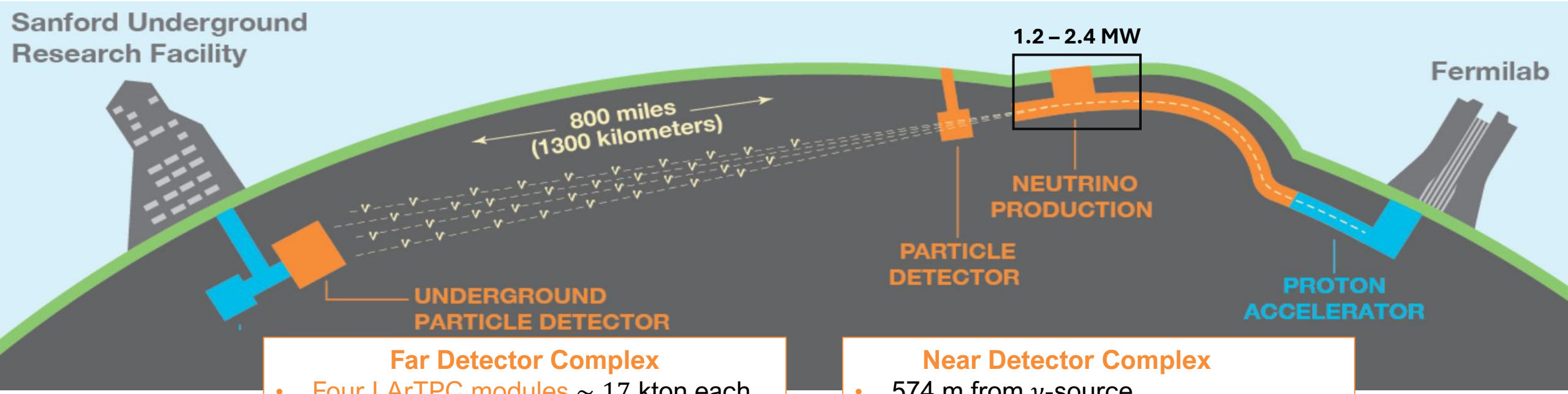
School of Science
Department of Physics and Astronomy
Master Degree in Physics

Particle tracking in DUNE-SAND detector: an Extended Kalman filter approach

Giulia Lupi

DUNE neutrino oscillation experiment



Far Detector Complex
- Four LArTPC modules ~ 17 kton each
- 1.5 km underground

Near Detector Complex
- 574 m from ν-source
- Three detectors: **SAND –** NDLAr - TMS

- **Primary program:** neutrino oscillation parameters θ_{13}, θ_{23}, potential CP violation δ_{CP}, mass ordering Δm^2_{32}
- **Beyond Standard Model Physics** (nucleon decay, CPT violation, sterile neutrino, non-standard interactions)
- ν_e flux measurements from **core-collapse Supernovae**
- Neutrino **cross-sections and nuclear effects**

The Near Detector

Beam measurements near the production point, limit systematic and nuclear effects to optimize oscillation analysis at the Far Detector (FD)

$$N_{\mathrm{X}}(E_{\mathrm{rec}}) = \int_{E_\nu} dE_\nu \, \Phi(E_\nu) \, P_{\mathrm{osc}}(E_\nu) \, \sigma_{\mathrm{X}}(E_\nu) \, R_{\mathrm{phys}}(E_\nu, E_{\mathrm{vis}}) \, R_{\mathrm{det}}(E_{\mathrm{vis}}, E_{\mathrm{rec}})$$

Initial neutrino flux — Interaction cross-section — Detector response

SAND
System for on-axis Neutrino Detection to monitor beam stability and study nuclear effects

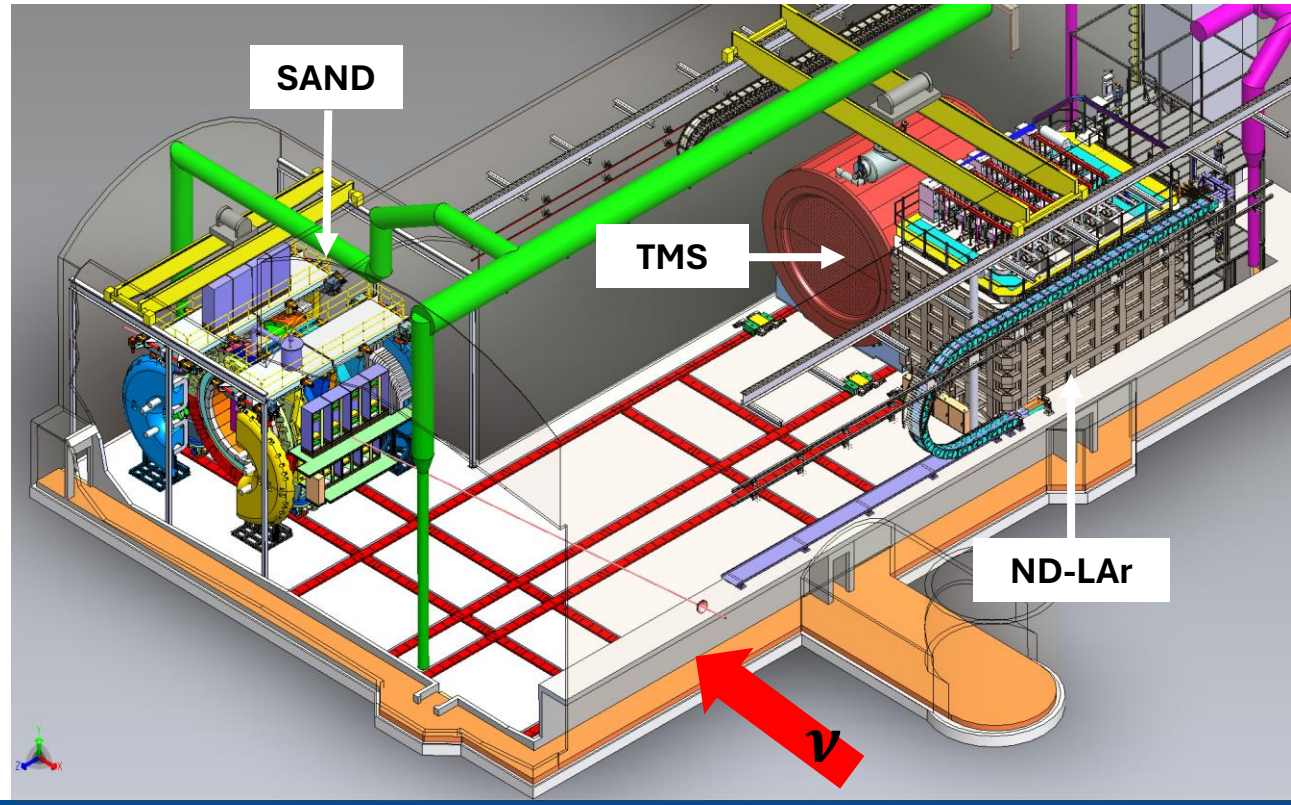
$\Phi(E_\nu), \sigma_H$

NDLar
Liquid Argon TPC for high statistics on ν-Ar interactions

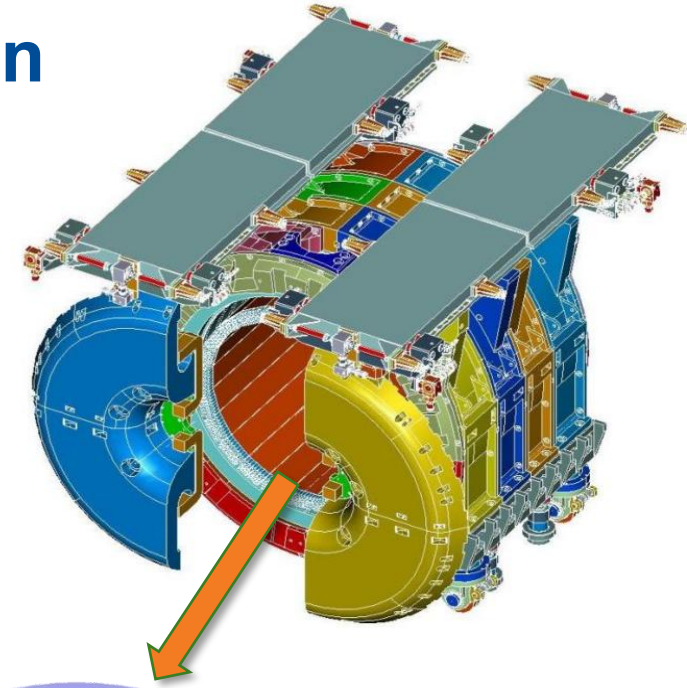
$\sigma_{Ar} \cdot R_{det}$

TMS
Magnetized steel range stack for measuring muon momentum and sign

σ_{Ar}



System for on-Axis Neutrino Detection

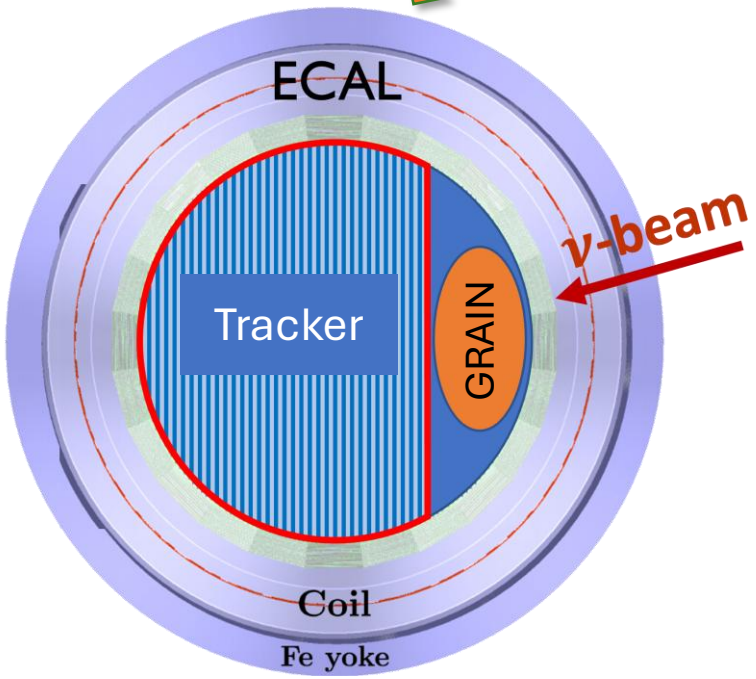


Multipurpose detector featuring:

- 0.6 T superconducting **solenoid magnet** from KLOE
- **Electromagnetic Calorimeter (ECAL)** made of lead and scintillating fibers $15X_0$
- **Inner low-density CH2, C target and tracker (STT/Drift)** $< 0.2\ g/cm^3$
- **GRAIN** Liquid Argon active target $\sim 1\ t$

Scientific Program

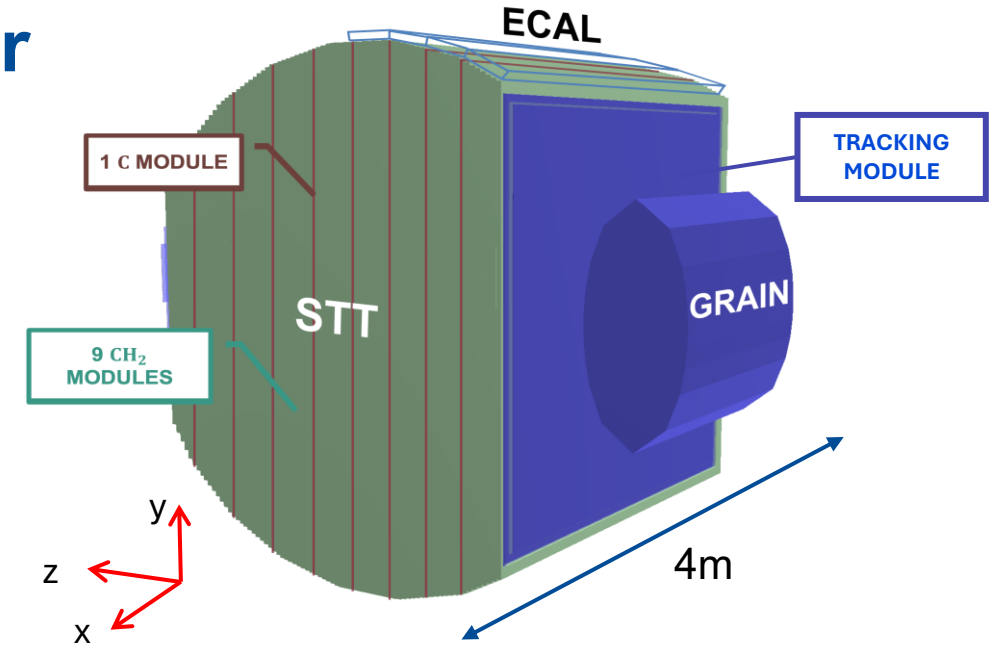
- **On-axis spectrum Monitoring :** Most deviations detectable on a weekly basis, with a sensitivity of $\sqrt{\Delta\chi^2} > 3$
- Independent **measurements of $\nu_{\mu,e}/\bar{\nu}_{\mu,e}$ fluxes** and **energy spectra**
- **Reduction of Systematic Uncertainties** : Inner target tracker with multiple nuclei targets to study nuclear effects
- **Extended ND Program** : EW physics, $\sin\theta_W$, isospin tests, QCD and form factors

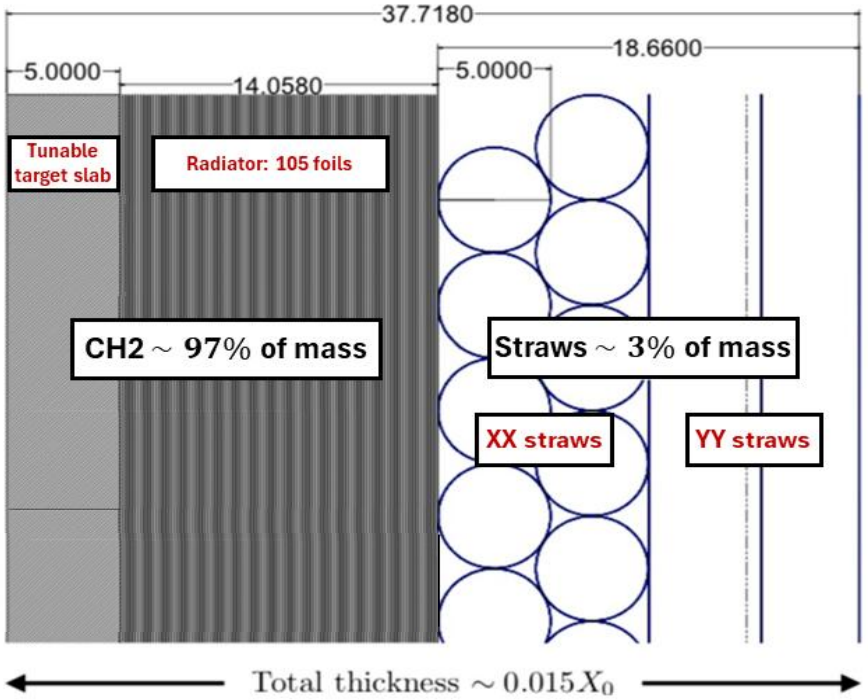


The Inner Tracker

The full geometry comprises 84 modules each containing

- **Target slabs** (**1 C** every **9 CH_2**)
- **Radiator** 105 polypropylene foils alternated with air gaps
- **Four Tracker layers** 5 mm diameter **straws tubes** filled with Xe/CO_2 gas at 1.9 atm arranged in XXYY



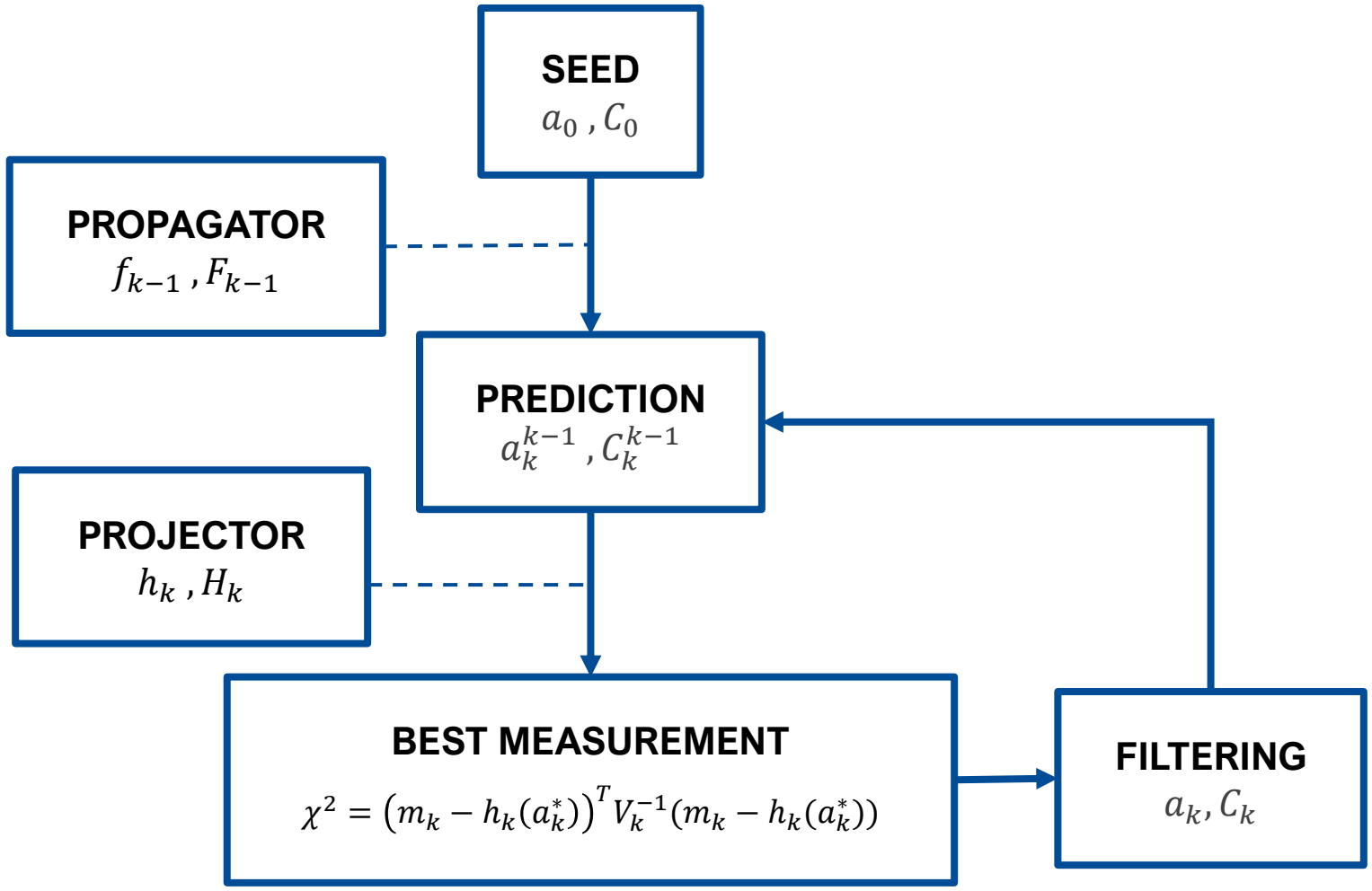


- 5 t fiducial mass to measure ν_μ **spectrum in one week**
- $< \mathbf{1}X_0$ to minimize secondary hadronic interactions
- Accurate reconstruction of transverse plane kinematics

