# *Catching neutrinos by the dipole*

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**** Muon collider probes of Majorana neutrino dipole moments and masses***
with **Natascia Vignaroli ,** arXiv:2409.02721 → JHEP 04 (2025) 008

**** Two portals to GeV sterile neutrinos : dipole versus mixing***
with **Enrico Bertuzzo ,** arXiv:2412.10101 → SciPost Phys. 18 (2025) 140

**Dark matter & Neutrinos**
*IHP Paris, 5-23 May 2025*

# Outline

* Generalities on the **dipole interaction of fermions**

* The dipole of **light (active) neutrinos** : from current bounds to possible collider signatures

* Sensitivity study at a **future muon collider**

**speculative**

* The dipole of **heavy (sterile) neutrinos** : its origin and its interplay with active-sterile mixing

* Sensitivity study at **beam-dump experiments**

# EFT for fermion-photon coupling

**Fermion bilinears with D-3 covariant derivatives** ($\psi_j$ are Weyl spinors) :

$$\mathcal{L}_{D=3} = \frac{1}{2}\,\psi_j\, m_{jk}\,\psi_k + h.c.$$

Fermion masses

$$\mathcal{L}_{D=4} = \psi_j^\dagger\, i\,\bar{\sigma}^\mu D_\mu\,\delta_{jk}\,\psi_k \quad \Rightarrow \quad Q_{jk} = q_j \delta_{ij}$$

Charge

$$\mathcal{L}_{D=5} = \frac{1}{2}\,\psi_j\,\sigma^{\mu\nu} F_{\mu\nu}\,\lambda_{jk}\,\psi_k + h.c. \quad \Rightarrow \begin{cases} \mu_{jk} = i\mathrm{Im}\lambda_{jk} \\ \epsilon_{jk} = i\mathrm{Re}\lambda_{jk} \end{cases}$$

Magnetic dipole

Electric dipole

$$\mathcal{L}_{D=6} = \psi_j^\dagger\,\bar{\sigma}^\mu(\partial^\nu F_{\mu\nu})\,\Delta_{jk}\,\psi_k \quad \Rightarrow \begin{cases} r^Q_{jk} = \mathrm{Im}\Delta_{jk} \\ a_{jk} = \mathrm{Re}\Delta_{jk} \end{cases}$$

Charge radius

Anapole

For recent studies see e.g.
**Atzori Corona, Cadeddu, Cargioli, Dordei, Giunti 2022 & 2024** (neutrino charged radius)
**Bose, Choudhury, Mondal, Sankar Ray 2023** (dark matter anapole)
**Herrera, Shoemaker 2024, Herrera, Huber 2024** (neutrino anapole)

# EFT for fermion-photon coupling

**Fermion bilinears with D-3 covariant derivatives** ($\psi_j$ are Weyl spinors) :

$$\mathcal{L}_{D=3} = \frac{1}{2}\,\psi_j\, m_{jk}\,\psi_k + h.c.$$

Fermion masses

$$\mathcal{L}_{D=4} = \psi_j^\dagger\, i\,\bar{\sigma}^\mu D_\mu\, \delta_{jk}\,\psi_k \quad \Rightarrow \quad Q_{jk} = q_j \delta_{ij}$$

Charge

$$\mathcal{L}_{D=5} = \frac{1}{2}\,\psi_j\,\sigma^{\mu\nu} F_{\mu\nu}\,\lambda_{jk}\,\psi_k + h.c. \quad \Rightarrow \begin{cases} \mu_{jk} = i\mathrm{Im}\lambda_{jk} \\ \epsilon_{jk} = i\mathrm{Re}\lambda_{jk} \end{cases}$$

Magnetic dipole

Electric dipole

**Naive Dimensional Analysis** of dipole (UV contribution) :

$$\boxed{\lambda \sim \frac{e}{16\pi^2}\frac{(\epsilon_\psi g_\star)^2}{m_\star} \simeq 10^{-7}\mu_B \left(\frac{\epsilon_\psi g_\star}{4\pi}\right)^2 \left(\frac{10\mathrm{TeV}}{m_\star}\right)}$$

Bohr magneton :

$$\mu_B \equiv \frac{e}{2m_e} \simeq \frac{300}{\mathrm{GeV}}$$

$m_* \equiv$ mass of UV states $\qquad \epsilon_\psi g_* \equiv \psi-$coupling to UV states

# Neutrinos: mass vs dipole

Two Weyl spinors : $\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$

Lepton number $U(1)_L \ : \quad L(\nu) = 1$

Flavour symmetry $SU(2)_F \ : \quad \nu \sim 2_F$

$$\mathcal{L} \supset \nu_\alpha^\dagger \, i\bar{\sigma}^\mu \partial_\mu \, \delta_{\alpha\beta} \, \nu_\beta$$

**Kinetic operator:** no electric charge, but global symmetry $U(1)_L$ x $SU(2)_F$

$$\mathcal{L} \supset \frac{1}{2} \nu_\alpha \, m_{\alpha\beta} \, \nu_\beta + h.c. \qquad (m_{\alpha\beta} = m_{\beta\alpha})$$

$$m \equiv m_A (\epsilon \sigma_A)$$

**Mass operator:** 3 independent entries, $\Delta L = 2$, breaks $SU(2)_F$

*$m_A$ real preserves residual $U(1)_F$ : one Dirac fermion*

*$m_A$ complex breaks also $U(1)_F$ : two Majorana fermions*

$$m_A \sim 3_F$$

$$\mathcal{L} \supset \frac{1}{2} \nu_\alpha \, \sigma^{\mu\nu} \, \lambda_{\alpha\beta} \, \nu_\beta \, F_{\mu\nu} + h.c. \qquad (\lambda_{\alpha\beta} = -\lambda_{\beta\alpha})$$

$$\lambda \sim 1_F$$

**Dipole operator:** 1 unique entry, $\Delta L = 2$, preserves $SU(2)_F$

$\Rightarrow$ **it is *technically natural* to have small mass and large dipole !**

# EFT for neutrino dipole

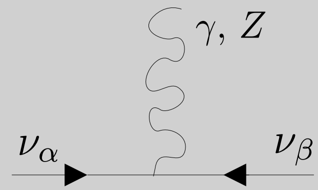
**dim = 7 , Δ L = 2** operators in the SM Effective Field Theory :

$$\mathcal{L}_7 = \sum_i C_i \mathcal{O}_i$$

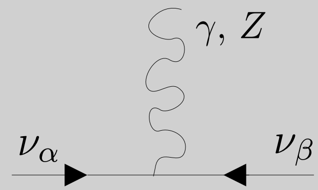
$$\lambda_{\alpha\beta} = -2ev^2 \left( C^B_{\alpha\beta} + 2C^W_{\alpha\beta} \right)$$

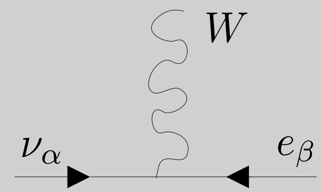
**Davidson Gorbahn Santamaria 2005**

$$(O_B)_{\alpha\beta} = g' \left( \overline{\ell^c_{L\alpha}} \epsilon H \right) \sigma^{\mu\nu} \left( H^T \epsilon \ell_{L\beta} \right) B_{\mu\nu}$$




$$(O_W)_{\alpha\beta} = ig\varepsilon_{abc} \left( \overline{\ell^c_{L\alpha}} \epsilon \sigma^a \sigma^{\mu\nu} \ell_{L\beta} \right) \left( H^T \epsilon \sigma^b H \right) W^c_{\mu\nu}$$




**Naive Dimensional Analysis :**

$$\frac{|\lambda_{\alpha\beta}|}{\mu_B} \simeq 2 \cdot 10^{-11} |c^B_{\alpha\beta} + 2c^W_{\alpha\beta}| \left( \frac{10\ \text{TeV}}{m_*} \right)^3 \left( \frac{g_*}{4\pi} \right)^4 \left( \frac{\epsilon_H}{1} \right)^2 \left( \frac{\epsilon_\alpha \epsilon_\beta}{10^{-3}} \right)$$

c's order one / $m_*$ mass of UV states / $g_*$ coupling among UV states

ε's couplings of Higgs & leptons to UV states :

*(have to be small to suppress lepton flavour violation, especially e-to-μ)*

# Some current bounds on ν dipole

* Solar neutrino elastic scattering on nuclei (photon exchange)

$$\lambda_\nu \simeq \Big(\sum_k |U_{ek}|^2 \sum_j |\lambda_{jk}|^2\Big)^{1/2} < 0.6 \cdot 10^{-11} \mu_B \ (90\% \, C.L.)$$

**Giunti Studenikin 1403.6344**

**XENON 2207.11330, LZ 2207.03764**

* Stellar energy loss (red giant branch of globular clusters)

$$\lambda_\nu \lesssim 0.1 \cdot 10^{-11} \mu_B \ (95\% \, C.L.)$$

**Capozzi Raffelt 2007.03694**

- indirect (neutrinos not observed)
- effective combination of flavours
- insensitive to lepton number
- which systematic uncertainty ?

