Higher-order QCD corrections in the Higgs into Hadrons

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In colaboration with Diogo Boito and Cristiane Y. London

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The Higgs decay into Hadrons

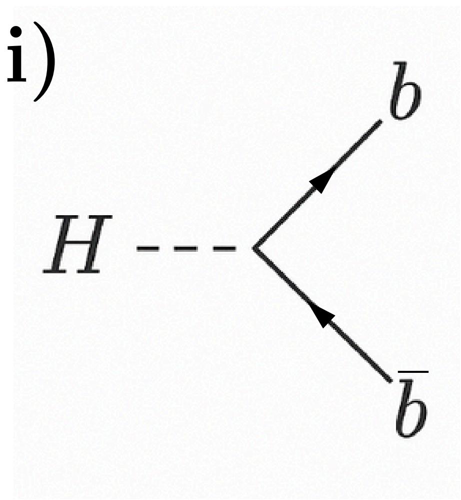
- The decays of the Standard Model(SM)Higgs boson into hadrons are dominated by quarks and gluons

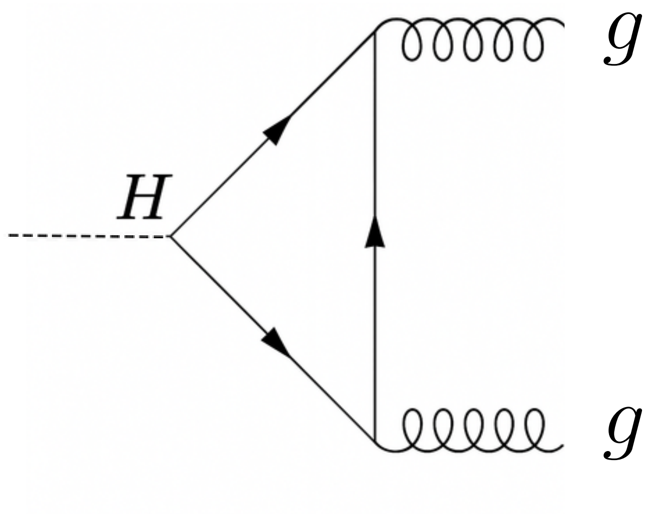
(approximately 70% of the Higgs boson decays are hadronic)

- No direct observation of physics beyond the SM–theory experiments require greater accuracy.

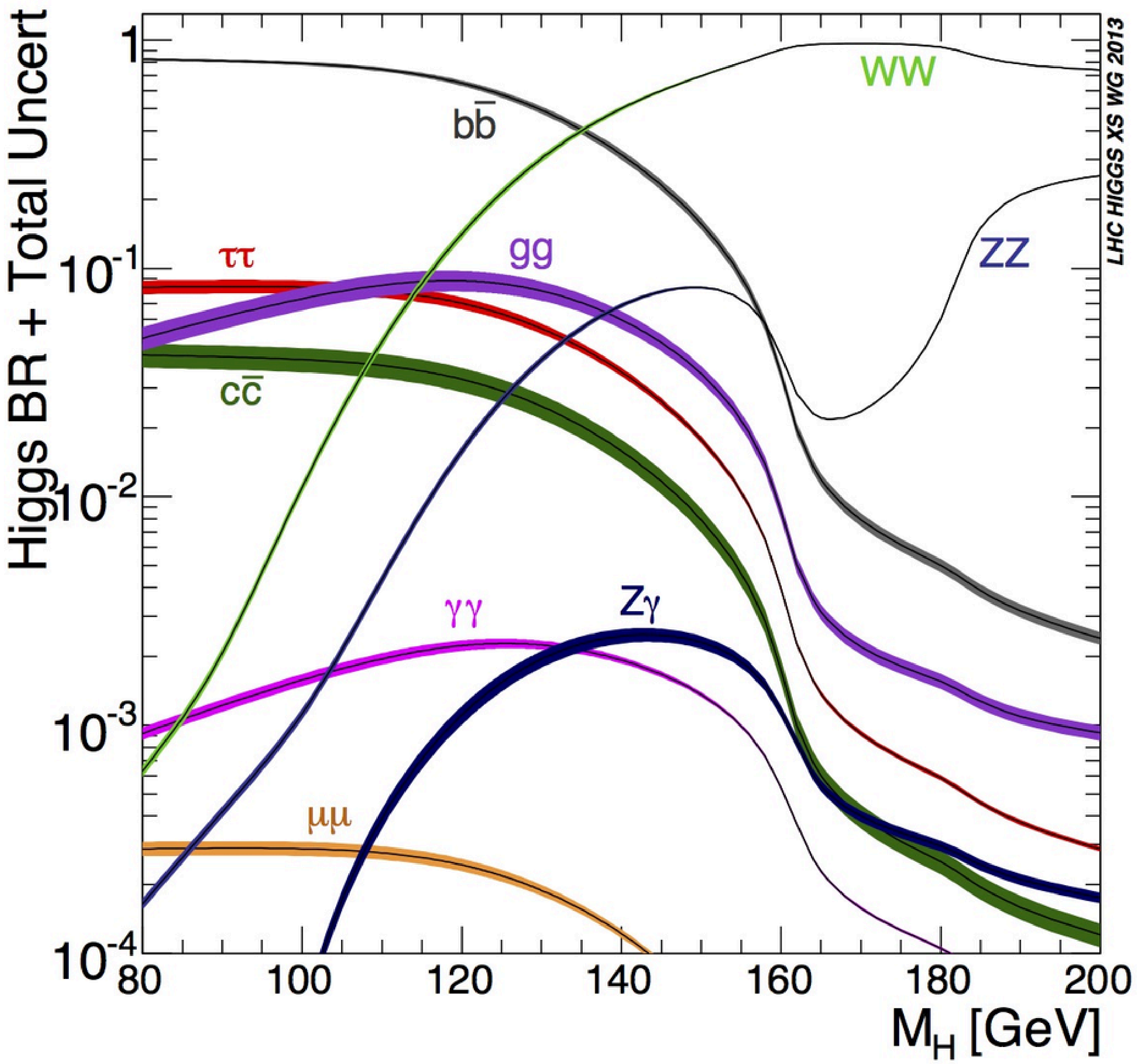
- **We present a model-independent estimate of the first unknown coefficients for these processes**

i)





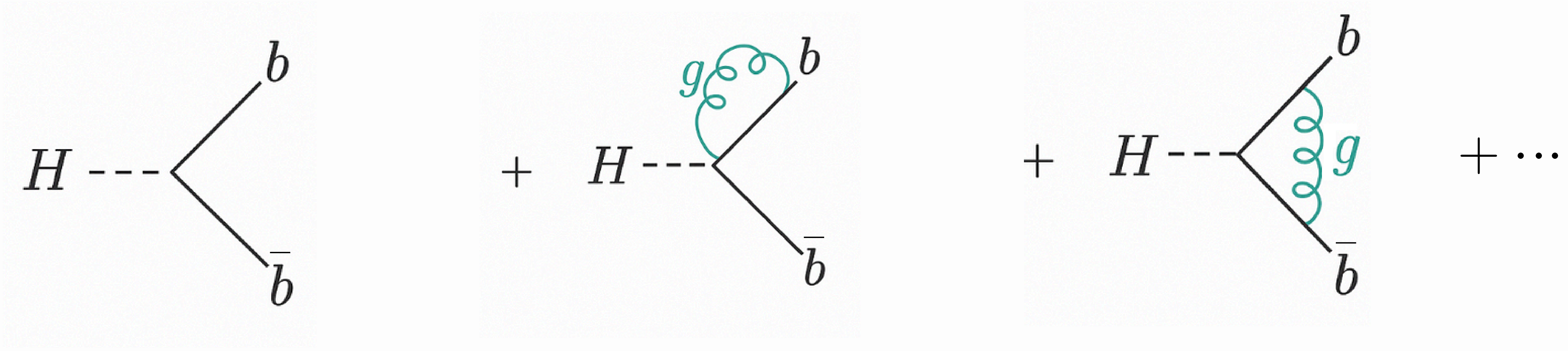
Standard Model of Particle Physics



[Particle Data Group, 2013]

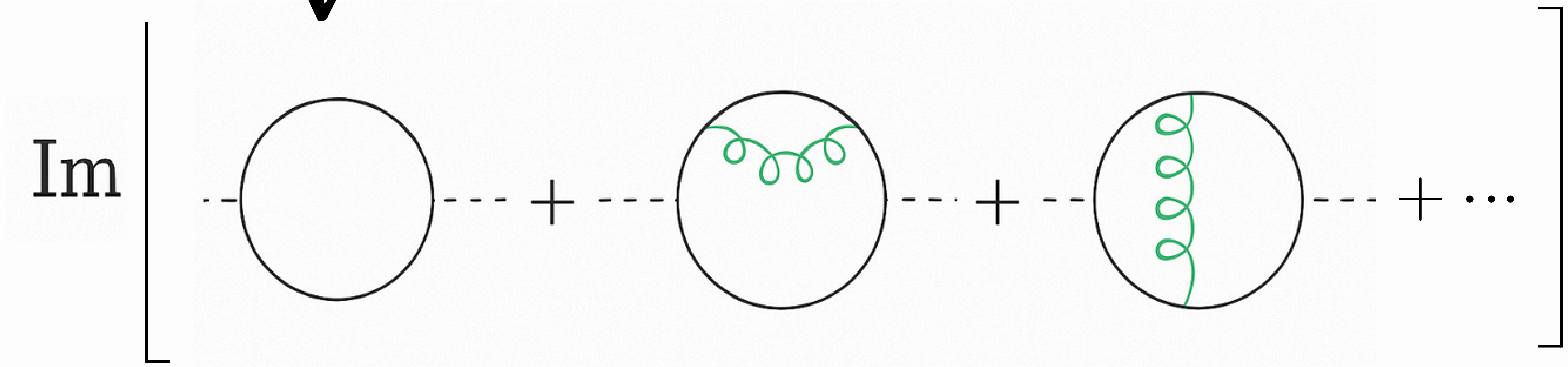
$H \to b\bar{b}$

This Higgs boson decay can be visualized diagramatically as:



$$\Gamma(H \to b\bar{b}) = \frac{1}{v^2\, m_H}\, \mathrm{Im}\,\Pi(s)$$

Related Through the optical theorem!



- Correlator of two scalar currents

$$\Pi(p^2) \equiv i \int d^4x\, e^{ipx} \langle \Omega | T\{j(x) j^\dagger(0)\} | \Omega \rangle$$

$$j(x) = m_q \, :\bar{q}_f(x) q_f(x):$$

- Imaginary part of the scalar correlator

$$\mathrm{Im}\,\Pi(s) = \frac{N_c}{8\pi} m_b^2(m_H)\, s \left[1 + \sum_{n=0}^{\infty} c_n\, a_s^n(m_H) \right]$$

for $n_f = 5$

$c_1 = \frac{17}{3}$ [Braaten, Leveille '80] [Sakai '80]

$c_2 = 29.146$ [Gorishny, et al '90]

$c_3 = 41.757$ [Chetyrkin '97]

$c_4 = -825.7$ [Baikov, Chetyrkin, Kühn '06]

- Correlator of two scalar currents

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- Imaginary part of the scalar correlator

$$\mathrm{Im}\,\Pi(s) = \frac{N_c}{8\pi} m_b^2(m_H)\, s \left[1 + \sum_{n=0}^{\infty} c_n \, \alpha_s^n(m_H) \right]$$

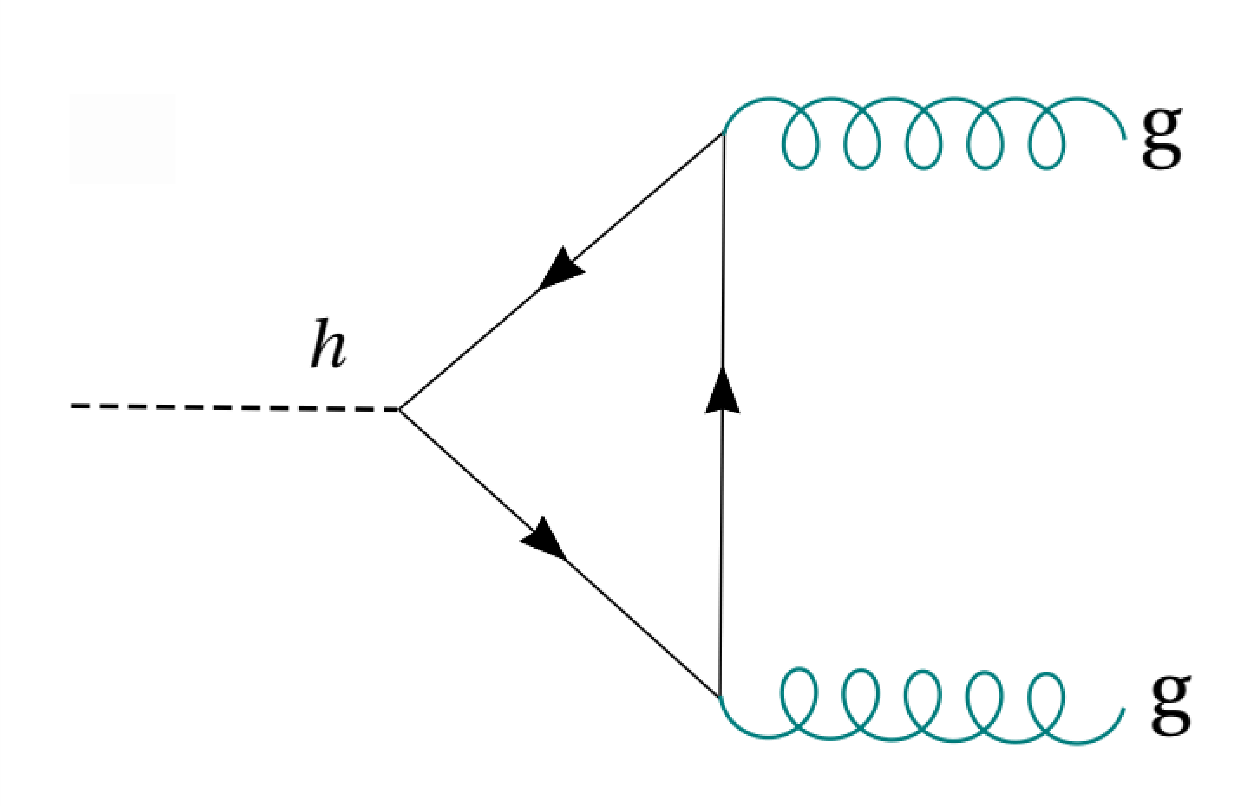
Each coefficient has an polynomial dependence on $\mathrm{n_f}$

