n or $\mathrm{Preper}_{k,n}$

near puncture of $B^2 \in \mathrm{Per}_n(0)$

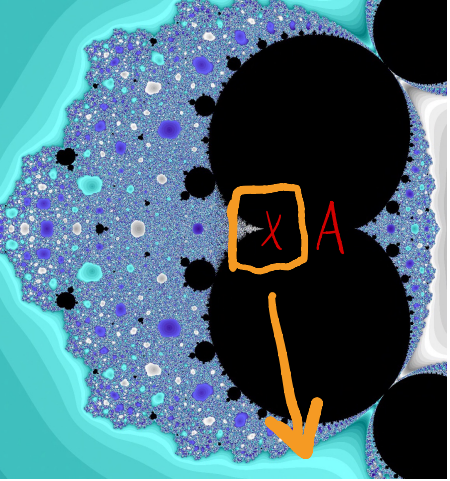
bubble rays in $\mathrm{Per}_2(0)$!

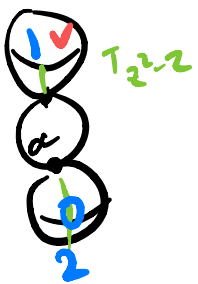


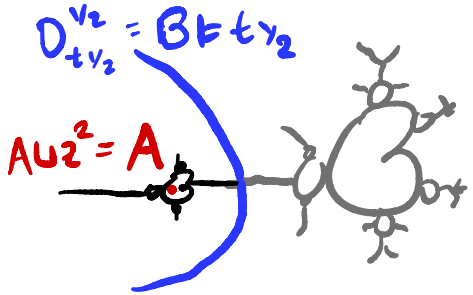


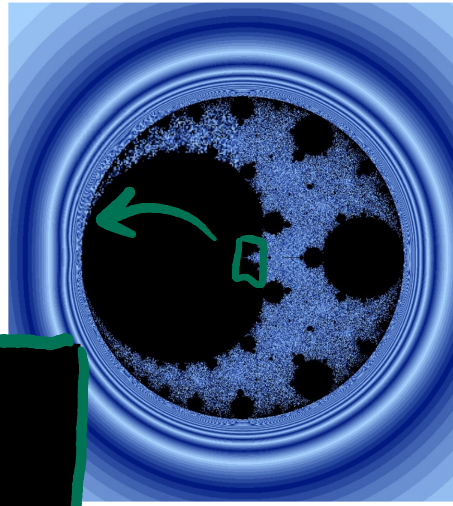
eg

(2) satellite case



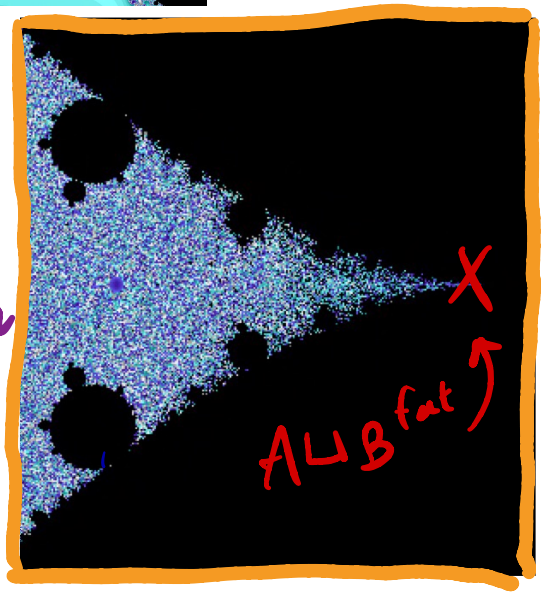




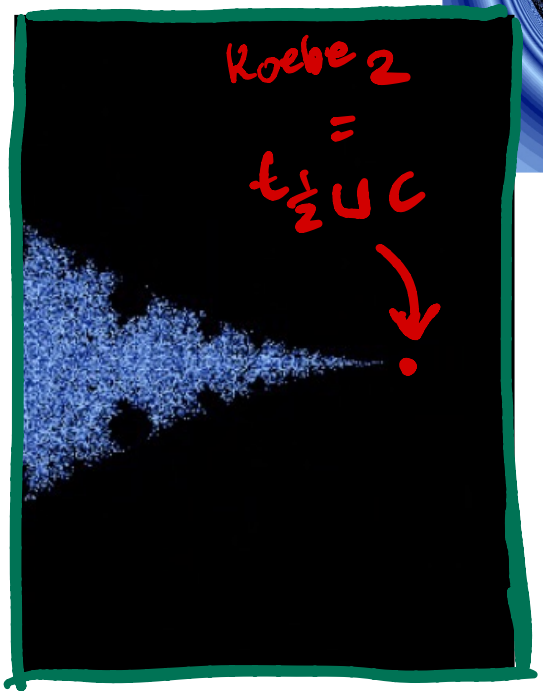


$Preper_{2,1}$

