Charged Lepton Flavour Violating Meson Decays in Seesaw Models

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Dark Matter and Neutrinos
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Neutrino Mass

- **Neutrino Oscillations $\Longrightarrow$ Non-zero neutrino masses.**

$$\mathcal{L}_Y = Y_\nu \, \bar{L}_L \, \tilde{H} \, \nu_R + h.c. \xrightarrow{\text{EW-SSB}} -(m_\nu \bar{\nu}_L \nu_R + h.c)$$

$$\boxed{m_\nu = \frac{Y_\nu \, v}{\sqrt{2}}}$$

- This requires an extreme fine-tuning, $\mathcal{O}\left(10^{-12}\right)$, of Y_ν to generate masses of sub-eV range. **Unnatural ! (S Pascoli Lectures) !**

- **To avoid this extreme fine-tuning, many neutrino mass mechanism has been proposed**
- **Of these, see-saw models are the most popular !**

Purpose of the work

Neutrino Oscillation → **Lepton flavour violation In neutrino sector**

- This necessarily leads to **Charged lepton flavour violation (CLFV)**

$$l_\alpha \xrightarrow{W} \nu_\alpha \xrightarrow{\text{oscillation}} \nu_\beta \xrightarrow{W} l_\beta$$

- Amount of **CLFV** depends on the details of the neutrino mass model

- In this work, we establish relations between **Branching Ratios of radiative CLFV and meson CLFV decays in seesaw models.**

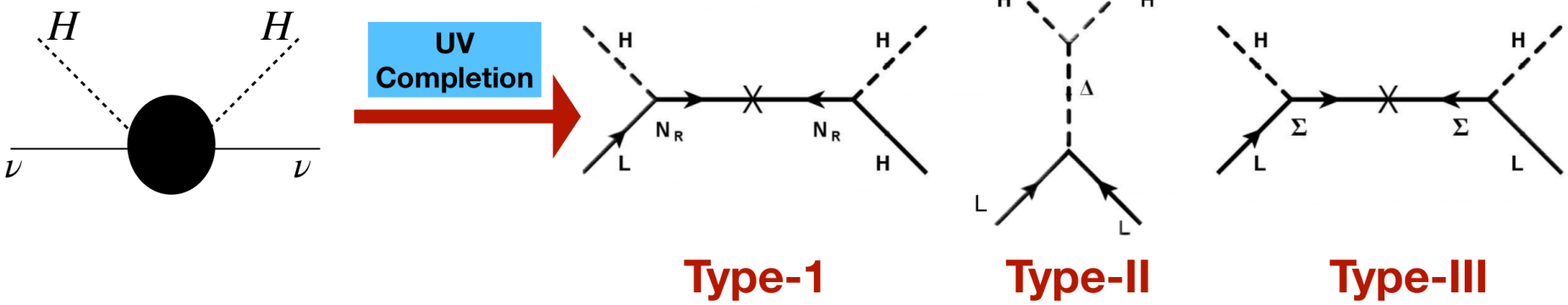
Seesaw Mechanism

What is the origin of tiny neutrino mass ?

- **Dimension-5 operator in SM fields → Neutrino mass**

$$\mathscr{L}_5 = \frac{g}{\Lambda}(L^T\sigma_2 H)\, C^\dagger\, (H^T\sigma_2 L) \xrightarrow{\text{EW -SSB}} \frac{gv^2}{2\Lambda}\nu_L^T C^\dagger \nu_L + \text{H.c.}$$

(Weinberg, PRD 22 (1980) 1694)



Seesaw Description : Type-I $N_R(1,0)$

Minkowski, 1977 Yanagida, 1979

Gell-Mann, Ramond, Slansky, 1979 Mohapatra, Senjanovic, 1980

$$\mathcal{L}_{\text{Type}-\text{I}} = -\overline{L}\, Y_D\, \widetilde{H}\, N_R \; - \; \frac{1}{2}\overline{N_R^c}\, M_R\, N_R + \text{ h.c.}$$

$$M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \text{ with } M_D = \frac{Y_D v_0}{\sqrt{2}}.$$

The mass matrix M can be diagonalised in two steps:

1. $$\mathbf{W^T} \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \mathbf{W} = \begin{pmatrix} m_\nu & 0 \\ 0 & M_N \end{pmatrix} = \mathbf{M_l},$$

2. $$\mathbf{U_l^T M_l U_l} = \begin{pmatrix} m_\nu^{diag} & 0 \\ 0 & M_N^{diag} \end{pmatrix} = \mathbf{diag(m_1, m_2, m_3, M_1, \ldots M_k)}.$$

Light-Heavy mixing

$$\mathbf{W} \equiv \begin{pmatrix} \sqrt{\mathbb{I} - RR^\dagger} & R \\ -R^\dagger & \sqrt{\mathbb{I} - R^\dagger R} \end{pmatrix} \simeq \begin{pmatrix} \mathbb{I} - \frac{1}{2} RR^\dagger & R \\ -R^\dagger & \mathbb{I} - \frac{1}{2} R^\dagger R \end{pmatrix}$$

W. Grimus and L. Lavoura (2000)

where $R = M_D^* M_R^{-1}$.

$$\mathbf{U_l} = \begin{pmatrix} U_\nu & 0 \\ 0 & U_N \end{pmatrix}$$

The complete diagonalising matrix

$$\mathbf{U} = \begin{pmatrix} U_{\nu\nu} & U_{\nu N} \\ U_{N\nu} & U_{NN} \end{pmatrix} = \begin{pmatrix} (\mathbb{I} - \frac{1}{2} RR^\dagger)\, U_\nu & R\, U_N \\ -R^\dagger U_\nu & (\mathbb{I} - \frac{1}{2} R^\dagger R)\, U_N \end{pmatrix}$$

$$\boxed{U_{PMNS} = \left(\mathbb{I} - \frac{1}{2} RR^\dagger\right) U_\nu = (\mathbb{I} + \eta)\, U_\nu} \implies \text{Non-Unitary}$$

We can take U_N to be Identity $\implies$ M_R is diagonal.