* Neutrino-to-antineutrino conversion in solar magnetic field

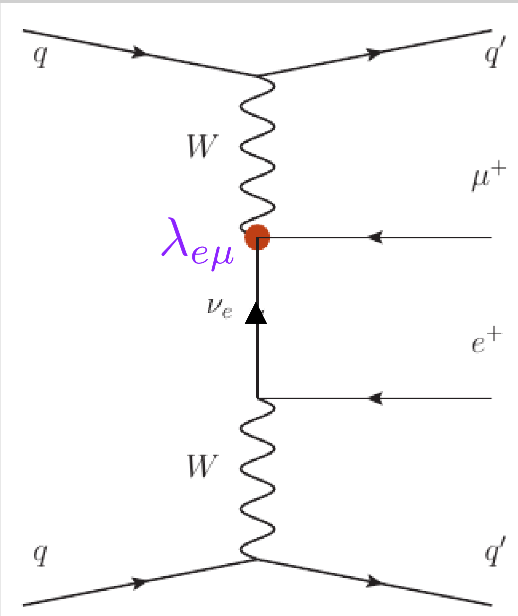
$$\lambda_\nu \lesssim 500 \cdot 10^{-11} \mu_B \left[\frac{kG}{B}\right] \ (90\% \, C.L.)$$

**KamLAND 2108.08527**
**Akhmedov Martínez-Miravé 2207.04516**

- sensitive to lepton number violation
- large uncertainty on value of solar magnetic field

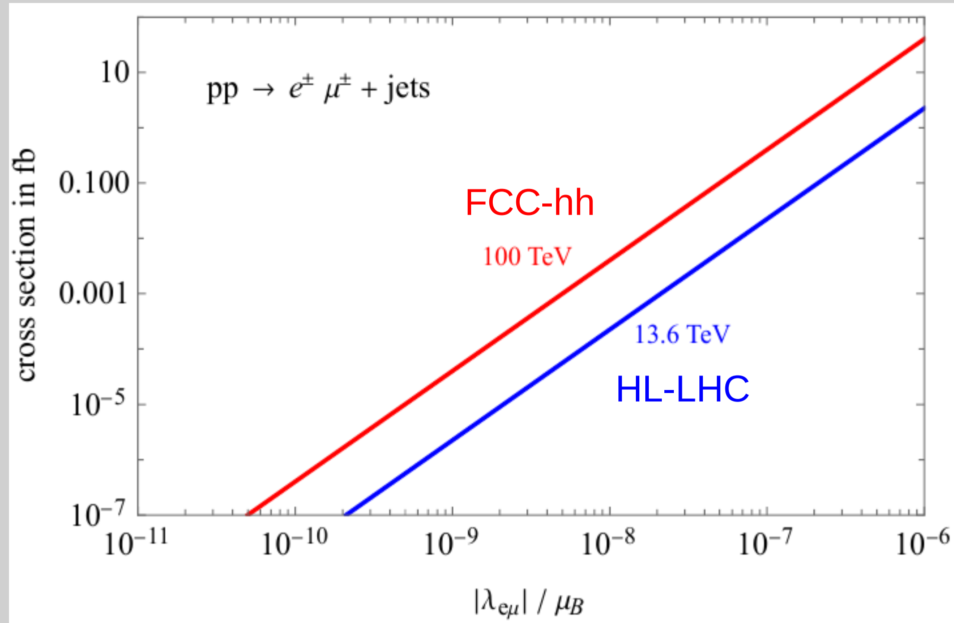
with N.Vignaroli

# ν dipole @ future hadron colliders




2 leptons
**same-sign**
**different-flavour**
+
2 jets

after acceptance cuts




**Recasting** LHC analysis for the same final state.

After selection cuts: **signal** efficiency ~ 0.5 and **background** (mostly from V V j j) ~ 10 fb [reducible]

**ATLAS 2403.15016**

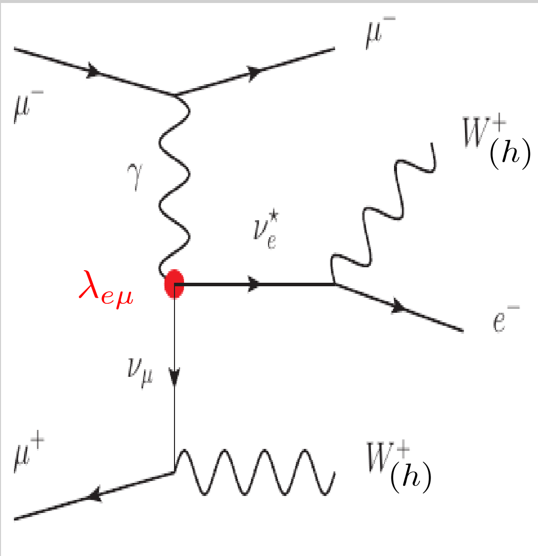
$$\mathrm{HL-LHC}\ \ \{3\ \mathrm{ab}^{-1}\} \qquad |\lambda_{e\mu}|/\mu_B < 2.2\cdot 10^{-7}\ [3.8\cdot 10^{-8}] \quad @\ 2\sigma$$

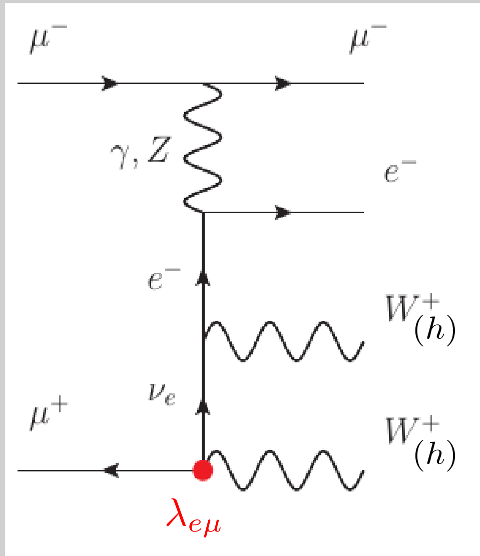
$$\mathrm{FCC-hh}\ \ \{30\ \mathrm{ab}^{-1}\} \qquad |\lambda_{e\mu}|/\mu_B < 3.8\cdot 10^{-8}\ [2.0\cdot 10^{-9}] \quad @\ 2\sigma$$