  $\delta p/p \leq 3\%$ $\quad$ $\delta\theta/\theta \leq 1.5\%$
- Separation of ν and $\bar{\nu}$ events for optimal charge ID
- Isolation of a **ν-H enriched sample**
- **minimise**

Extended Kalman Filter algorithm

- Track fitting estimates a track **state vector** a_k from discrete **measurements** m_k
- The Kalman Filter evaluates the most optimal a_k of a linear system **iteratively** using first-order expansion and adding new m_k at each iteration
- An **extended** version of the algorithm can handle **non-linear systems** such as charged particles moving in the SAND inner volume



EKF implementation for SAND

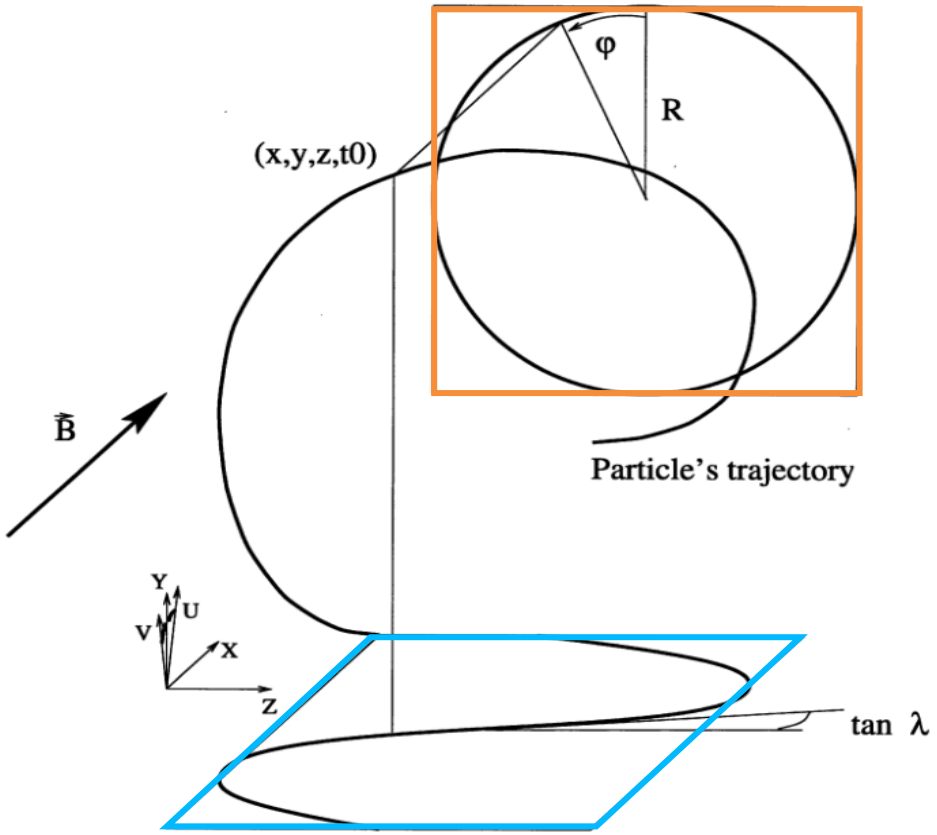
Non–linear dynamic track model including:

Magnetic field along the *x*-axis

- **Sinusoidal** in the *xz*-plane
- **Circular trajectory** in the *yz*-plane

STT passive targets

- Energy loss between subsequent STT planes
- Multiple Coulomb Scattering



Trajectory parametrized as helix of variable radius

$$\boldsymbol{a_k} \longrightarrow \begin{pmatrix} x \\ y \\ q/R \\ \tan\lambda \\ \phi \end{pmatrix}$$

Tracking modules consist of straw tubes layers

$$\boldsymbol{m_k} \longrightarrow m_x = \begin{pmatrix} x \\ \theta_{xz} \end{pmatrix} \quad m_y = \begin{pmatrix} y \\ \theta_{yz} \end{pmatrix}$$

Algorithm validation with DUNE framework

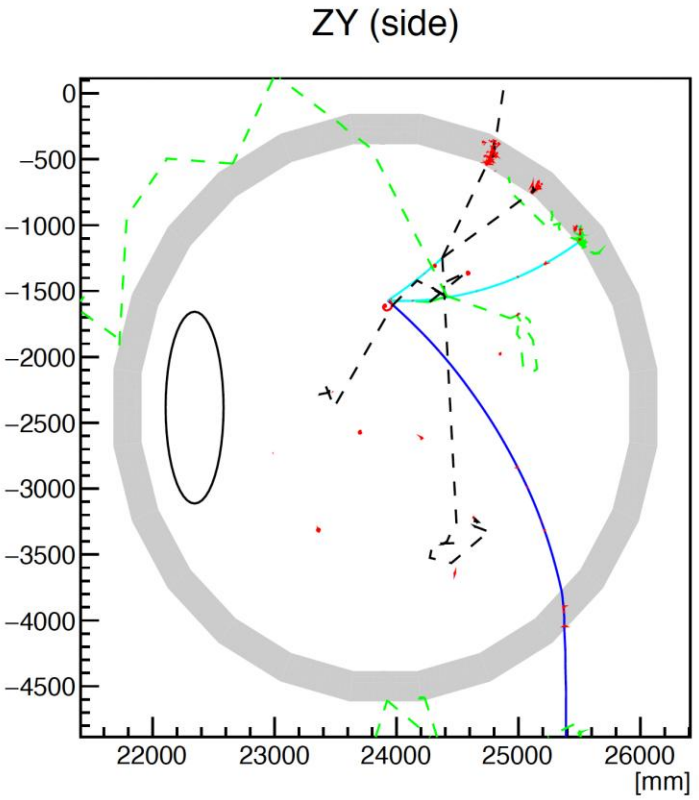
FLUX MODEL

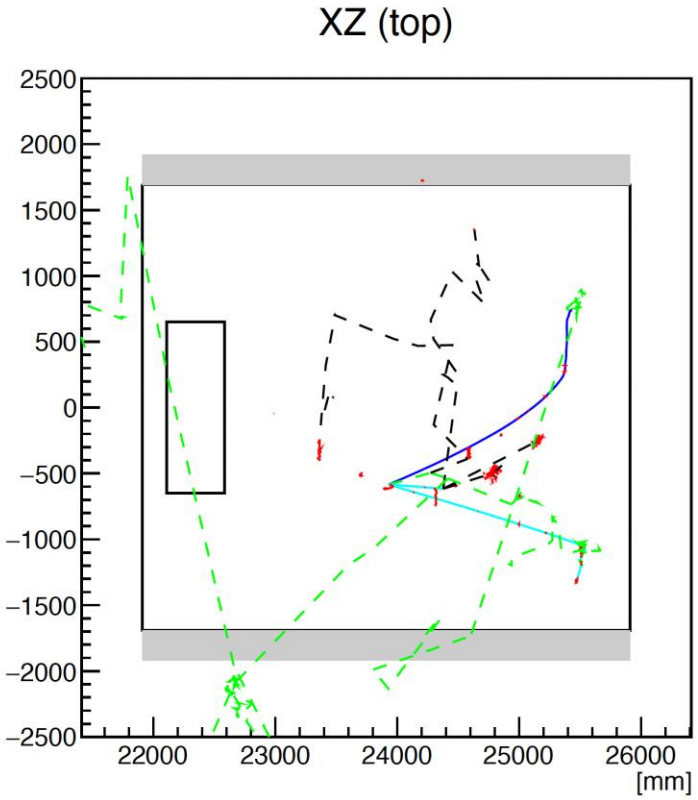
SAND GEOMETRY
GDML files

GENIE PRODUCTION
ν_μ CC IN SAND tracker volume
+
PROPAGATION (EDepSim)

DIGITIZATION

SELECTION and
RECONSTRUCTION





Neutrino CC events reconstruction

PRODUCTION

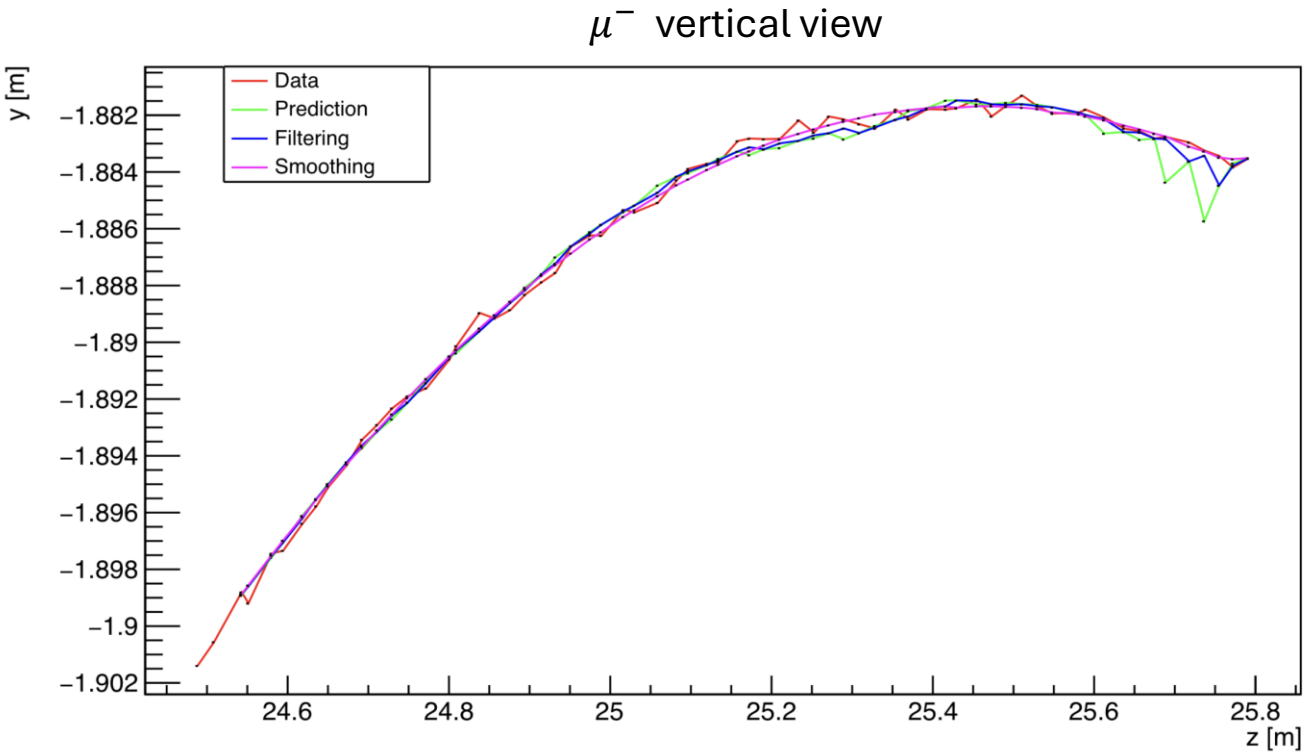
- 10^6 ν_μCC events in STT

SAMPLE SELECTION

- Muon, proton or pion
- Minimum of 10 measurement points per track
- Vertex inside STT fiducial volume
- $\sqrt{y^2 + z^2} < 1.85\ m \qquad |x| < 1.5\ m$

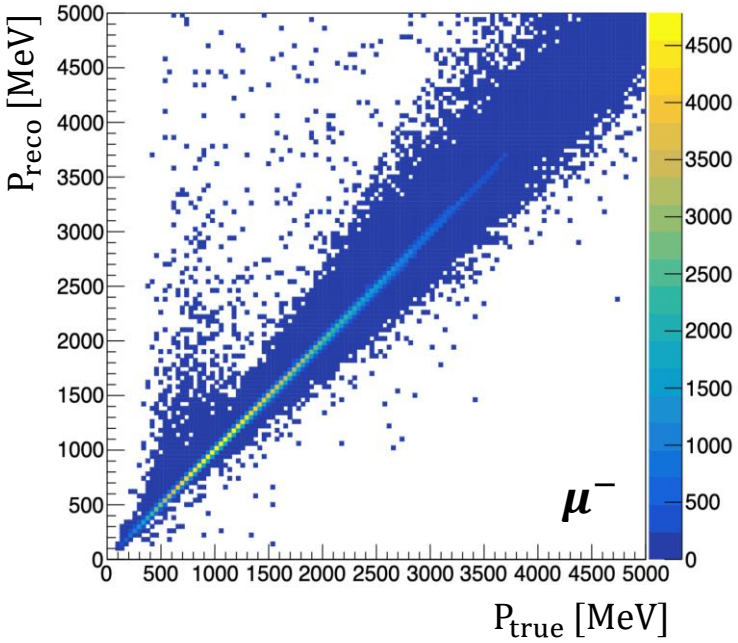
RECONSTRUCTION

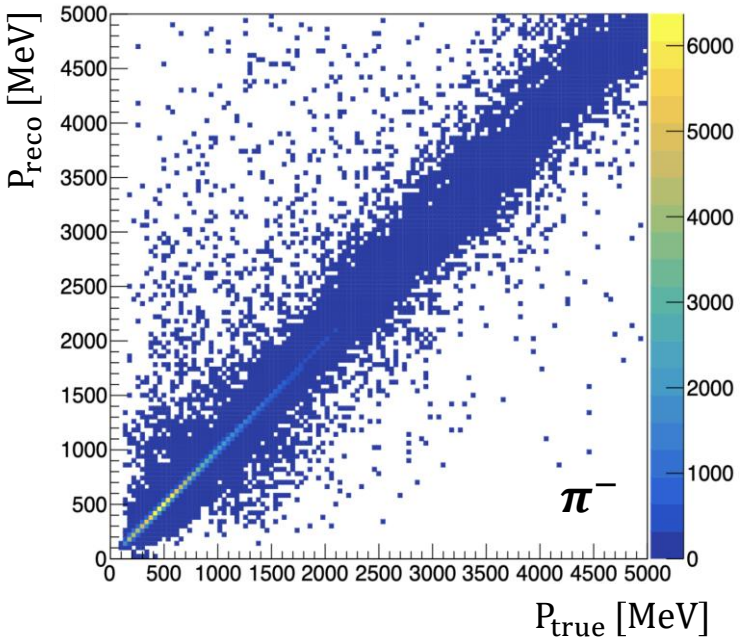
- Track reconstruction with KF
- Vertex position reconstruction

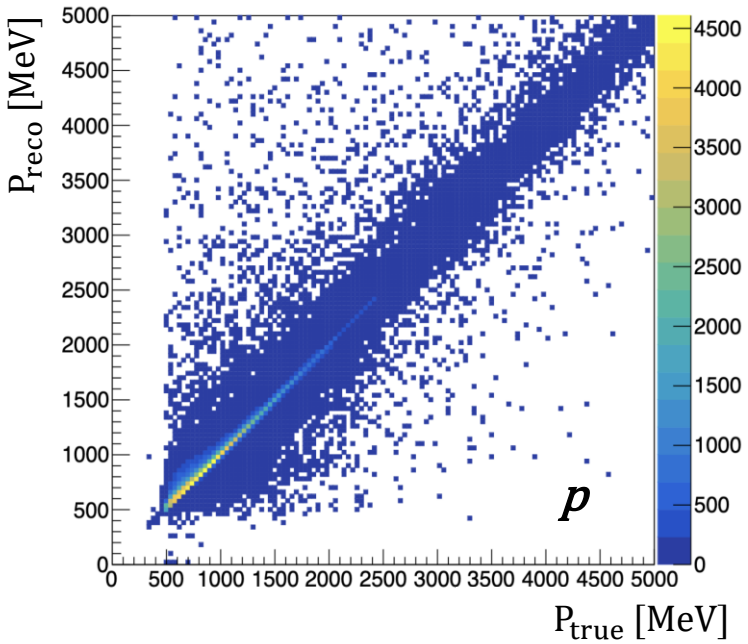


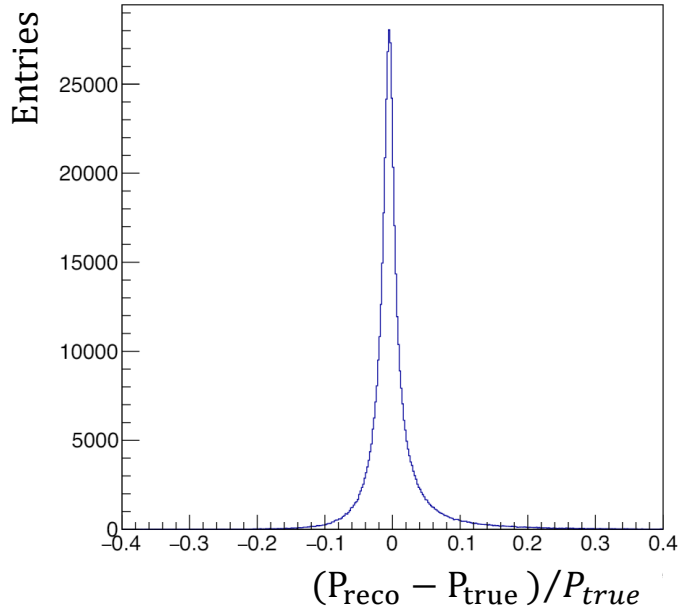
- Poistion residuals $\mu_{xy} < 10^{-6}$ m RMS < 0.6 mm
- Direction residuals $\mu_{\tan\lambda} < 10^{-5}$ RMS < 0.01
  $\mu_\Phi < 1.5$ mrad RMS < 0.06 mrad
- Momentum resolution 5-8%
- Preliminary vertexing algorithm $\mu_{xy} < 3$ mm $\mu_z < 5$ mm