Radiative $\mu \to e\gamma$ Decay

$$\nu_\alpha = [U_\nu(1+\eta)]_{\alpha i}\nu_i + R_{\alpha K}N_K]$$

Active flavour

Light-heavy mixing in Type-I and Type-III

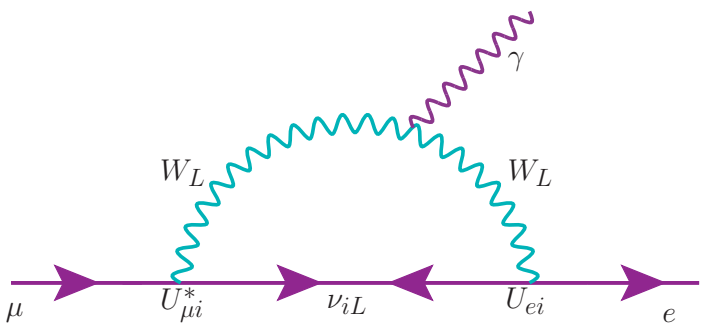
- **Charged current**

$$\mathscr{L}^{\nu}_{cc} = -\frac{g}{2\sqrt{2}}\bar{l}_\alpha\gamma_\mu(1-\gamma_5)\,[(1+\eta)U_\nu]_{\alpha i}\,\nu_i W^\mu + h.c.$$

$$\mathscr{L}^{N}_{cc} = -\frac{g}{2\sqrt{2}}\bar{l}_\alpha\gamma_\mu(1-\gamma_5)\,R_{\alpha K}\,N_K W^\mu + h.c$$

- Flavor changing **neutral current in Type-III**

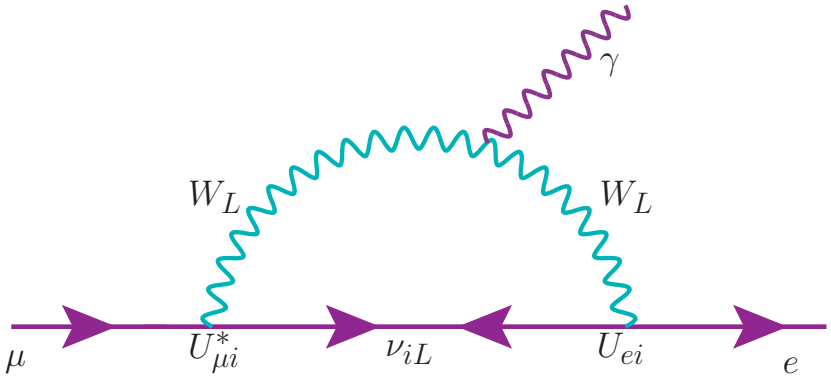
$$\mathscr{L}^{l}_{FCNC} = -\frac{g}{2\cos\Theta_W}(R_{\alpha K}R^{\dagger}_{K\beta})\bar{l}_\alpha\gamma_\mu(1-\gamma_5)l_\beta Z^\mu + h.c.$$

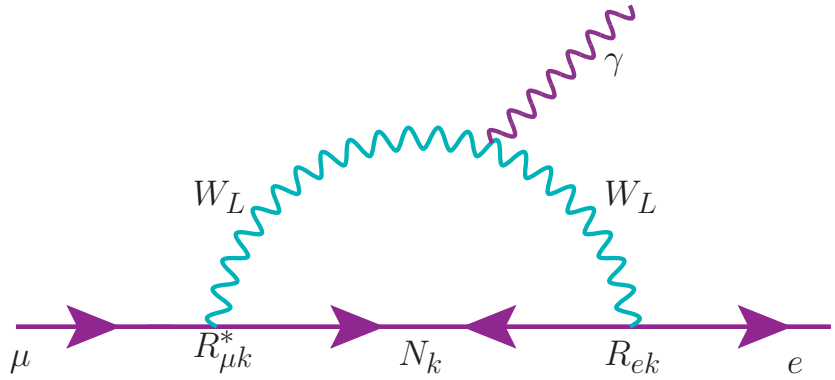


Petcov,77 Marciano, Sanda, 77

$$BR(\mu \to e+\gamma) = \frac{3\alpha_{\rm em}}{32\pi}\left|\frac{\Delta m^2_{\rm sol}}{M_W^2}U_{e2}U^*_{\mu 2} + \frac{\Delta m^2_{\rm atm}}{M_W^2}U_{e3}U^*_{\mu 3}\right|^2 < 10^{-54}$$

Strong GIM suppression!





Observable effects!!

Now the GIM cancellation is not exact due to the non-unitary nature of PMNS matrix.

$$Br\left(\mu \to e\,\gamma\right) = \frac{3\alpha_{em}}{32\pi}\left|T\right|^2.$$

$$T = \sum_{j=1}^{3}\left[(1+\eta)U_\nu\right]_{ej}\left[(1+\eta)U_\nu\right]^*_{\mu j} F\left(\frac{m_j^2}{M_W^2}\right) + \sum_k R_{ek}R^*_{\mu k} F\left(\frac{M_k^2}{M_W^2}\right).$$

Ibarra, Molinaro, and Petcov (2011)

Light-heavy mixing:Constraint on R (Theoretical)

- **Casas Ibarra** parameterisation (Casas, Ibarra (2001))

