

Tangent to the Identity Germs and Affine Surfaces

Rapport sur les contributions

ID de Contribution: 1

Type: **Non spécifié**

Affine surfaces (1)

lundi 10 février 2025 09:00 (1 heure)

The goal of this series of talks is to present some elements of the dynamics and topology of affine surfaces, with a focus on two subclasses: translation surfaces and dilation surfaces in the compact case.

We will begin with translation surfaces, introducing their deformation space and explaining a fundamental connection between their dynamics and the dynamics within this space, known as Masur's criterion. This criterion is a key ingredient in proving that unique ergodicity is generic for translation surfaces.

After becoming familiar with this subclass of affine surfaces, we will turn to a more general class: dilation surfaces. While these surfaces share many structural similarities with translation surfaces, their dynamics and topology generally exhibit significantly different behaviors. We will discuss their expected generic dynamics (which represent the simplest behavior from a dynamical perspective) and present a beautiful result by Veech on their triangulations.

If time permits, we will conclude by exploring some non-generic intermediate dynamical behaviors, such as the emergence of strange attractors, which can appear even in simple examples of dilation surfaces.

Orateur: FOUGERON, Charles

ID de Contribution: 2

Type: **Non spécifié**

Local dynamics of biholomorphisms (1)

lundi 10 février 2025 10:30 (1 heure)

The goal of this series of lecture is to present the relation between the dynamics of germs $f : (\mathbb{C}^2, 0) \rightarrow (\mathbb{C}^2, 0)$ tangent to the identity, the real-time dynamics of homogeneous vector fields in \mathbb{C}^2 and the dynamics of the geodesic flow on affine surfaces.

In the first talk, we will review the dynamics of germs $f : (\mathbb{C}, 0) \rightarrow (\mathbb{C}, 0)$, in particular, the Leau-Fatou flower theorem.

Orateur: BUFF, Xavier (Institut de Mathématiques de Toulouse)

Tangent to the Id... / Rapport sur les contributions

TBA

ID de Contribution: 3

Type: **Non spécifié**

TBA

lundi 10 février 2025 14:00 (50 minutes)

Orateur: REIS, Helena

ID de Contribution: 4

Type: **Non spécifié**

Classification of polar punctures of meromorphic connections.

lundi 10 février 2025 15:00 (50 minutes)

(Joint work with Xavier Buff) Given a meromorphic connection with a pole of degree >1 near a puncture of a Riemann surface, we introduce a sequence of asymptotic values and use it to define an invariant that allows for a complete local classification of those objects, up to local conformal isomorphism. We also provide a geometric description.

Orateur: CHÉRITAT, Arnaud (CNRS/Institut de Mathématiques de Toulouse)

ID de Contribution: 5

Type: **Non spécifié**

Parabolic implosion in dimension 2

lundi 10 février 2025 16:30 (50 minutes)

Parabolic implosion is a tool for studying the dynamics of perturbations of a map with a fixed point tangent to the identity, or more generally with at least one eigenvalue which is a root of unity. We will start by surveying classical parabolic implosion in dimension one, and then we will explain an ongoing work on parabolic implosion of germs tangent to the identity in dimension 2.

Joint work with Lorena Lopez-Hernanz and J. Raissy.

Orateur: ASTORG, Matthieu (Université d'Orléans, IDP)

ID de Contribution: 6

Type: **Non spécifié**

Local dynamics of biholomorphisms (2)

mardi 11 février 2025 09:00 (1 heure)

The goal of this series of lecture is to present the relation between the dynamics of germs $f : (\mathbb{C}^2, 0) \rightarrow (\mathbb{C}^2, 0)$ tangent to the identity, the real-time dynamics of homogeneous vector fields in \mathbb{C}^2 and the dynamics of the geodesic flow on affine surfaces.

In the second lecture, we will explain how, to each germ $f : (\mathbb{C}^2, 0) \rightarrow (\mathbb{C}^2, 0)$ tangent to the identity, one can associate a homogeneous vector field in \mathbb{C}^2 and a meromorphic affine surface modeled on the Riemann sphere minus finitely many points.

Orateur: BUFF, Xavier (Institut de Mathématiques de Toulouse)

ID de Contribution: 7

Type: **Non spécifié**

Affine Surfaces (2)

mardi 11 février 2025 10:30 (1 heure)

The goal of this series of talks is to present some elements of the dynamics and topology of affine surfaces, with a focus on two subclasses: translation surfaces and dilation surfaces in the compact case.

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After becoming familiar with this subclass of affine surfaces, we will turn to a more general class: dilation surfaces. While these surfaces share many structural similarities with translation surfaces, their dynamics and topology generally exhibit significantly different behaviors. We will discuss their expected generic dynamics (which represent the simplest behavior from a dynamical perspective) and present a beautiful result by Veech on their triangulations.