Zone of ignorance



$\sim$

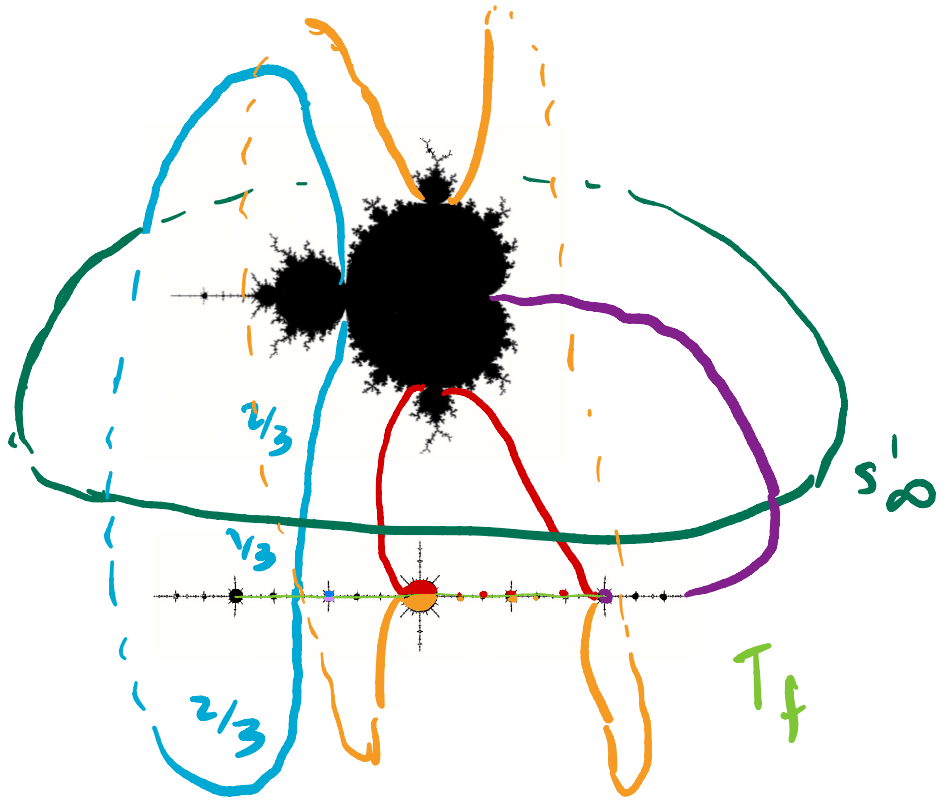


ej

(1) primitive case

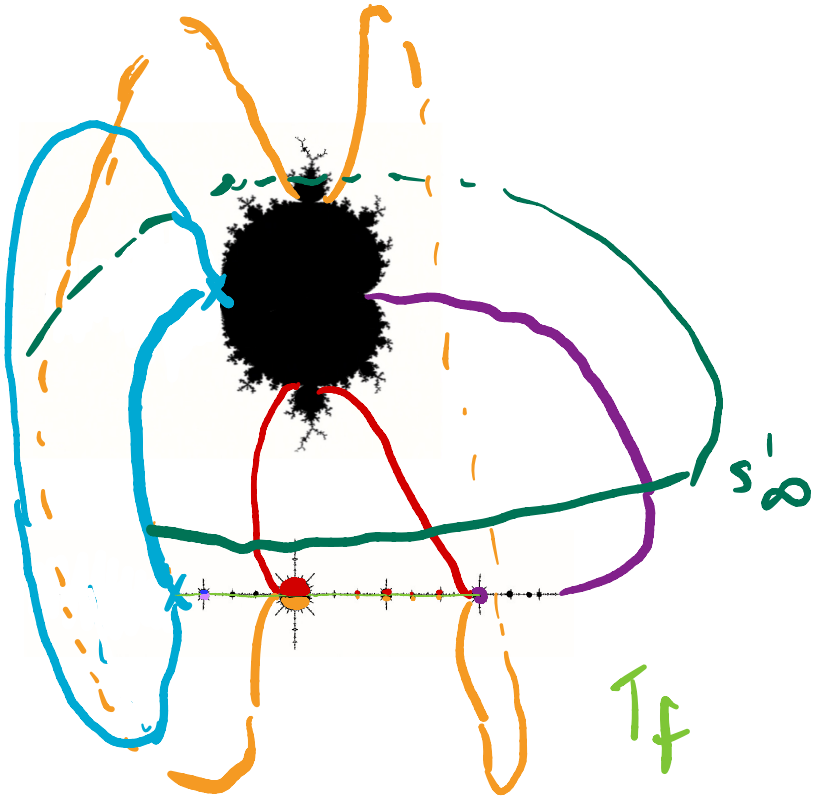
Candidate Family 2: "slit recompletions"

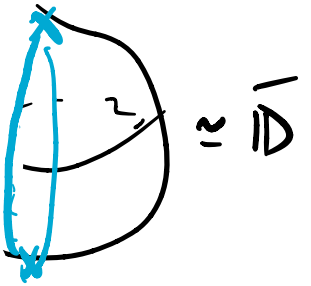
$\hat{M}_f \stackrel{?}{=} M \cup K_f$



Candidate Family 2: "slit recompletions"

$\hat{M}_f \stackrel{?}{=} M^* \sqcup K_f^*$





Candidate Family 2: "slit recompletions"