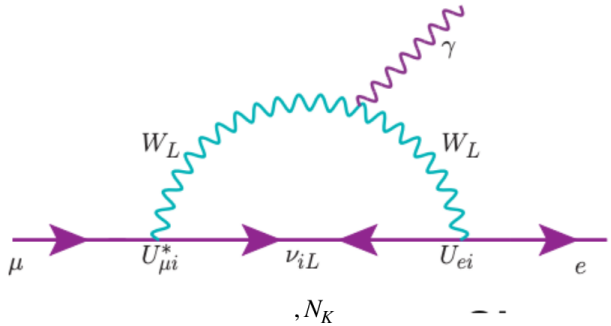
$$R = M_D^* M_R^{-1} = \sqrt{m_\nu^{dia}}\, U O^* \sqrt{M_R^{-1}}$$

- There is a lot of freedom in parameterizing the unphyscial matrix O
- Due to this freedom, **the light-heavy mixing matrix will only have a mild dependence on** M_R
- The strongest constraints on R come from experiments

Constraint on R from $\mu \to e\gamma$

- For **K=2, nearly degenerate** case : **Type-I**

$$M_2 = M_1(1+Z),\ Z << 1$$



$$B\,.\,R\,.(\mu \to e\gamma) \approx \frac{3\alpha}{32\pi}\left(\frac{2+Z}{1+Z}\right)^2 |R_{e1}R^*_{\mu 1}|^2 [F(X_1) - F(0)]^2$$

(Ibarra, Molinaro, Petcov(2011))

with

$$F(X_K) = \frac{10 - 43X_K + 78X_K^2 - 49X_K^3 + 4X_K^4 + 18X_K^3 \log X_K}{3(X_K - 1)^4}\ ; \qquad X_K = \left(\frac{M_K}{M_W}\right)^2$$

Bounds on $|R_{ek}R^{\star}_{\mu k}|$

LFV Decays	Present Exp. Bound
$\mathrm{Br}(\mu \to e\gamma)$	4.2×10^{-13} MEG (2016)
$\mathrm{Br}(\tau \to e\gamma)$	3.3×10^{-8} BaBar (2009)
$\mathrm{Br}(\tau \to \mu\gamma)$	4.4×10^{-8} Belle (2021)

Type-I	$M_1 = 100$ GeV	$M_1 = 1$ TeV
$\lvert R_{e1}R^{\star}_{\mu 1}\rvert$	3.43×10^{-5}	1.17×10^{-5}
$\lvert R_{e1}R^{\star}_{\tau 1}\rvert$	9.62×10^{-3}	3.28×10^{-3}
$\lvert R_{\tau 1}R^{\star}_{\mu 1}\rvert$	11.1×10^{-3}	3.79×10^{-3}

- The constraints on the seesaw parameter $|R_{e1}R^{\star}_{\mu 1}|$ exhibit mild dependence on the masses of heavy particles through loop functions.

Type-II Seesaw

J. Schechter and J. W. F. Valle 1980

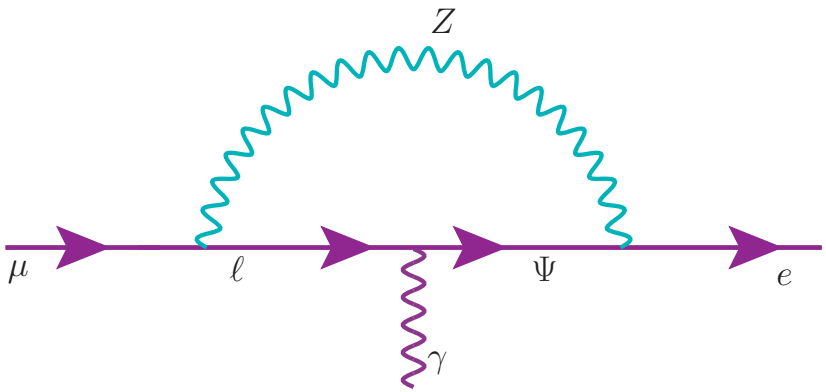
G. Lazarides, Q. Shafi, and C. Wetteric 1981

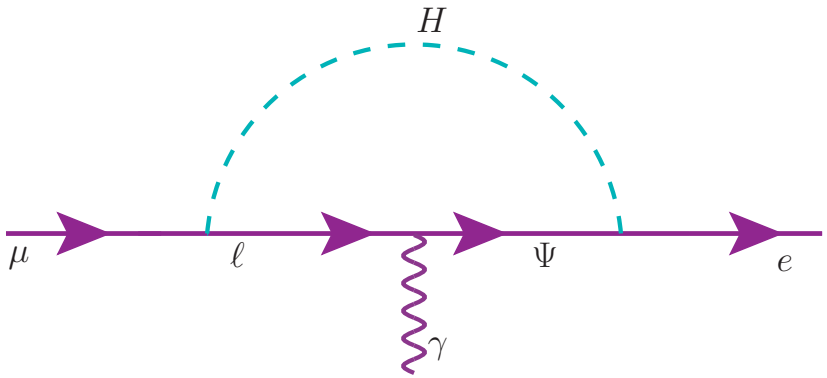
- Here an $SU(2)_L$ scalar triplet field (Δ) with hypercharge $Y = 2$, is added to the SM.

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+\sqrt{2} \end{pmatrix}$$

- The radiative CLFV decays are mediated by the charged and doubly charged components of the scalar triplet. These particles do not couple to quarks and hence play no role in the CLFV decays of mesons.