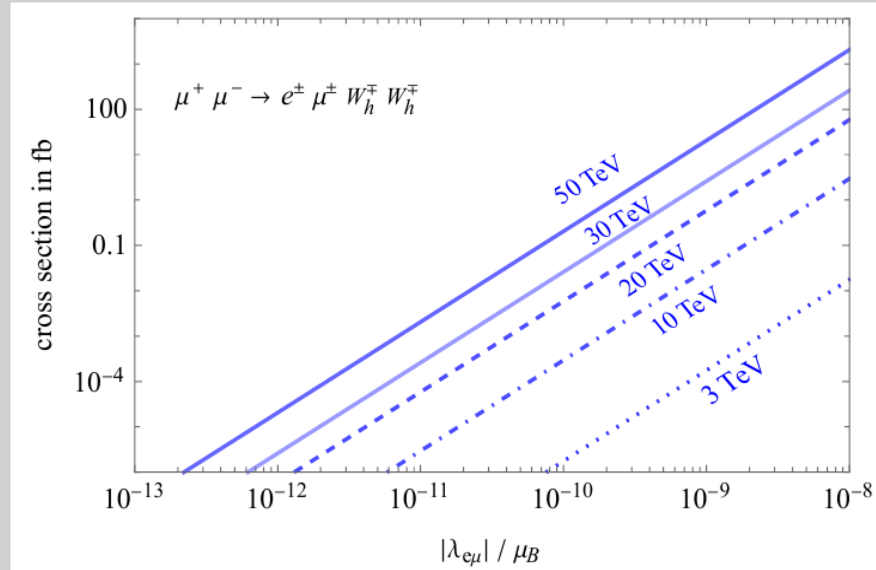
Sensitivity about **2-3 orders of magnitude weaker than current bounds**

with N.Vignaroli

# ν dipole @ future muon collider (I)




Illustrative for ~ 100 diagrams

Including acceptance cuts
& reconstruction efficiency

2 leptons (**same-sign different-flavour)**
+ 2 fat jets (**W into hadrons**)

Clean, unambiguous signal of
lepton number and flavour violation

Achievable integrated luminosity
for 5-years data taking :

$$\mathcal{L} = 10 \left( \frac{\sqrt{s}}{10\,\mathrm{TeV}} \right)^2 \mathrm{ab}^{-1}$$

Signal **simulation & reconstruction** (Madgraph, Pythia8, FastJet, ...)

Same for background (mostly from **W W μ⁺ μ⁻ & W W W W**) ~ 1 ab [reducible]

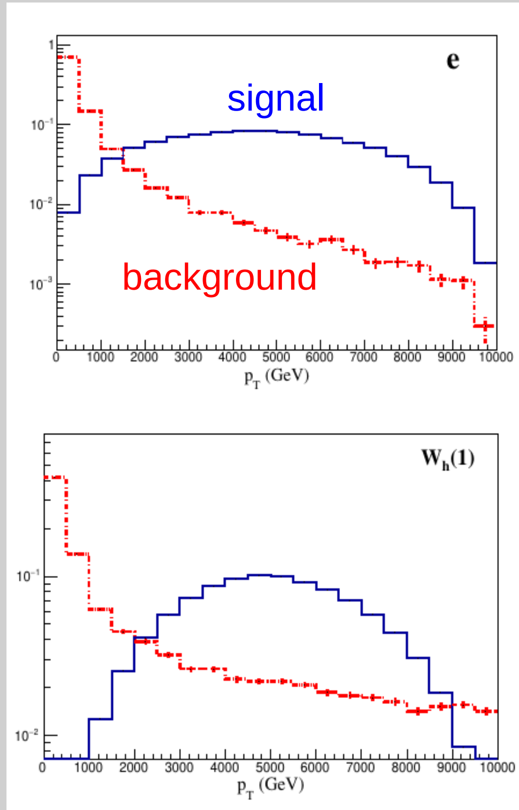
# ν dipole @ future muon collider (II)

Improving discrimination by **kinematic variables**

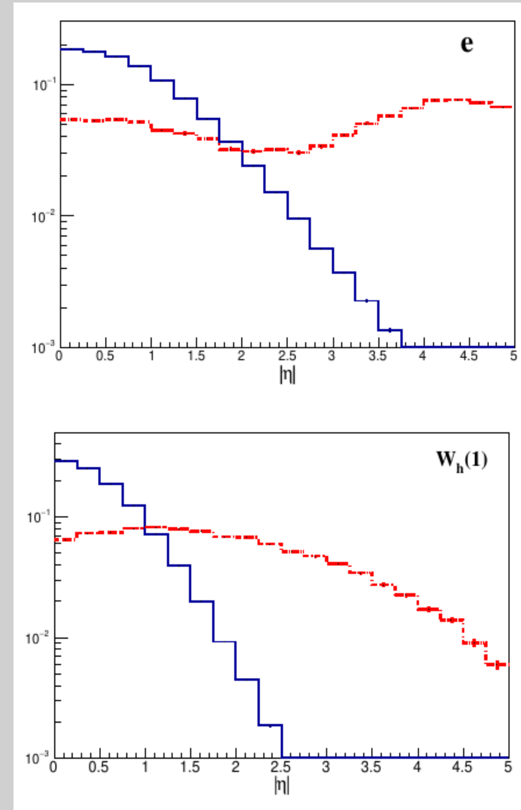
Signal events have **higher $p_T$** & are **more central**

With tailored cuts, **background down to ~ 0.1 ab**

transverse momentum distribution




rapidity distribution




| $\sqrt{s}$ | 3 TeV | 10 TeV | 20 TeV | 30 TeV | 50 TeV |
|---|---|---|---|---|---|
| $\frac{\lvert\lambda_{e\mu}\rvert}{\mu_B}$ | $5.4\ [4.8] \cdot 10^{-9}$ | $1.5\ [1.1] \cdot 10^{-10}$ | $1.9\ [1.2] \cdot 10^{-11}$ | $6.6\ [3.9] \cdot 10^{-12}$ | $1.5\ [0.8] \cdot 10^{-12}$ |

@ $2\sigma$

further background rejection, improved lepton identification

**~ current laboratory bound**

# Summary and perspective (part I)

- **First study** of projected collider sensitivities to the $\nu$ dipole

- **Muon collider:** one can probe in clean and direct way the $\nu$ dipoles (including $e\tau$ and $\mu\tau$ cases) as well as other sources of L-violation (e.g. $\nu$ masses)

- **Flavour model-building:** induce large $\lambda_{e\mu}$ while suppressing other e-to-$\mu$ transitions

- **Beyond EFT:** new particles responsible for large $\lambda_{e\mu}$ may be directly produced

# Dipole for sterile neutrinos

**SM + 2 sterile neutrinos $N_{1,2}$**

with masses $M_{1,2}$

below EW scale

$$\begin{aligned}\mathcal{L}_N &= i{N_i}^\dagger\bar{\sigma}^\mu\partial_\mu\delta_{ij}N_j \\ &- \left(\frac{1}{2}N_i\mathcal{M}_{ij}N_j - \tilde{H}^\dagger\ell_\alpha Y_{\alpha i}N_i + h.c.\right) \\ &+ \frac{1}{2}\left(d\,N_i\sigma^{\mu\nu}\epsilon_{ij}N_j\,B_{\mu\nu} + h.c.\right) + \ldots\end{aligned}$$

dipole Wilson coefficient

hypercharge field strength

**<u>U(1)$_L$ conserving limits</u>** <u>(as neutrino masses are tiny):</u>

(i) $L(N_{1,2}) = 0$ : no Yukawa, no active-sterile mixing, $M_1 < M_2$ arbitrary, $N_1$ dark matter candidate