- **We predict the coefficient of $\mathcal{O}(\alpha_s^5)$**

for $n_f = 5$

$c_1 = \frac{17}{3}$	$c_2 = 29.146$	$c_3 = 41.757$	$c_4 = -825.7$
[Braaten, Leveille '80] [Sakai '80]	[Gorishny, et al '90]	[Chetyrkin '97]	[Baikov, Chetyrkin, Kühn '06]

$\mathbf{H \to gg}$

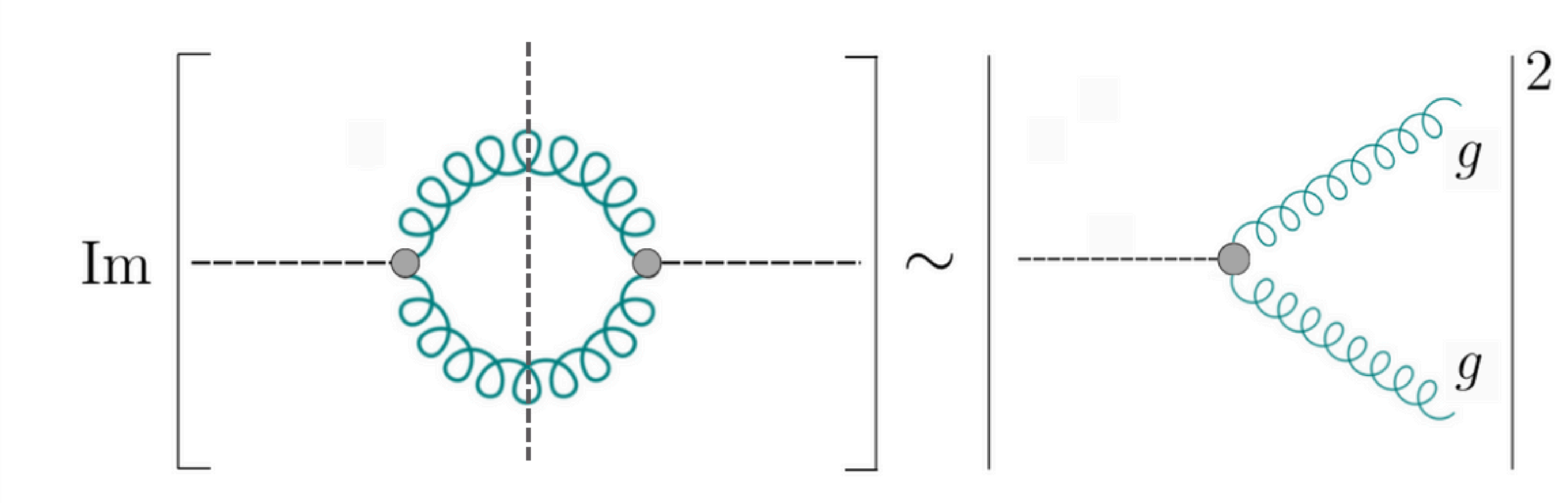


five coefficients are known exactly !

$$\Gamma_{H\to gg} = \frac{m_h^3}{72\pi v^2}\left(\frac{\alpha_s}{\pi}\right)^2 (1 + c_1 a_s + c_2 a_s^2 + \cdots)$$

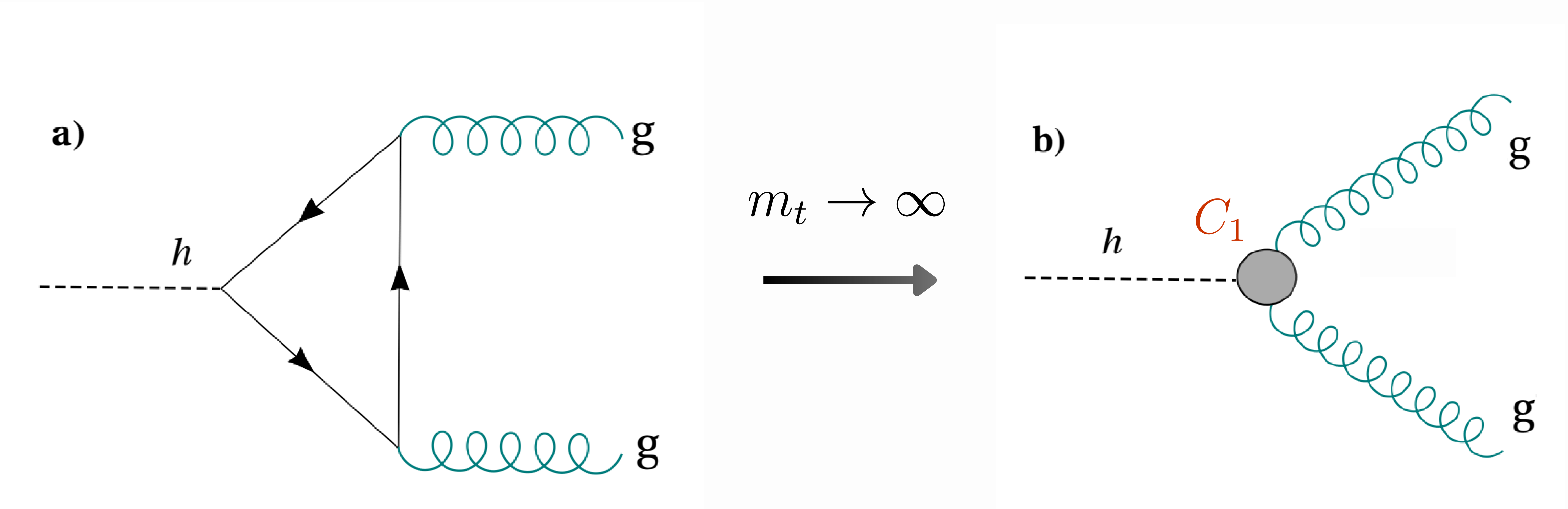
$$\Gamma_{H\to gg} = \frac{\sqrt{2}G_F}{M_H}|C_1|^2\,\mathrm{Im}\Pi_{G^2}(-M_H^2 - i\delta)$$

Through optical theorem



$\mathrm{H} \rightarrow \mathrm{gg}$

- We work at the heavy-top limit,where the quark-top is integrate out



Effective Lagrangian $\longrightarrow$ $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}(n_f)} - 2^{1/4} G_{\text{F}}^{1/2} C_1 \, h \, G_a^{\mu\nu} G_{\mu\nu}^a$

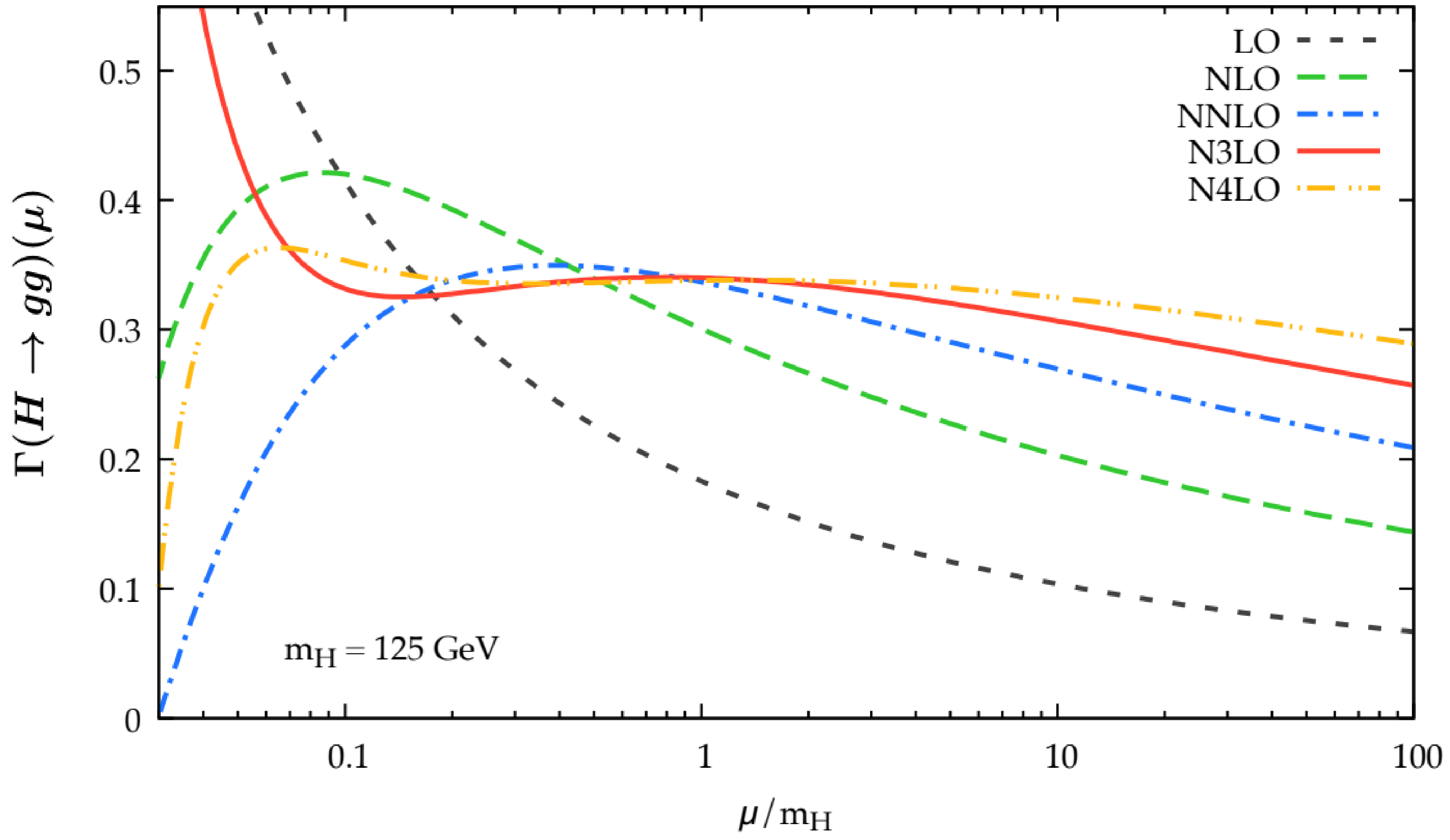
$H \to gg$

[HERZOG, ET AL. JHEP 08 (2017)]

$$\Gamma_{H\to gg} = \frac{\sqrt{2}G_F}{M_H} |C_1|^2 \mathrm{Im}\Pi_{G^2}(-M_H^2 - i\delta)$$

- The combination of these two quantities makes it possible

$$\Gamma^{(n_f)}_{h\to gg}(m_t) = \Gamma_0[1 + \underbrace{\alpha_s(7.5586 - 0.37136n_f)}_{\text{NLO}} + \underbrace{\alpha_s^2(0.091368n_f^2 - 4.82489n_f + 37.3523)}_{\text{NNLO}} + \underbrace{\alpha_s^3(-0.01746n_f^3 + 1.70814n_f^2 - 34.5769n_f + 144.901)}_{\text{N3LO}} + \cdots]$$



[BONVINI, ARXIV:2006.16293 (2020)]

- **This process is known up to $\mathcal{O}(\alpha_s^6)$**
- **We predict the coefficient of $\mathcal{O}(\alpha_s^7)$ using the the decay with in the large-β_0**