$\mu \to e\gamma$ Decay Type-III Seesaw





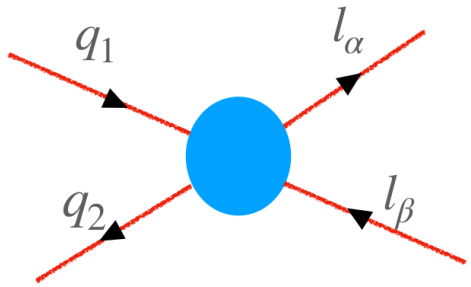
$$\text{Br}\,(\mu \to e\gamma) \cong \frac{3\alpha_{em}}{32\pi} \sum_k |R_{ek} R^{\star}_{\mu k}|^2 \left[-2.23 + A(x_k) + B(y_k) + C(z_k)\right]^2$$

Type-III	$M_{\Sigma_1} = 100$ GeV	$M_{\Sigma_1} = 1$ TeV
$\lvert R_{e1} R^{*}_{\mu 1}\rvert$	3.69×10^{-6}	5.39×10^{-6}
$\lvert R_{e1} R^{\star}_{\tau 1}\rvert$	1.03×10^{-3}	1.51×10^{-3}
$\lvert R_{\tau 1} R^{*}_{\mu 1}\rvert$	1.19×10^{-3}	1.74×10^{-3}

CLFV Meson Decays

- **For the transition** $q_1 \to q_2 l_\beta^+ l_\alpha^-$ (Becirevic, Jaffredo, Pinheiro, Sumnesari(2024))

$$H_{eff} = \frac{4G_F}{\sqrt{2}} \sum_{j=u,c,t} V_{jq_1}^* V_{jq_2} [C_9 O_9 + C_{10} O_{10}]$$



With

$$O_9 = \frac{\alpha}{8\pi} [\bar{q}_2 \gamma^\mu (1 - \gamma_5) q_1][l_\alpha \gamma_\mu l_\beta]$$

$$O_{10} = \frac{\alpha}{8\pi} [\bar{q}_2 \gamma^\mu (1 - \gamma_5) q_1][l_\alpha \gamma_\mu \gamma_5 l_\beta]$$

$$\Rightarrow \quad C_9 = -C_{10}$$

CLFV Meson decay in Type-I

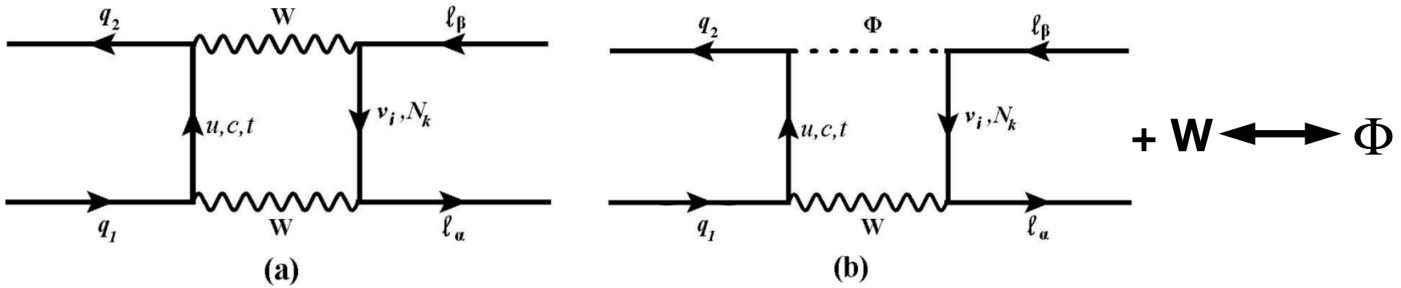
The effective Hamiltonian for all diagrams in Type-I seesaw:

$$\mathcal{H}_{eff} = f_{Type-I}\left[\overline{q_2}\gamma^\mu(1-\gamma_5)q_1\right]\underbrace{\left[\bar{\ell}_\alpha\gamma^\mu(1-\gamma_5)\ell_\beta\right]}_{\text{V-A}}$$

$$f^a_{Type-I} = \frac{G_F^2 M_W^2}{8\pi^2}\sum_{j=u,c,t} V^*_{j\,q_1} V_{j\,q_2} x_j \sum_k \underbrace{R_{e\,k} R^*_{\mu\,k}}\, \mathcal{I}_1(x_j, x_k)$$

- The amplitude depends on the same set of seesaw parameters as the radiative CLFV decays.

  - **Type-I**



$$\rightarrow \quad C_{9(I)} \cong X_t[\underbrace{R_{\alpha 1}R^*_{\beta 1}}]\left[\frac{1}{2\sin^2\theta_W} I_1(X_t, X_1)\right]$$

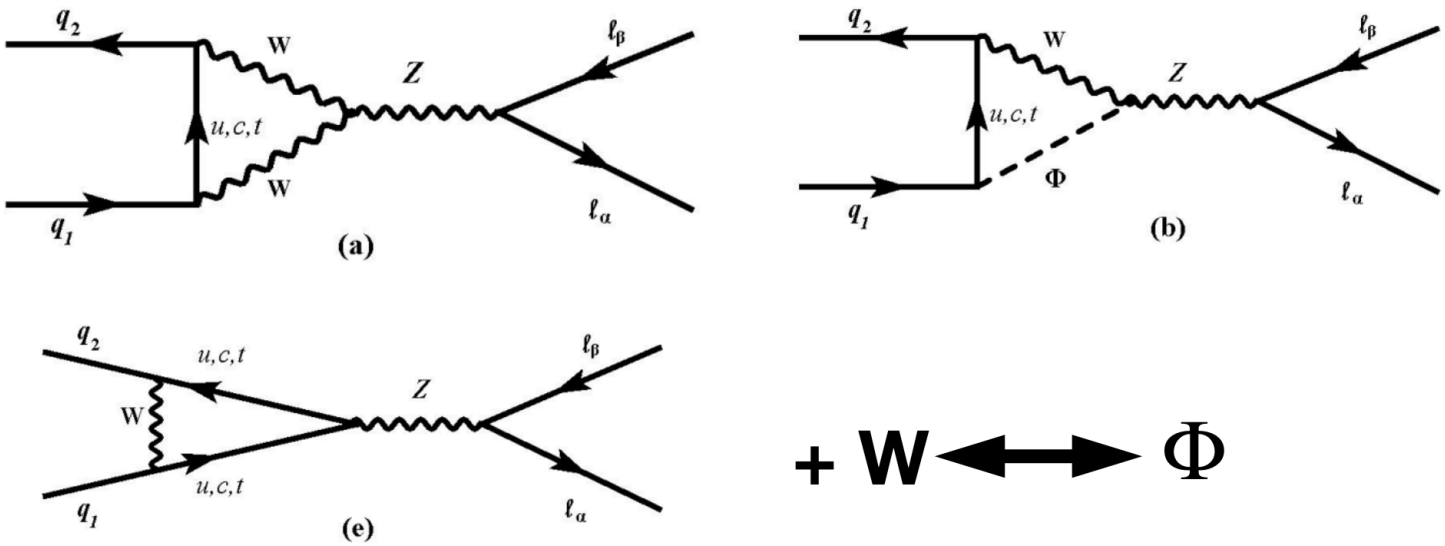
CLFV Meson decay in Type-III

- The effective Hamiltonian for all diagrams in Type-III seesaw:

$$\mathcal{H}_{eff} = f_{Type-III}\left[\overline{q_2}\gamma^{\mu}(1-\gamma_5)q_1\right]\underbrace{\left[\bar{\ell}_{\alpha}\gamma^{\mu}(1-\gamma_5)\ell_{\beta}\right]}_{\text{V-A}}$$

- All the six triangle diagrams in Type-III have comparable contributions.

- **Additional contribution in Type-III:**



$$C_{9(III)} \cong X_t[R_{\alpha 1}R^{*}_{\beta 1}]\, I_{tot}(X_t, X_1)$$

CLFV in K decays

- The amplitude for purely leptonic kaon decay $K_L \to \mu^+ e^-$ is the sum of the amplitudes for $K^0 \to \mu^+ e^-$ and $\overline{K^0} \to \mu^+ e^-$.

$$|K_L\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K^0}\rangle), \qquad CP|K^0\rangle = -|\overline{K^0}\rangle.$$

$$Br(K_L \to \mu^+ e^-) = 2\tau_K \frac{G_F^2 \alpha^2}{32\pi^3} f_K^2 m_K m_\mu^2 \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 [\mathcal{R}e(V_{ts} V_{td}^*)]^2 |C_9|^2$$

- The semi-leptonic decay simplifies to, after integrating over the phase space,

$$Br\left(K^+ \to \pi^+ \mu^+ e^-\right) = a_9^K |C_9|^2, \qquad a_9^K = 1.94 \times 10^{-13}$$

CLFV in B decays

- For purely leptonic decays,

$$Br(B_s \to \ell_\beta^+ \ell_\alpha^-) = \tau_{B_s} \frac{G_F^2 \alpha^2}{32\pi^3} f_{B_s}^2 m_{B_s} m_{\ell_\beta}^2 \left(1 - \frac{m_{\ell_\beta}^2}{m_{B_s}^2}\right)^2 |V_{tb} V_{ts}^*|^2 |C_9|^2$$

- The semi-leptonic decays simplify to, after integrating over the phase space,

$$Br\left(B \to M' \ell_\beta^+ \ell_\alpha^-\right) = 2a_9 \left|C_9\right|^2 \times 10^{-9}$$

$$Br\left(B \to V \ell_\beta^+ \ell_\alpha^-\right) = 2(a_9 + b_9) \left|C_9\right|^2 \times 10^{-9}$$

D. Becirevic et.al (2407.19060)

Upper Bounds on CLFV Meson Decays

Similarly, we can relate B.R. of other radiative and meson CLFV decays.

Decay	Exp. Limit	Type-I M=100(1000)GeV	Type-III M=100(1000)GeV
$Br(K_L \to e\mu)$	6.3×10^{-12}	$4.32\,(1.11) \times \mathbf{10^{-20}}$	$1.67\,(3.37) \times \mathbf{10^{-19}}$
$Br(K^+ \to \pi^+ e\mu)$	1.1×10^{-10}	$5.48\,(1.42) \times 10^{-22}$	$2.12\,(4.28) \times 10^{-21}$
$Br(B_s \to e\mu)$	5.4×10^{-9}	$2.64\,(0.68) \times \mathbf{10^{-19}}$	$1.01\,(2.06) \times \mathbf{10^{-18}}$
$Br(B^+ \to K^+ e\mu)$	1.8×10^{-8}	$1.05\,(0.27) \times 10^{-16}$	$4.09\,(8.27) \times 10^{-16}$
$Br(B^0 \to K^{\star 0} e\mu)$	1.0×10^{-8}	$2.14\,(0.55) \times \mathbf{10^{-16}}$	$0.82\,(1.67) \times \mathbf{10^{-15}}$
$Br(B_s \to \phi\, e\mu)$	1.6×10^{-8}	$2.23\,(0.57) \times \mathbf{10^{-16}}$	$0.86\,(1.74) \times \mathbf{10^{-15}}$
$Br(B^+ \to \pi^+ e\mu)$	9.2×10^{-8}	$3.68\,(0.95) \times 10^{-18}$	$1.42\,(2.87) \times 10^{-17}$
$Br(B^0 \to \rho^0 e\mu)$	3.2×10^{-6}	$8.33\,(2.15) \times 10^{-18}$	$3.22\,(6.50) \times 10^{-17}$

Table: Predictions of various CLFV meson decays in seesaw models.

Summary

- **Type-I :** Meson CLFV B.R. $\sim 10^{-3} \times$ Radiative CLFV B.R.
- **Type-II :** Meson CLFV B.R. is negligibly small ($\sim 10^{-50}$) due to exact GIM cancellation
- **Type-III :** Similar to type-I with additional contribution from FCNC of Z-boson to charged lepton

Meson CLFV B.R. $\sim 10^{-2} \times$ Radiative CLFV B.R.

Conclusion

- Models addressing neutrino masses also predict CLFV process, which are experimentally testable
- In three basic seesaw models -

Meson CLFV B.R. $<$ **Radiative CLFV B.R.**

- Can distinguish between different types of seesaw
- If experiments find **meson CLFV B.R. is greater than radiative CLFV decay**

→ **neutrino mass generatuion is not simple seesaw.**

Thank You!