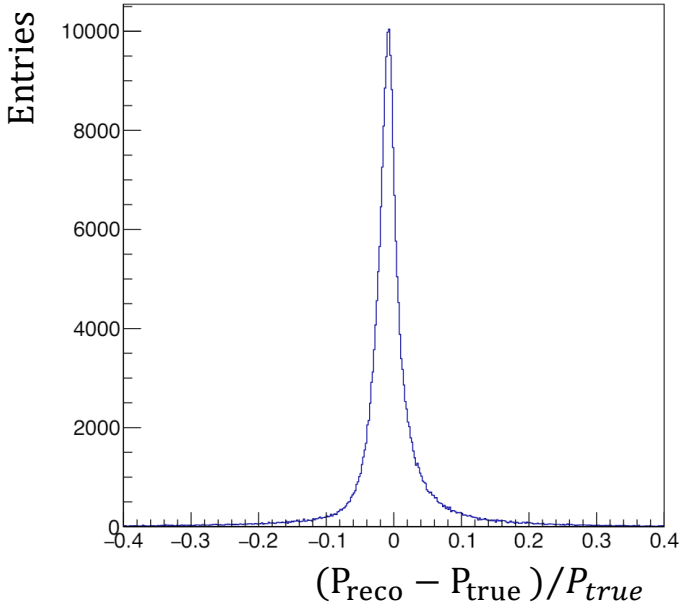
Momentum reconstruction

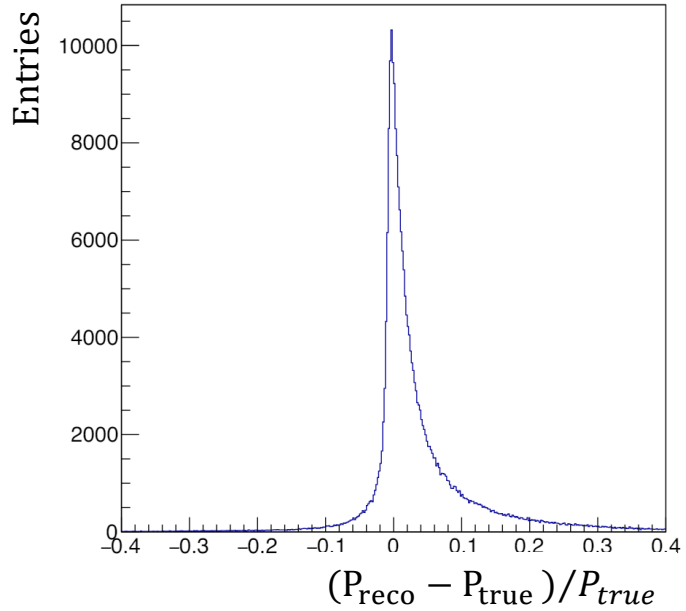




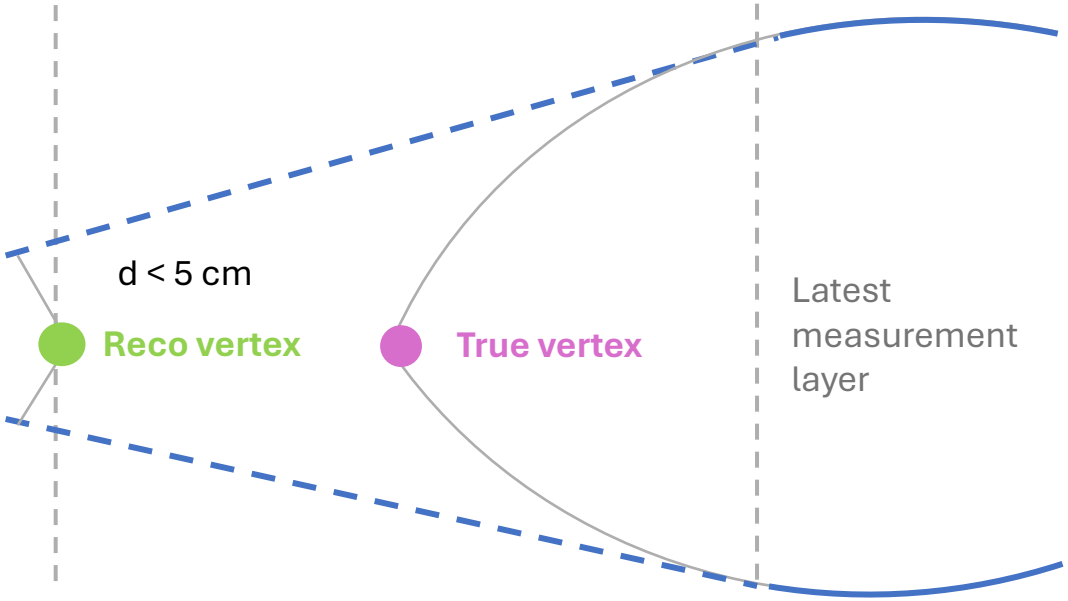






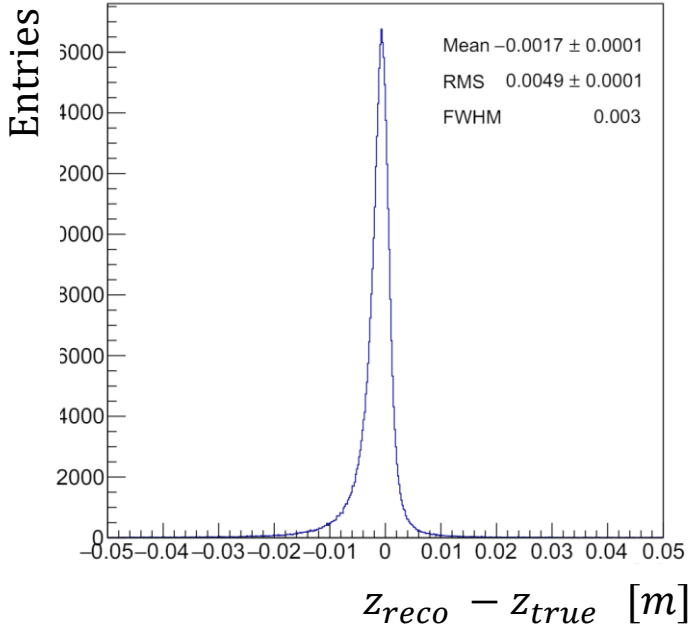


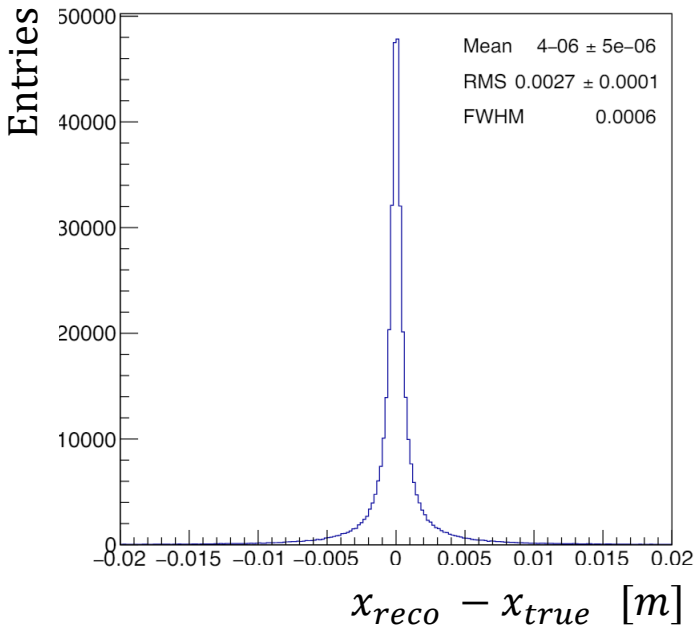
Vertex reconstruction



A **2-prong vertex** is defined as the 3D point with the smallest impact parameter between the two straight lines backwards propagated from the latest measurement layer

2-prong vertices are merged into a **multi-prong vertex** if within 5 cm





	Mean [mm]	RMS [mm]	FWHM [mm]
x, y	< 0.1	3	0.6
z	~ -2	5	3

Meets requirement since it is smaller than target width

Conclusions

RESULTS

- Fully **developed EKF algorithm** for SAND
- Analytically computed and **validated** a non-linear model
- Reconstruction performances on $\boldsymbol{\nu_\mu}$**-CC events**
- Preliminary reconstruction of vertex position

PERSPECTIVES

- Remove any **dependency on MC-truth**
- Integrate other SAND components for a **comprehensive event reconstruction**
- Full neutrino energy reconstruction
- **Physics studies -** ν-H cross section

THANKS FOR THE ATTENTION!

BACKUP

Reconstruction results

Trajectory residuals

	x [mm]		y [mm]		$\tan\lambda$		ϕ [mrad]	
	Bias	RMS	Bias	RMS	Bias	RMS	Bias	RMS
μ^-	$< 10^{-6}$	0.4	$< 10^{-6}$	0.2	$< 10^{-6}$	0.006	1.1	3.5
π^+/p	$< 10^{-6}$	0.7	$< 10^{-5}$	0.3	$< 10^{-5}$	0.01	1.5	6.9

$\Delta P/P$

	μ^-	π^+	p
Bias	-0.002	-0.004	0.034
RMS	4.8%	6.3%	7.9%
FWHM	2%	2.8%	2.4%

Energy loss implementation

Energy loss from layer *k-1* to layer *k* computed as:

$$\Delta E_k = \left.\frac{dE}{dx}\right|_{Bethe-Bloch} \times \Delta Z_k \, [g \cdot cm^{-2}]$$

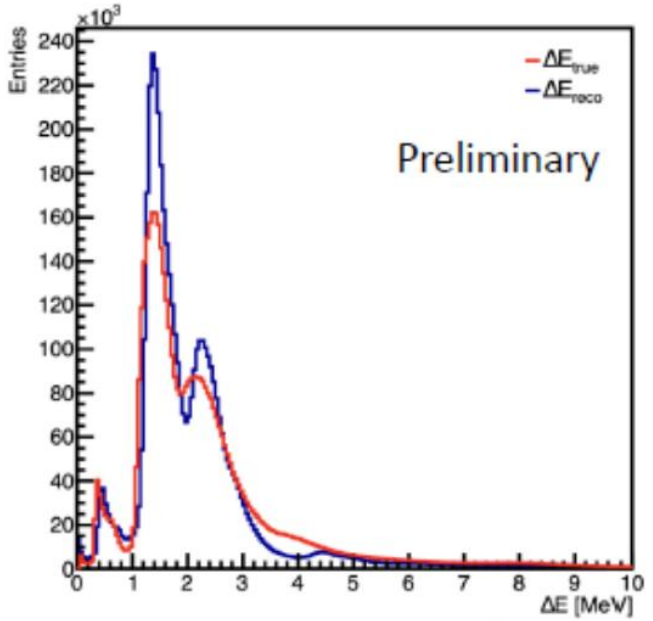
$$\hookrightarrow \Delta Z_k = \rho \cdot L \left[\frac{g}{cm^2}\right]$$

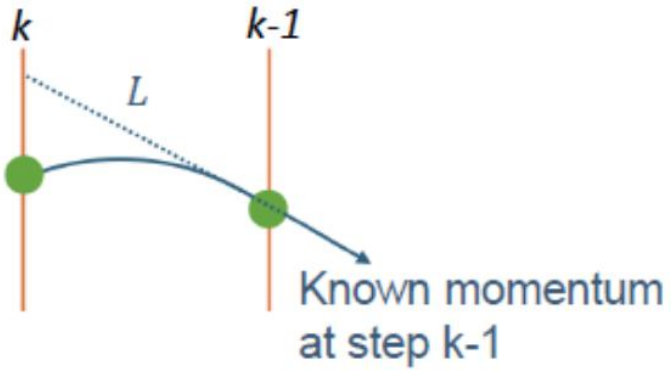
MCS angle computed as:

$$\theta_k = \frac{13.6 \text{ MeV}}{\beta cp} \cdot q \cdot \sqrt{\Delta Z_k / X_0} \, [1 + 0.038 \ln(\Delta Z_k / X_0)]$$

Where:

- materials information taken from geometry file
- L computed as a straight line from measurement layer *k-1* to *k*
- direction taken from trajectory state of previous step



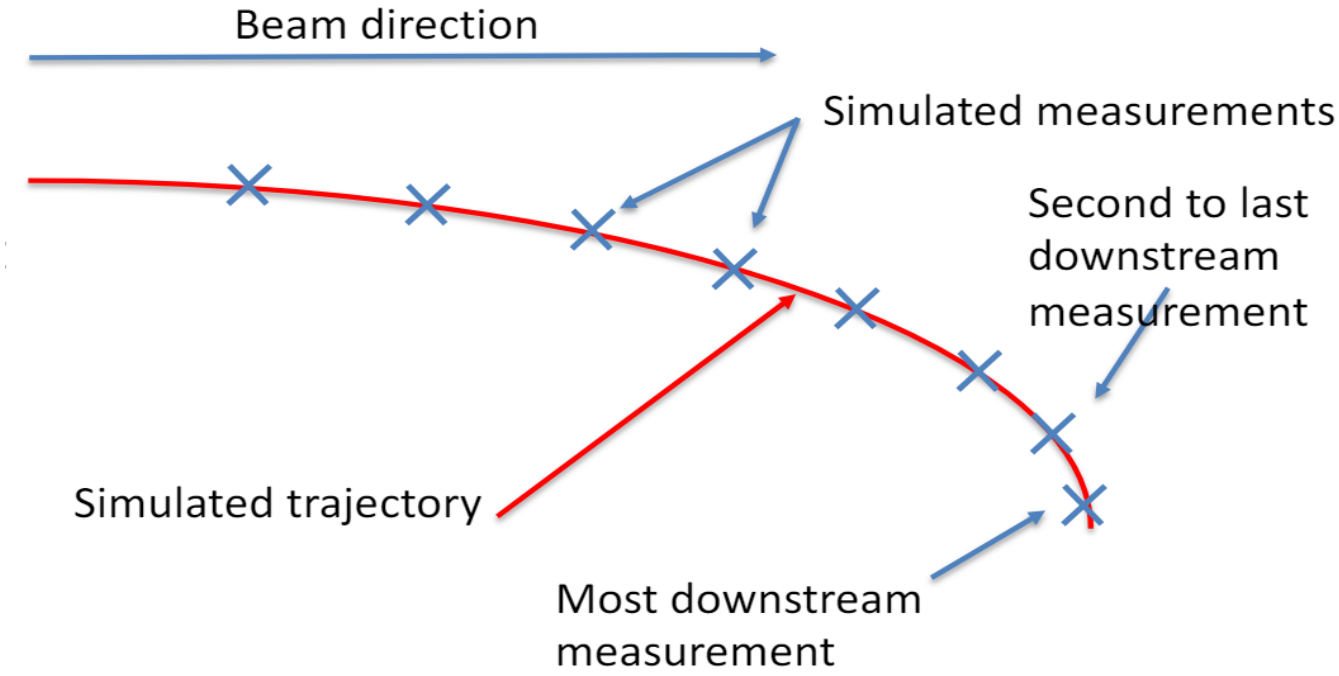


Seeding

$$\begin{pmatrix} x \\ y \\ q/R \\ \tan\lambda \\ \phi \end{pmatrix}$$

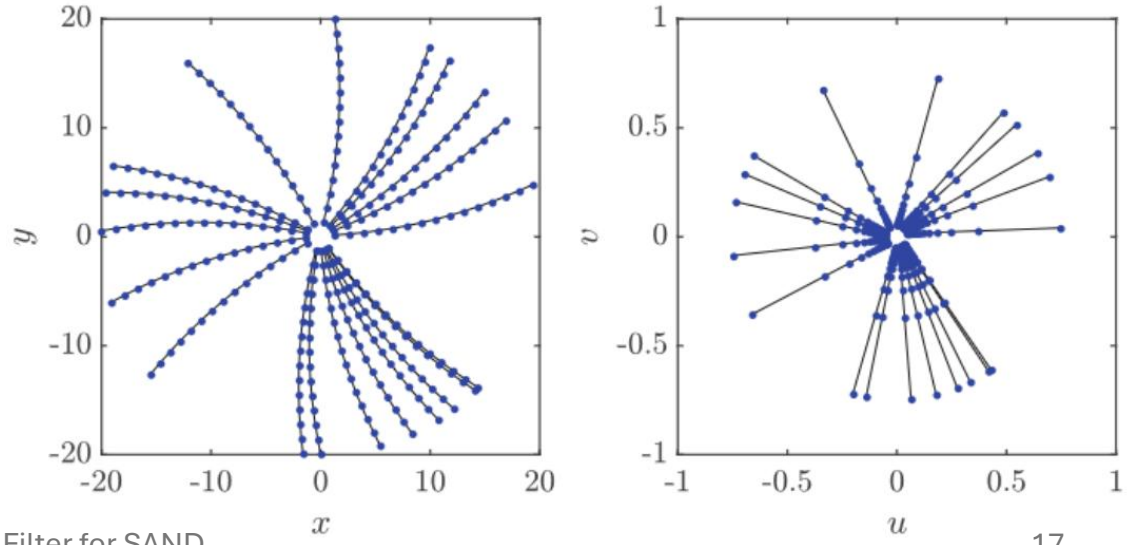
Position of the most downstream measurement

Angles in both planes (xz, yz) between two most downstream measurements

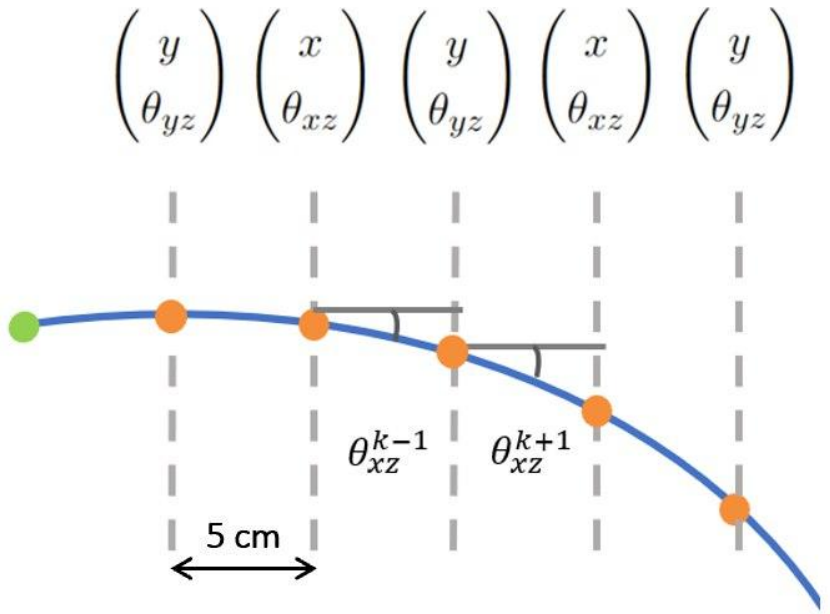


Conformal transformation of last few measurements:

1. change of coordinate
2. circle described as a straight line
3. circle radius and center obtained from fit of the line