The large-β_0 limit

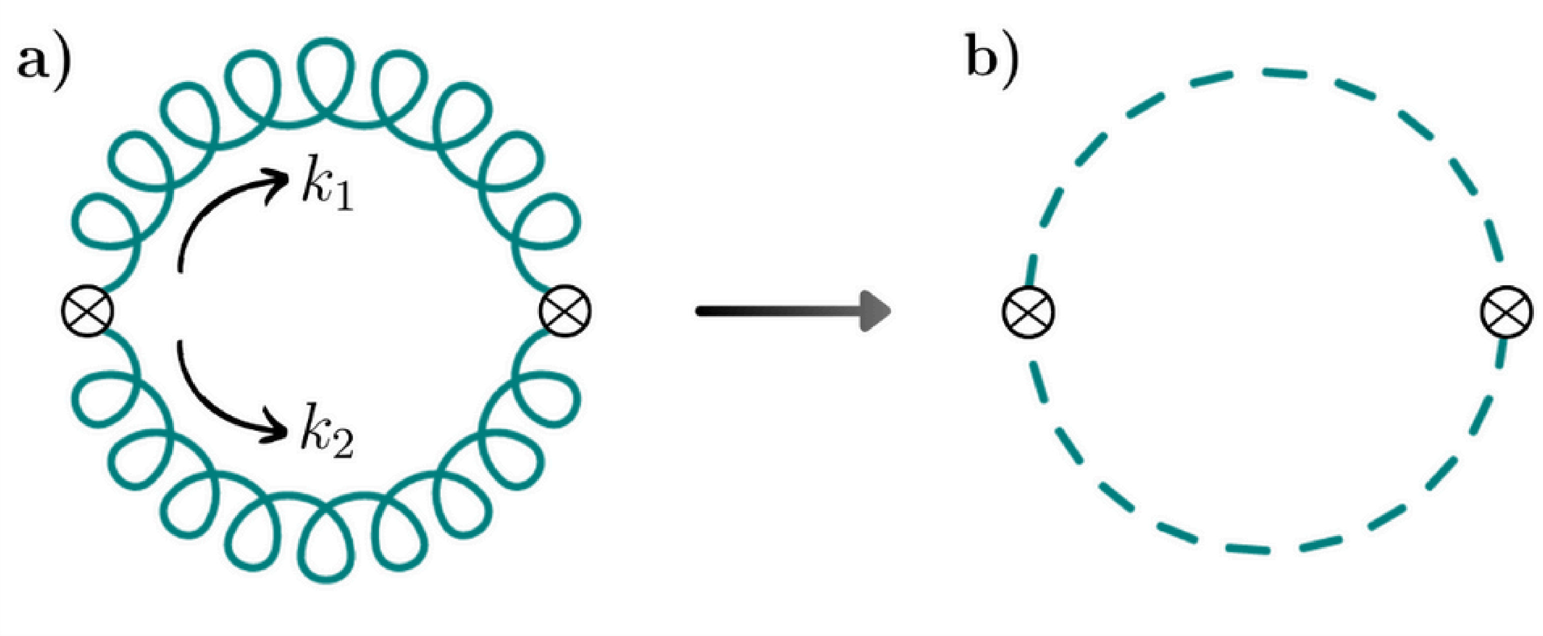
[JAMIN, MATTHIAS, ARXIV:1202.1169, 2012]

Taking into account a usual quantity in perturbation theory:

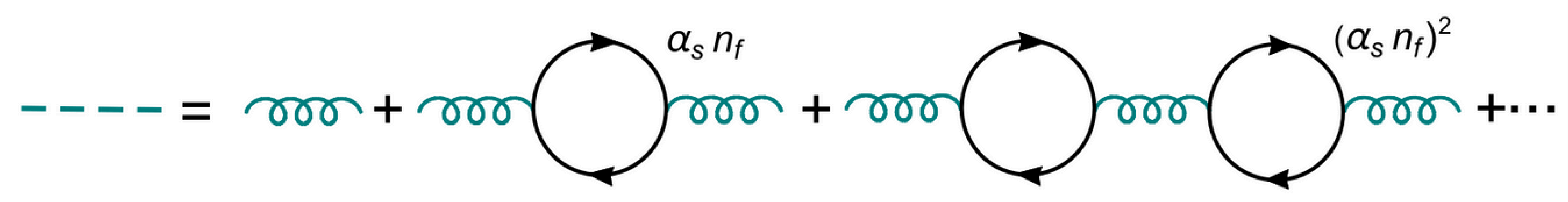
$$f(\alpha_s) = 1 + \sum_{n=0}^{\infty}\sum_{k=0}^{n} r_{n,k} n_f^k \alpha_s^{n+1}$$

large-β_0 limit: where the perturbative series is known in all orders (we use it as a laboratory)

Where the non-Abelianization procedure is necessary !



Resumed gluon Progator



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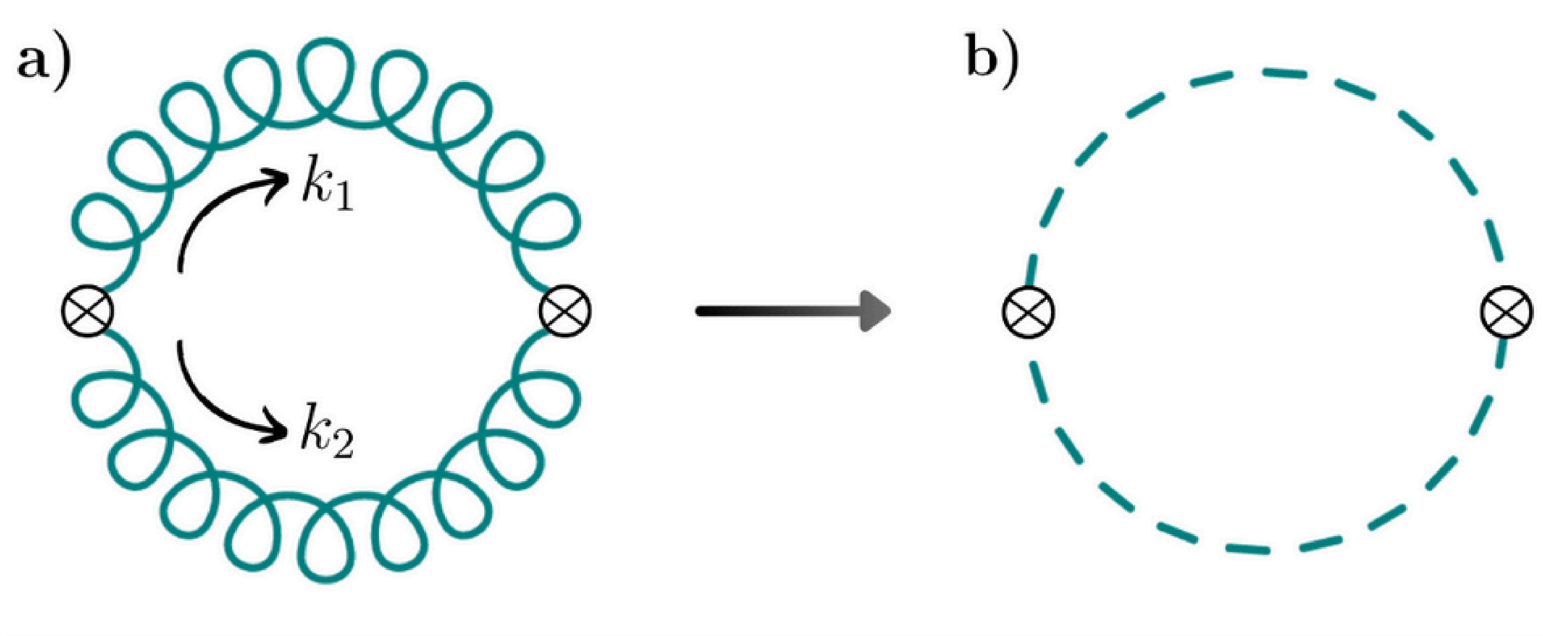
maximum

$$n_f \to n_f - \frac{33}{2}$$

Where the non-Abelianization procedure is necessary !



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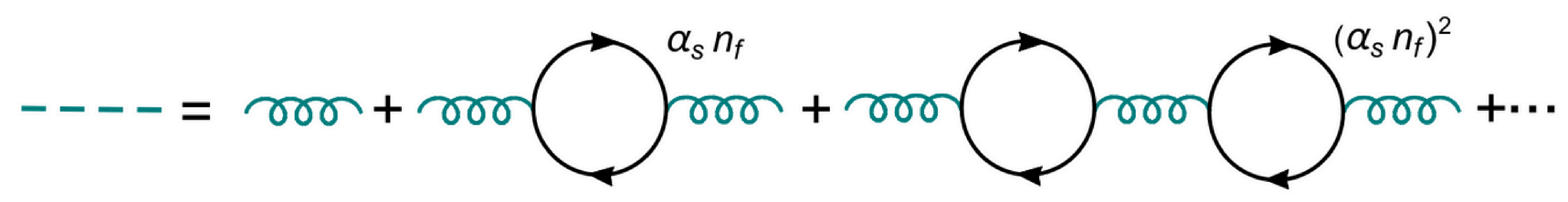

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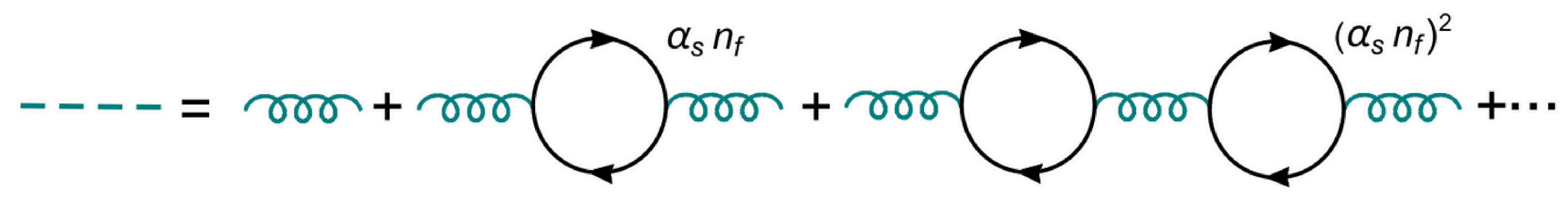


$$f(Q) = \sum_{n=0}^{\infty} c_n(Q,Q)\alpha_s^n(Q) \equiv \sum_{n=0}^{\infty} c_n \alpha_s^n$$

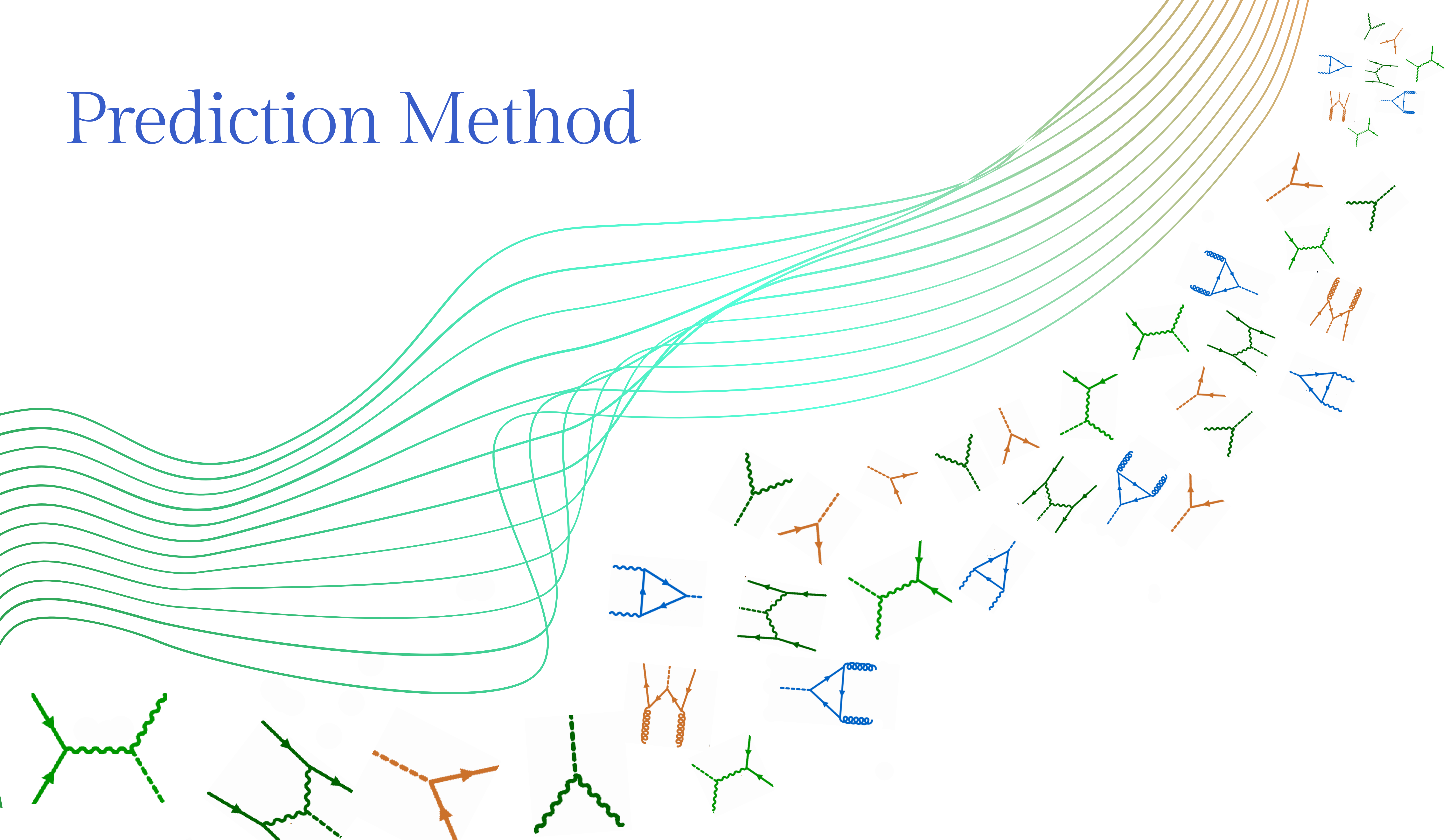
$$c_n \sim n!$$

Borel Transform

$$B[u] = \sum_{n=0}^{\infty} c_n \frac{u^n}{n!}$$

The Borel space singularities are known as *renormalons and govern the behavior of the series!*