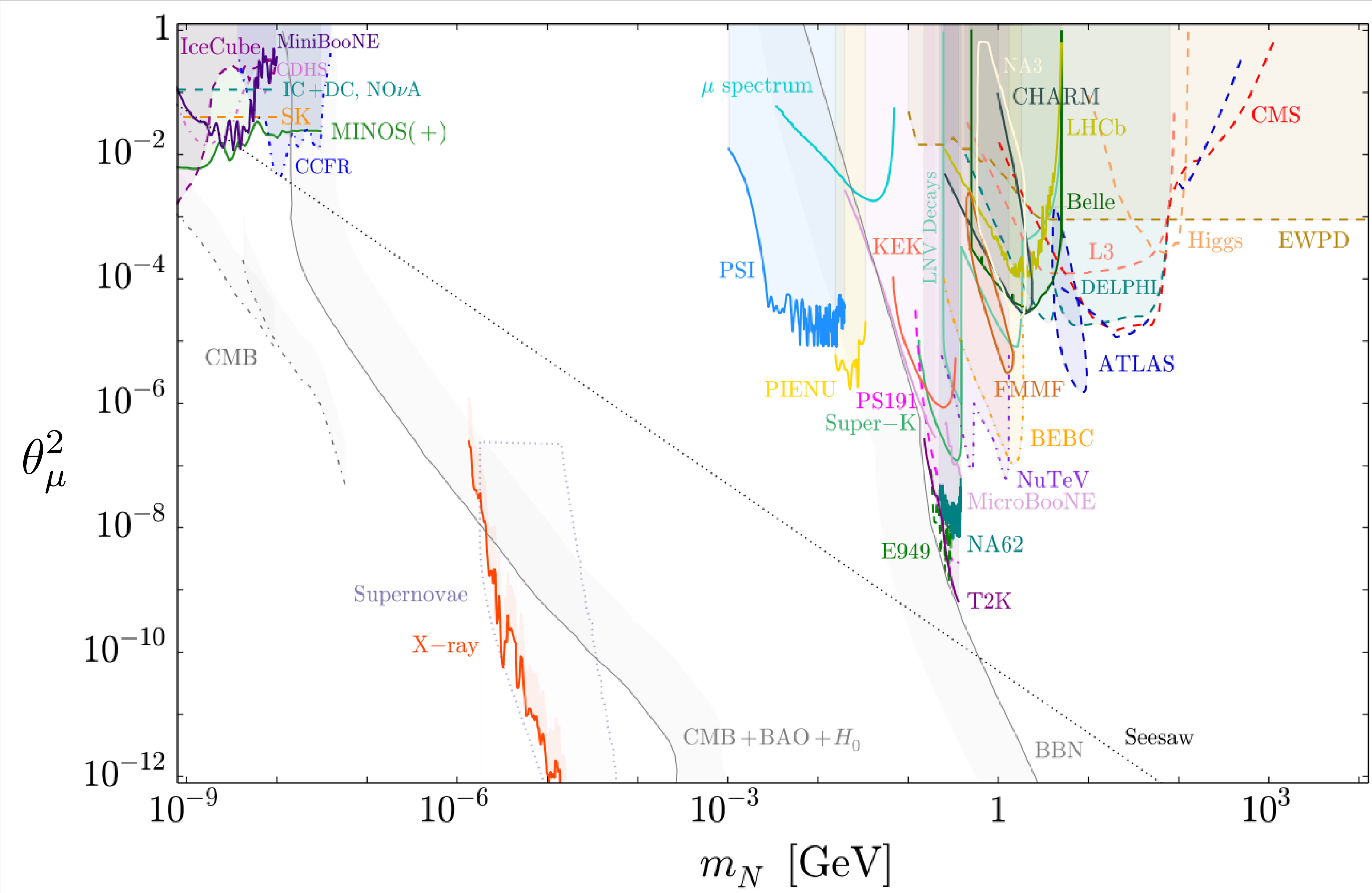
**Chu Pradler Semmelrock 2018** **Barducci Bertuzzo Taoso Toni 2022**

(ii) $L(N_1) = - L(N_2) = 1$ : $N_1$ & $N_2$ combine into one Dirac neutrino

$$M_\nu = \left(\begin{array}{c|cc} 0 & 0 & m \\ \hline 0 & 0 & M \\ m & M & 0 \end{array}\right) \qquad m_\alpha \equiv Y_{\alpha 2}\frac{v}{\sqrt{2}} \qquad \theta_\alpha \simeq \frac{m_\alpha}{\sqrt{2}M} \qquad M_1 = M_2 \simeq M$$

# Bounds on active-sterile mixing

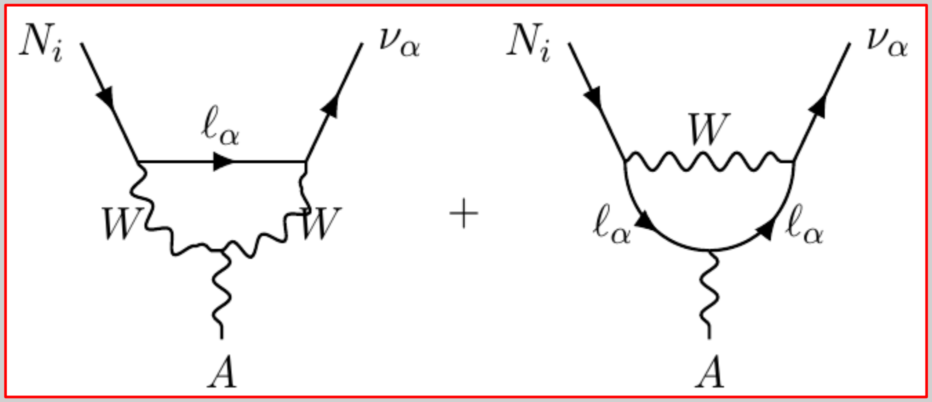
**https://www.hep.ucl.ac.uk/~pbolton/index.html**




Similar plots for mixing with electron or tau neutrino ...

# Dipole from electroweak loops

**Even in absence of dim-5 dipole,** sterile neutrinos acquire a dipole coupling to photon :




$$d_{i\alpha}^{EW} = \frac{3e}{16\pi^2}\,\frac{M_i}{v^2}\,\theta_{i\alpha}$$

**R.Shrock 1982**
**C.Giunti A.Studenikin 2014**

This effect is well-known for **keV-neutrino dark matter** : strong bounds from X-ray searches

**"White paper on keV sterile neutrino DM" 2016**

We realised such effect is also **observable for GeV-neutrinos @ intensity frontier !**

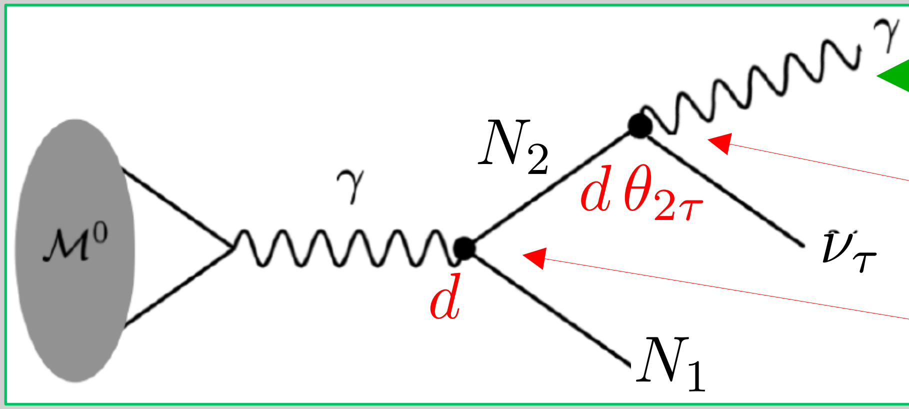
$$d_{i\alpha}^{total} = d_{i\alpha}^{D=5} + d_{i\alpha}^{EW} \qquad (d_{i\alpha}^{D=5} = d\,\cos\theta_w\,\theta_{i\alpha})$$

relative size : $\dfrac{(\epsilon_N g_*)^2}{m_*}$ vs $\dfrac{M_i}{v^2}$

# Sterile production & decay

**Mesons from protons on target** (beam-dump experiments SHiP, NA62, CHARM, …)
**or from p-p collisions** (forward experiments FASER 2, …) decay into sterile neutrinos

**Barducci Bertuzzo Taoso Toni Ternes 2024**




Photons with E > 1 GeV can be seen in the detector calorimeter

sterile decay in the detector

sterile production in the target

**GeV sterile neutrinos are long-lived** (thus reaching the far detector) :

* one loop suppression

* NP/EW scale suppression

* phase space (small $M_2 - M_1$)