$\hat{M}_f$

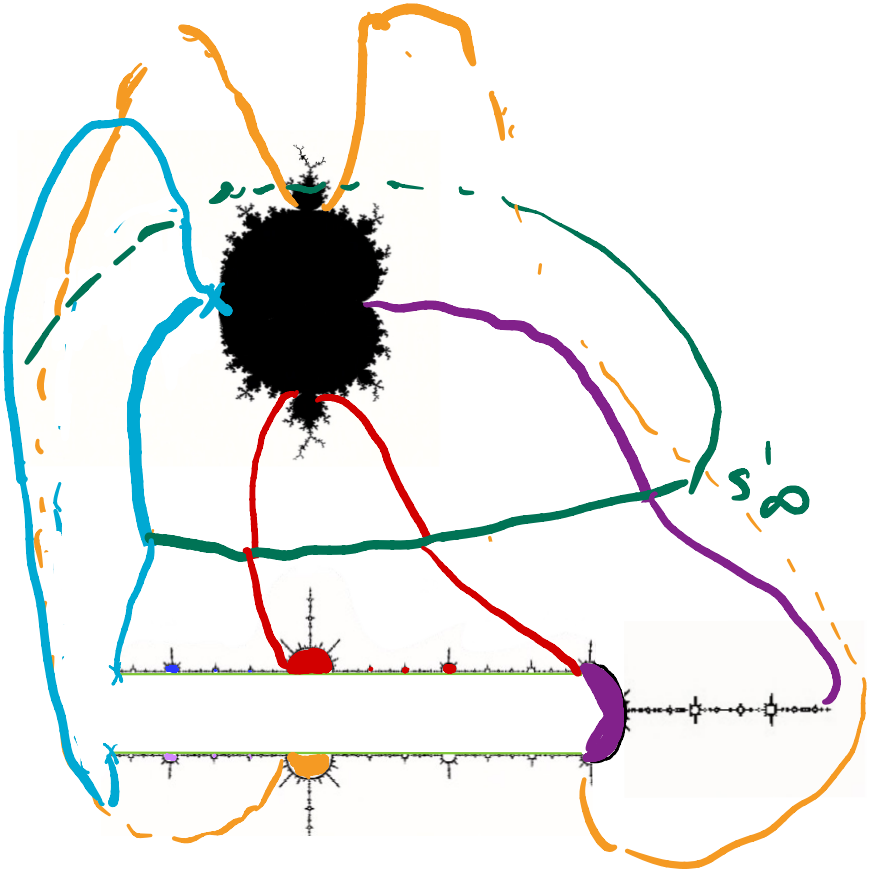
$\stackrel{?}{=}$

M^*

$\sqcup$

$K_f^* - T_f$

$K_f^s := K_f^* - T_f$

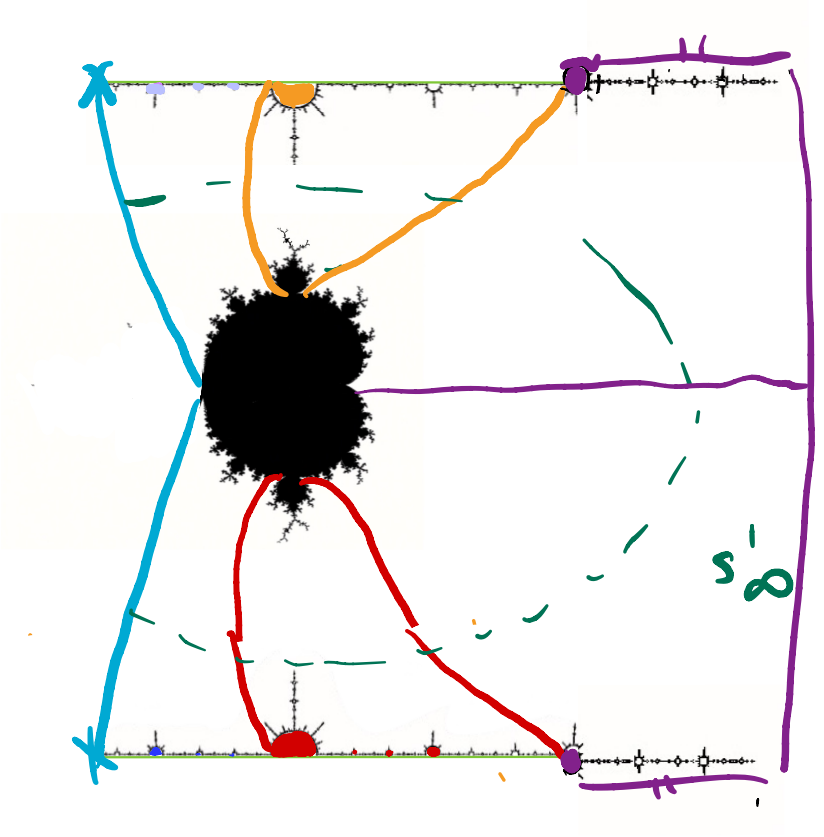


$\overline{\mathbb{D}}$ slit

cut along T_f b/c capture path homotopy class matters rel. P_f

Candidate Family 2: "slit recompletions"

$\hat{M}_f \stackrel{?}{=} M^* \sqcup K_f^s$



Candidate Family I · "slit recompletions"

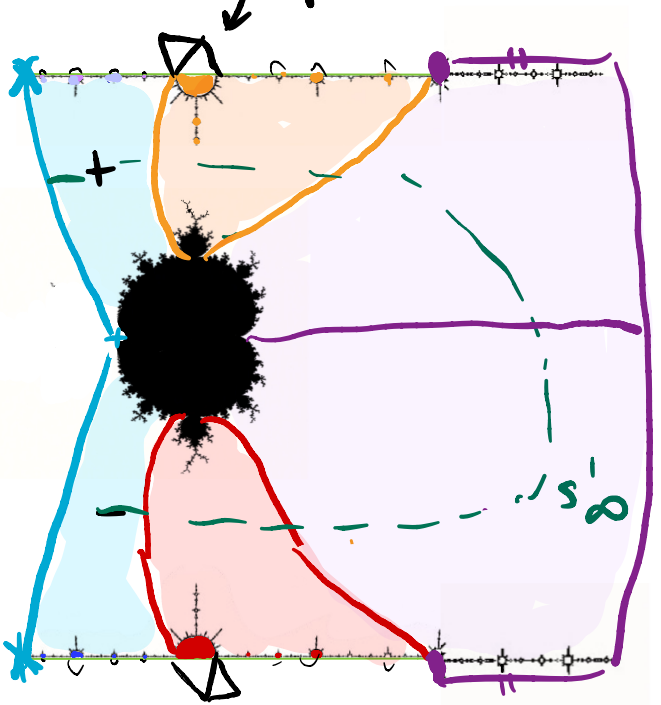
$\hat{M}_f$

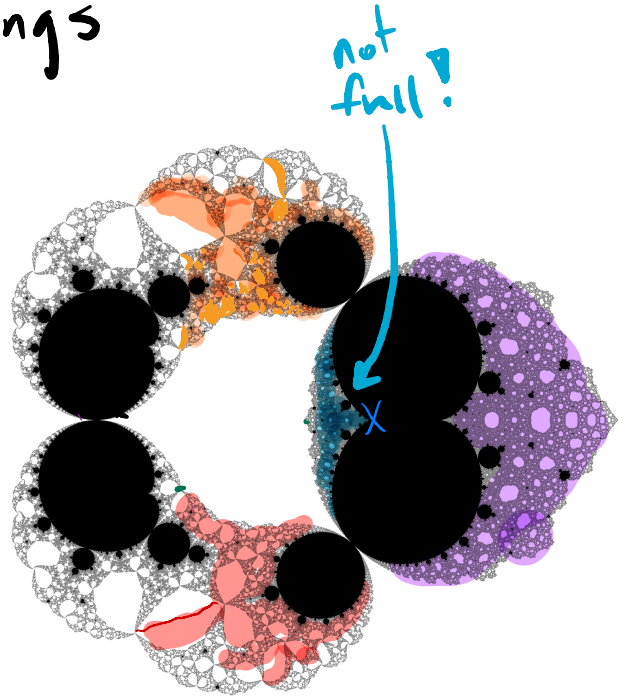
$?\ =$

M^*

$\sqcup$

K_f^s





break into multiple slides

Shaw Jung

Candidate Family I · "slit recompletions"