Prediction Method



Padé Approximants (PA)

$$P_N^M(u) = \frac{a_0 + a_1 u + \cdots + a_M u^M}{1 + b_1 u + \cdots + b_M u^N}$$

Dlog Padé Approximants

$$\mathrm{Dlog}_N^M(u) = f(0)\exp\left[\int du\, \mathrm{P}_N^M(u)\right]$$

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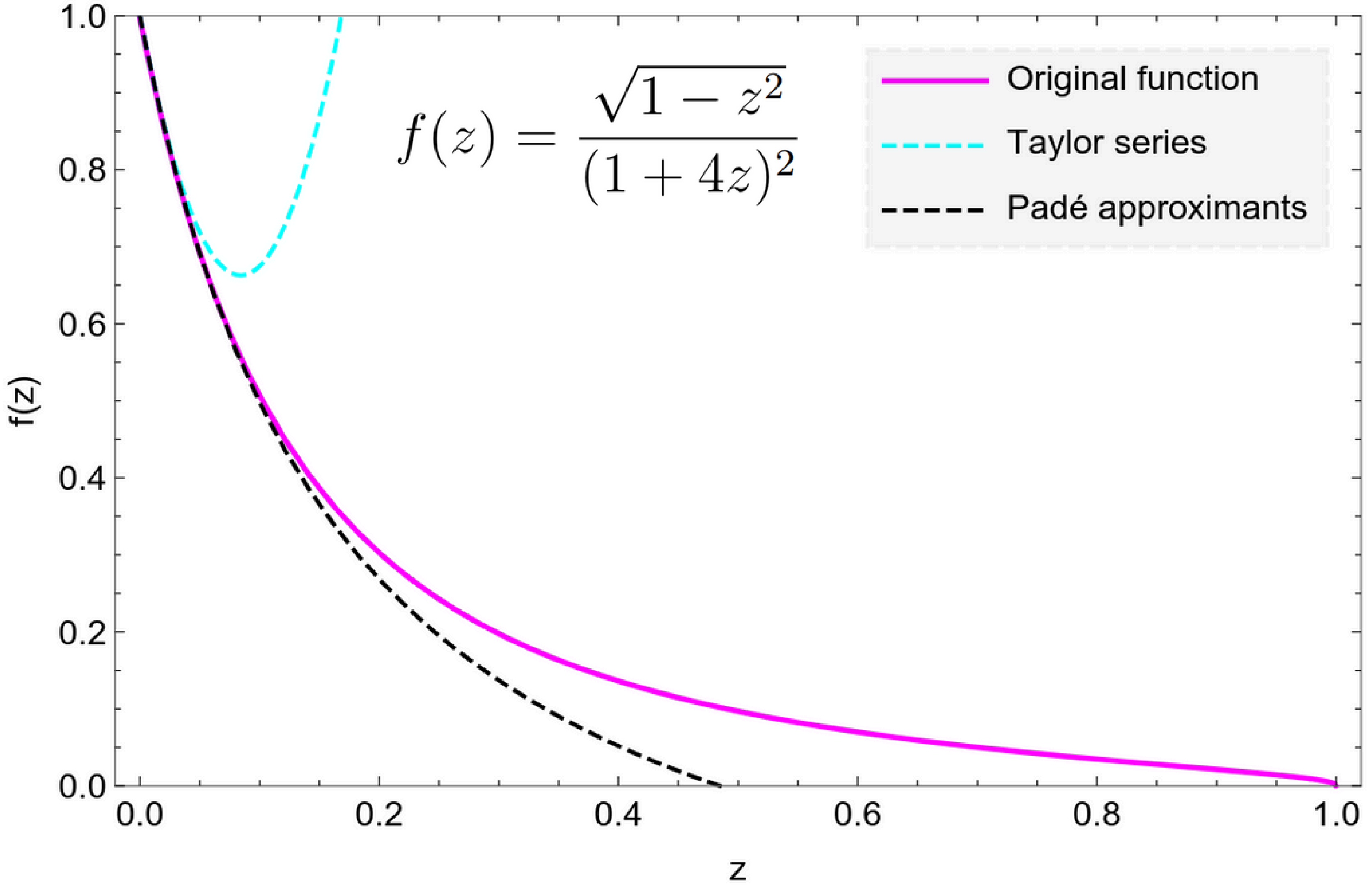
[BOITO, MASJUAN, OLIANI, 2018]



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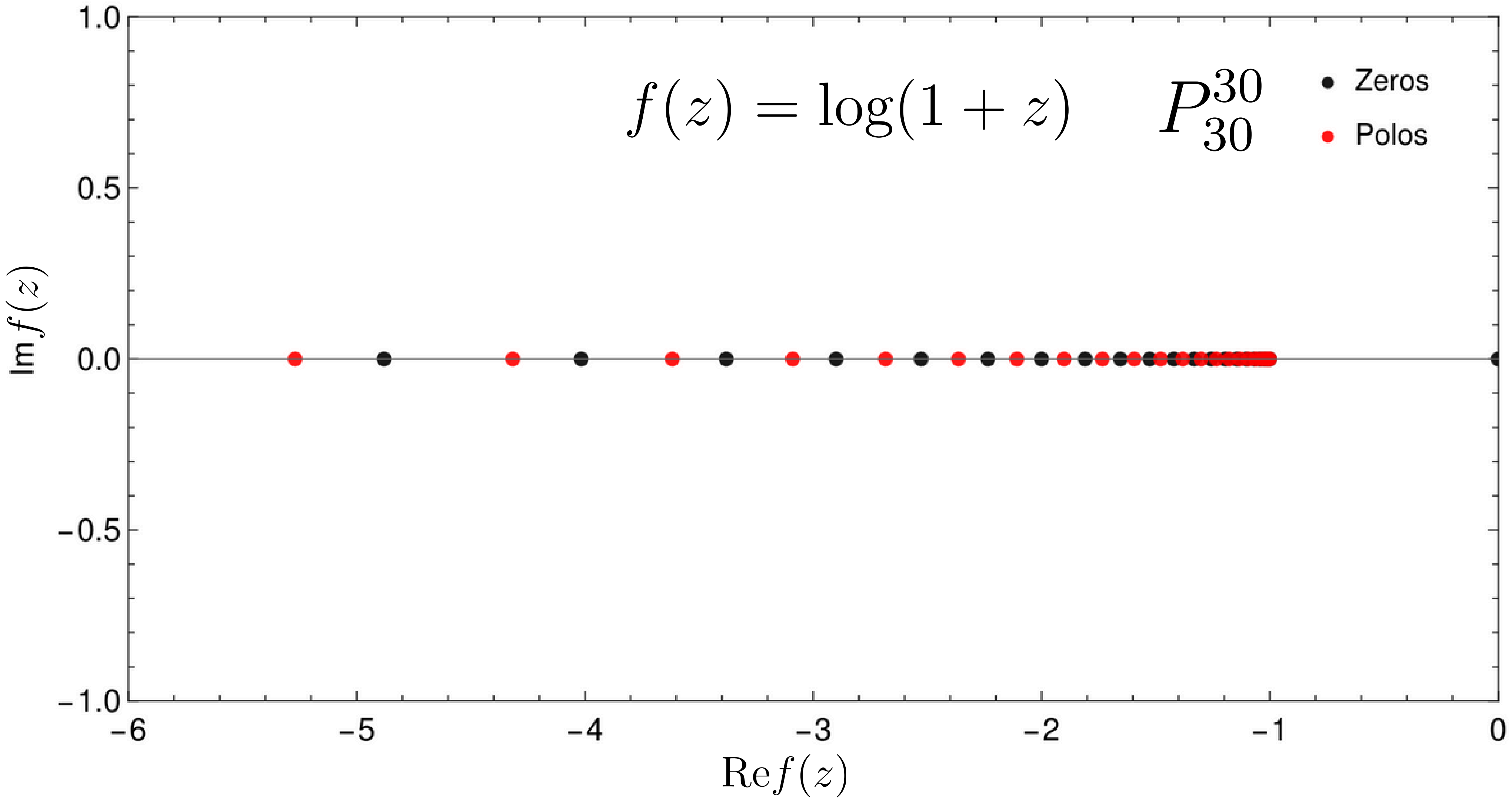


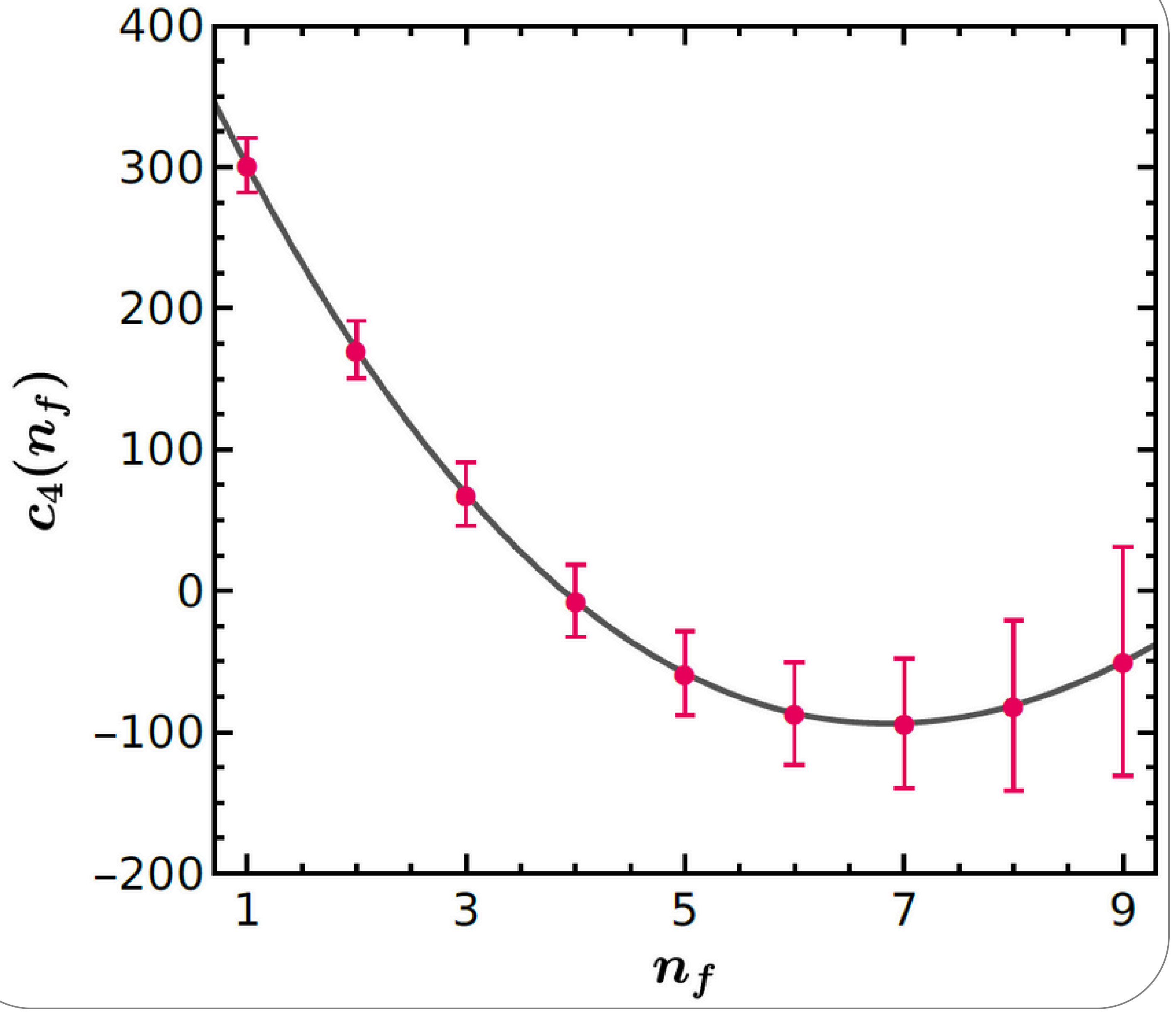
[ELLIS, JOHN R., KARLINER, M. . SAMUEL, M. A., 1996]

We will postdict the last known coefficient of $\Gamma(\mathrm{H} \to \mathrm{gg})$

$$c_4(n_f) = c_4^{(0)} + c_4^{(1)} n_f + c_4^{(2)} n_f^2 + c_4^{(3)} n_f^3 + c_4^{(4)} n_f^4$$

[HERZOG, ET AL. JHEP 08 (2017)]

Remains fixed!