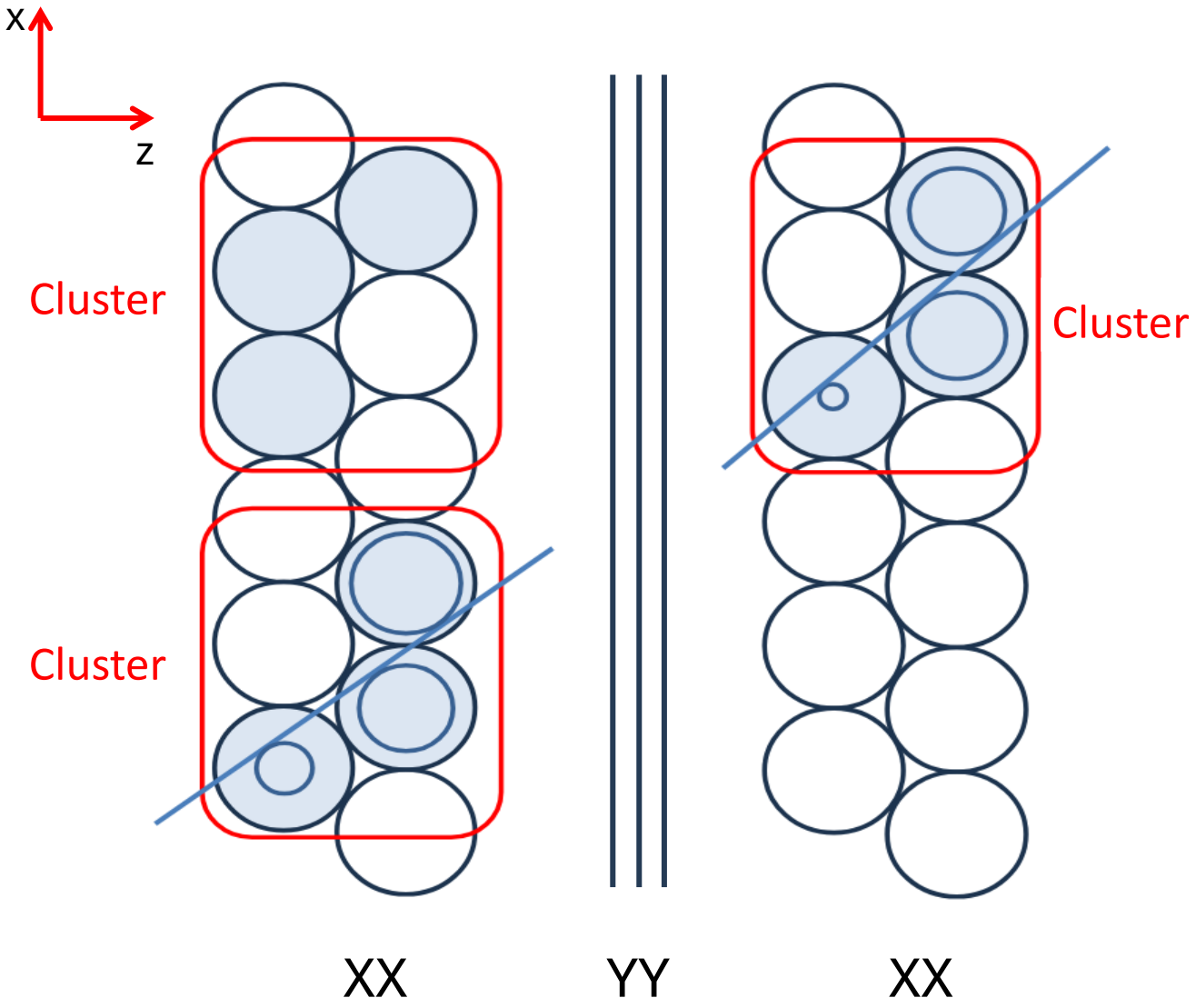
Fast digitization



- Trajectory sampled at a **discrete interval of 5 cm** from the **interaction point**
  - The MC-data at each point provides position and momentum
- Measurements alternate between horizontal and vertical measurements layers → **not the real pattern!**
- Measurement vector parameters smeared to reproduce expected STT point resolutions $\sigma_{x,y} = 200\,\mu m$ $\sigma_{\theta} = 200\,mrad$

Real digitization

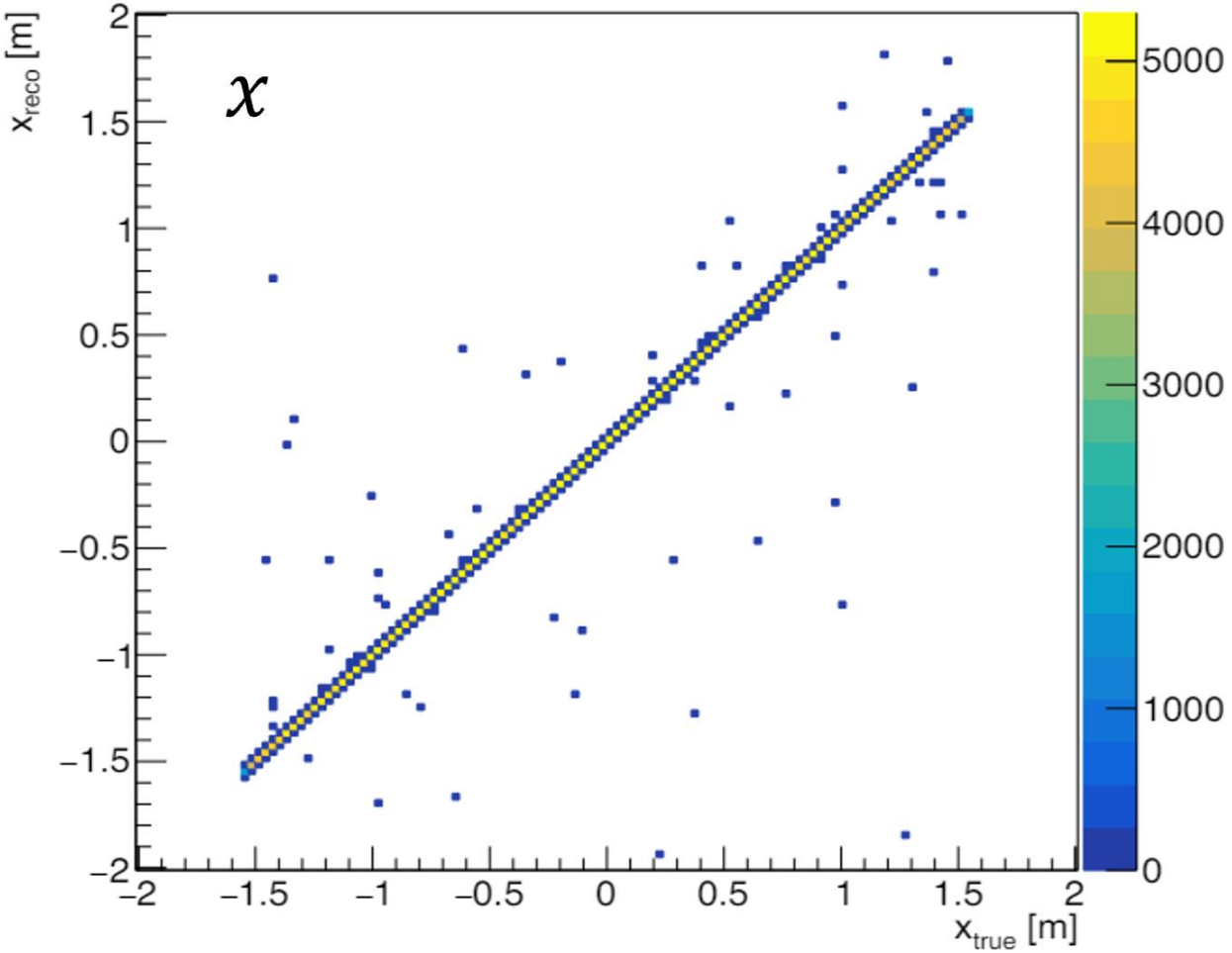
- Input tube-digits are grouped in plane
- Within plane adjacent tube-digits are clustered
- Reconstruct radius for tube-digits
- Evaluate common tangents and take the best one according to the likelihood
- Clusters are reconstructed: m, q, t_0, quality
- **Pair of vertical and horizontal measurements of each STT module**