* small active-sterile mixing

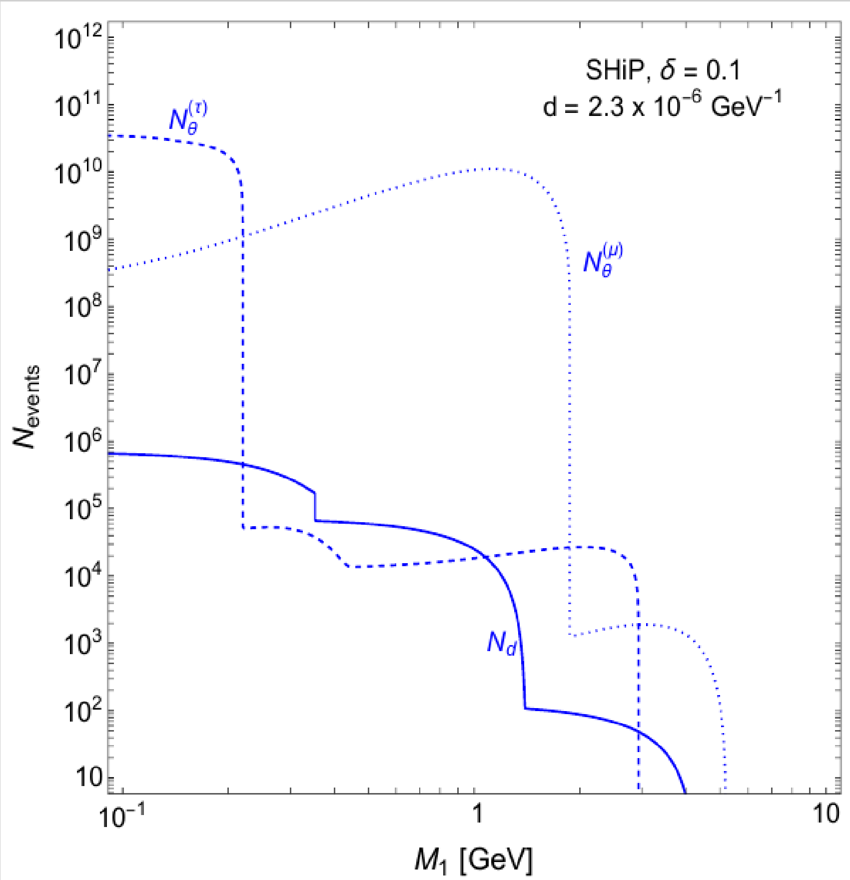
$$\Gamma(N_2 \to N_1\gamma) = \frac{|d|^2\ \cos^2\theta_w}{8\pi} M_2^3 \left(1 - \frac{M_1^2}{M_2^2}\right)^3$$

$$\Gamma(N_2 \to \nu_\alpha\gamma) = \frac{1}{8\pi}\left|d\ \cos\theta_w \sqrt{\frac{M_2}{M_1}} - \frac{3e}{16\pi^2}\frac{M_2}{v^2}\right|^2 |\theta_{2\alpha}|^2 M_2^3$$

$$\Gamma(N_1 \to \nu_\alpha\gamma) = \frac{1}{8\pi}\left|d\ \cos\theta_w \sqrt{\frac{M_1}{M_2}} + \frac{3e}{16\pi^2}\frac{M_1}{v^2}\right|^2 |\theta_{1\alpha}|^2 M_1^3$$

with E.Bertuzzo

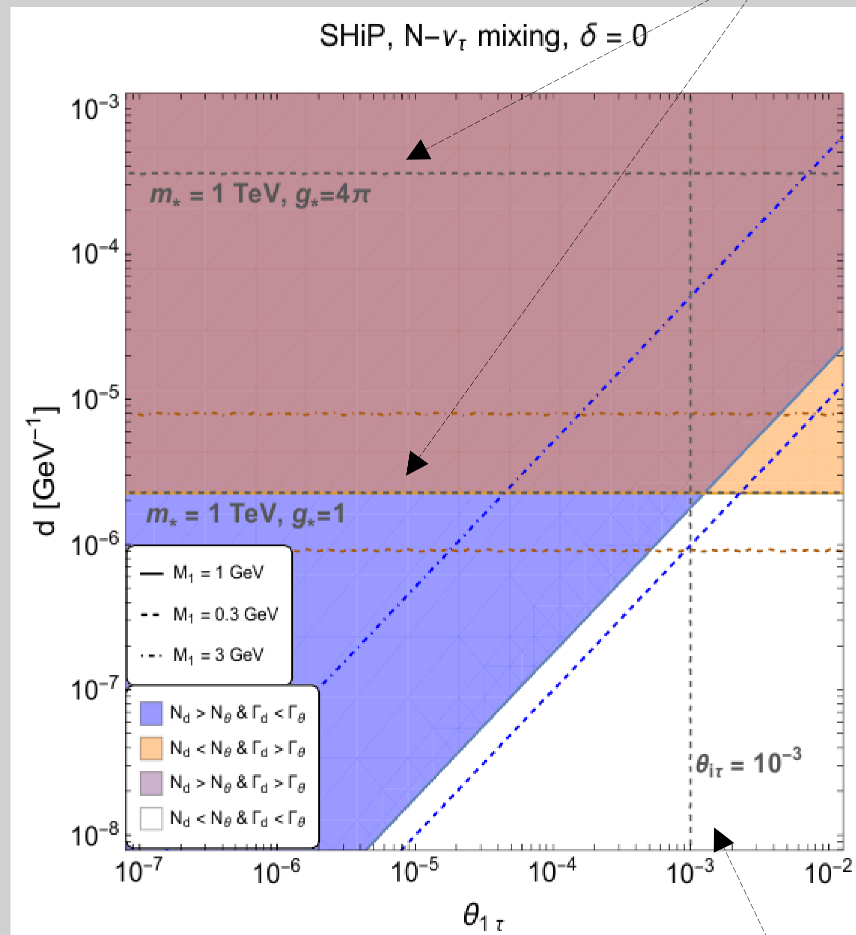
# Dipole versus mixing




## SHiP experiment

$$N_{POT} = 6 \cdot 10^{20}$$
$$E_{POT} = 400\text{GeV}$$
$$L_{initial} = 33\text{m}$$
$$L_{final} = 83\text{m}$$