- We applied the following approximants:

$$P_2^3(\alpha_s), P_3^2(\alpha_s), P_1^3(u), P_2^2(u), \mathrm{Dlog}_1^2(\alpha_s), \mathrm{Dlog}_1^2(u) \text{ and } \mathrm{Dlog}_2^1(u)$$

$$\Gamma^{n_f}_{h\to gg}(\mu_i) = \Gamma_0 K^{(n_f)}_{\mu_i}$$

$$K^{(1)}_{\mu_t} = 1 + 7.188498\alpha_s + 32.65167\alpha_s^2 + 112.015\alpha_s^3 + 298.873\alpha_s^4 + \cdots$$
$$K^{(3)}_{\mu_t} = 1 + 6.445775\alpha_s + 23.74728\alpha_s^2 + 56.0755\alpha_s^3 + 62.4363\alpha_s^4 + \cdots$$
$$\vdots$$

[HERZOG, ET AL. JHEP 08 (2017)]

We will postdict the last known coefficient of $\Gamma(H \to gg)$

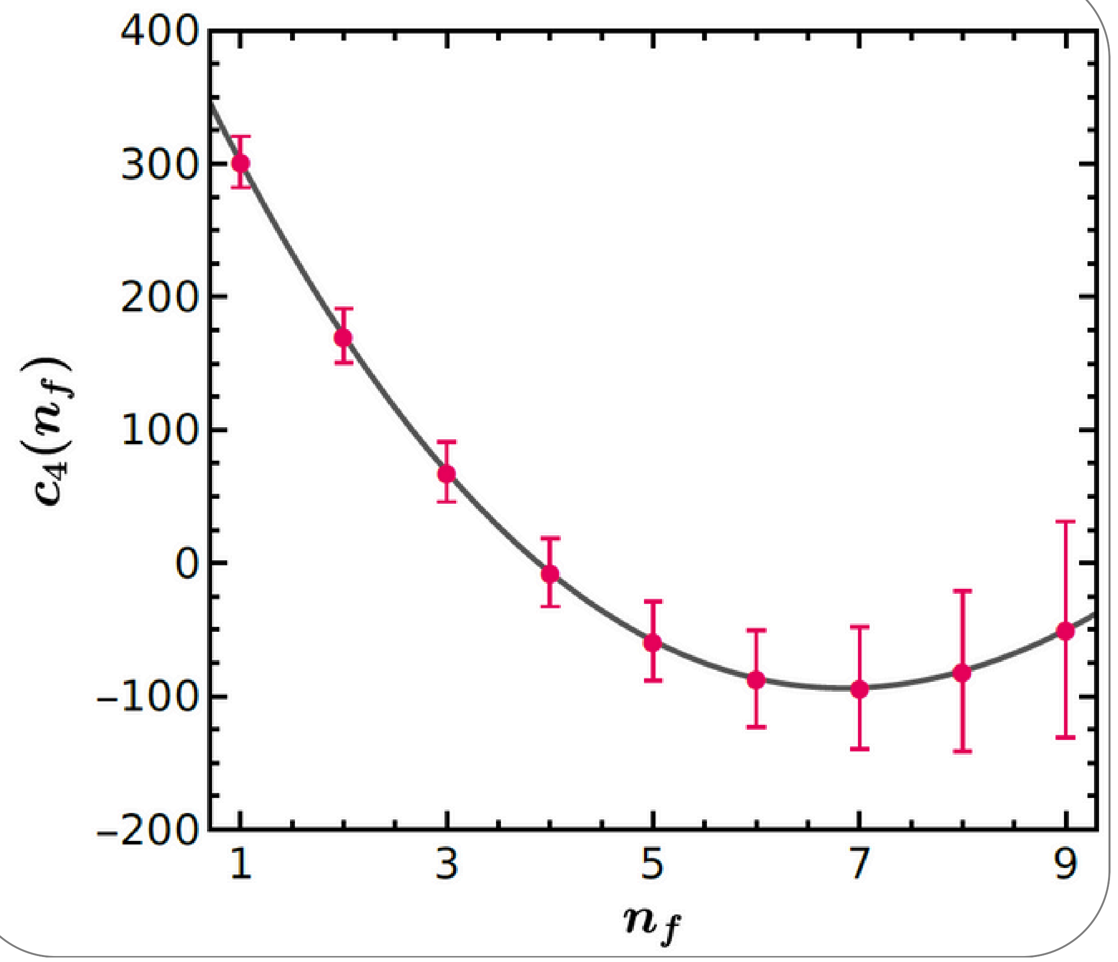
Agreement!

$$c_4(n_f) = 462.628 - 178.466\, n_f + 16.554\, n_f^2 - 0.440714\, n_f^3 + 0.00258089\, n_f^4$$

$$c_4(n_f) = (462 \pm 17) - (176 \pm 3) n_f + (16.1 \pm 0.9) n_f^2 - (0.3 \pm 0.1) n_f^3 + 0.00258089 n_f^4$$

- We applied the following approximants:

$P_2^3(\alpha_s), P_3^2(\alpha_s), P_1^3(u), P_2^2(u), \mathrm{Dlog}_1^2(\alpha_s), \mathrm{Dlog}_1^2(u)$ and $\mathrm{Dlog}_2^1(u)$



Preliminary Results

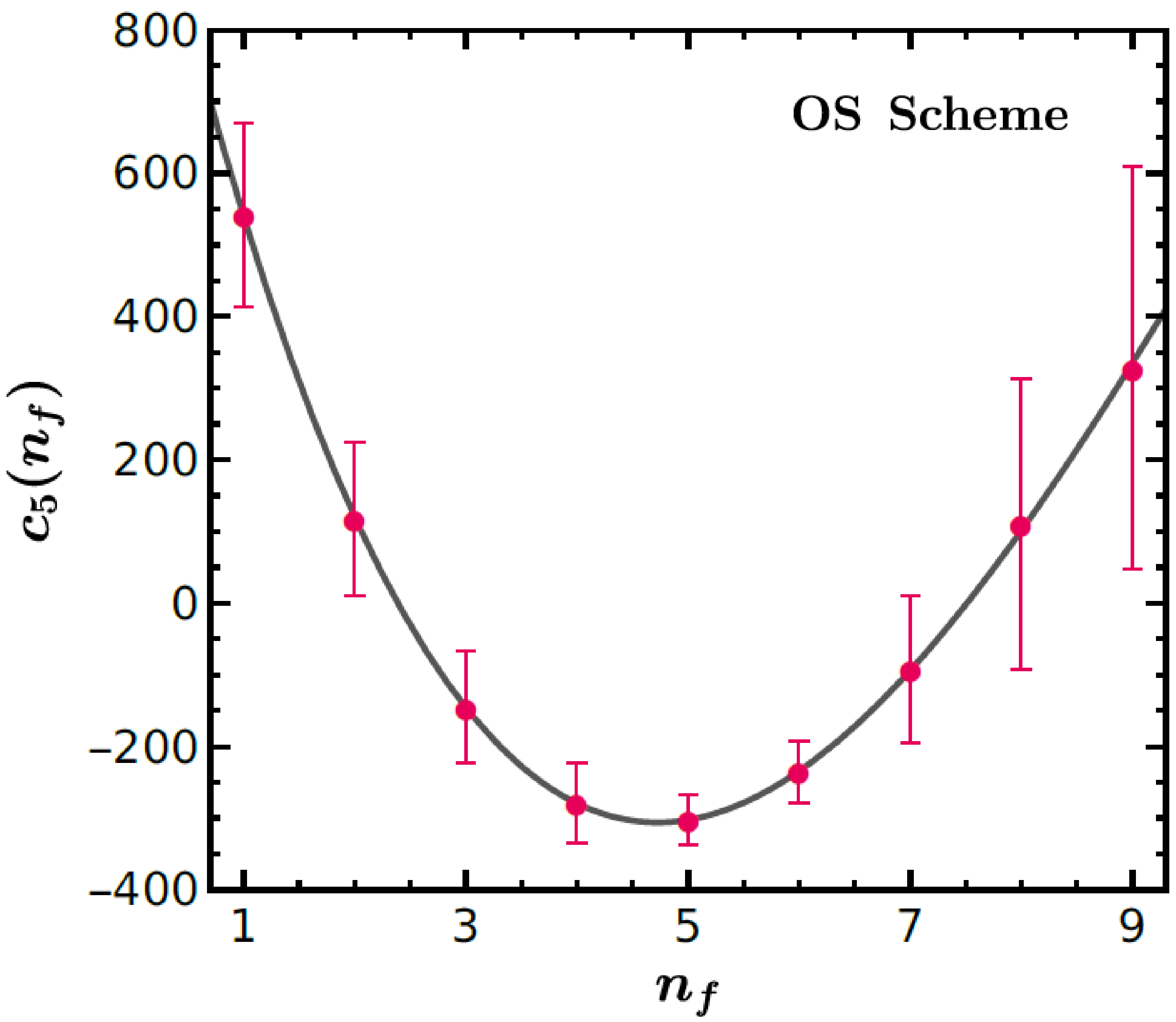
Results for H → gg (OS scheme)

$$\Gamma(H \to gg) = \frac{m_H^3 \alpha_s^2}{72\pi^3 v^2}\left[1 + 5.7031\,\alpha_s + 15.512\,\alpha_s^2 + 12.666\,\alpha_s^3 - 69.329\,\alpha_s^4 + c_5^{\mathrm{OS}}(5)\alpha_s^5 + \cdots\right]$$

$$c_5^{\mathrm{OS}}(5) = -302 \pm 35$$

Predicted coefficient

$$c_5^{\mathrm{OS}}(n_f) = (1137 \pm 121) - (694 \pm 116)\,n_f + (101 \pm 51)\,n_f^2 - (4 \pm 8)\,n_f^3 + (0.06 \pm 0.37)\,n_f^4 - 0.000282079\,n_f^5$$



Estimated decay width

$$\Gamma(H \to gg) = 0.3384 \pm (0.0063)_{\alpha_s} \pm (0.0009)_{m_H} \pm (0.0005)_{\mu} \pm (0.0001)_{c_5}\ \mathrm{MeV}$$

Results for $\mathrm{H} \to \mathrm{b}\bar{\mathrm{b}}$

$$\Gamma(H \to b\bar{b}) = \frac{N_c\, m_q^2}{8\pi}\, s \left[1 + 1.8038\,\alpha_s + 2.9532\,\alpha_s^2 + 1.3468\,\alpha_s^3 - 8.4771\,\alpha_s^4 + p_5(5)\,\alpha_s^5 + \cdots\right]$$

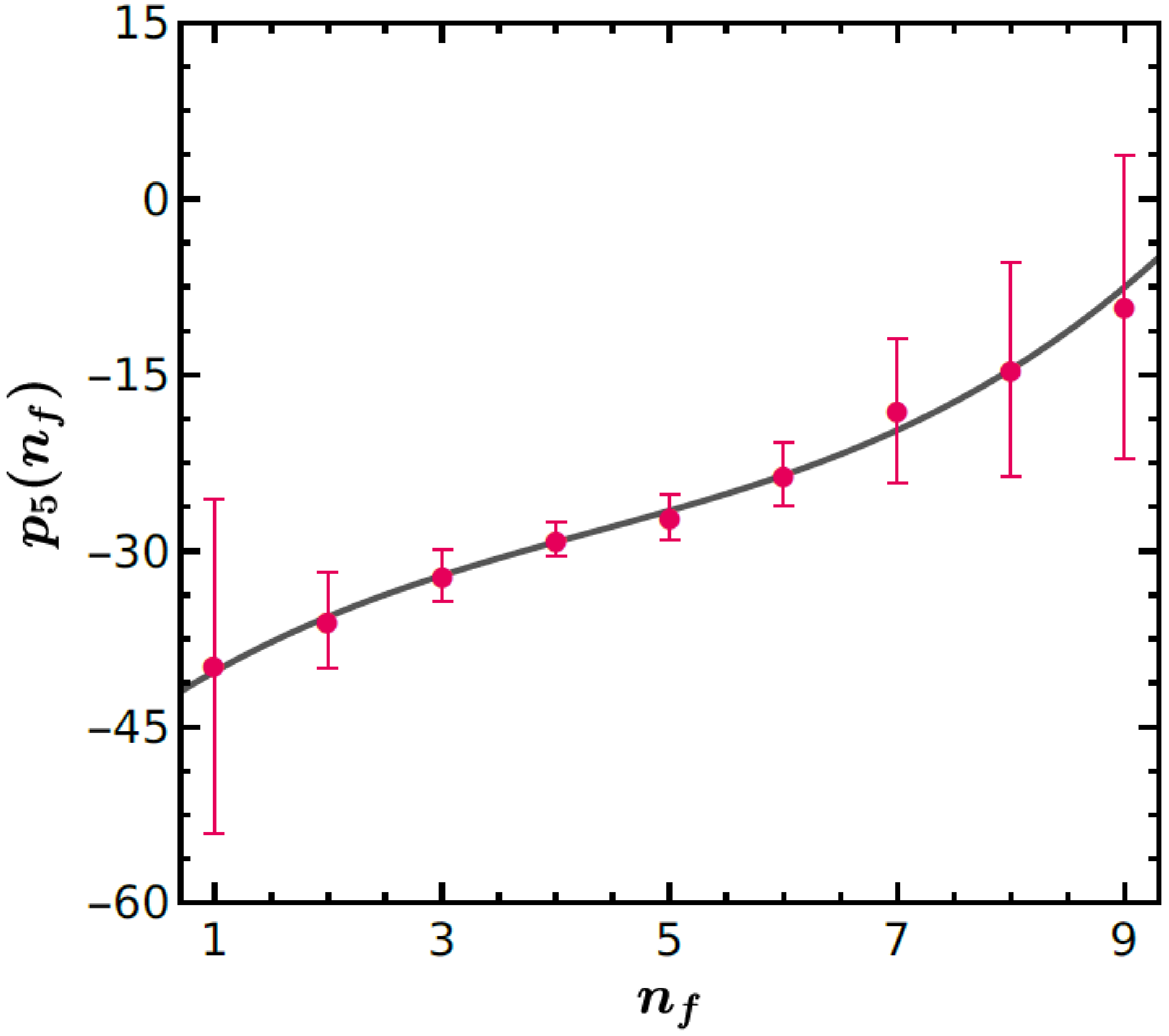
Predicted coefficient

$$p_5(n_f) = -(47 \pm 20) + (7 \pm 13)\, n_f - (1 \pm 3)\, n_f^2 + (0.08 \pm 0.18)\, n_f^3 + 7.09 \times 10^{-6}\, n_f^4$$