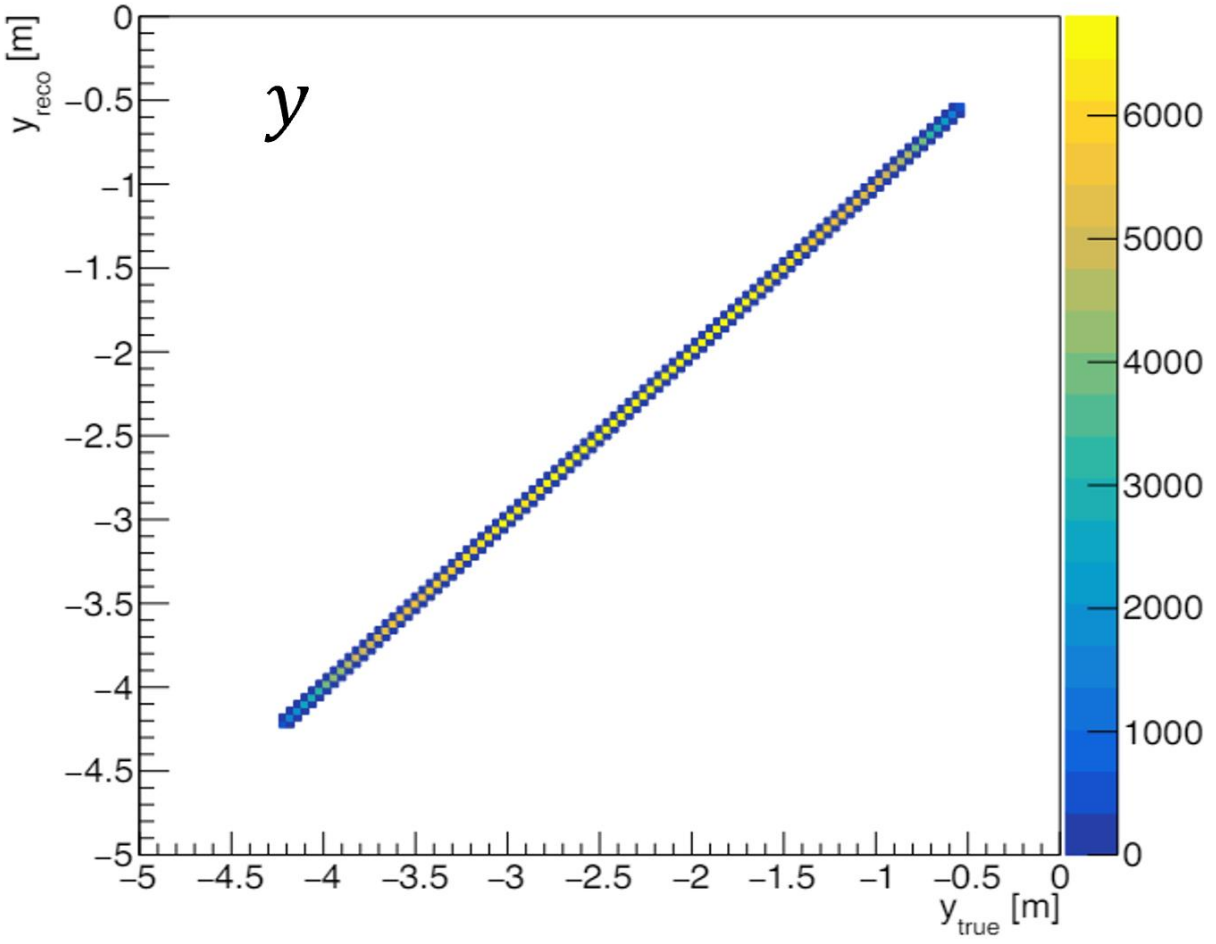
Position reconstruction - muons

$$\begin{pmatrix} x \\ y \\ q/R \\ \tan\lambda \\ \phi \end{pmatrix}$$

Horizontal plane

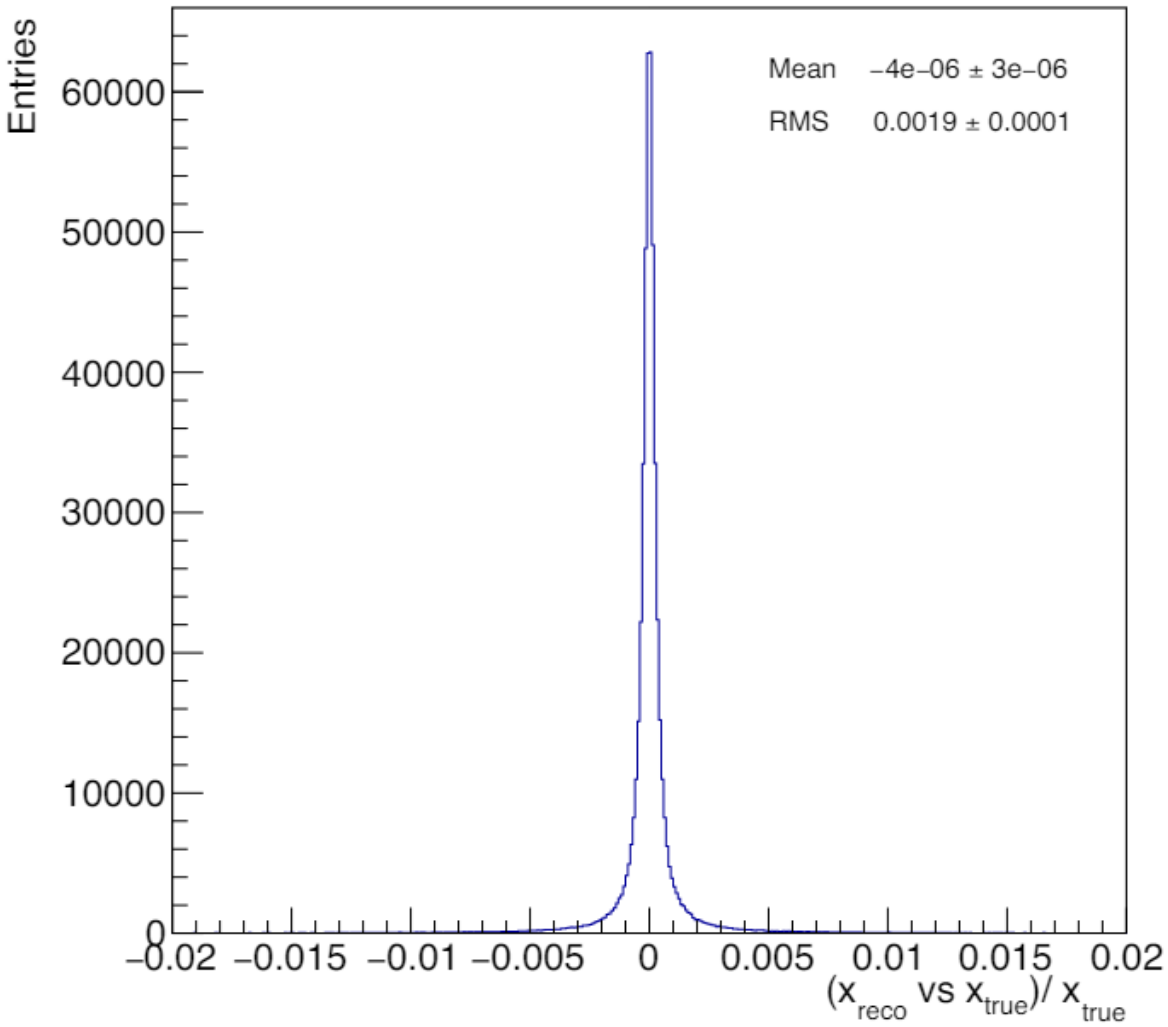


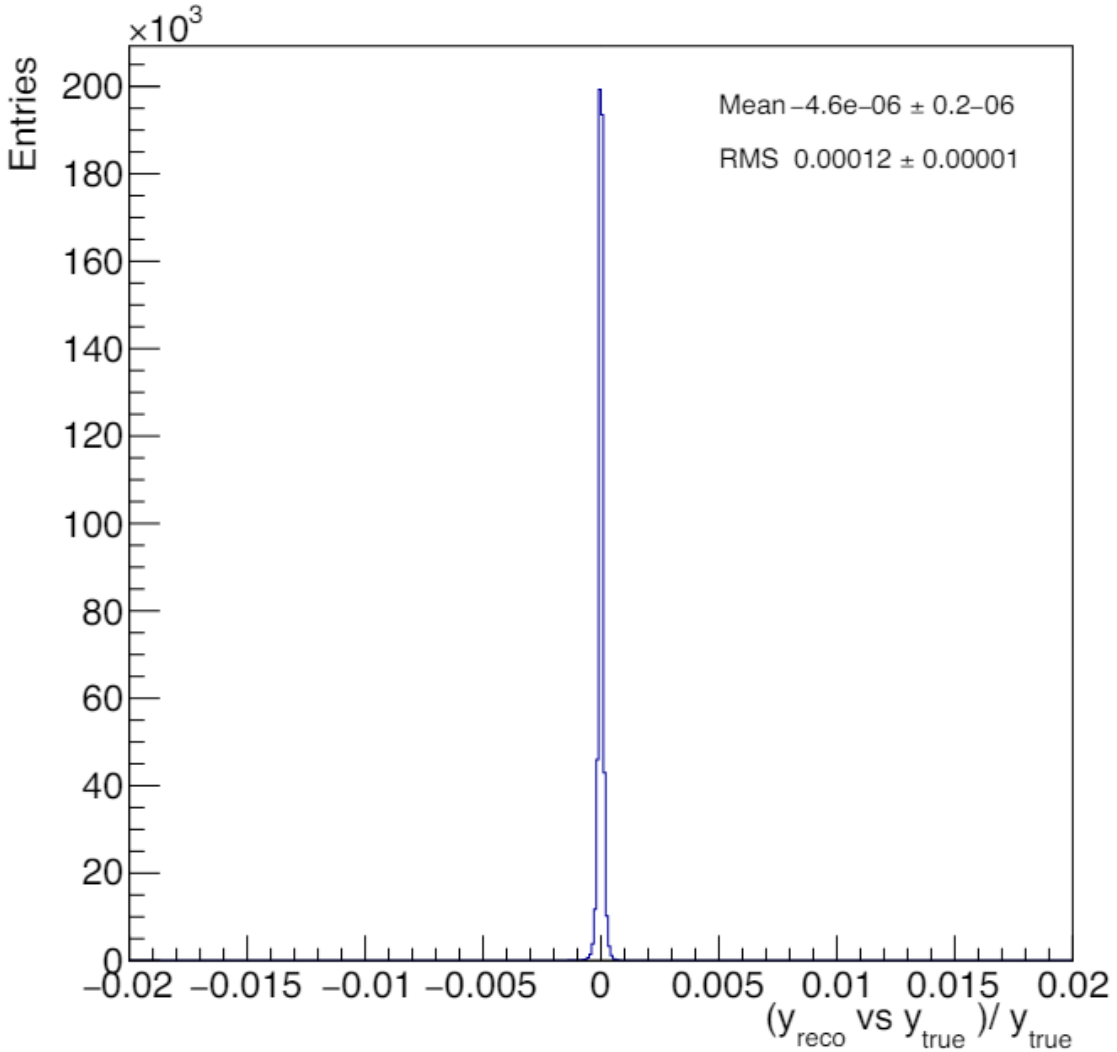
Vertical plane



Residual on track parameters - muons

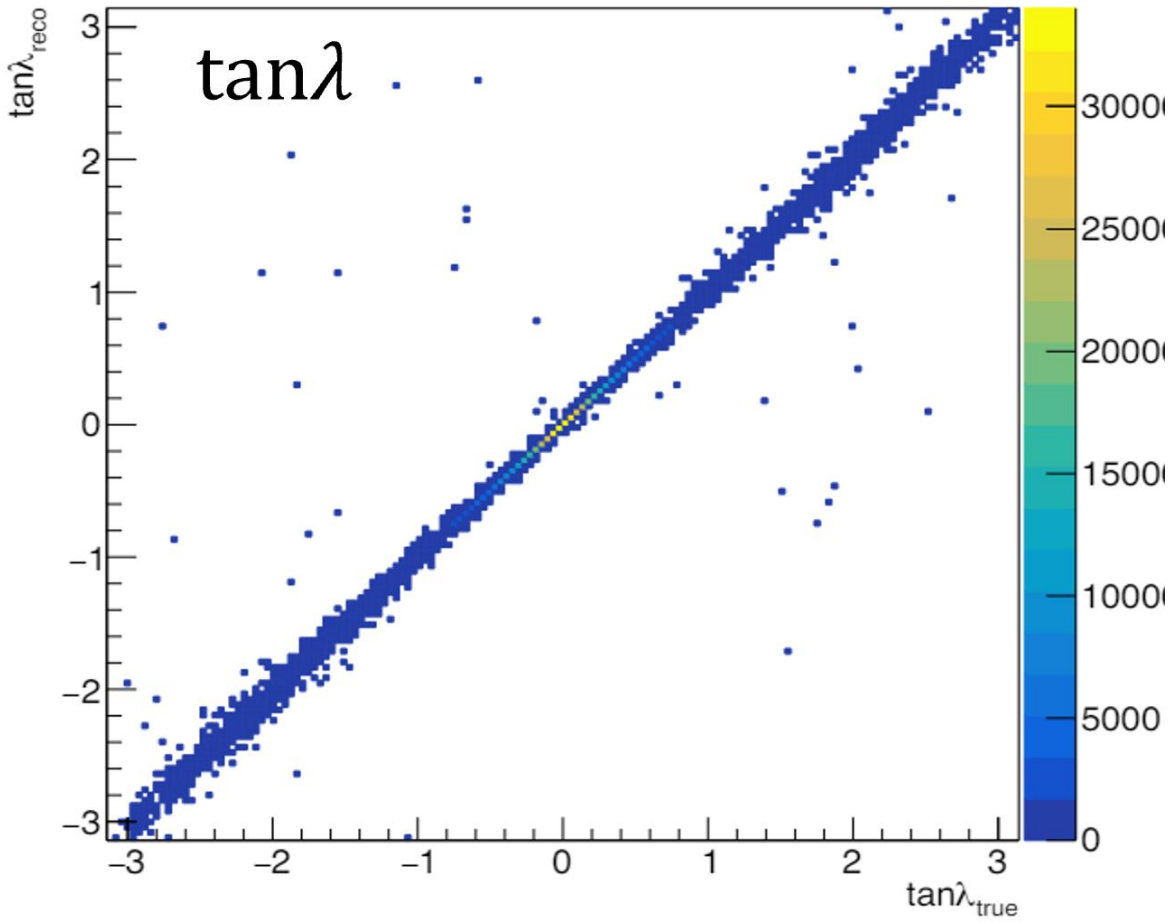
$$\begin{pmatrix} x \\ y \\ q/R \\ \tan\lambda \\ \phi \end{pmatrix}$$

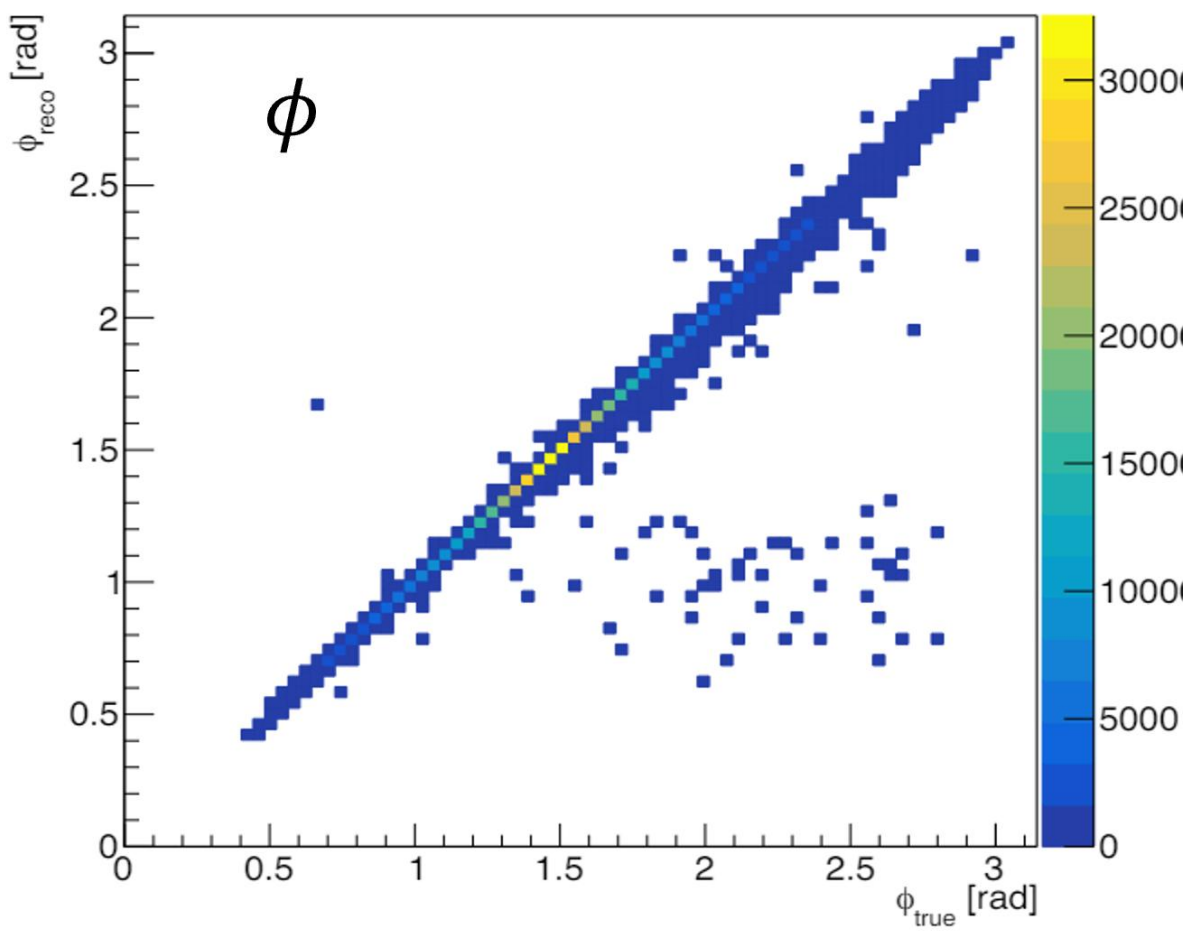




Angles reconstruction - muons

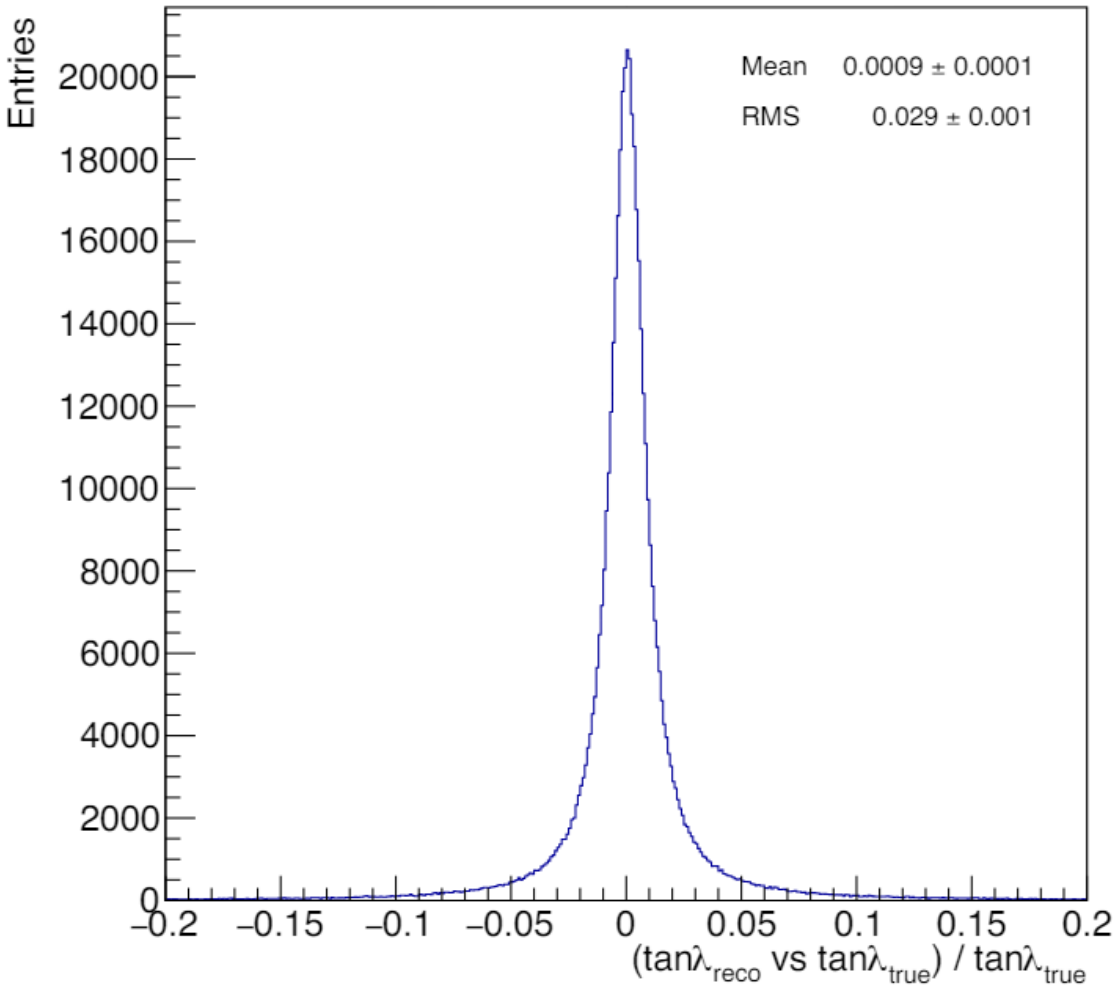
$$\begin{pmatrix} x \\ y \\ q/R \\ \tan\lambda \\ \phi \end{pmatrix}$$