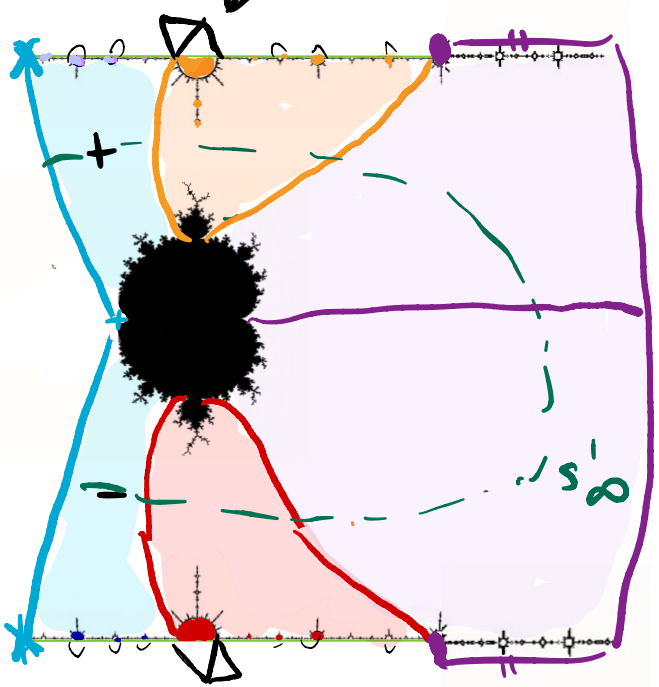
$\hat{M}_f$

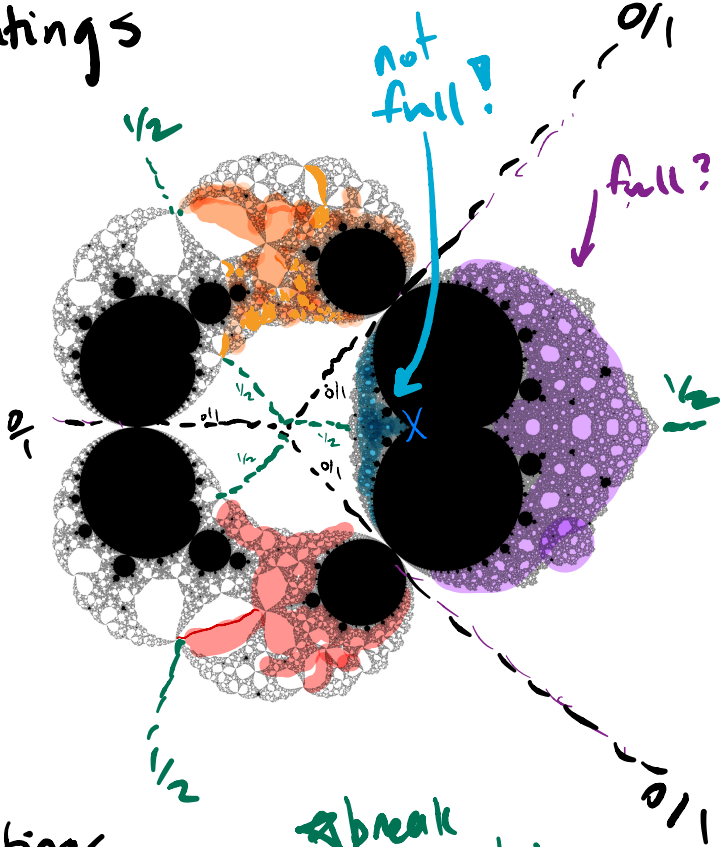
$\stackrel{?}{=}$

M^*

$\sqcup$

K_f^s



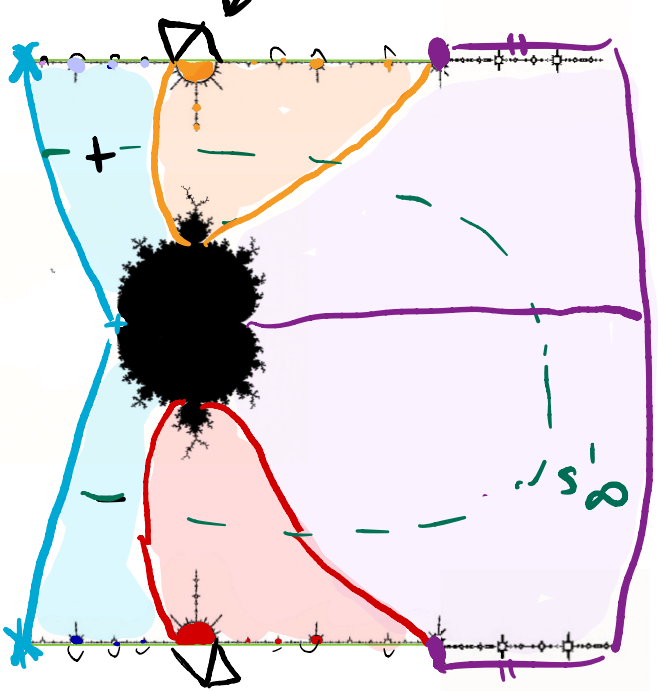


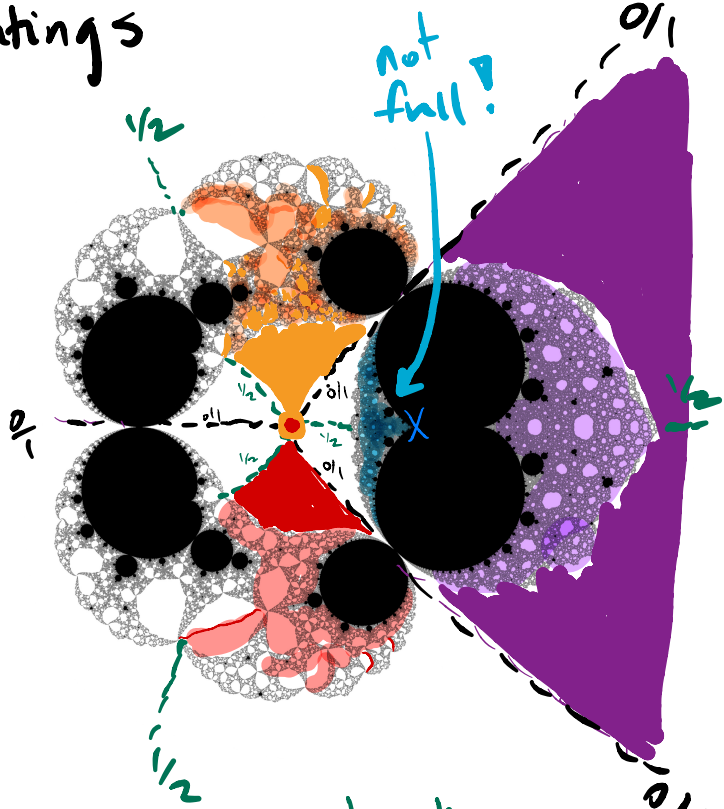
break into multiple slides

Shiwen Jung

Candidate Family I · "slit recompletions"

$\hat{M}_f \overset{?}{=} M^* \sqcup K_f^s$





break into multiple slides

Shaw Jung

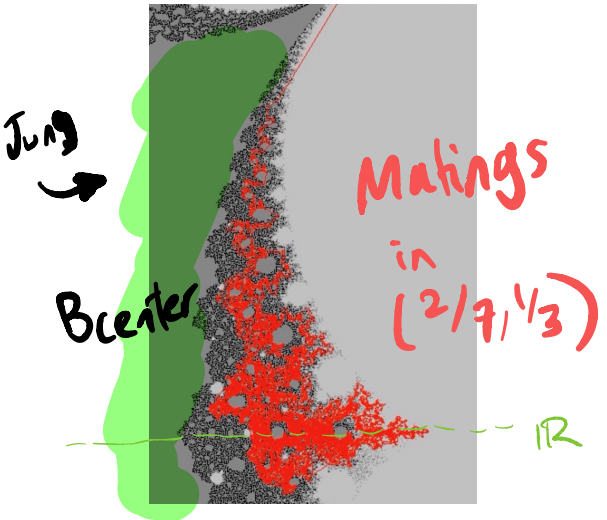
Candidate Family 1 · "slit recompletions"

