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Control theory and splitting methods

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Our goal is to highlight some of the deep links between numerical splitting methods and control theory. We consider evolution equations of the form $x' = f_0(x) + f_1(x)$, where f_0 encodes a non-reversible dynamic, so that one is interested in schemes only involving forward flows of f_0. In this context, a splitting method can be interpreted as a trajectory of the control-affine system $x'(t) = f0(x(t)) + u(t)f1(x(t))$, associated with a control u which is a finite sum of Dirac masses. The general goal is then to find a control such that the flow of $f_0 + u(t)f_1$ is as close as possible to the flow of $f_0 + f_1$. Using this interpretation and classical tools from control theory, we revisit well-known results concerning numerical splitting methods, and we prove a handful of new ones, with an emphasis on splittings with additional positivity conditions on the coefficients. First, we show that there exist numerical schemes of any arbitrary order involving only forward flows of f_0 if one allows complex coefficients for the flows of f_1. Equivalently, for complex-valued controls, we prove that the Lie algebra rank condition is equivalent to the small-time local controllability of a system. Second, for real-valued coefficients, we show that the well-known order restrictions are linked with so-called "bad"Lie brackets from control theory, which are known to yield obstructions to small-time local controllability. We use our recent basis of the free Lie algebra to precisely identify the conditions under which high-order methods exist.

This is a joint work with Adrien Laurent and Frédéric Marbach.

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