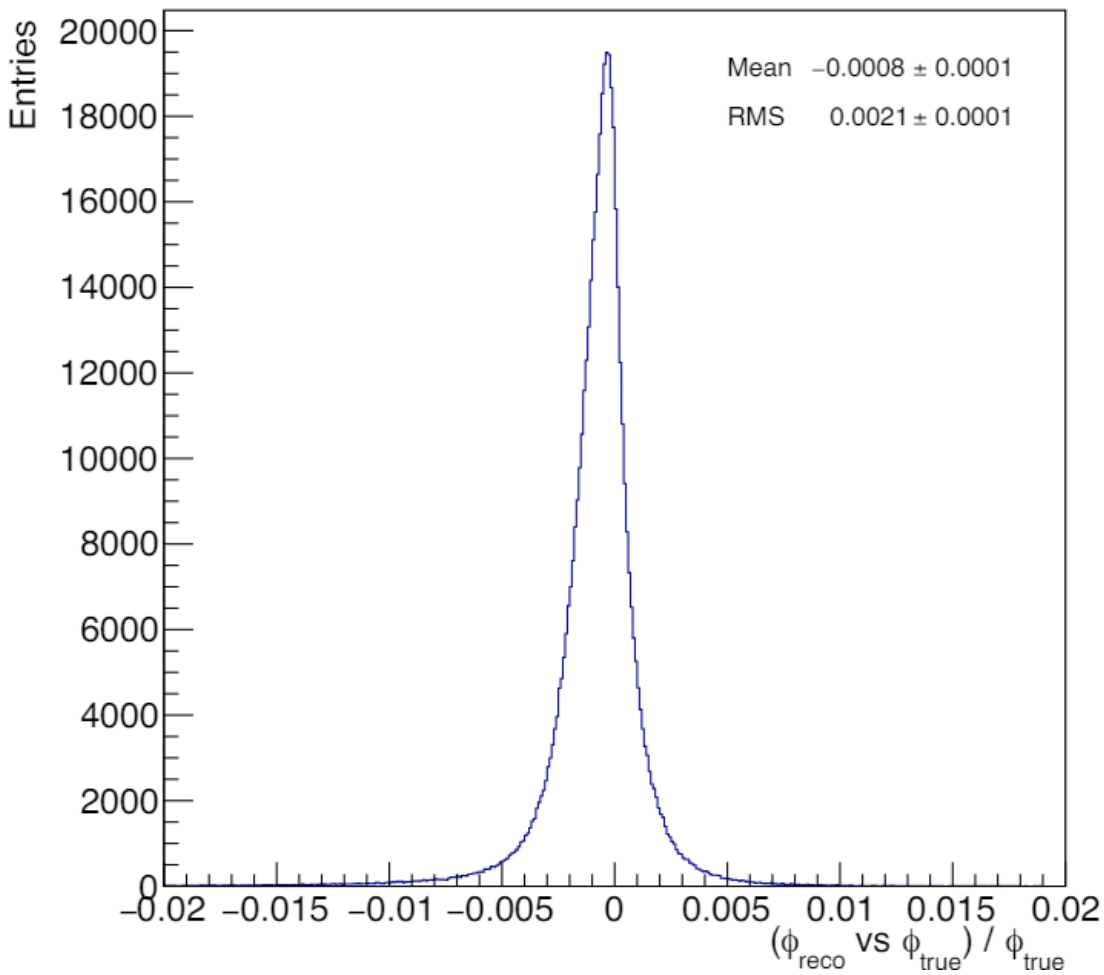




Residual on track parameters - muons

$$\begin{pmatrix} x \\ y \\ q/R \\ \tan\lambda \\ \phi \end{pmatrix}$$

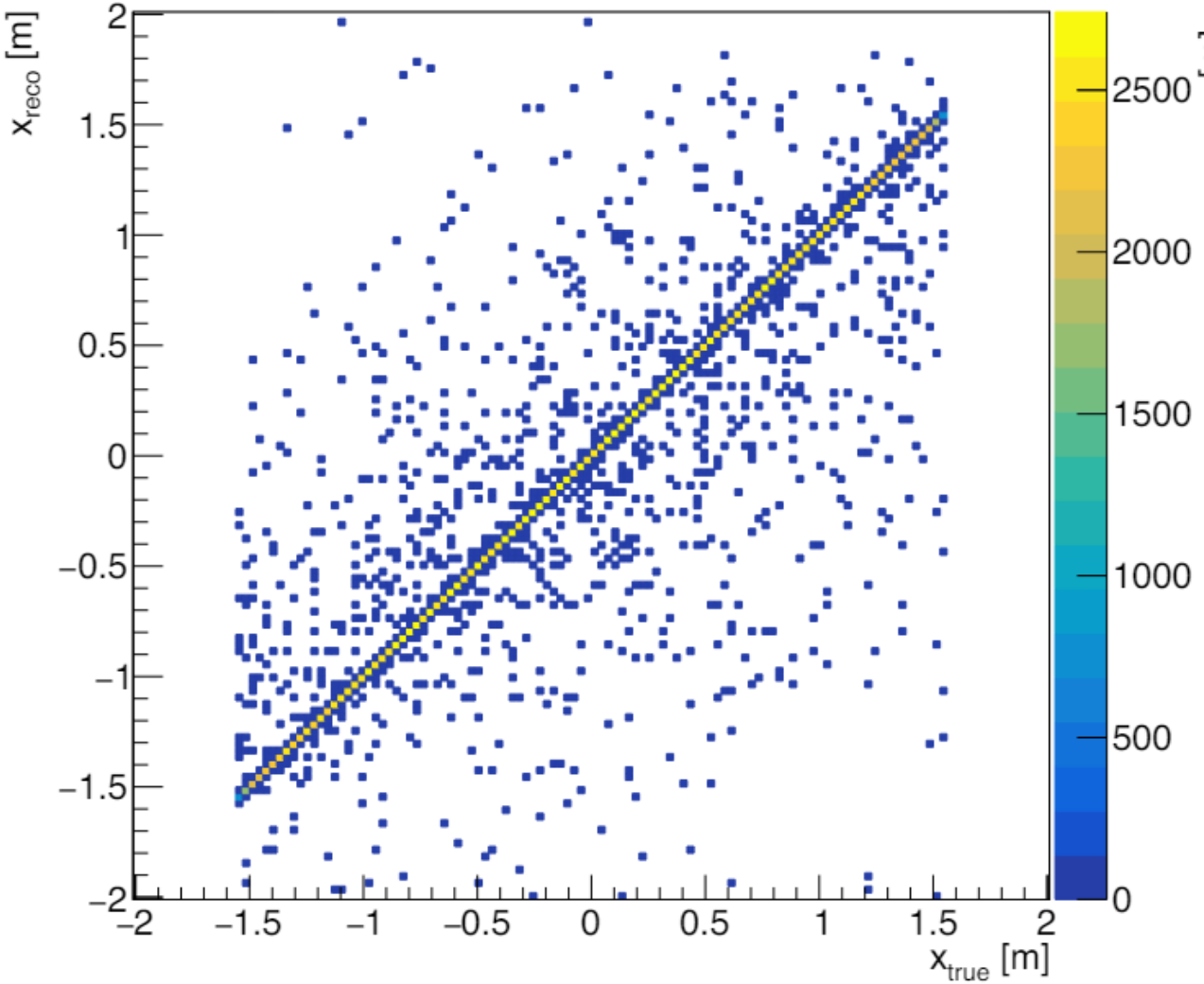




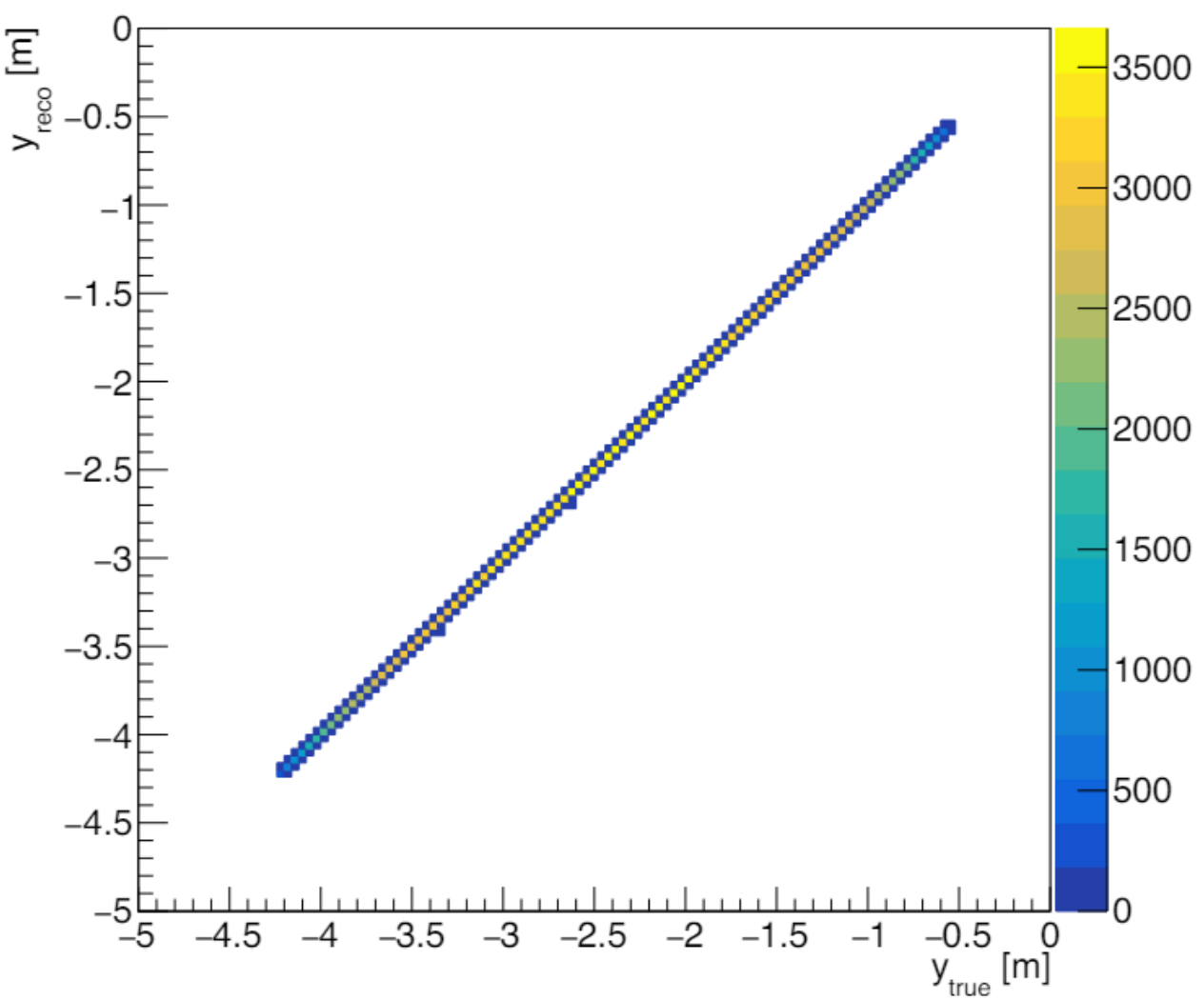
Position reconstruction - protons

$$\begin{pmatrix} x \\ y \\ q/R \\ \tan\lambda \\ \phi \end{pmatrix}$$

Horizontal plane

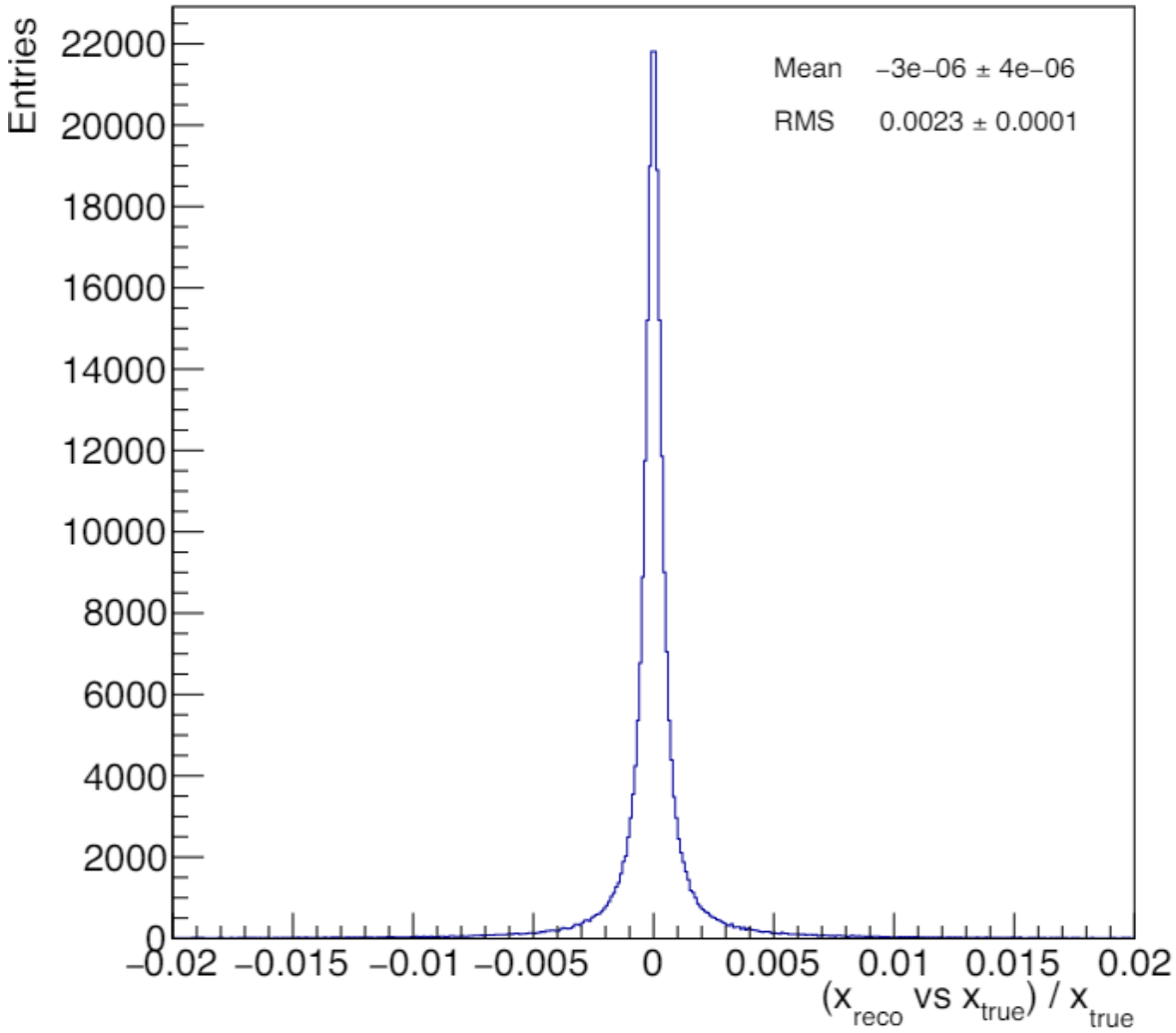


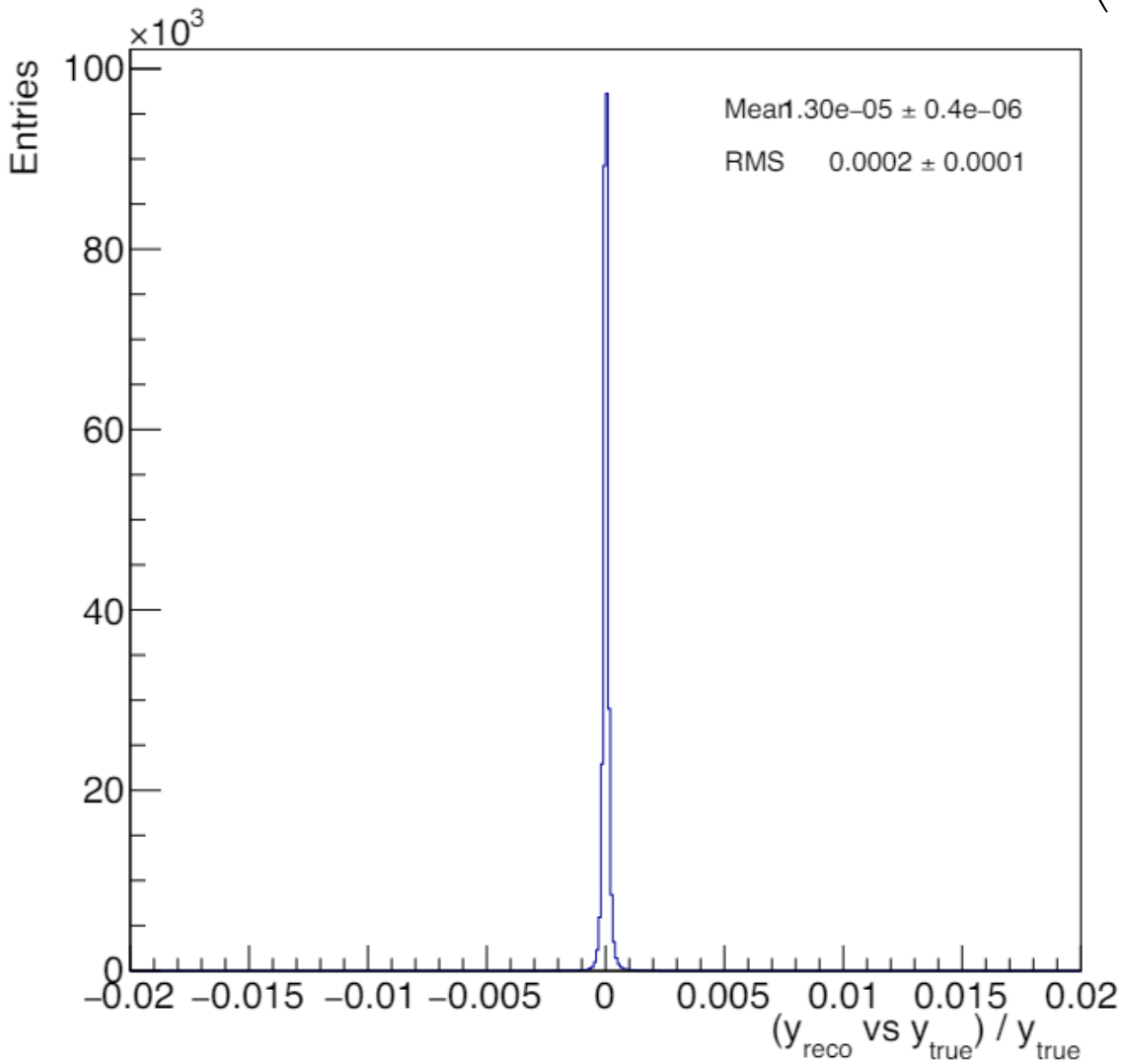
Vertical plane



Residual on track parameters - protons

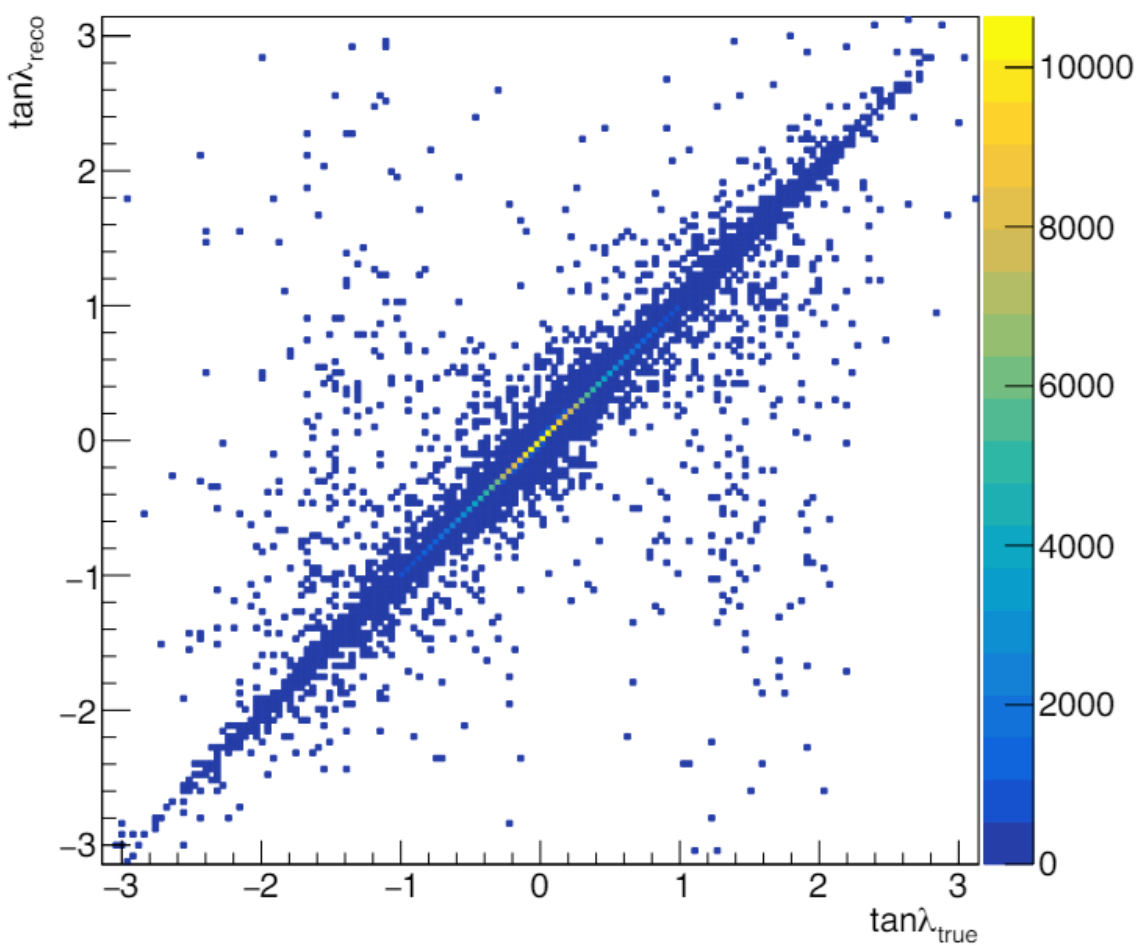
$$\begin{pmatrix} x \\ y \\ q/R \\ \tan\lambda \\ \phi \end{pmatrix}$$

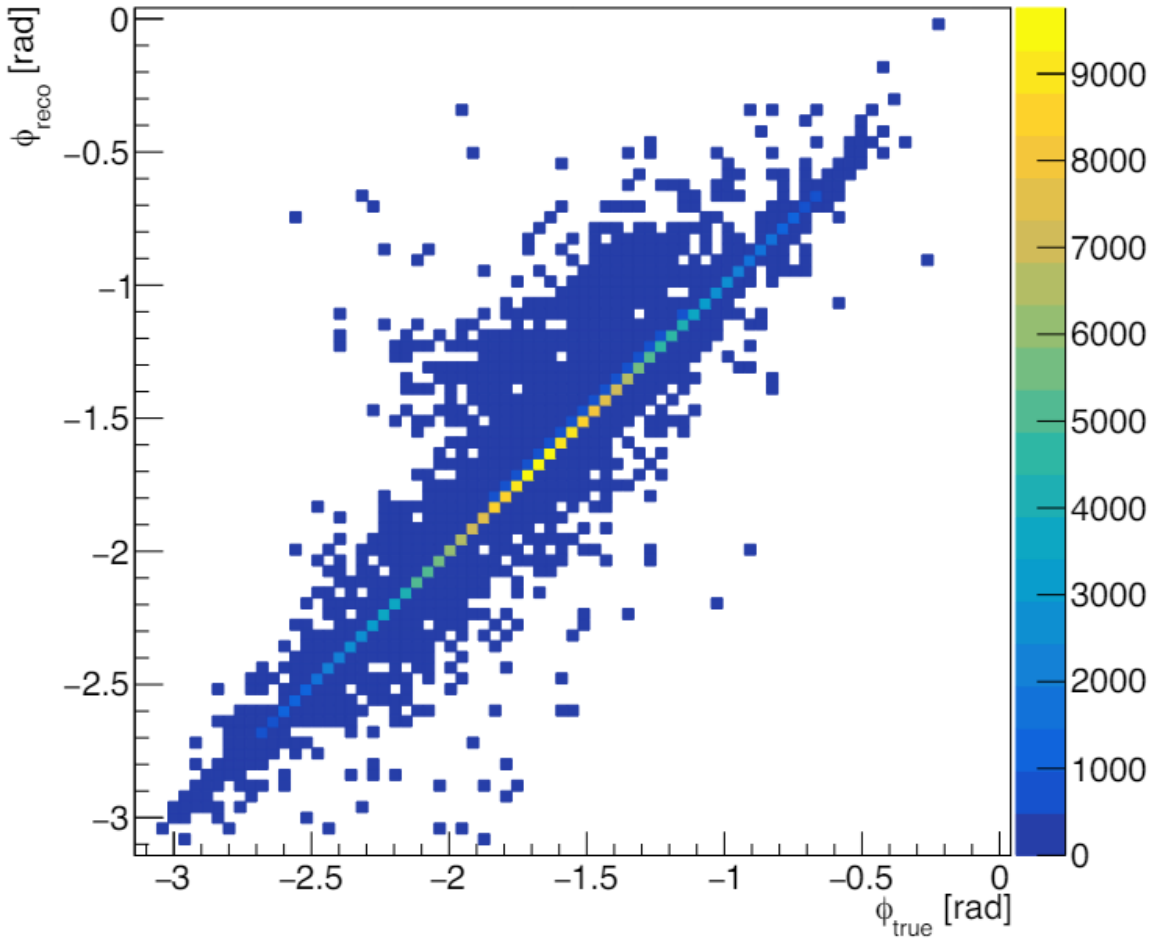




Angles reconstruction - protons

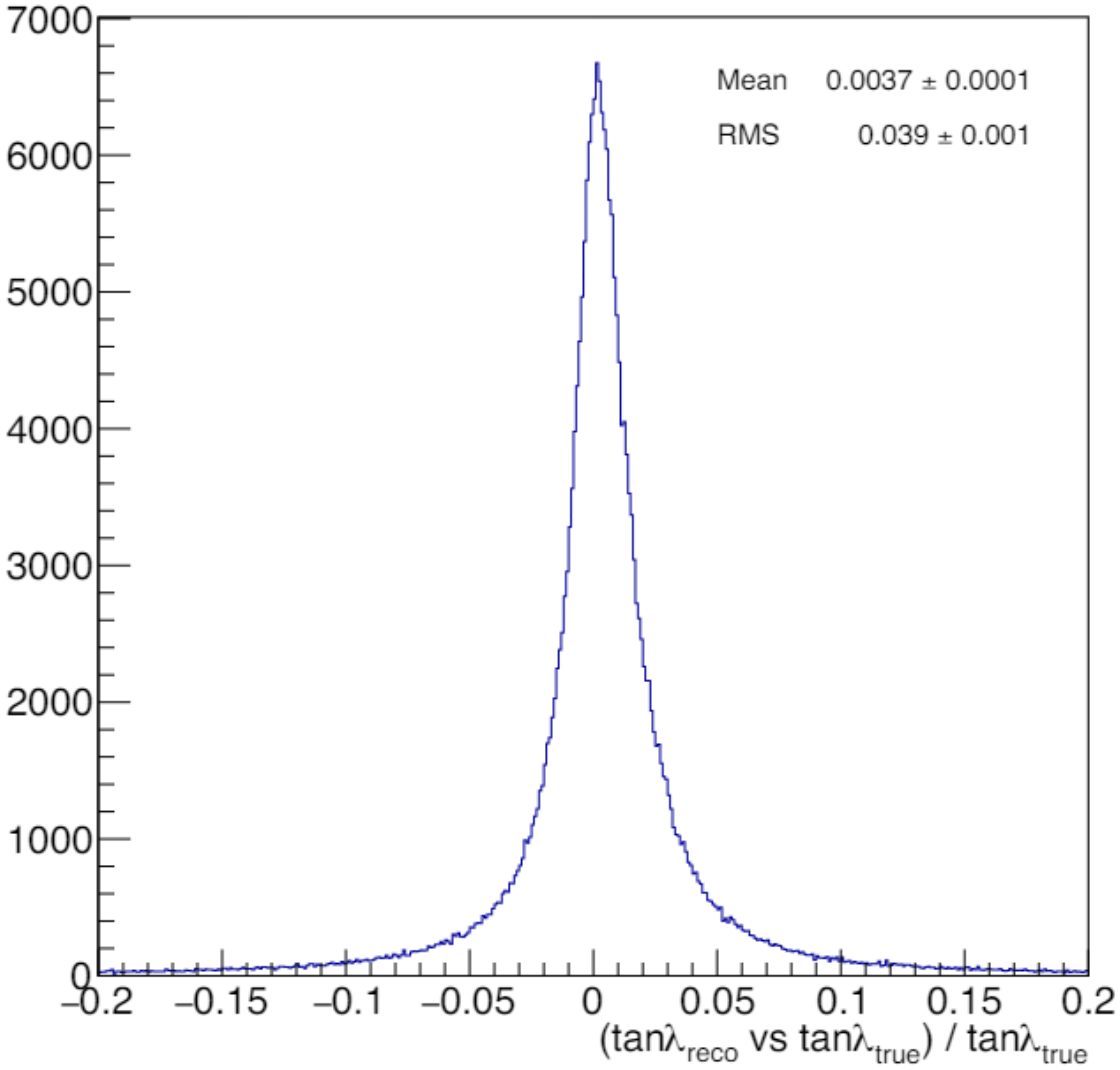
$$\begin{pmatrix} x \\ y \\ q/R \\ \tan\lambda \\ \phi \end{pmatrix}$$