Theory upper bounds on dipole



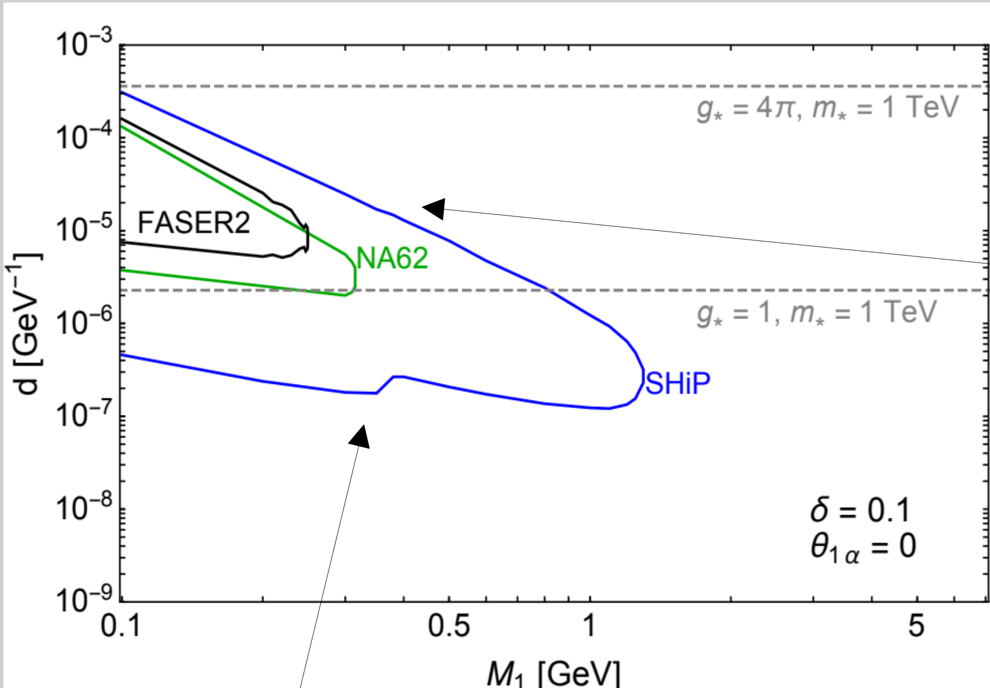

Exp upper bound on mixing

**Sterile neutrino production** from meson decays, both via dipole & via active-sterile mixing

$$N_{events} = N_d + N_\theta$$
$$\Gamma_{sterile} = \Gamma_d + \Gamma_\theta$$

with E.Bertuzzo

# Future experimental sensitivity (I)




**Dipole only : no mixing, $N_1$ stable**

$$N_2 \to N_1 \gamma$$

2 σ contours assuming no background

$N_2$ decays before the detector

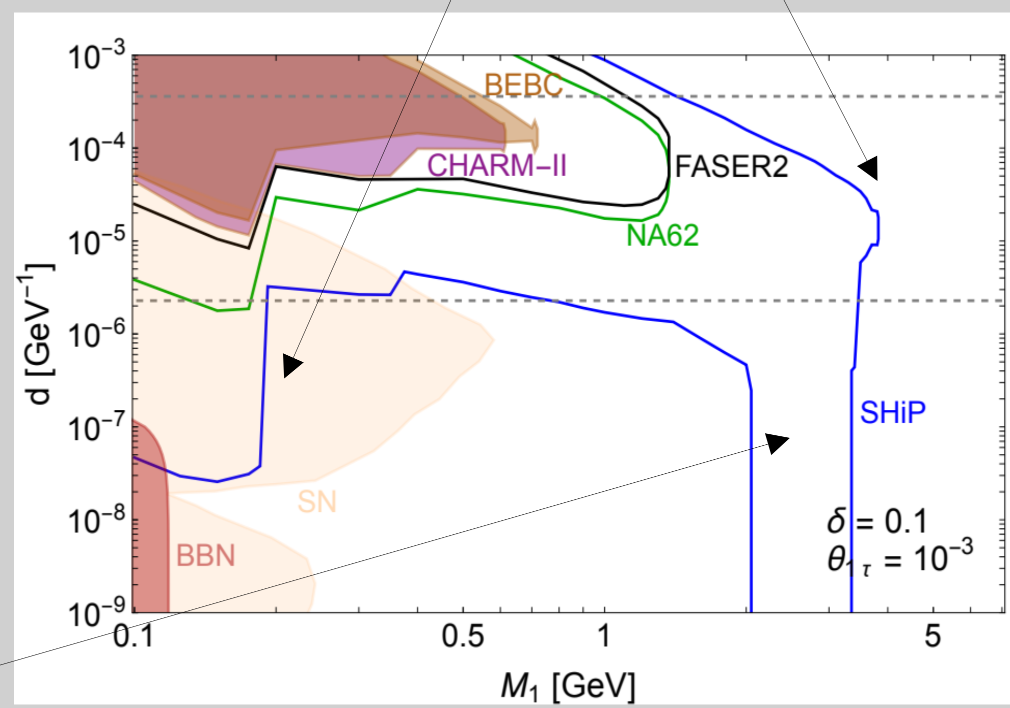
$N_2$ decays after the detector

$$\delta \equiv \frac{M_2 - M_1}{M_2 + M_1}$$

Thresholds due to meson masses

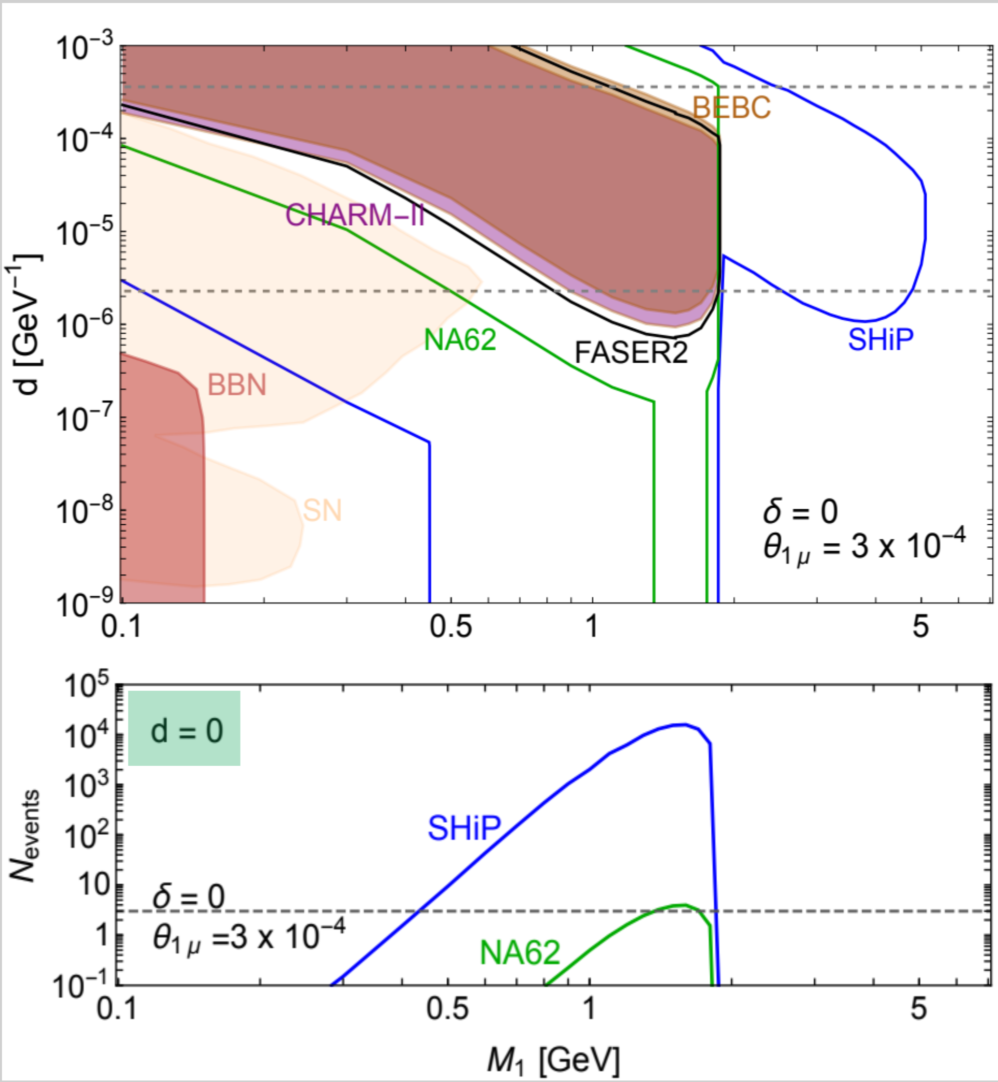
**Switching on mixing with τ flavour**

$$N_2 \to \nu_\tau \gamma$$
$$N_2 \to N_1 \gamma$$
$$N_1 \to \nu_\tau \gamma$$




Signal dominated by EW dipole

# Future experimental sensitivity (II)



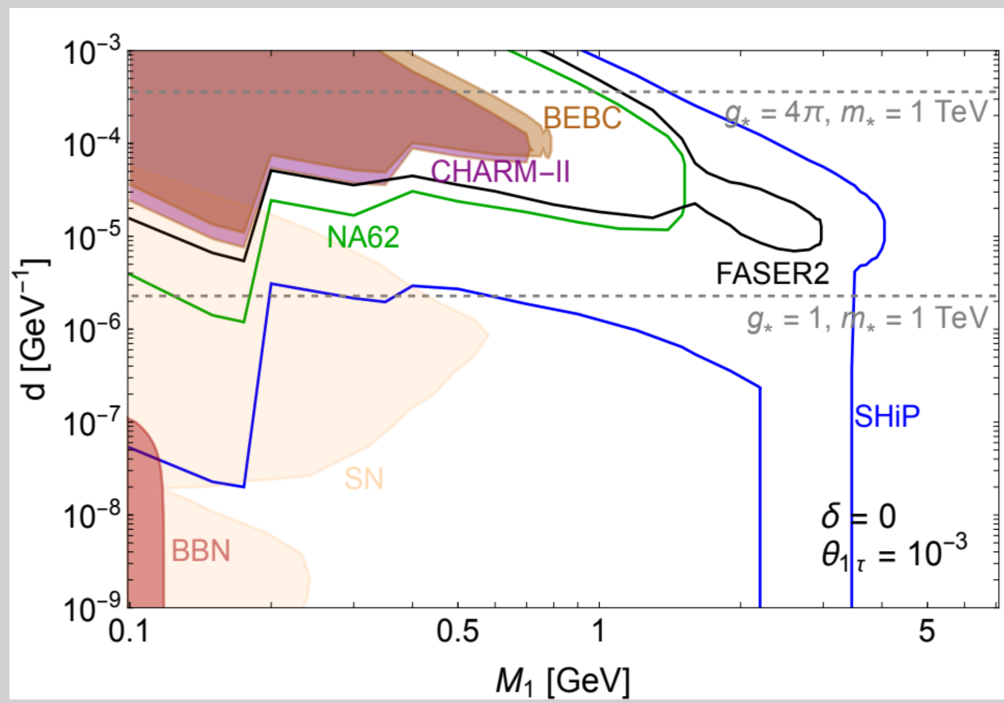

$N_D \to \nu_\mu \gamma$ **Mixing with μ flavour**

N production suppressed, as
**allowed mixing is smaller for μ flavour**

N production channels involving μ
remain **open at higher masses**

U(1)$_L$ limit
$M_1 = M_2$

**Mixing with τ flavour** $N_D \to \nu_\tau \gamma$




SHiP can observe GeV-sterile neutrinos via
**photons produced by EW loops**

with E.Bertuzzo

# Future experimental sensitivity (III)

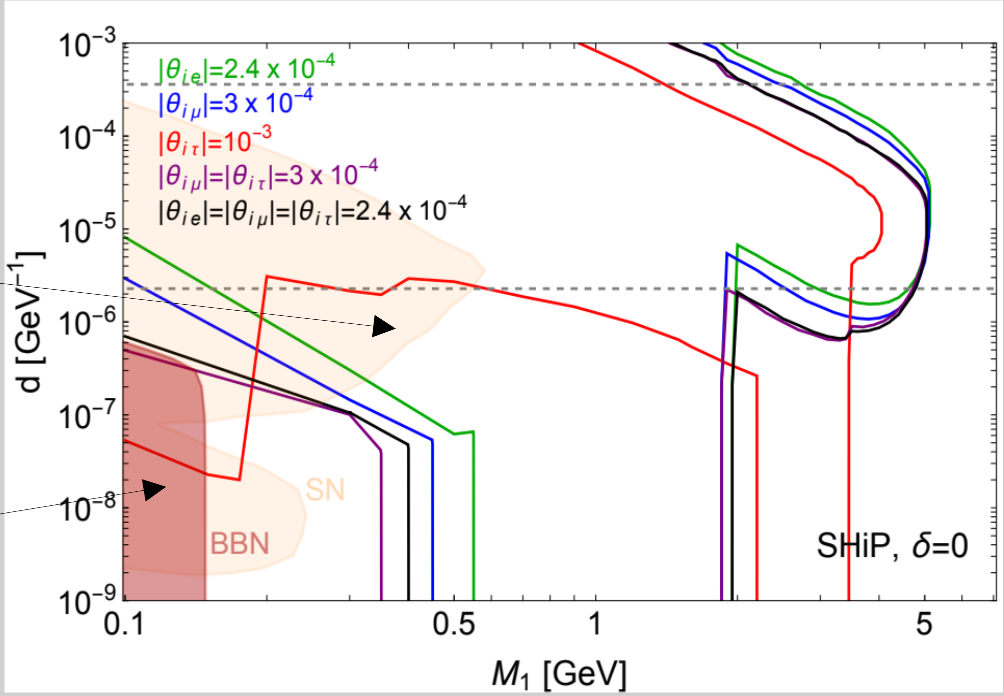
**Magill Plestid Pospelov Tsai 2018**

$$N_D \to \nu_{e,\mu,\tau}\gamma$$

**Comparing flavours**

Produced N's alter star cooling rate by draining energy

Late N-decays alter yield of primordial nuclei




* Signal for **tau** mixing complementary to signal for e-mu mixing

* Similar sensitivity to **electron** and **muon** mixing

* In order to explain *large mixing in oscillations*, one has to take **comparable muon and tau mixing** or even **equal mixing for all flavours**

⇒ possible to *correlate the dipole signal with flavour constraints*