Estimated decay width

$$\Gamma(H \to b\bar{b}) = 2.38092 \pm (0.00981)_{m_b} \pm (0.00385)_{\alpha_s} \pm (0.00209)_{m_H} \pm (0.00032)_{\mu} \pm (0.00007)_{p_5} \ \ \mathrm{MeV}$$

$p_5(5) = -27 \pm 2$



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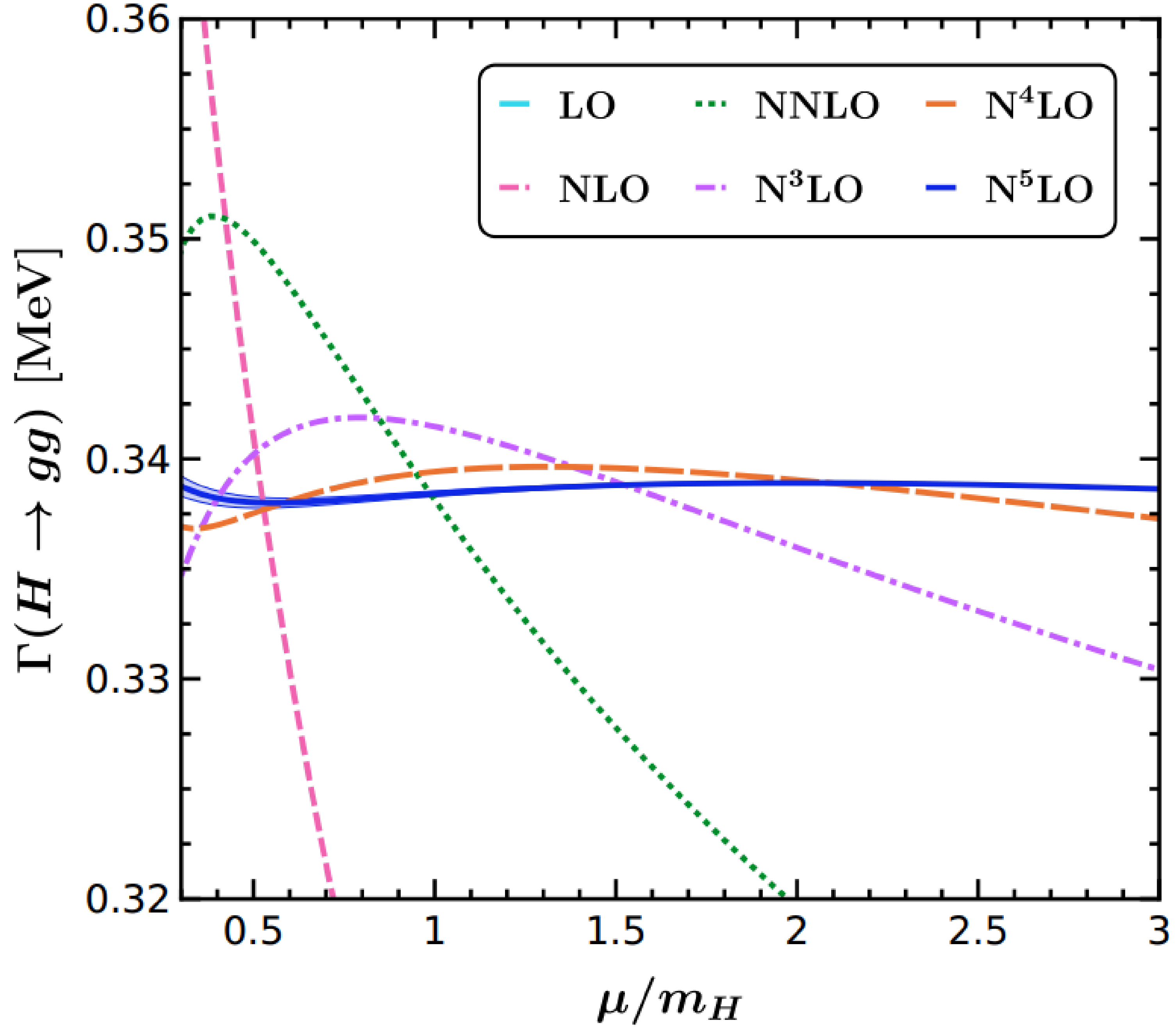

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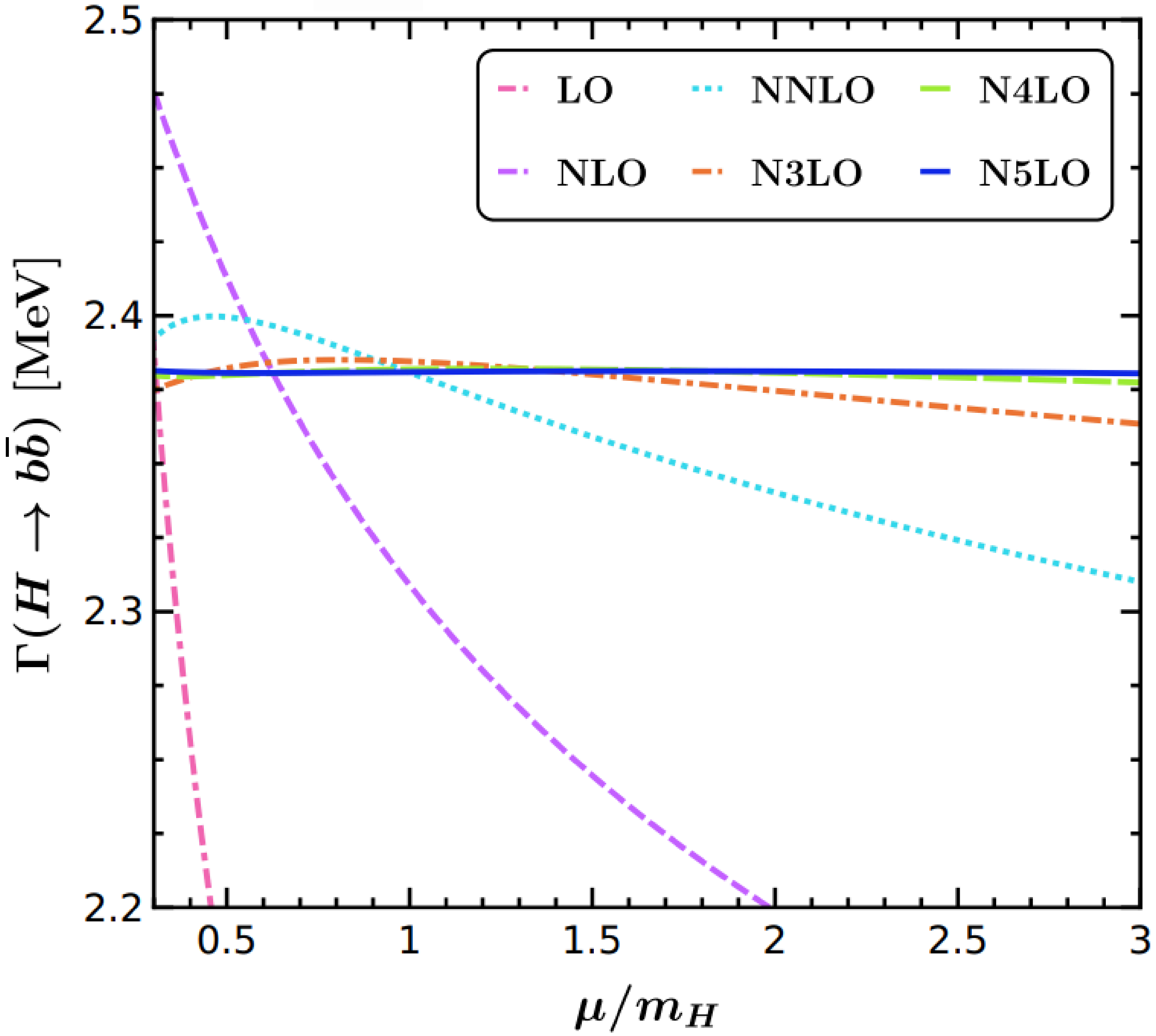

$$\Gamma(H \to gg) = 0.3384 \pm (0.0063)_{\alpha_s} \pm (0.0009)_{m_H} \pm (0.0005)_{\mu} \pm (0.0001)_{c_5} \ \text{MeV}$$

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Conclusion

- We firstly determine the $\Gamma_{H\to gg}$ in the large $-\,\beta_0$ limit

- Model independent method to obtain higher-order coefficients as a function of the number of flavors

  Full QCD Results ($n_f = 5$)

- We performed estimates of the first unknown coefficients of $\Gamma_{H\to gg}$ and $\Gamma_{H\to b\bar{b}}$

$$c_5^{OS}(5) = -302 \pm 35 \qquad p_5(5) = -27 \pm 2$$

- Limiting factors in the precision of decay rates are the uncertainty in the quark masses, Higgs mass and α_s

Thank you!

Backup Slides

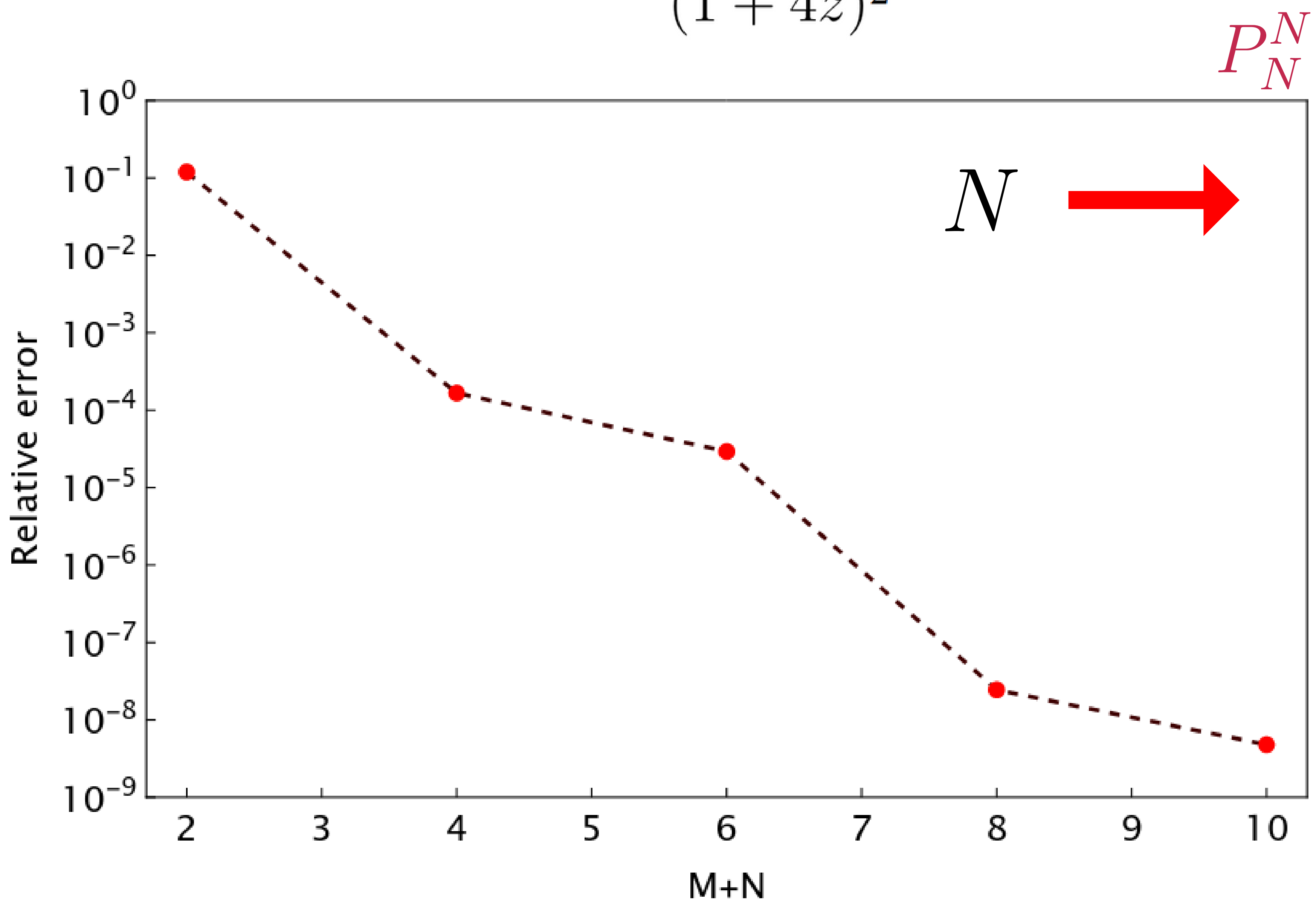
Padé Approximant (PA)

$$P_N^M(u) = \frac{a_0 + a_1 u + \cdots + a_M u^M}{1 + b_1 u + \cdots + b_M u^N}$$

The quantitative way to analyze our estimates is the relative error:

$$\sigma_{\rm rel} = \left| \frac{c_n^P - c_n}{c_n} \right|$$

$$f(z) = \frac{\sqrt{1-z^2}}{(1+4z)^2}$$



- Pomerenke's Theorem: Let $f(z)$ be an analytic function

$$\lim_{N\to\infty} P_N^{\lambda N} = f(z)$$

where $\lambda \neq 0$ and $\lambda \neq \infty$.