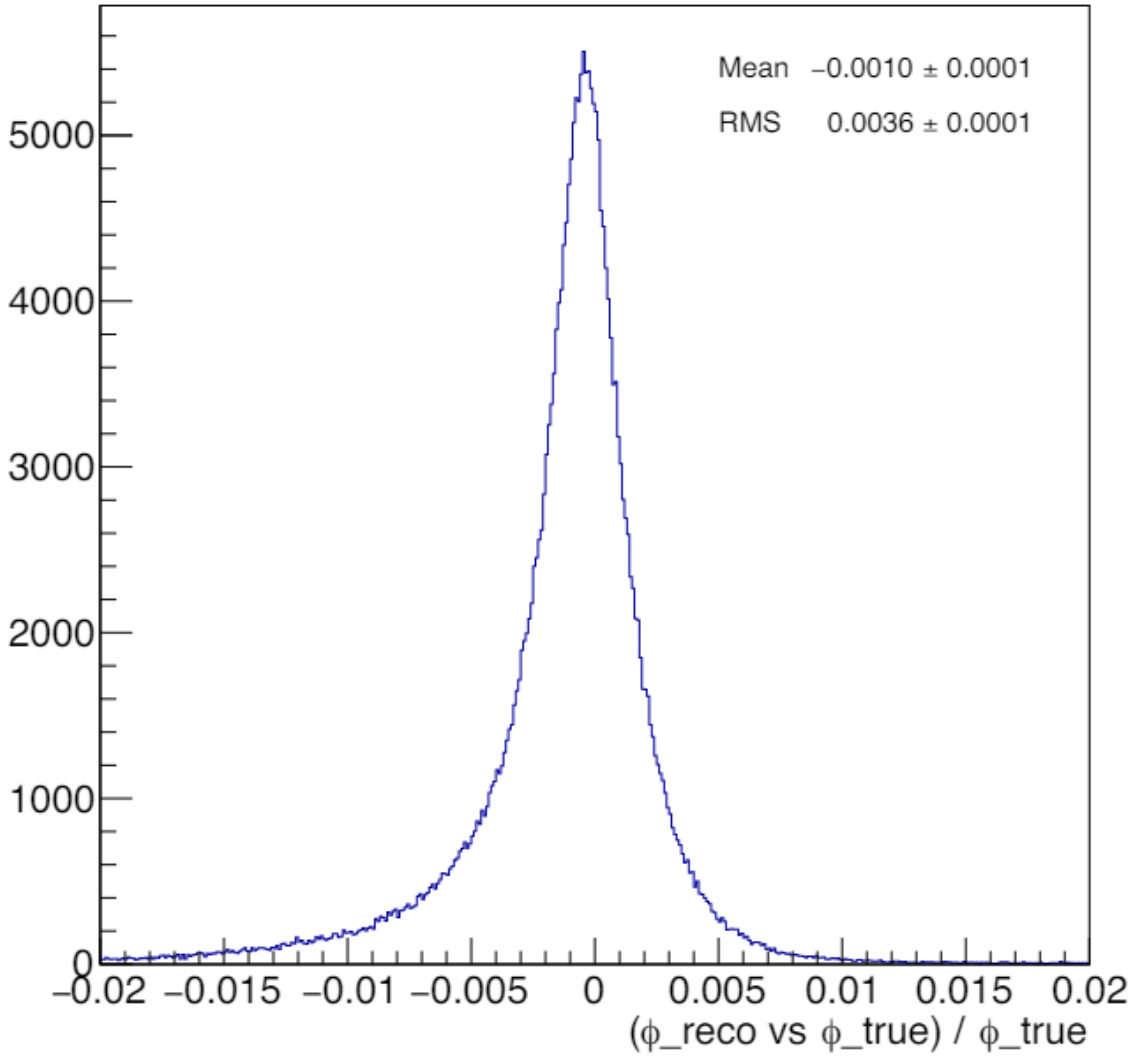




Residual on track parameters - protons

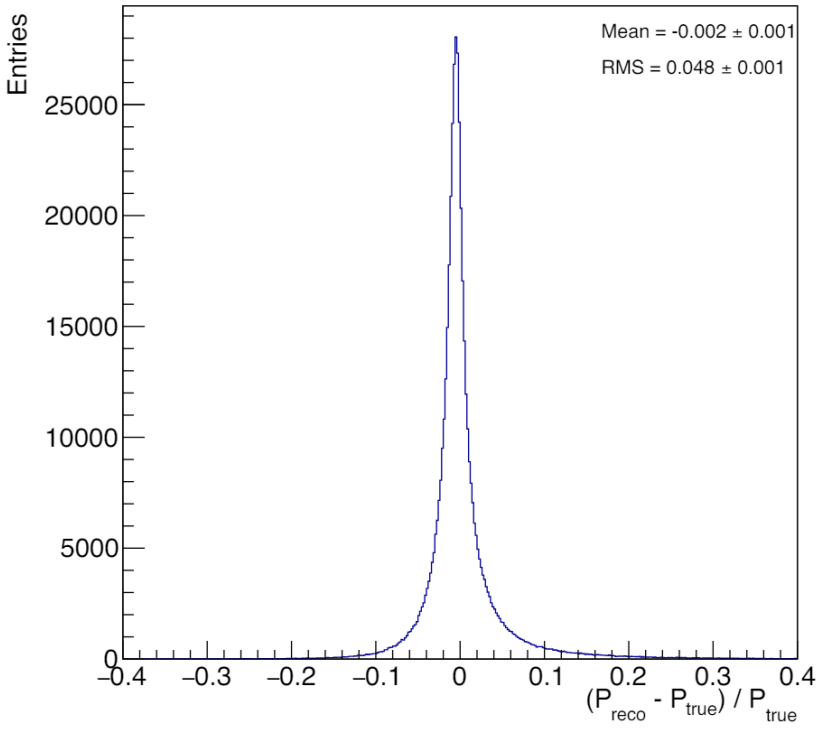
$$\begin{pmatrix} x \\ y \\ q/R \\ \tan\lambda \\ \phi \end{pmatrix}$$



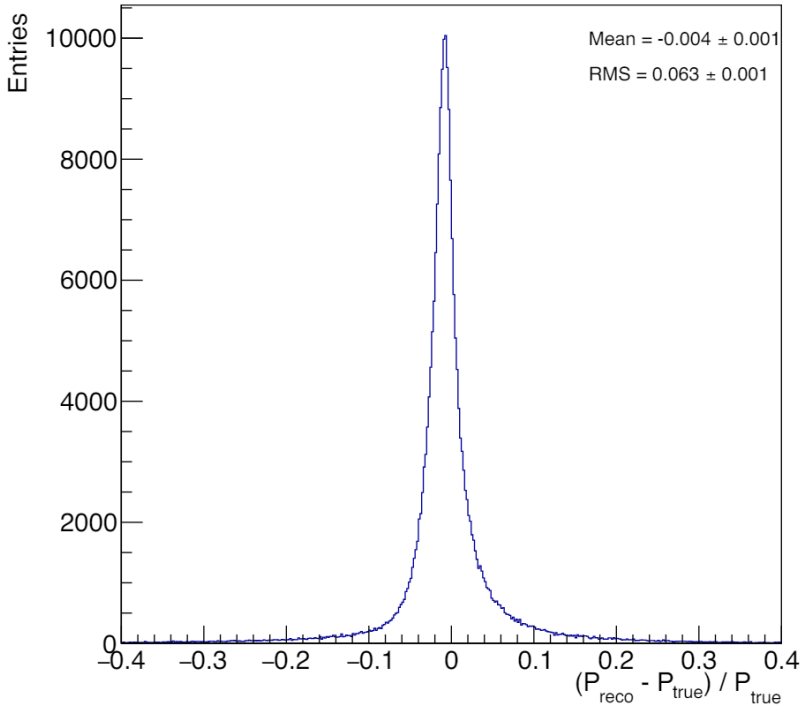


Residuals on momentum reconstruction

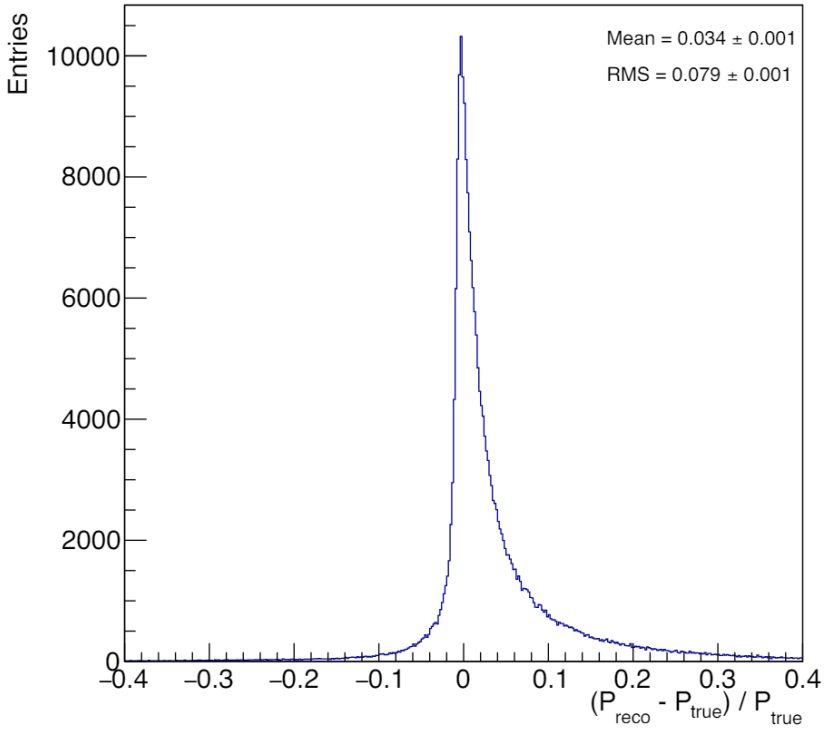
μ^-



π^+

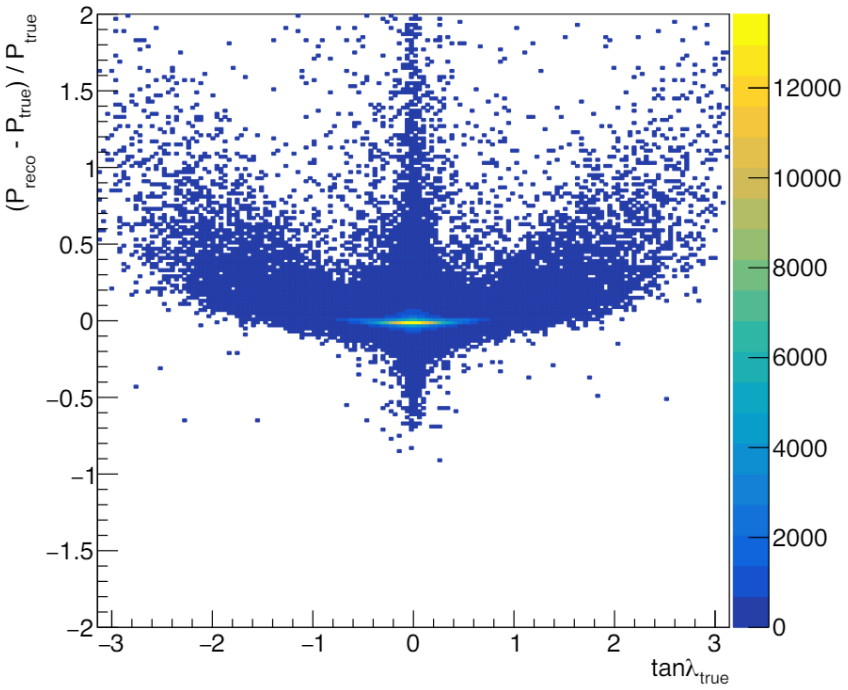


p

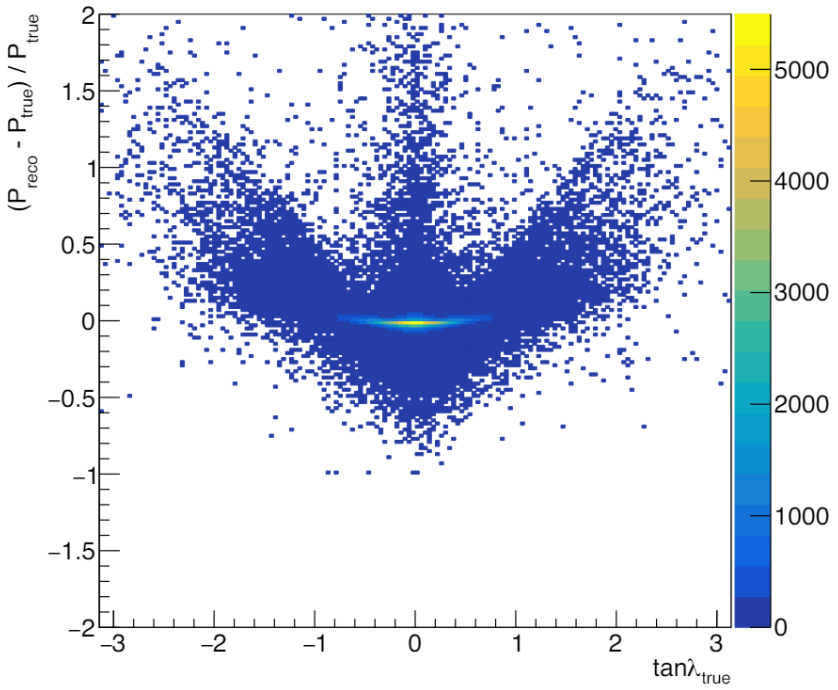


Momentum and dip angle correlation

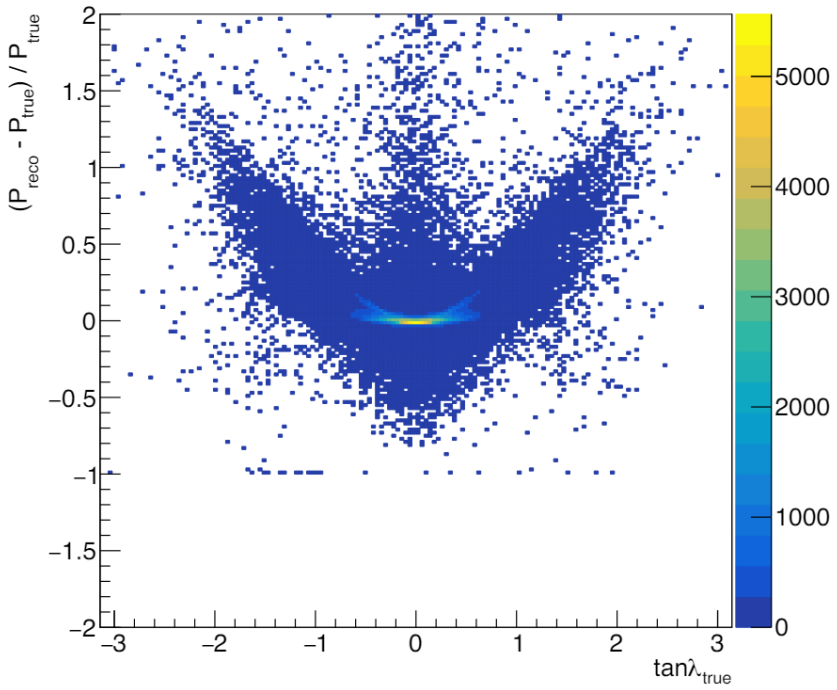
μ^-



π^+



p



Standard consistency checks - 1

Innovation vector $r_k = m_k^{pred} - m_k^{true}$

$$g(i)_k = \frac{r(i)_k}{\sqrt{C(i)_k}}$$

r_k^i = i-th element of the innovation at step k

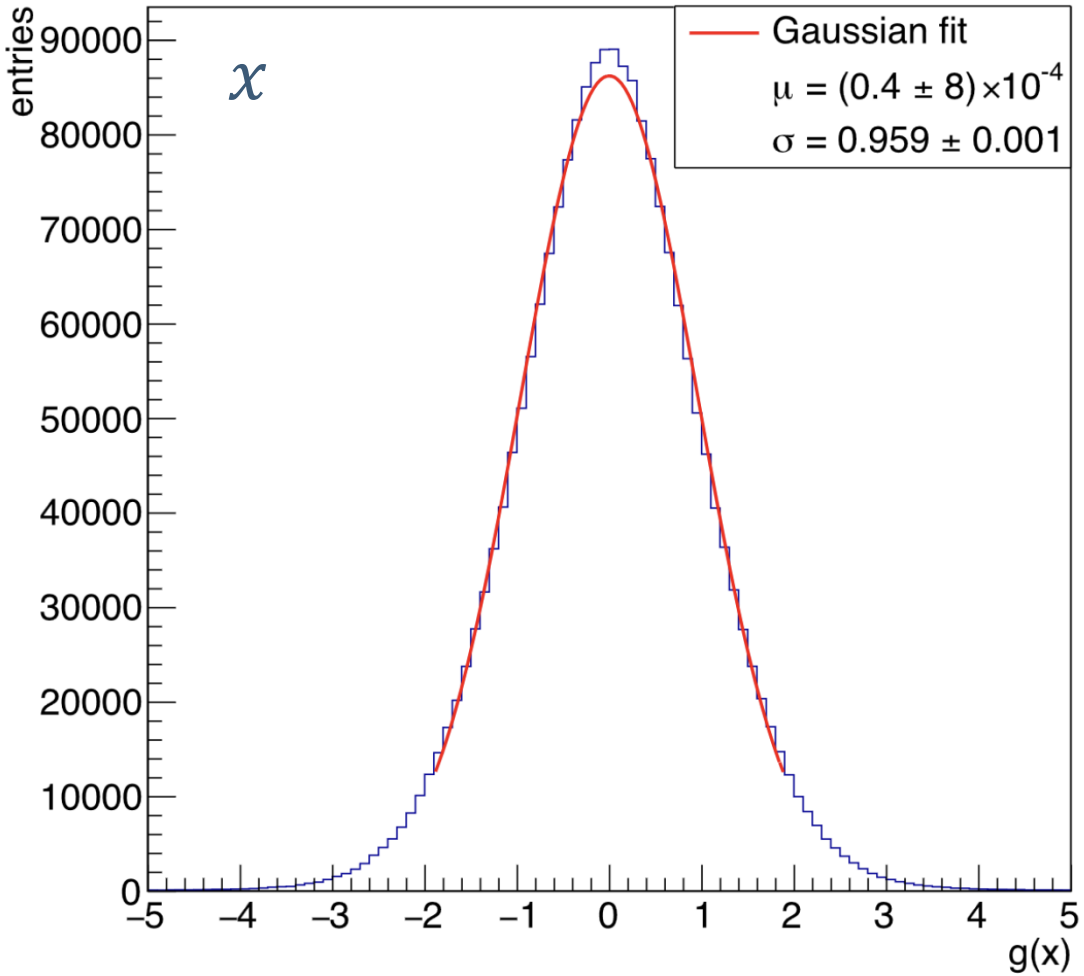
C_k^i = corresponding element of the measurement covariance matrix