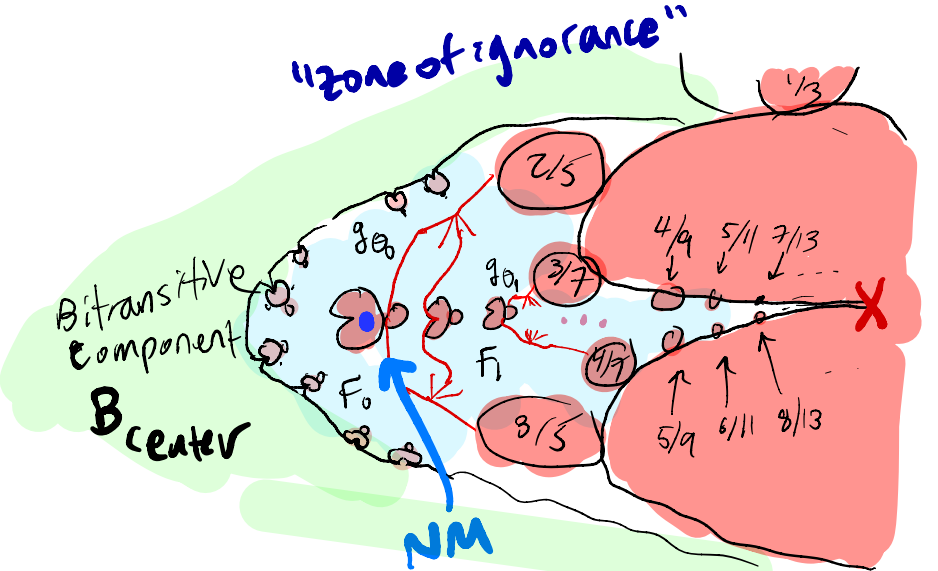
Prop (D.) No direct matings on $\partial B_{center} \cap \mathcal{Z}$

- slit model precise — predicts* everything not a mating w/ Airplane
  - but more t- d- to prove cf Jung! ex
- type C slit components don't recombine & Julia-type points

in progress:

show no indirect either





in progress:

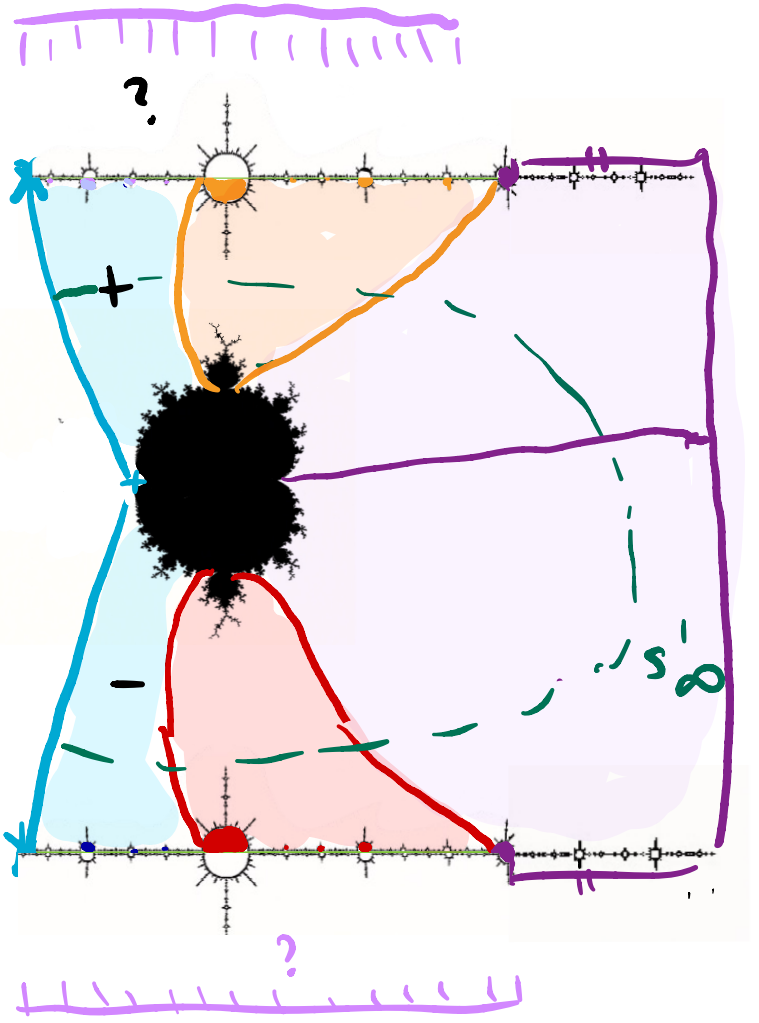
$$\hat{M}_A \simeq \overline{\mathbb{D}} - \{-1\}$$

Find $\tilde{\lambda}_A$ lamination

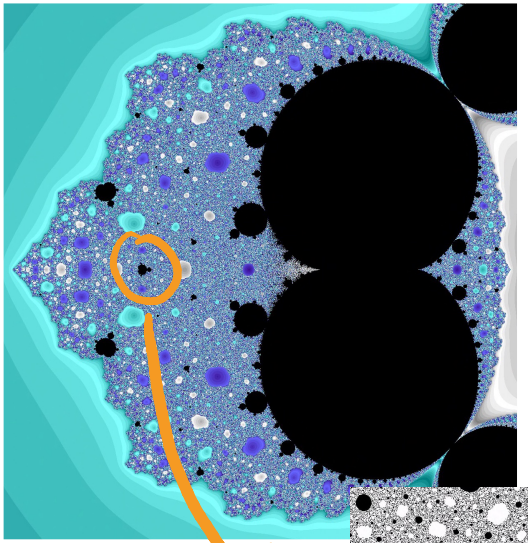
such that

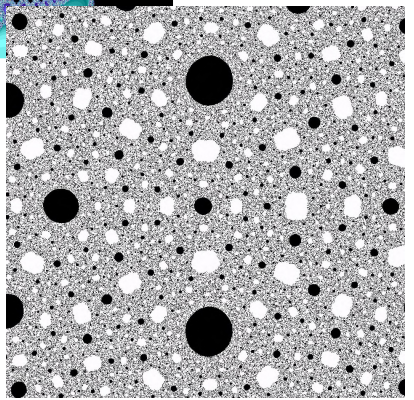
$$\hat{M}_A = \overline{\mathbb{D}} / \tilde{\lambda}_A$$