- Froissart doublets: This behavior arises due to the numerator and denominator zeros being very close together, which effectively acts as an order reduction.

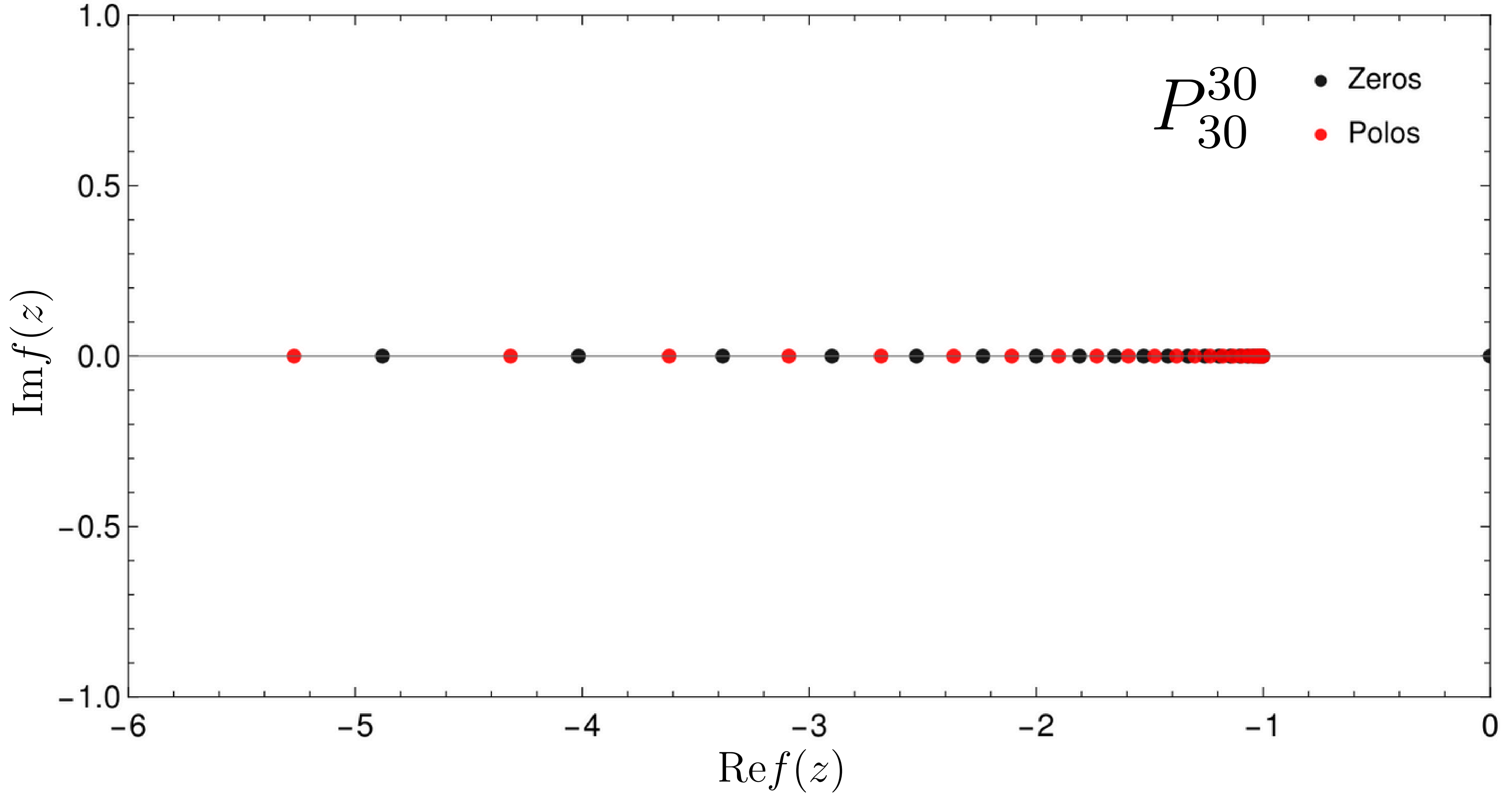
Dlog Padé Approximants

Taking into account a function with cuts:

$$f(u) = \frac{A(u)}{(p-u)^{\gamma}} + B(u)$$

Padé approximants do not reproduce functions with branch cuts very well!

$f(z) = \log(1+z)$



Dlog Approximant

$$F(u) = \frac{d}{du}\log[f(u)] \sim \frac{\gamma}{p-u}$$

$$\mathrm{Dlog}^{M}_{N} = f(0)\exp\left[\int du\, P^{M}_{N}(u)\right]$$

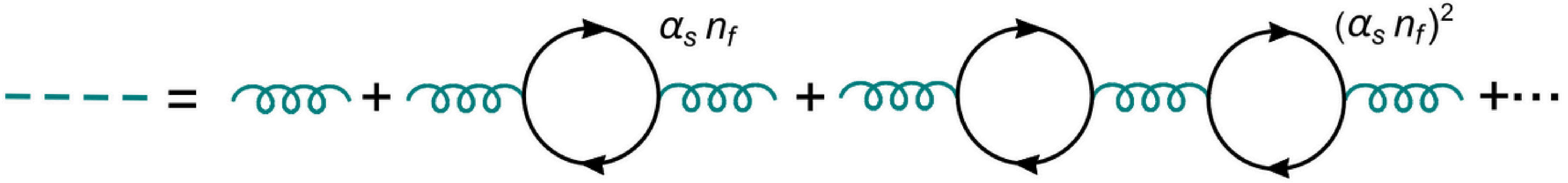
The large-β_0 limit

$$n_f \to n_f - \frac{33}{2}$$

Taking into account a usual quantity in perturbation theory:

$$f(\alpha_s) = 1 + \sum_{n=0}^{\infty}\sum_{k=0}^{n} r_{n,k} n_f^k \alpha_s^{n+1}$$

maximum

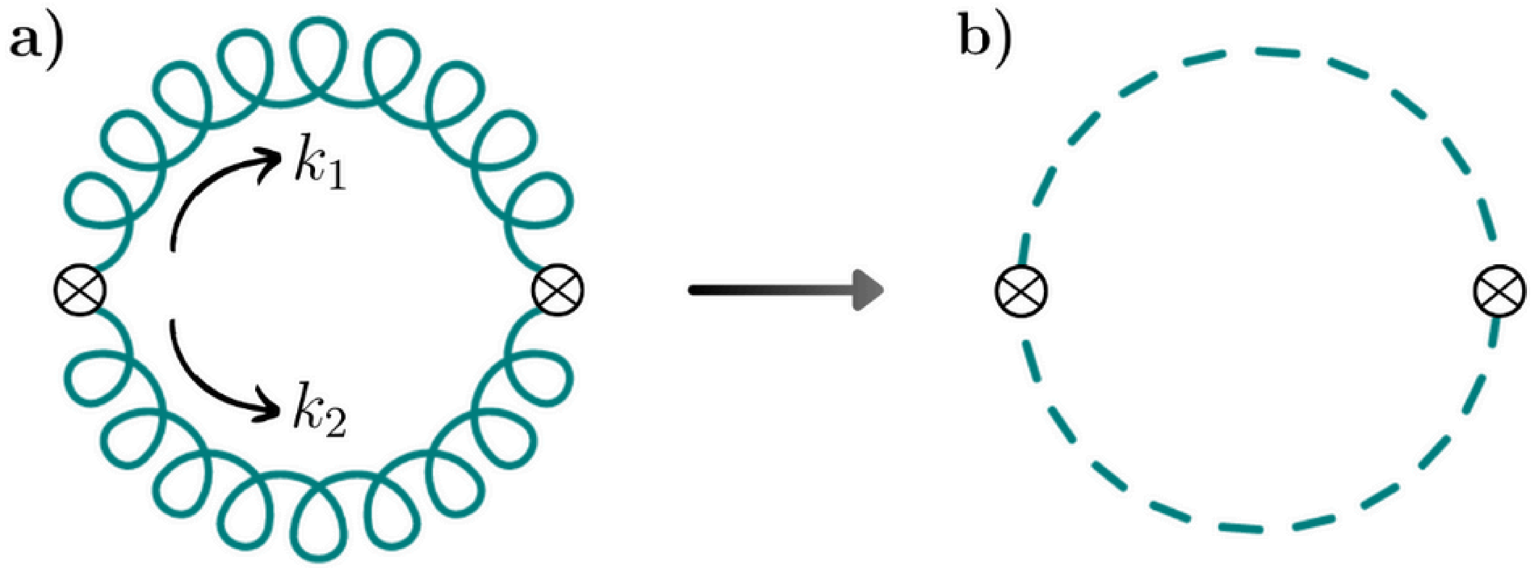


large-β_0 limit: where the perturbative series is known in all orders (we use it as a laboratory)

Where the non-Abelianization procedure is necessary !

$$D_{\mu\nu}^{ab}(k^2) = -\frac{i\,\delta^{ab}}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{1+\Pi_1(k^2)} + (-i)\xi\frac{k_\mu k_\nu}{k^4}$$

$$\frac{-g_{\mu\nu}}{k^2+i\eta} \to (-\mu^2 e^{-C})\frac{-g_{\mu\nu}}{(k^2+i\eta)^{1+u}}$$



Borel transform

In cases where the perturbative regime is valid

$$f(Q) = \sum_{n=0}^{\infty} c_n(Q,Q)\alpha_s^n(Q) \equiv \sum_{n=0}^{\infty} c_n \alpha_s^n$$

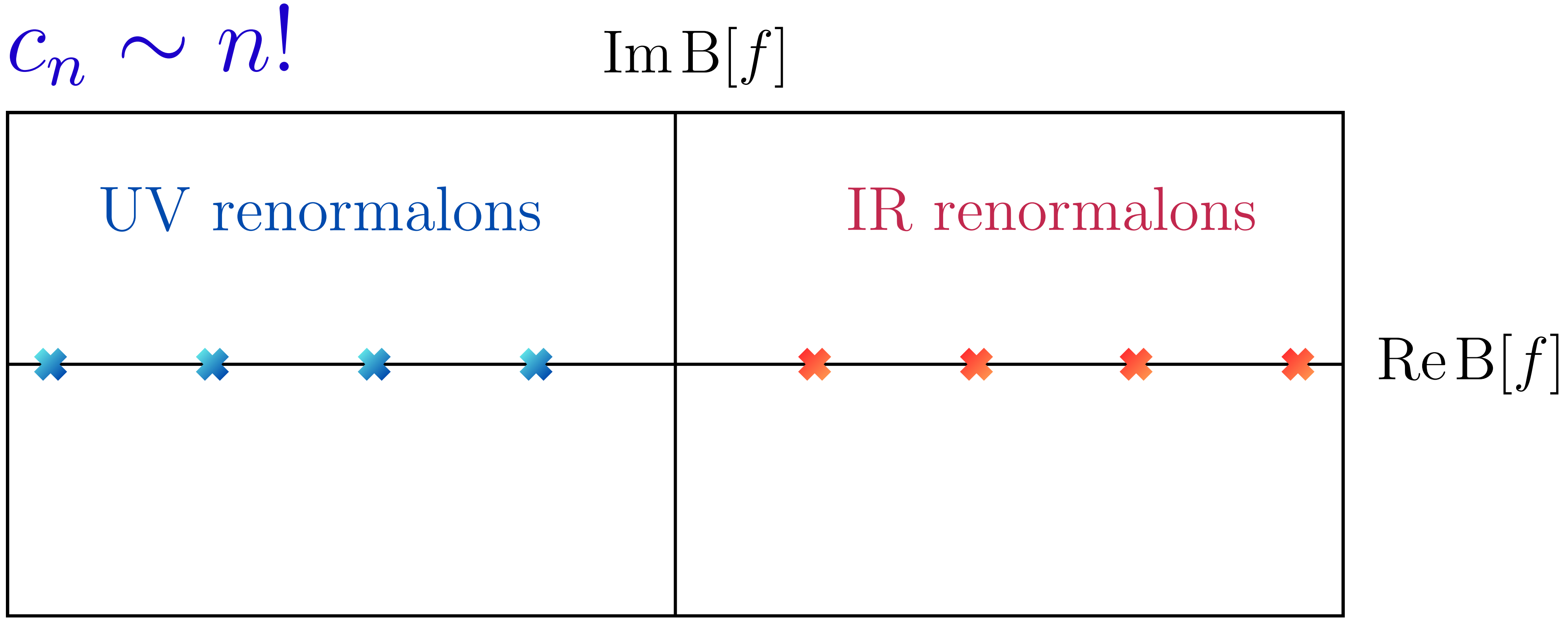
Borel Transform

$$B[f] = f(0)\delta(t) + \sum_{n=0}^{\infty} c_n \frac{t^n}{n!}$$

The Borel space singularities are known as *renormalons!*

$$\tilde{f}(\alpha_s) = \left(\frac{1}{\beta_0}\right) \int_0^{\infty} du\, e^{-u/\beta_0 \alpha_s} B[f](u)$$