Backup!

Remarks

- Other neutrino mass models can also be explored to study CLFV decays and establish limits. For an example, the mass model with $\mathcal{A}_4$ symmetry predicts $Br(K^0 \to \bar{\mu}e) = 3.5 \times 10^{-11}$, which is just below the current experimental sensitivity ($< 5.4 \times 10^{-9}$).

  R. Korrapati, J. More, U. Rahaman, and S. U. Sankar (2021)

- Beyond the three canonical seesaw models, extended variants such as the **inverse seesaw** and **linear seesaw**, which operate at lower scales might enhance the branching ratios.
- If a CLFV meson decay is observed with a branching ratio larger than radiative CLFV decay in future experiments, it would indicate that an alternative mechanism, beyond the seesaw models, is responsible for neutrino mass generation.

For light neutrino exchange

$$\sum_{j=1}^{3}[(1+\eta)U_\nu]_{ej}\,[(1+\eta)U_\nu]^*_{\mu j}\,F\left(\frac{m_j^2}{M_W^2}\right) \simeq \left(\eta_{e\mu}+\eta^*_{\mu e}\right)F(0)$$

$$\simeq -\sum_k R_{ek}R^*_{\mu k}F(0).$$

$$\boxed{\mathrm{Br}\,(\mu\to e\gamma) = \frac{3\alpha_{em}}{32\pi}\sum_k |R_{ek}R^\star_{\mu k}|^2\,[F(x_k)-F(0)]^2}$$

$$F(x_k) = \frac{10-43x_k+78x_k^2-49x_k^3+4x_k^4+18x_k^3\log(x_k)}{3(x_k-1)^4}, \qquad x_k = (\frac{M_k}{M_W})^2$$

We work in the limit where $k = 2$ and the two heavy RH neutrinos are nearly degenerate *i.e.* $M_2 = M_1(1+z)\,, z \ll 1$.

$$M_D = \overbrace{iU_\nu^* \sqrt{\hat{m}}}^{\substack{\text{low energy} \\ \text{measurables}}} \quad \overbrace{O\sqrt{M_R}}^{\substack{\text{high energy} \\ \text{free parameters}}}$$

Casas, Ibarra, 2001

$$R = iU_\nu \sqrt{\hat{m}}\, O^* \sqrt{M_R}^{-1}$$

For **NH**

$$R \simeq -\frac{e^{-iw} e^{\xi}}{2} \sqrt{\frac{m_3}{|M_1|}} \begin{pmatrix} U_{e3} + i\sqrt{\frac{m_2}{m_3}}\, U_{e2} & \pm\left(U_{e3} + i\sqrt{\frac{m_2}{m_3}}\, U_{e2}\right)/\sqrt{1+z} \\ U_{\mu 3} + i\sqrt{\frac{m_2}{m_3}}\, U_{\mu 2} & \pm\left(U_{\mu 3} + i\sqrt{\frac{m_2}{m_3}}\, U_{\mu 2}\right)/\sqrt{1+z} \\ U_{\tau 3} + i\sqrt{\frac{m_2}{m_3}}\, U_{\tau 2} & \pm\left(U_{\tau 3} + i\sqrt{\frac{m_2}{m_3}}\, U_{\tau 2}\right)/\sqrt{1+z} \end{pmatrix}.$$

Ibarra, EM, Petcov, 2010

$R_{\alpha 2} = \pm i \frac{1}{\sqrt{1+z}} R_{\alpha 1}$

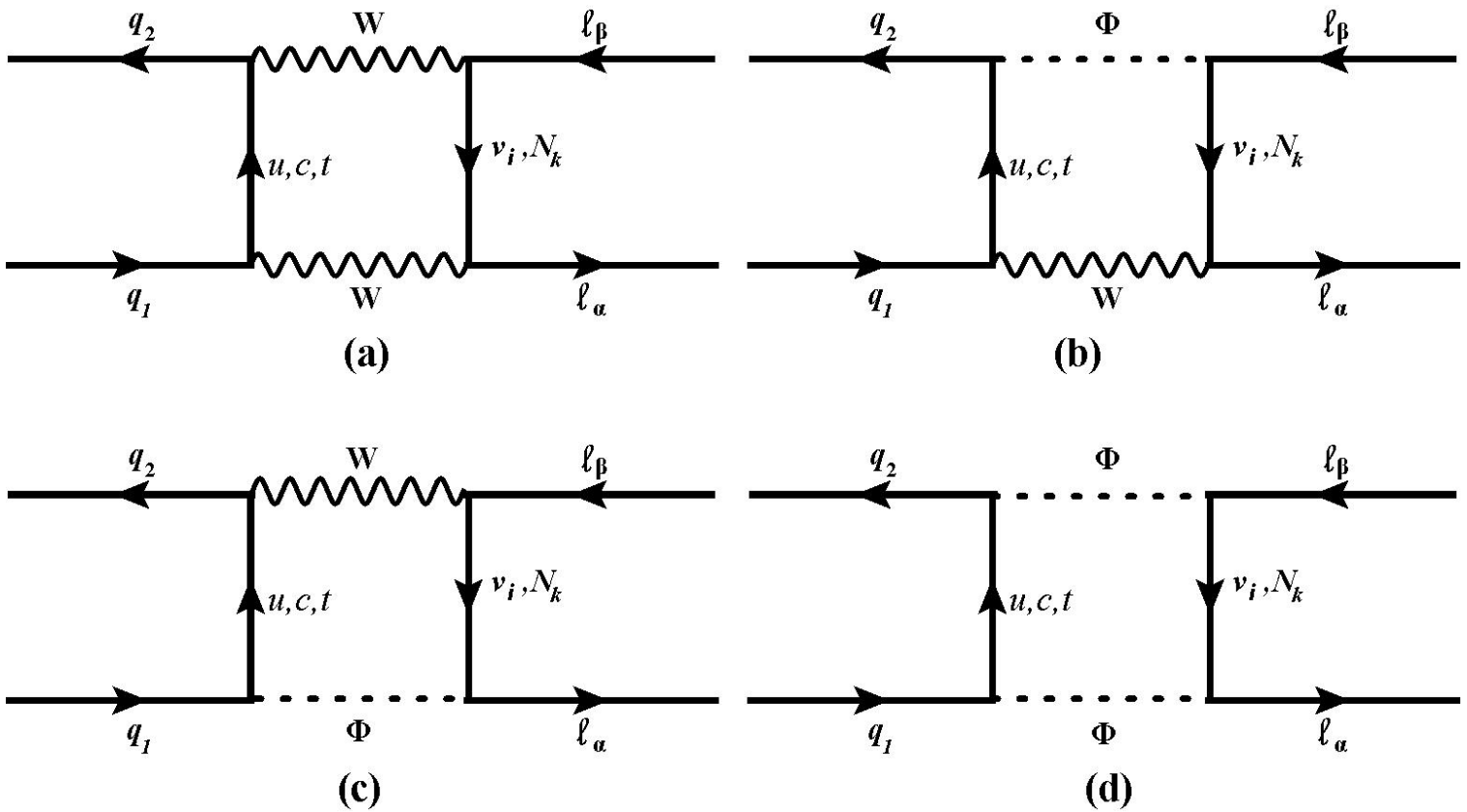
$$\boxed{\text{Br}\,(\mu \to e\gamma) \cong \frac{3\alpha_{em}}{32\pi} \left(\frac{2+z}{1+z}\right)^2 |R_{e1} R_{\mu 1}^\star|^2 \left[F(x_1) - F(0)\right]^2}$$