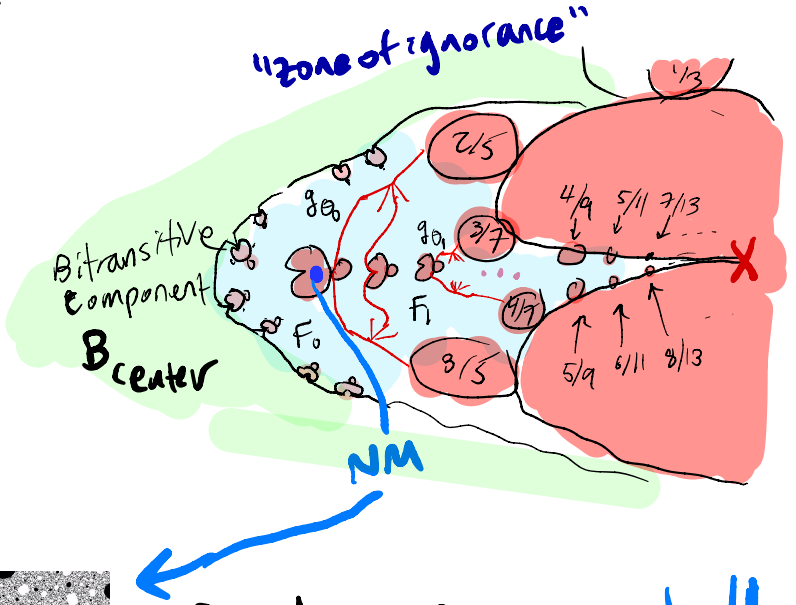
"QML for $\mathrm{Per}_3(0)$"



Wittner's OG example:







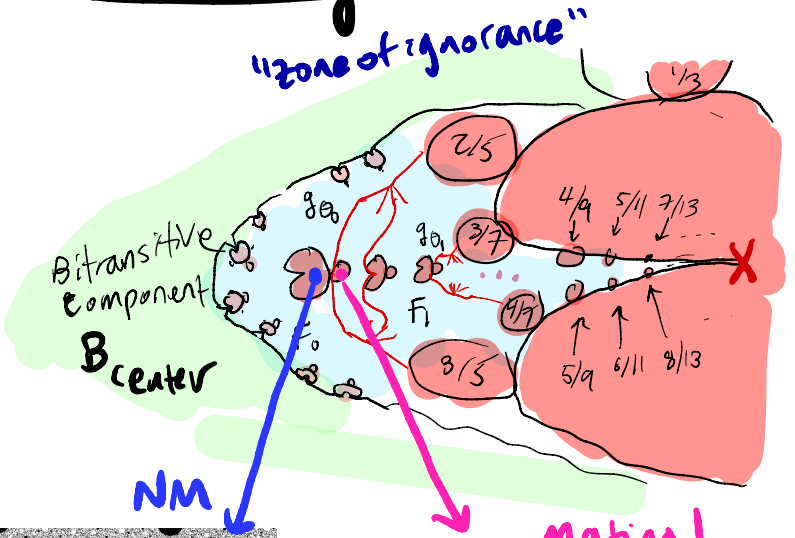
Real map with critical

- 3-cycle
- 4-cycle

} only 4 airplane matings w/ 4-cycles

[$\hat{\mathbb{C}}$ J_f is a carpet] (and none fit the bill)

Candidate Family 2: satellite-untunings

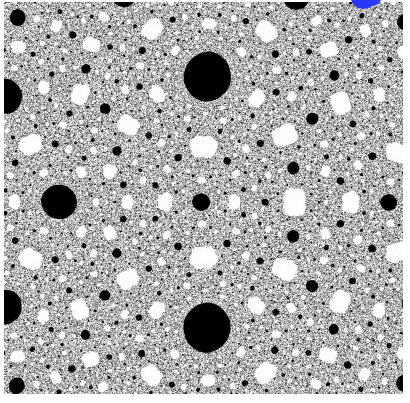


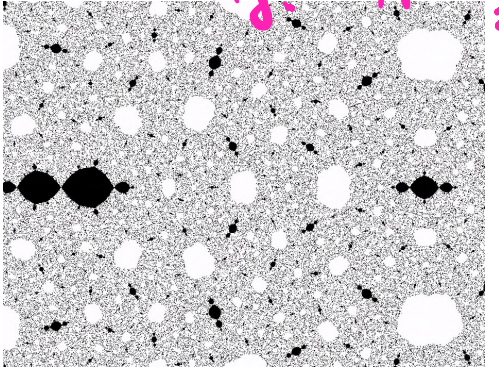
NM

mating!