# Summary and perspective (part II)

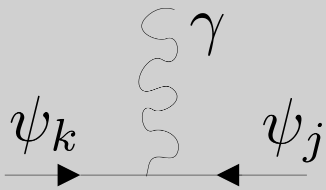
- Sterile fermions could be discovered through **photon signal**
- **SHiP will mark jump in sensitivity** in the 0.1-10 GeV range, probing
  - dipole from heavy new physics, $1/d \sim 10^8$ GeV
  - dipole from EW loops
- **Discriminating power:** N dark matter or N-$\nu$ mixing ? which lepton flavour ? how many N's ?
- Sterile dipole worth to explore in **other mass ranges** or in **different experimental setups** (neutrino long baseline ...)

# Waiting for some $\nu$ light

# Back-up material

**LE RAYON VERT**
**Jules Verne, 1882**
**Eric Rohmer, 1986**

# Fermion couplings to a photon




$$\overline{u_j}\,\Lambda_\mu^{jk}(q)\,u_k\,\epsilon^\mu(q)$$


**C.Giunti A.Studenikin 2014**
**review on**
**¨Neutrino electromagnetic interactions¨**


From **Lorentz + U(1) gauge** invariance :

Q - charge

Anapole

$$\Lambda_\mu^{jk}(q) \quad = \quad \left(\gamma_\mu - \frac{q_\mu q_\nu}{q^2}\gamma^\nu\right)\left[f_Q^{jk}(q^2) + q^2 f_A^{jk}(q^2)\gamma_5\right]$$
$$-i\Sigma_{\mu\nu}q^\nu\left[f_M^{jk}(q^2) + i f_E^{jk}(q^2)\gamma_5\right]$$

Magnetic dipole

Electric dipole

For ***real* photon emission ($q^2 = 0$)** form factors reduce to constant ‘moments’

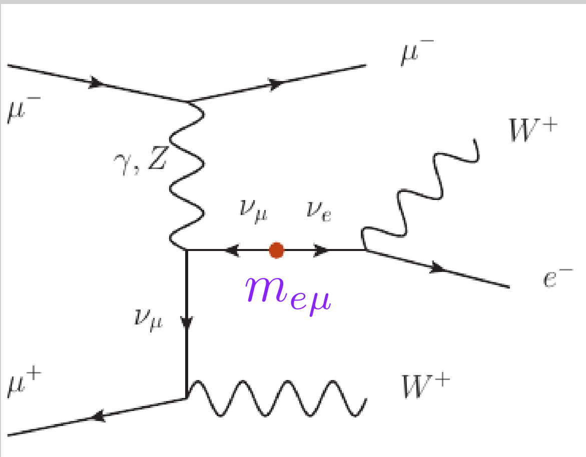
For ***Majorana* fermions ($\psi = \psi^c$)** no *diagonal* charges nor dipoles :

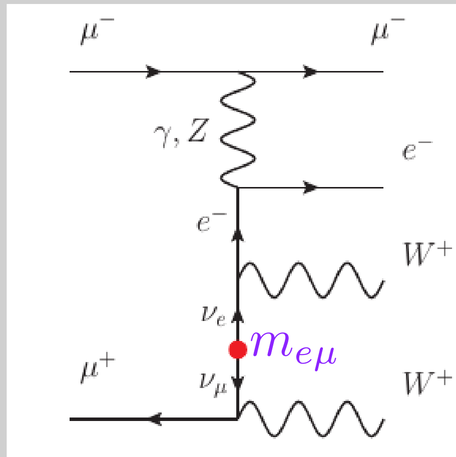
$$f_{Q,M,E} = -f^T_{Q,M,E} \quad \Rightarrow \quad f^{kk}_{Q,M,E} = 0$$

with N.Vignaroli

# ν mass @ future muon collider

**As a spin-off of the dipole analysis,** we can also set bounds on Majorana neutrino masses !




Same-sign leptons from $\mathcal{L} \supset \frac{1}{2}\,\nu_\alpha\, m_{\alpha\beta}\, \nu_\beta$

$$0\nu 2\beta - \text{decay} \ : \ \boxed{m_{ee} \lesssim 0.1 \text{ eV}}$$

$$\text{LHC (FCC}-\text{hh)} \ : \ \boxed{m_{e\mu,\mu\mu} \lesssim 10 \ (1) \text{ GeV}}$$