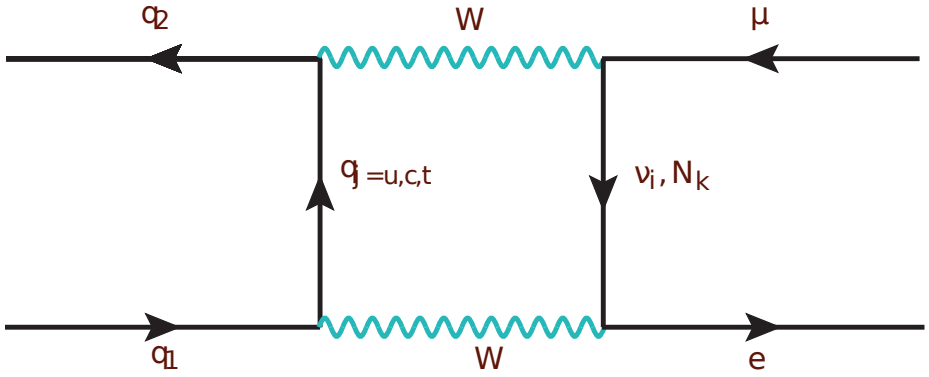
Effective Hamiltonian for CLFV meson decays

For the transition $q_1 \to q_2 \bar{\ell}^+_\beta \ell^-_\alpha$,

$$H_{eff} = \frac{4 G_F}{\sqrt{2}} \sum_{j=u,c,t} V^*_{j\,q_1} V_{j\,q_2} \left[C_9\, O_9 + C_{10}\, O_{10} \right],$$

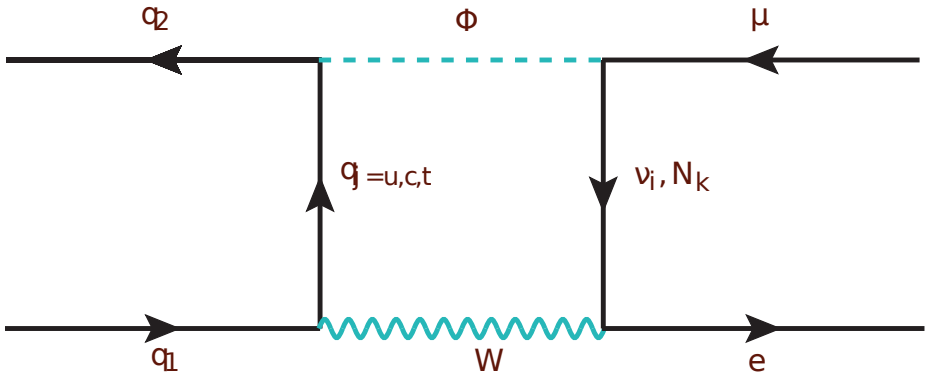
$$O_9 = \frac{\alpha}{8\pi} [\bar{q}_2\, \gamma^\mu\, (1-\gamma_5)\, q_1][\bar{\ell}_\alpha\, \gamma_\mu\, \ell_\beta], \quad O_{10} = \frac{\alpha}{8\pi} [\bar{q}_2 \gamma^\mu\, (1-\gamma_5)\, q_1][\bar{\ell}_\alpha \gamma_\mu \gamma_5\, \ell_\beta].$$