If time permits, we will conclude by exploring some non-generic intermediate dynamical behaviors, such as the emergence of strange attractors, which can appear even in simple examples of dilation surfaces.

Orateur: FOUGERON, Charles

ID de Contribution: 8

Type: **Non spécifié**

Formal transversality of the infinitesimal generator along the fixed point set

mardi 11 février 2025 14:00 (50 minutes)

Given a tangent to the identity germ of holomorphic diffeomorphism ϕ , we consider the map $P \mapsto \phi_P$ that associates to any fixed point P of ϕ near the origin the germ ϕ_P of ϕ at P . Such germs are not in general tangent to the identity. Given the infinitesimal generator X of ϕ , a formal vector field, it is natural to ask whether we can define “infinitesimal generators” X_P of ϕ_P for $P \in \text{Fix}(\phi)$ and if the dependence of X_P on $P \in \text{Fix}(\phi)$ is analytic. In other words, we are asking whether X is formally transversal along $\text{Fix}(\phi)$.

We introduce a weak and a strong concept of formal transversality for X . On the one hand, we will see that X is always formally transversal along $\text{Fix}(\phi)$ in the weak sense. On the other hand, there are examples where X is not formally transversal along $\text{Fix}(\phi)$ in the strong sense. We will discuss the gap between the weak and the strong concepts and provide a characterization of strong formal transversality. This is a joint work with Rudy Rosas.

Orateur: RIBON, Javier

ID de Contribution: 9

Type: **Non spécifié**

On translation surfaces defined by meromorphic differentials

mardi 11 février 2025 15:00 (50 minutes)

A meromorphic one form with poles on a Riemann surface defines naturally a translation surface of infinite area. In this talk, after seeing briefly how such structures appear naturally when studying usual translation surfaces, we will describe the orbits of the geodesic flow and show how we can use this result to classify the connected components of the corresponding moduli space.

Orateur: BOISSY, Corentin (Université de Toulouse, IMT)

ID de Contribution: **10**

Type: **Non spécifié**

Local dynamics of biholomorphisms (3)

mardi 11 février 2025 16:30 (1 heure)

The goal of this series of lecture is to present the relation between the dynamics of germs $f : (\mathbb{C}^2, 0) \rightarrow (\mathbb{C}^2, 0)$ tangent to the identity, the real-time dynamics of homogeneous vector fields in \mathbb{C}^2 and the dynamics of the geodesic flow on affine surfaces.

In the third lecture, we will study the geodesic flow of meromorphic affine surfaces modeled on compact Riemann surfaces minus finitely many points.

Orateur: BUFF, Xavier (Institut de Mathématiques de Toulouse)

ID de Contribution: 11

Type: **Non spécifié**

Affine Surfaces (3)

mercredi 12 février 2025 09:00 (1 heure)

The goal of this series of talks is to present some elements of the dynamics and topology of affine surfaces, with a focus on two subclasses: translation surfaces and dilation surfaces in the compact case.

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If time permits, we will conclude by exploring some non-generic intermediate dynamical behaviors, such as the emergence of strange attractors, which can appear even in simple examples of dilation surfaces.

Orateur: FOUGERON, Charles

ID de Contribution: 12

Type: **Non spécifié**

Resolution of the infinitesimal generator and parabolic manifolds in dimension 3

mercredi 12 février 2025 10:20 (50 minutes)

Through explicit examples introduced by Samuele Mongodi and myself, we will see how the resolution of singularities of vector fields of McQuillan and Panazzolo, and the resolution along separatrices of Lopez-Hernanz, Ribon, Sanz-Sanchez and Vivas, intervene in the study of parabolic manifolds for tangent to the identity germs in dimension 3.

Part of the talk is based on a work in progress with Samuele Mongodi and André Belotto Da Silva.

Orateur: RUGGIERO, Matteo (Université Paris Cité - IMJ-PRG)

ID de Contribution: 13

Type: **Non spécifié**

Local dynamics of reduced saddle-node biholomorphisms

mercredi 12 février 2025 11:20 (50 minutes)

We study the dynamics on a full neighborhood of the origin for a biholomorphism F in \mathbb{C}^2 that is of the reduced saddle-node type. For these type of diffeomorphisms we will show that there exist connected domains with the origin in their boundary which are either stable for F or for its inverse, and that outside these domains every point is either fixed or has a finite orbit. This is a work in progress in collaboration with Lorena Lopez-Hernanz and Rudy Rosas.