**ATLAS 2305.14931**
**Fuks Neundorf Peters Ruiz Saimpert 2012.09882**

For $\mathbf{m_{e\mu}}$ same analysis as for the dipole $\boldsymbol{\lambda_{e\mu}}$

For $\mathbf{m_{\mu\mu}}$ similar selection, but ~ 5 times larger background (W W $\mu^+$ $\mu^-$)

| $\sqrt{s}$ | 3 TeV | 10 TeV | 20 TeV | 30 TeV | 50 TeV |
|---|---|---|---|---|---|
| $\lvert m_{e\mu}\rvert$ | 110 [100] MeV | 3.2 [2.5] MeV | 0.50 [0.31] MeV | 140 [92] keV | 36 [20] keV |
| $\lvert m_{\mu\mu}\rvert$ | 300 [140] MeV | 10 [3.5] MeV | 1.5 [0.44] MeV | 420 [130] keV | 84 [28] keV |

@ $2\sigma$

**Improvement up to 5 orders of magnitude !**

Still far above the neutrino mass scale ...

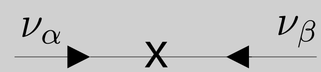
# SM EFT for ν dipoles

**Δ L = 2** operators in the SM effective field theory :

dimension-5 $\mathcal{L}_5 = C^5 \mathcal{O}_5$

$$m_{\alpha\beta} = C^5_{\alpha\beta} v^2$$

$$(\mathcal{O}_5)_{\alpha\beta} = (\overline{\ell^c_{L\alpha}} \epsilon H)(H^T \epsilon l_{L\beta})$$

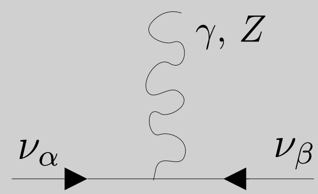



dimension-7 $\mathcal{L}_7 = \sum_i C_i \mathcal{O}_i$

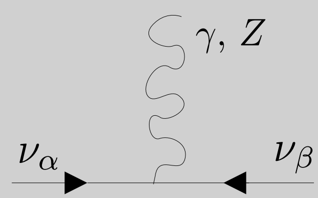
$$\lambda_{\alpha\beta} = -2ev^2 \left( C^B_{\alpha\beta} + 2C^W_{\alpha\beta} \right)$$

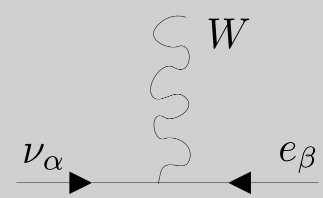
**Davidson Gorbahn Santamaria 2005**

$$(O_B)_{\alpha\beta} = g' \left( \overline{\ell^c_{L\alpha}} \epsilon H \right) \sigma^{\mu\nu} \left( H^T \epsilon \ell_{L\beta} \right) B_{\mu\nu}$$

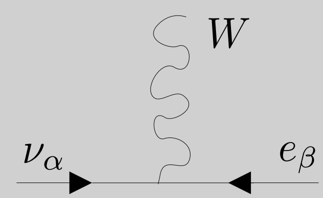



$$(O_W)_{\alpha\beta} = ig\varepsilon_{abc} \left( \overline{\ell^c_{L\alpha}} \epsilon \sigma^a \sigma^{\mu\nu} \ell_{L\beta} \right) \left( H^T \epsilon \sigma^b H \right) W^c_{\mu\nu}$$




$$(O_{\mathcal{M}})_{\alpha\beta} = g \left( \overline{\ell^c_{L\alpha}} \epsilon \sigma^{\mu\nu} \ell_{L\beta} \right) \left( H^T \epsilon \sigma^a H \right) W^a_{\mu\nu}$$




+ several non-dipole operators

# Bounds on active-sterile mixing (II)

**In the U(1)$_L$ limit** $m_\nu = 0$ , still mixing is constrained by a variety of **lepton precision measurements**

* **Indirect bounds** (on sterile neutrinos heavier than charged leptons) :

$$s_\alpha \equiv \sqrt{2}\theta_\alpha$$

$$s_e^2 \lesssim 10^{-3}\,, \qquad s_\mu^2 \lesssim 10^{-3}\,, \qquad s_\tau^2 \lesssim 3\cdot 10^{-3}$$

$$\text{if} \quad s_e \simeq s_\mu\,, \quad \text{then} \quad s_{e,\mu}^2 \lesssim 10^{-5}\ [10^{-7}]$$

**Coy Frigerio 2019,2022**

* **Direct searches** for sterile neutrinos :

$$\begin{aligned}
&\text{for} \quad 2\ \text{GeV} \lesssim M_D \lesssim 80\ \text{GeV}\,, && s_{e,\mu}^2 \lesssim 10^{-5}\,, && s_\tau^2 \lesssim 10^{-5}\,,\\
&\text{for} \quad 0.5\ \text{GeV} \lesssim M_D \lesssim 2\ \text{GeV}\,, && s_{e,\mu}^2 \lesssim 10^{-7}\,, && s_\tau^2 \lesssim 10^{-6}\,,\\
&\text{for} \quad 0.2\ \text{GeV} \lesssim M_D \lesssim 0.5\ \text{GeV}\,, && s_{e,\mu}^2 \lesssim 10^{-9}\,, && s_\tau^2 \lesssim 10^{-5}\,.
\end{aligned}$$

**Snowmass Review 2203.08039**

**Departing from the U(1)$_L$ limit** & requiring that oscillation data are reproduced by ***$N_{1,2}$* seesaw** :

$$\begin{aligned}
&\text{normal ordering } (m_1 < m_2 < m_3): && \hat{s}_e^2 \lesssim 0.1\,, \quad 0.25 \lesssim \hat{s}_\mu^2 \lesssim 0.85\,, \quad \hat{s}_\tau^2 \simeq 1 - \hat{s}_\mu^2\,,\\
&\text{inverted ordering } (m_3 < m_1 < m_2): && 0.05 \lesssim \hat{s}_e^2 \lesssim 0.95\,, \quad \hat{s}_\mu^2 \simeq \hat{s}_\tau^2 \simeq 0.5(1 - \hat{s}_e^2)\,,
\end{aligned}$$

**Blennow Fernández-Martínez Hernández-García Lopéz-Pavón Marcano Naredo-Tuero 2306.01040**

$$\hat{s}_\alpha^2 \equiv \frac{s_\alpha^2}{\sum_\beta s_\beta^2}$$

# Theoretical estimate of ν dipole

**Naive Dimensional Analysis :**

UV theory characterised by one mass scale $m_*$ and one coupling $g_*$

$$\lambda_{\alpha\beta} = c_{\alpha\beta} \frac{e}{(4\pi)^2} \frac{v^2}{m_*^3} (g_* \epsilon_H)^2 (g_* \epsilon_\alpha)(g_* \epsilon_\beta)$$

*c*'s are order one / ε's quantify couplings of Higgs and lepton doublets to UV states

$$\boxed{\frac{|\lambda_{\alpha\beta}|}{\mu_B} \simeq 2 \cdot 10^{-11} |c^B_{\alpha\beta} + 2c^W_{\alpha\beta}| \left(\frac{10\ \mathrm{TeV}}{m_*}\right)^3 \left(\frac{g_*}{4\pi}\right)^4 \left(\frac{\epsilon_H}{1}\right)^2 \left(\frac{\epsilon_\alpha \epsilon_\beta}{10^{-3}}\right)}$$

expected size of ν dipole

typical collider energy

need to suppress lepton flavour violation especially e-to-μ

Neutrino dipoles close to current bounds correspond to **multi-TeV new physics**

If new physics below collider energy, then **EFT approach breaks down**