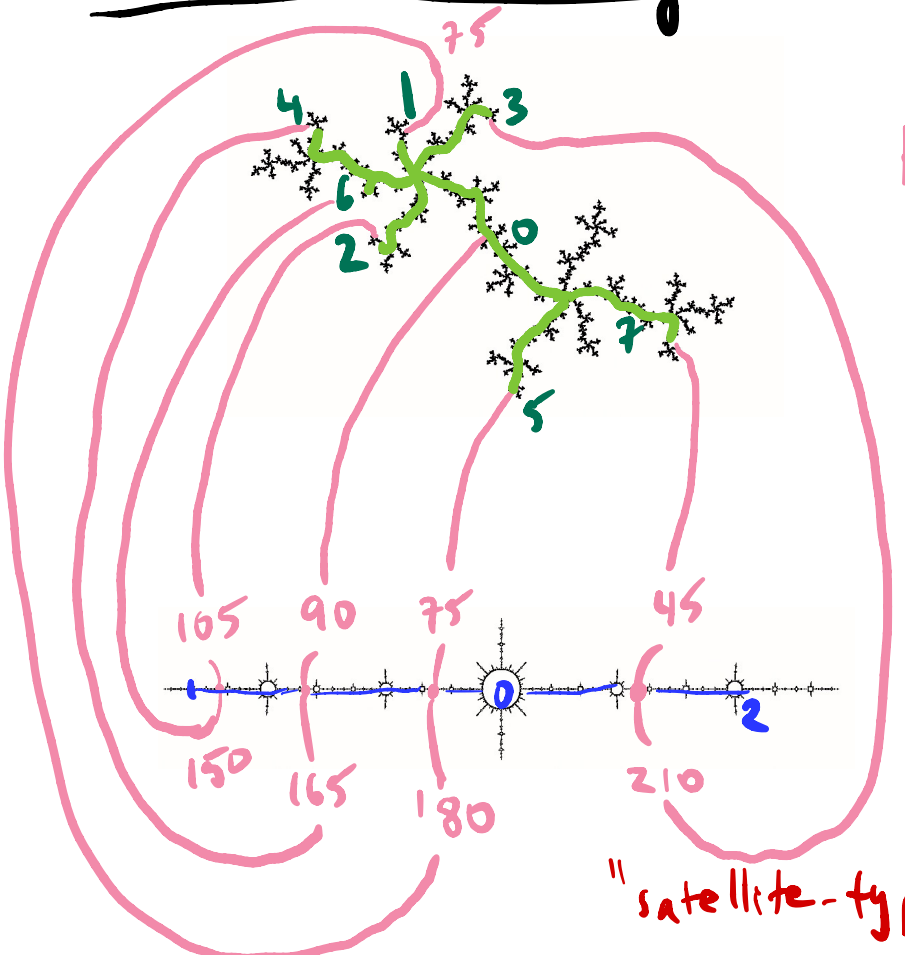
$\cong B$

$Au \frac{g_{75}}{255} = Au \frac{g_{-75}}{255}$



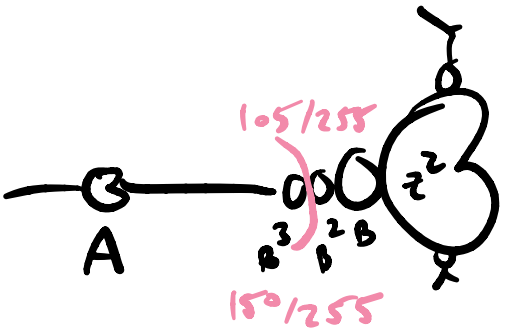


Candidate Family 2: satellite-untunings



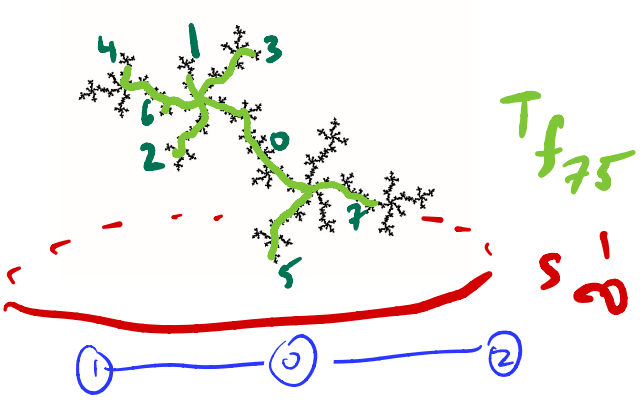
$k_{\frac{75}{255}}$ } 8-cycle in $L_{2/5}$

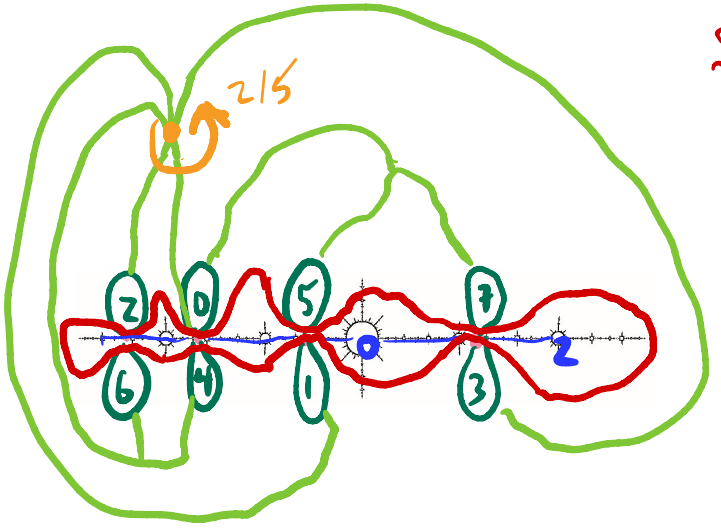
$k_{\bar{\Lambda}}$



"satellite-type point"

Candidate Family 2: satellite-untunings

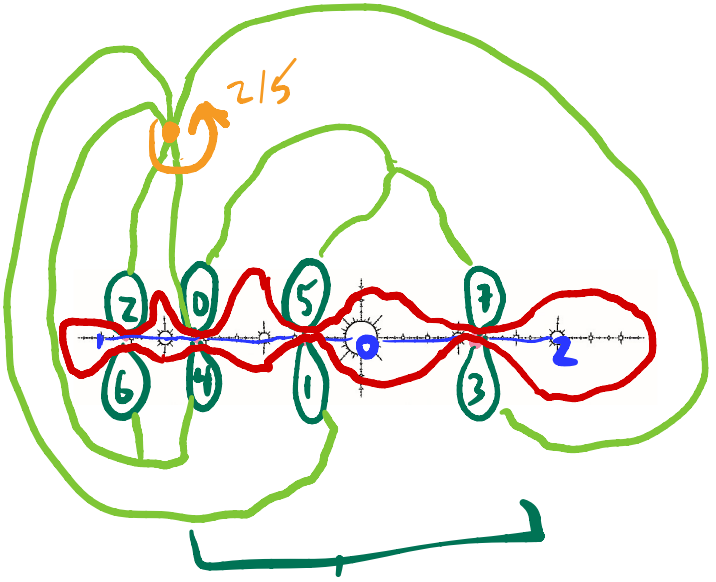




untuning
small
Basilica

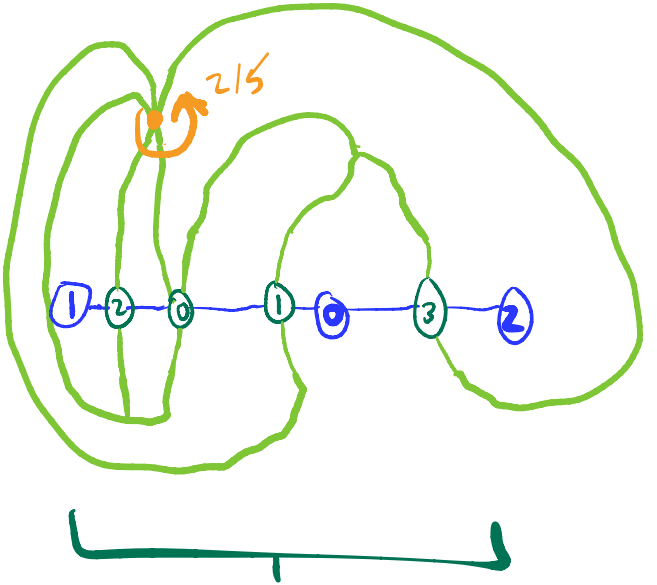
Candidate Family 2: satellite-untunings

lost original equator!
(but are there more?)