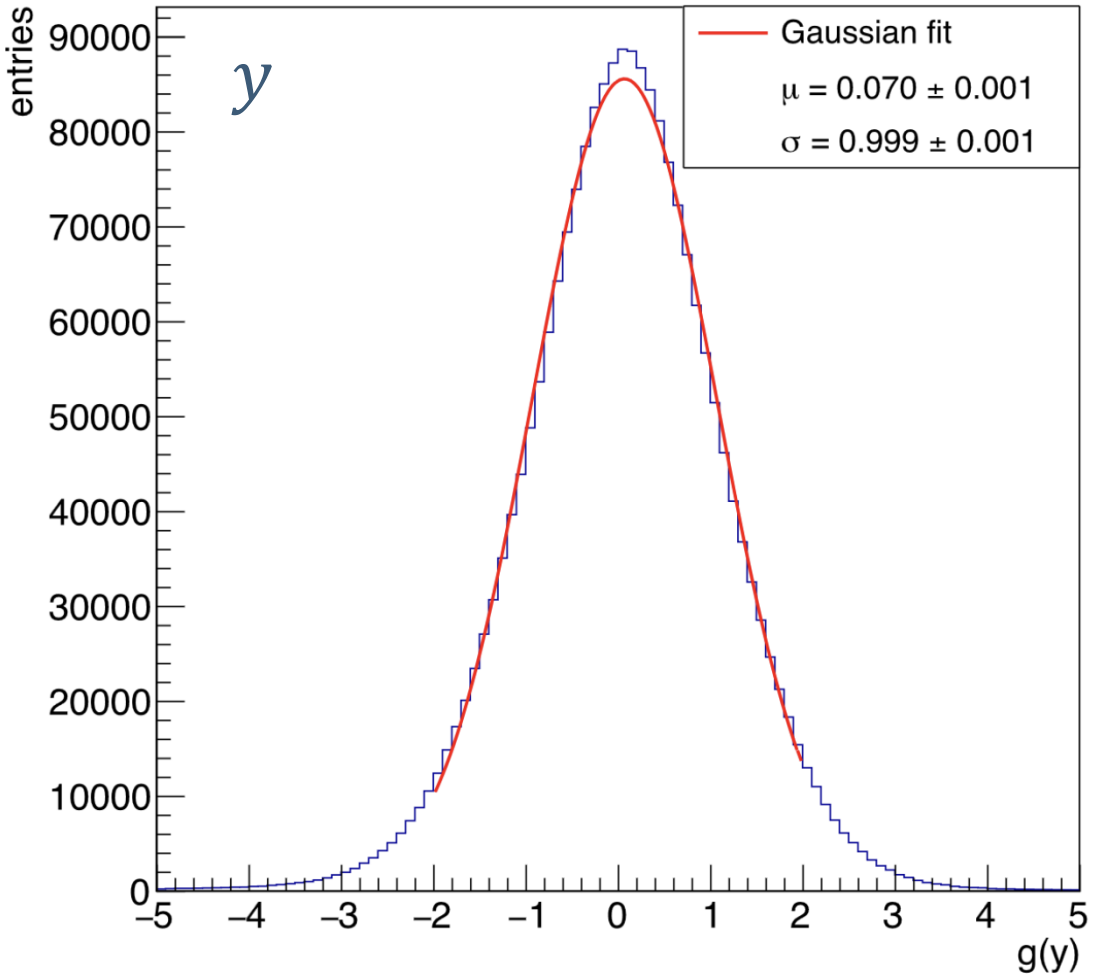
If the prediction is correct
each $\boldsymbol{g^i}$ should be distributed as a **standard gaussian distribution** ($\boldsymbol{\mu = 0,\ \sigma = 1}$)

Distributions of *g* - positions

$$m_x = \begin{pmatrix} x \\ \theta_{xz} \end{pmatrix} \quad m_y = \begin{pmatrix} y \\ \theta_{yz} \end{pmatrix}$$

$$g(pos)_k = \frac{r(pos)_k}{\sqrt{C(pos)_k}}$$

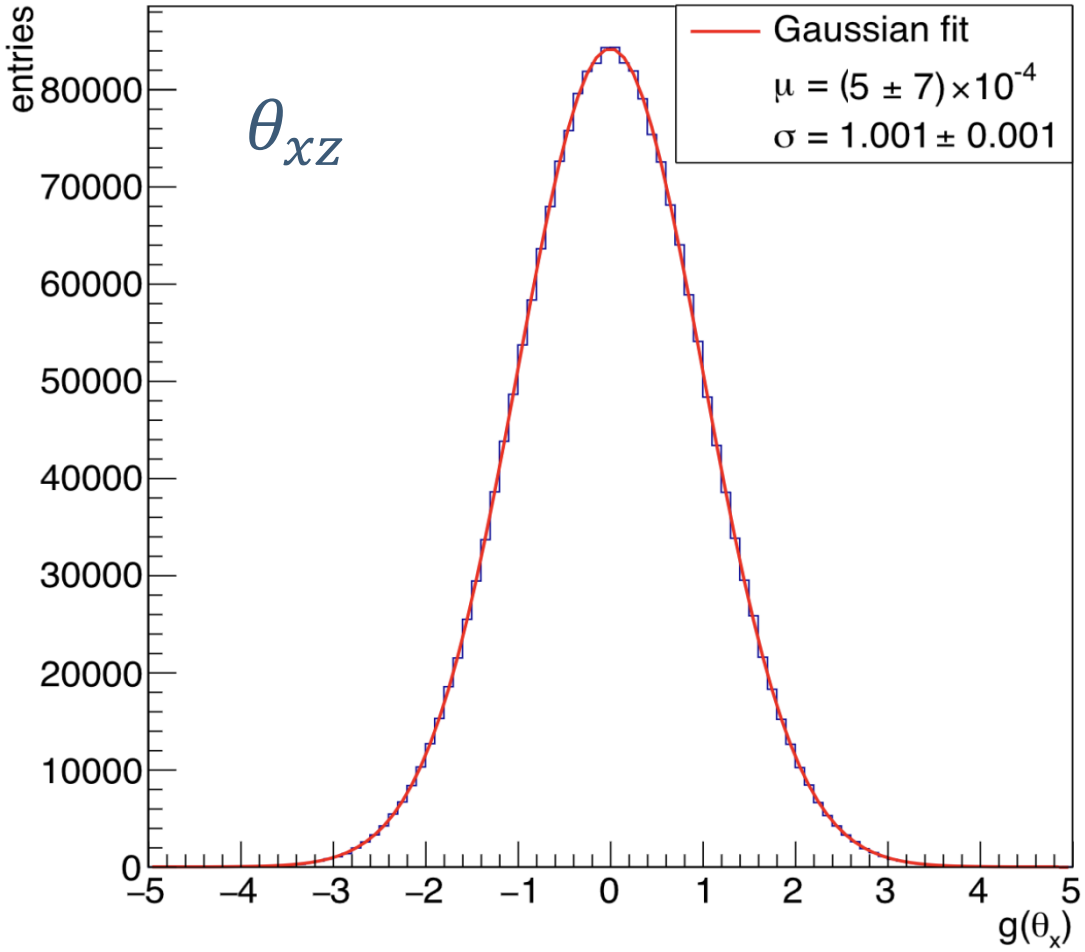


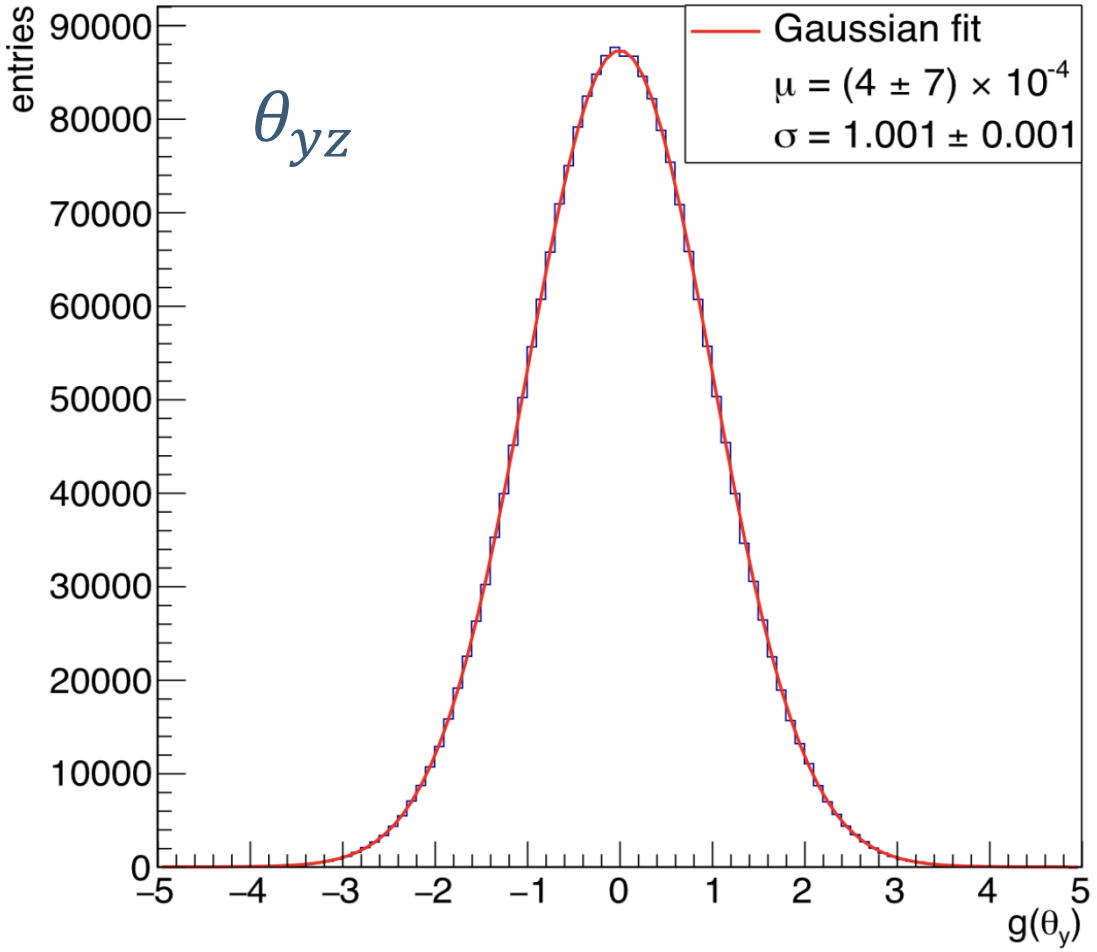


Distributions of *g* - angles

$$m_x = \begin{pmatrix} x \\ \theta_{xz} \end{pmatrix} \quad m_y = \begin{pmatrix} y \\ \theta_{yz} \end{pmatrix}$$

$$g(\theta)_k = \frac{r(\theta)_k}{\sqrt{C(\theta)_k}}$$





Standard consistency checks - 2

Innovation vector $r_k = m_k^{pred} - m_k^{true}$

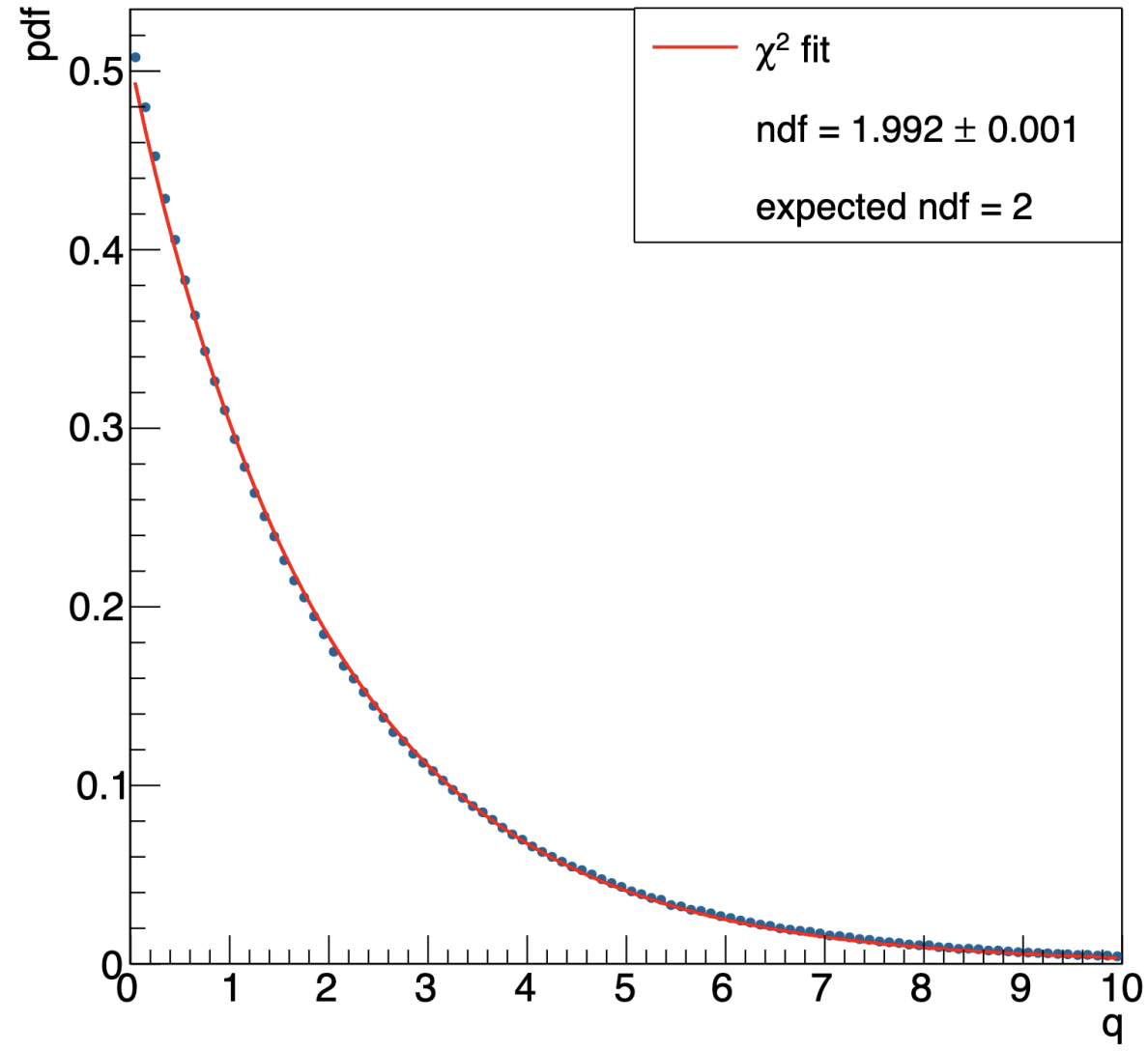
$$q_k = r_k^T C_k^{-1} r_k$$

r_k = innovation vector at step k

C_k = corresponding measurement covariance matrix

If the prediction is correct
$\boldsymbol{q}$ should be a $\boldsymbol{\chi^2}$ random variable with $\boldsymbol{rank(r) = 2}$ ndf

q-distribution



KF implementation

State vector propagation from z_{k-1} *to* z_k

$$f_{k-1}: \quad \begin{aligned} x_k &= x_{k-1} + \tilde{R}_{k-1} \cdot \tan\lambda_{k-1} \cdot (\phi_k - \phi_{k-1}) \\ y_k &= y_{k-1} + R_{k-1} \cdot (\sin\phi_k - \sin\phi_{k-1}) \\ 1/\tilde{R}_k &= 1/\tilde{R}_{k-1} + \kappa \cdot \Delta(1/R_{k-1}) \\ \tan\lambda_k &= \tan\lambda_{k-1} \\ \cos\phi_k &= \cos\phi_{k-1} + (z_k - z_{k-1})/R_{k-1} \end{aligned}$$

$$= -\frac{(1/R)^2}{0.3 \cdot B} \cdot \sqrt{1+\tan^2\lambda} \cdot \sqrt{1 + \frac{m^2(1/R)^2}{(0.3 \cdot B)^2}(1+\tan^2\lambda)} \cdot \Delta E$$

Prediction of measurement vector m_k from propagated state vector

$$h_k^{(x)} = \begin{pmatrix} x_k \\ -q \cdot \arctan\dfrac{\tan\lambda_k}{\sin\phi_k} \end{pmatrix}$$

$$h_k^{(y)} = \begin{pmatrix} y_k \\ \phi_k + q \cdot \dfrac{\pi}{2} \end{pmatrix}$$

Measurement projection to state vector

$$H_k^{(x)} = \begin{pmatrix} 1 & 0 & 0 & 0 & -\dfrac{\tan\lambda_k}{\tan\phi_k} \cdot \dfrac{\kappa}{\sin\phi_k \cdot \left[1+\left(\frac{\tan\lambda_k}{\sin\phi_k}\right)^2\right]} \\ 0 & 0 & 0 & \dfrac{\kappa}{\sin\phi_k \cdot \left[1+\left(\frac{\tan\lambda_k}{\sin\phi_k}\right)^2\right]} & 0 \end{pmatrix}$$

$$H_k^{(y)} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$