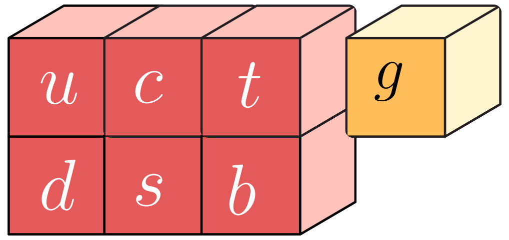
$c_n \sim n!$



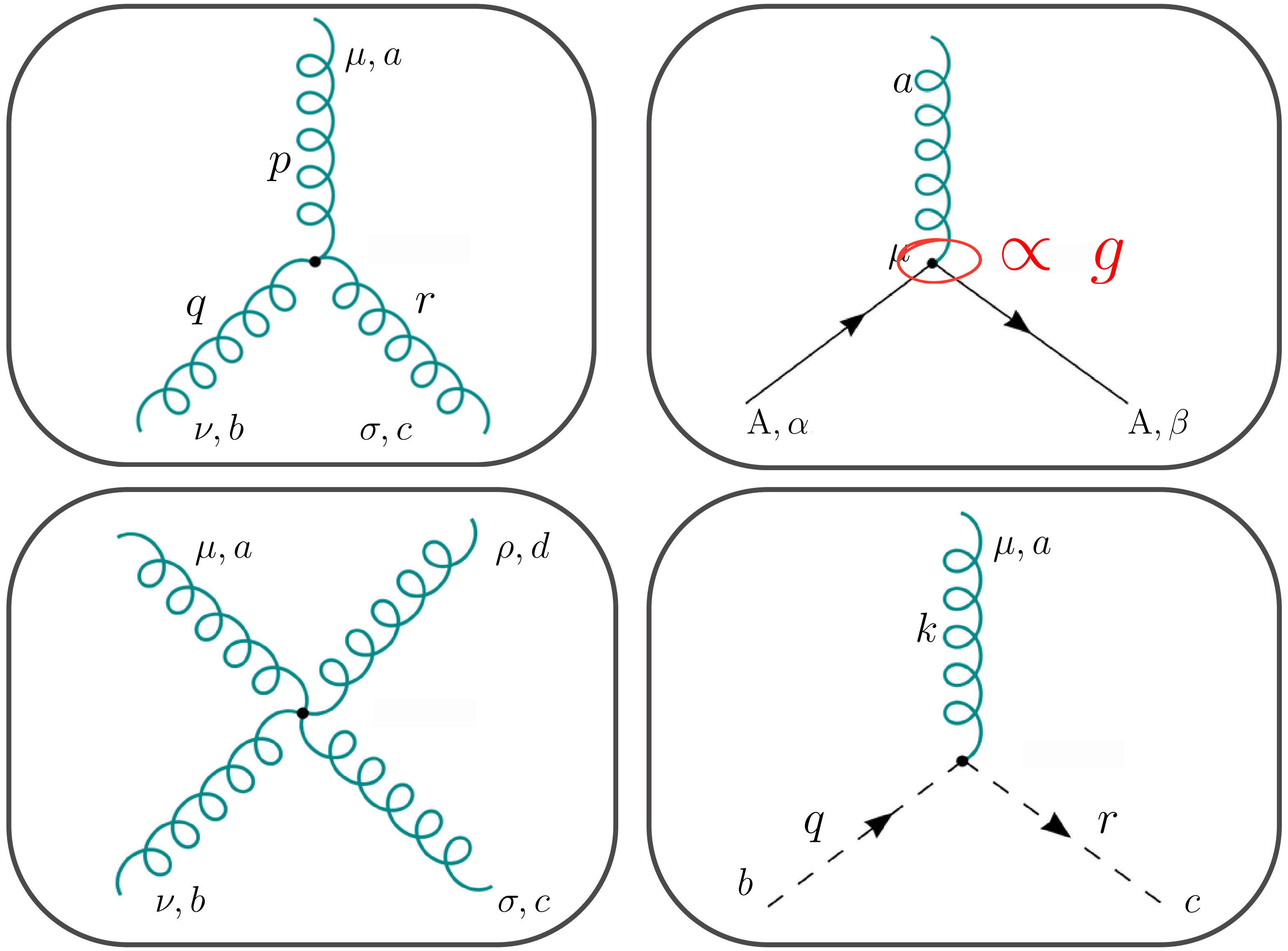
- If the coefficients have alternating signs, the singularities appear in the negative part of the axis (UV singularities)
- In the case where the coefficients have a fixed sign, the irregularities appear in the real part of the axis (IR renormalons)

Quantum Chromodynamics

$\alpha_s = g^2/4\pi$



Fundamental interactions of QCD



- Beta function **five loops**

$$\beta(a_s) = -\mu \frac{da_s}{d\mu} \equiv \sum_{n=0}^{\infty} \beta_n a_s^{n+2}$$

- Gamma function

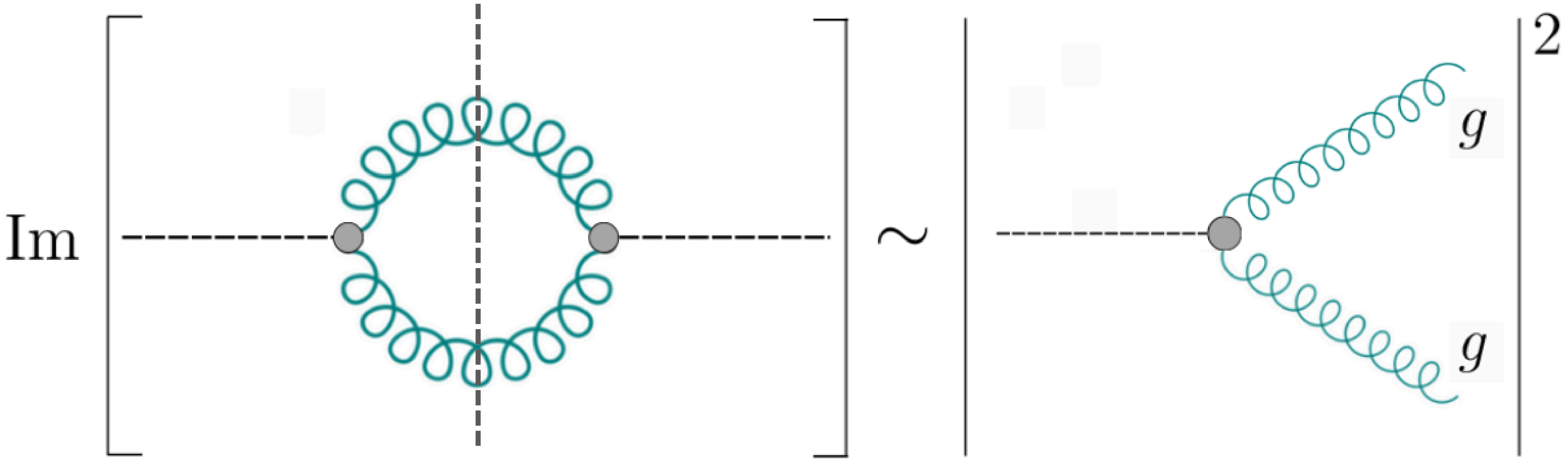
$$\gamma(a_s) \equiv -\frac{\mu}{m}\frac{dm}{d\mu} = \sum_{n=1}^{\infty} \gamma_n a_s^n$$

Renormalization Group Equation !

- Relation between strong coupling with n_f and $n_l + 1$ active flavours

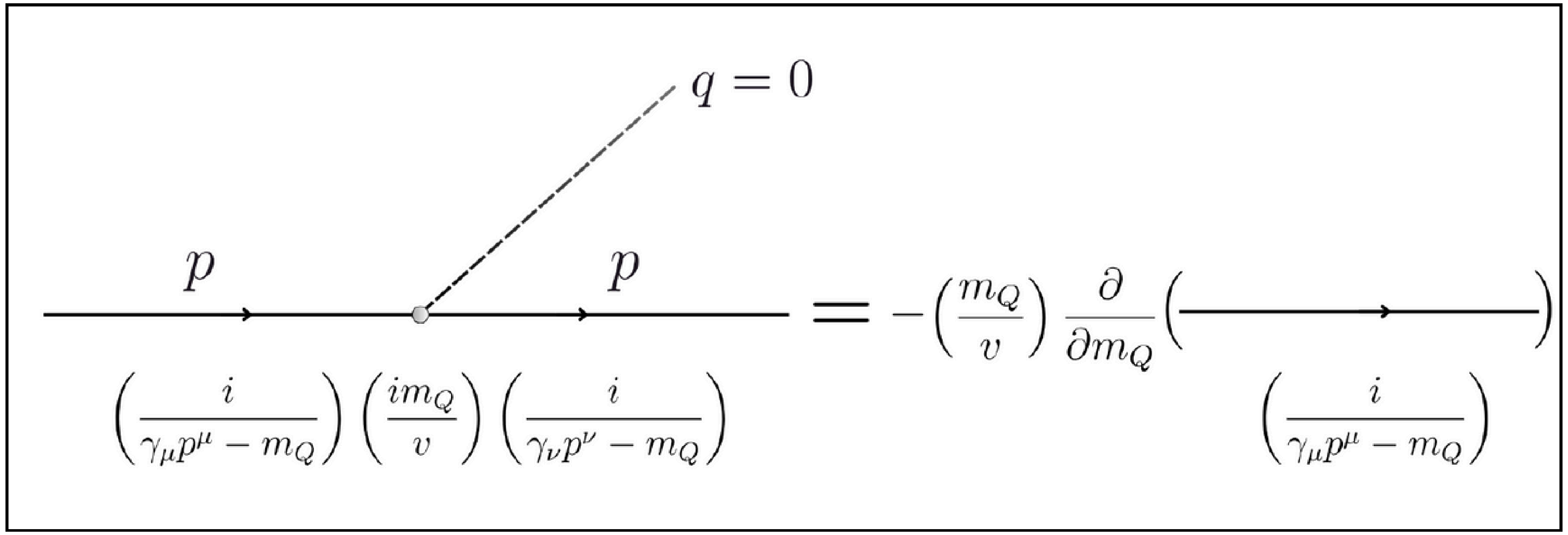
$$\alpha_s^{(n_l)}(\mu) = \zeta_g^2(\mu, \alpha_s^{(n_f)}, m)\alpha_s^{(n_f)}(\mu)$$

Optical theorem

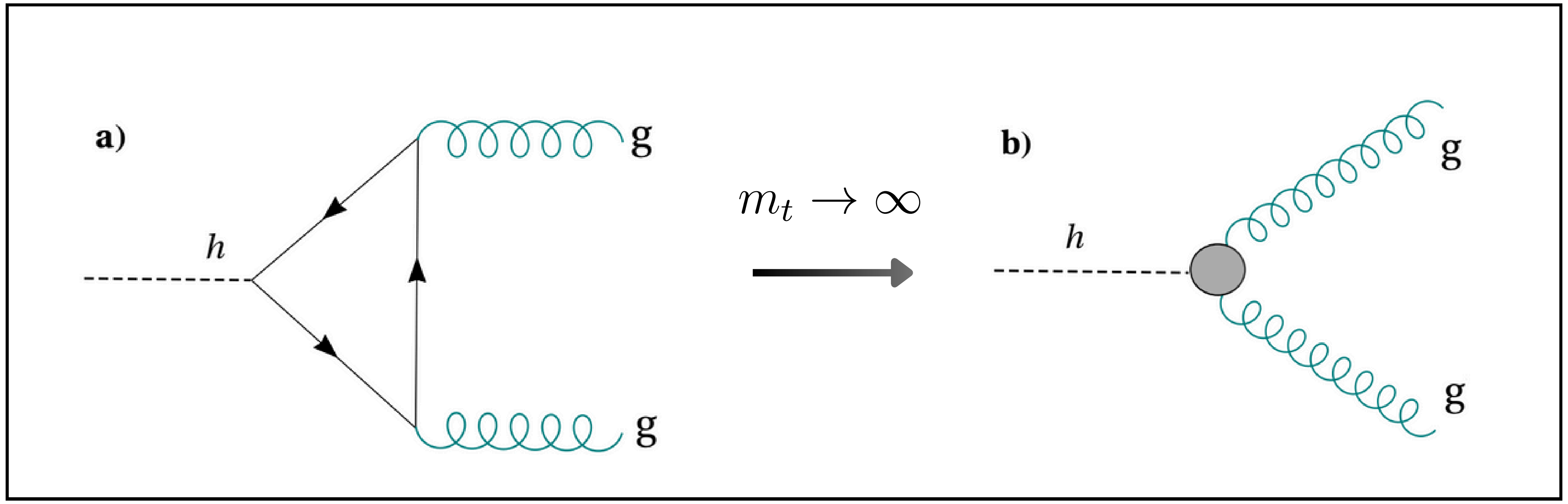


$$-2\mathrm{Im}\left(\overline{\sum_n}\mathcal{M}_{i\to i}\right) = 2\sqrt{k^2}\sum_n \Gamma(a\to n) = 2\sqrt{k^2}\,\Gamma(a\to all)$$

Low energy Theorem



$$\lim_{q\to 0}\mathcal{M}(X\to Y+h)=\frac{m_Q^0}{v_0}\frac{\partial}{\partial m_Q^0}\mathcal{M}(X\to Y)$$



$$C_1=-\frac{1}{2}m_t^2\frac{\partial}{\partial m_t^2}\ln\zeta_g^2(\mu,\alpha_s^{(n_f)},m_t)$$