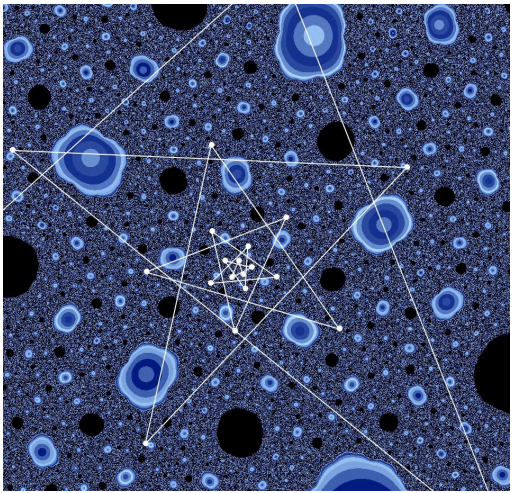
encoding for
Wittner's map $\in B$

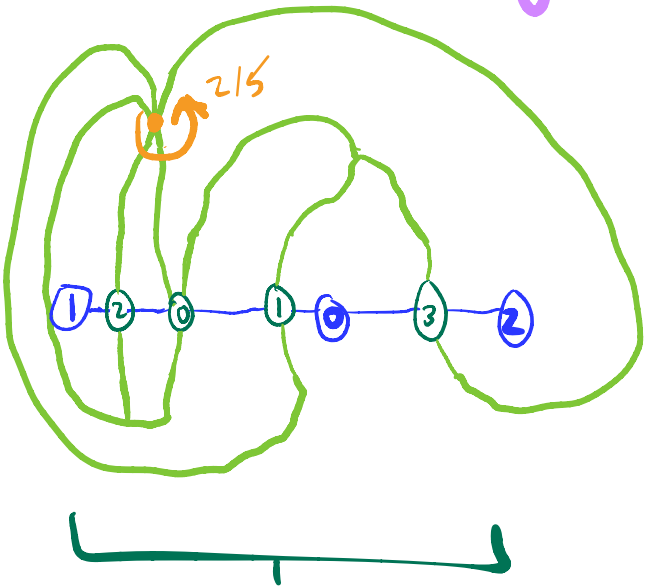
untuning
small
Basilica



encoding for
Wittner's map

Candidate Family 2: satellite-untunings





encoding for Wittner's map

Q: can we define a "combinatorial rotation #" for maps w/ J_f carpet?

Candidate Family 2: satellite-untunings

Prop: $\exists^{\infty} W_n \in Per_3(\circ)$ st

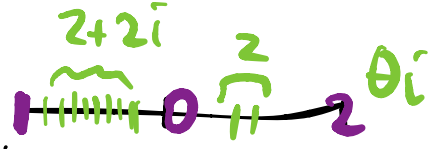
a) W_0 is wittners map

b) W_i has free period $4+2i$

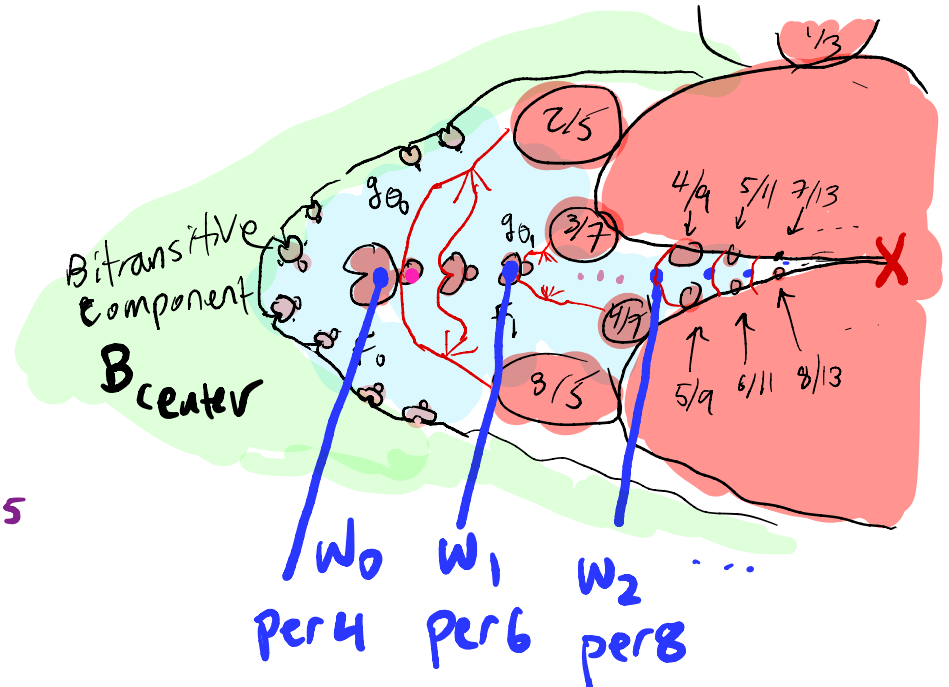
c) $W_i \sharp B = A \cup g_{\theta_i}$
$= A \cup g_{-\theta_i}$

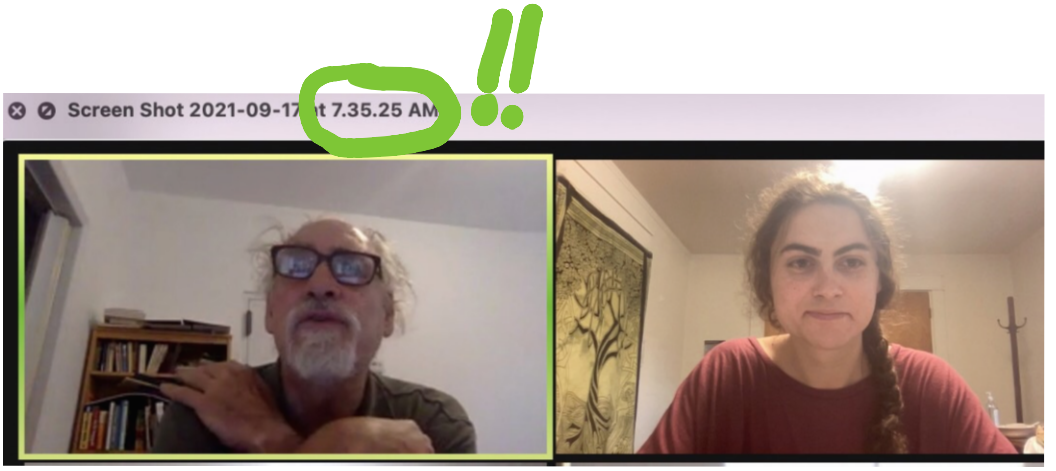
for $\theta_i = \frac{4^{i+2}-1}{3(4^{i+2}+1)}$ $t L_{\frac{i+2}{2i+5}}$

d) "seem to have" fp wl rot $\neq 2i+5$

e) 

Tf





Happy $(3^4-1)^{st}$!
Hamal ☺