Orateur: VIVAS, Liz

ID de Contribution: 14

Type: **Non spécifié**

Local dynamics of biholomorphisms (4)

jeudi 13 février 2025 09:00 (1 heure)

The goal of this series of lecture is to present the relation between the dynamics of germs $f : (\mathbb{C}^2, 0) \rightarrow (\mathbb{C}^2, 0)$ tangent to the identity, the real-time dynamics of homogeneous vector fields in \mathbb{C}^2 and the dynamics of the geodesic flow on affine surfaces.

In the fourth lecture, we will explain how, using the dynamics of the geodesic flow on affine surfaces, one can recover several known results on the existence of parabolic domains for germs $f : (\mathbb{C}^2, 0) \rightarrow (\mathbb{C}^2, 0)$ tangent to the identity.

Orateur: BUFF, Xavier (Institut de Mathématiques de Toulouse)

ID de Contribution: 15

Type: **Non spécifié**

Affine Surfaces (4)

jeudi 13 février 2025 10:30 (1 heure)

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If time permits, we will conclude by exploring some non-generic intermediate dynamical behaviors, such as the emergence of strange attractors, which can appear even in simple examples of dilation surfaces.

Orateur: FOUGERON, Charles

ID de Contribution: 16

Type: Non spécifié

Parabolic manifolds of analytic diffeomorphisms along an invariant formal curve

jeudi 13 février 2025 14:00 (50 minutes)

Let $F : (\mathbb{C}^n, 0) \rightarrow (\mathbb{C}^n, 0)$ be a germ of a holomorphic diffeomorphism and let Γ be a formal curve at 0, invariant for F . Under certain sharp conditions on the restricted diffeomorphism $F|_{\Gamma}$, we show that there exists a finite non-empty family of complex submanifolds of $\mathbb{C}^n \setminus \{0\}$, invariant for F and entirely composed of orbits which converge to the origin and have flat contact with Γ (parabolic manifolds). In a second part of the talk, we adapt this result for the case of a germ of a real analytic diffeomorphism $F : (\mathbb{R}^n, 0) \rightarrow (\mathbb{R}^n, 0)$, where we can show, moreover, that each parabolic manifold of the family is foliated by real parabolic curves of F .

These results are obtained in collaboration with L. López-Hernández, J. Ribón, J. Raissy and L. Vivas.

Orateur: SANZ, Fernando

ID de Contribution: 17

Type: **Non spécifié**

Singularities and flow completeness of infinite type translation surfaces

jeudi 13 février 2025 15:00 (50 minutes)

Translation surfaces are (very) particular type of affine surfaces where transition maps are translations. Though, any affine surface admits a cover which is a translation surface (possibly of infinite type). The goal of my talk is to introduce a metric invariant for singularities of infinite type translation surfaces due to Bowman-Valdez and explain how it is related to the completeness of the translation flow.

Orateur: DELECROIX, Vincent (CNRS, Université de Bordeaux)

ID de Contribution: 18

Type: **Non spécifié**

Horn maps of semi-parabolic Hénon maps

jeudi 13 février 2025 16:30 (50 minutes)

We prove that horn maps associated to quadratic semi-parabolic fixed points of Hénon maps, first introduced by Bedford, Smillie, and Ueda, satisfy a weak form of the Ahlfors island property. As a consequence, two natural definitions of their Julia set (the non-normality locus of the family of iterates and the closure of the set of the repelling periodic points) coincide. As another consequence, we also prove that there exist small perturbations of semi-parabolic Hénon maps for which the Hausdorff dimension of the forward Julia set J_+ is arbitrarily close to 4. This is a joint work with M. Astorg.

Orateur: BIANCHI, Fabrizio

ID de Contribution: 19

Type: **Non spécifié**

Affine Surfaces (5)

vendredi 14 février 2025 09:00 (1 heure)

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If time permits, we will conclude by exploring some non-generic intermediate dynamical behaviors, such as the emergence of strange attractors, which can appear even in simple examples of dilation surfaces.

Orateur: FOUGERON, Charles

ID de Contribution: **20**Type: **Non spécifié**

Local dynamics of biholomorphisms (5)

vendredi 14 février 2025 10:30 (1 heure)

The goal of this series of lecture is to present the relation between the dynamics of germs $f : (\mathbb{C}^2, 0) \rightarrow (\mathbb{C}^2, 0)$ tangent to the identity, the real-time dynamics of homogeneous vector fields in \mathbb{C}^2 and the dynamics of the geodesic flow on affine surfaces.

In the last lecture, we will try to formulate open problems concerning the dynamics of germs $f : (\mathbb{C}^2, 0) \rightarrow (\mathbb{C}^2, 0)$ tangent to the identity, the real-time dynamics of homogeneous vector fields in \mathbb{C}^2 and the dynamics of the geodesic flow on affine surfaces.

Orateur: BUFF, Xavier (Institut de Mathématiques de Toulouse)