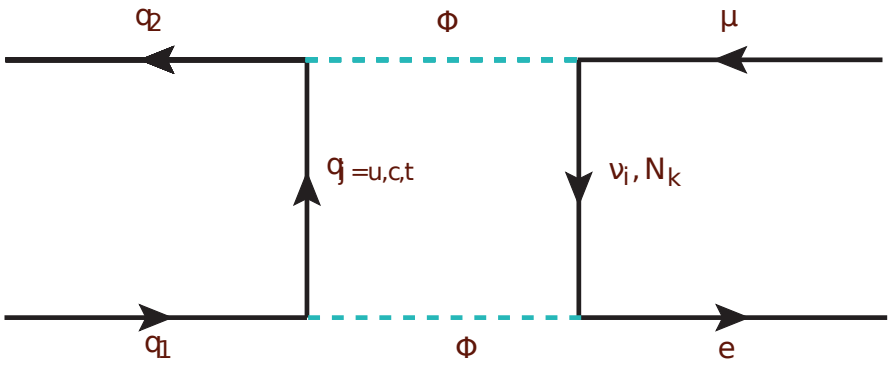
$$f^{a}_{Type-I} = \frac{G_F^2 M_W^2}{8\pi^2} \sum_j V^*_{j\,q_1} V_{j\,q_2} x_j \sum_k R_{e\,k} R^*_{\mu\,k} x_k \mathcal{I}^a(x_j\,, x_k)$$

$$C^a_9 = x_j \sum_k R_{e\,k} R^*_{\mu\,k} x_k \mathcal{I}^a(x_j\,, x_k)$$



$$f^{b}_{Type-I} = \frac{G_F^2 M_W^2}{8\pi^2} \sum_j V^*_{j\,q_1} V_{j\,q_2} x_j \sum_k R_{e\,k} R^*_{\mu\,k} U_{\mu k} x_k \mathcal{I}^b(x_j\,, x_k)$$

$$C^b_9 = x_j \sum_k R_{e\,k} R^*_{\mu\,k} x_k \mathcal{I}^a(x_j\,, x_k)$$



$$f^{c}_{Type-I} = \frac{G_F^2 M_W^2}{8\pi^2} \sum_j V^*_{jq_1} V_{jq_2} x_j \sum_k R_{ek} R^*_{\mu k} U_{ek} U^*_{\mu k} x_k \mathcal{I}^c(x_j, x_k)$$

$$C^c_9 = x_j \sum_k R_{ek} R^*_{\mu k} x_k \mathcal{I}^a(x_j, x_k)$$

The effective Hamiltonian for these transitions can be expressed as,

$$T \cong f_i \left[\overline{q_1}\gamma^\mu \left(1-\gamma_5\right) q_2\right]\left[\overline{e}\gamma^\mu \left(1-\gamma_5\right) \mu\right] \tag{1}$$

where i =(Type-I, Type-II, Type-III) and q_1, q_2 are quarks.

$$f_{Type-I} = f^a_{Type-I} + f^b_{Type-I} + f^c_{Type-I} \tag{2}$$

$$f^a_{Type-I} = \frac{G_F^2 M_W^2}{8\pi^2} \sum_j V^*_{j\,q_1} V_{j\,q_2} x_j \sum_k R_{e\,k} R^*_{\mu\,k} x_k \mathcal{I}^a(x_j\,, x_k) \tag{3}$$

$$f^b_{Type-I} = \frac{G_F^2 M_W^2}{8\pi^2} \sum_j V^*_{j\,q_1} V_{j\,q_2} x_j \sum_k R_{e\,k} R^*_{\mu\,k} U_{\mu k} x_k \mathcal{I}^b(x_j\,, x_k) \tag{4}$$

$$f^c_{Type-I} = \frac{G_F^2 M_W^2}{8\pi^2} \sum_j V^*_{j\,q_1} V_{j\,q_2} x_j \sum_k R_{e\,k} R^*_{\mu\,k} U_{ek} U^*_{\mu k} x_k \mathcal{I}^c(x_j\,, x_k) \tag{5}$$

where $x_j = \left(\frac{m_j}{M_W}\right)^2$ and $x_k = \left(\frac{M_k}{M_W}\right)^2$.

The Loop integral functions are

$$\mathcal{I}^a(x_j\,, x_k) = \frac{-\ln x_j}{(x_j - x_k)(1-x_j)^2} + \frac{\ln x_k}{(x_j - x_k)(1-x_j)^2} - \frac{1}{(1-x_j)(1-x_k)},$$

$$\mathcal{I}^b(x_j\,, x_k) = \mathcal{I}^c(x_j\,, x_k) = \frac{-x_j \ln x_j}{(x_j - x_k)(1-x_j)^2} + \frac{x_k \ln x_k}{(x_j - x_k)(1-x_j)^2} - \frac{1}{(1-x_j)(1-x_k)}. \tag{6}$$

Type-III Seesaw $\Sigma(3,0)$

R. Foot, H. Lew, X. G. He, and G. C. Joshi 1988 E. Ma 1998

$$-\mathcal{L}_{\text{Type- III}} = \frac{1}{2} Tr\overline{\Sigma}_k (M_\Sigma)_{k\ell} \Sigma^c_\ell + \sqrt{2}\tilde{H}^\dagger \overline{\Sigma}_k (Y_\Sigma)_{k\alpha} L_{\alpha L} + \text{ h.c}$$

$$\Sigma_k = \begin{pmatrix} \Sigma^0_k/\sqrt{2} & \Sigma^+_k \\ \Sigma^-_k & -\Sigma^0_k\sqrt{2} \end{pmatrix}$$

- The neutral component of the triplet Σ^0 mixes with neutrinos making the PMNS mixing matrix non-unitary.
- The charged Dirac spinor $\Psi \equiv \Sigma^{+c}_R + \Sigma^-_R$ mixes with charged leptons.

$$\mathcal{L}_{\text{M}} = -\left(\overline{\ell}_L \;\; \overline{\Psi}_L\right) \begin{pmatrix} m_\ell & Y^\dagger_\Sigma v \\ 0 & M_\Sigma \end{pmatrix} \begin{pmatrix} \ell_R \\ \Psi_R \end{pmatrix} - \frac{1}{2} \left(\overline{\nu^c_L} \;\; \overline{\Sigma^0}\right) \begin{pmatrix} 0 & M_D \\ M^T_D & M_\Sigma \end{pmatrix} \begin{pmatrix} \nu_L \\ \Sigma^{0c} \end{pmatrix}$$

Abada, Biggio, Bonnet, Gavela, and Hambye 2008

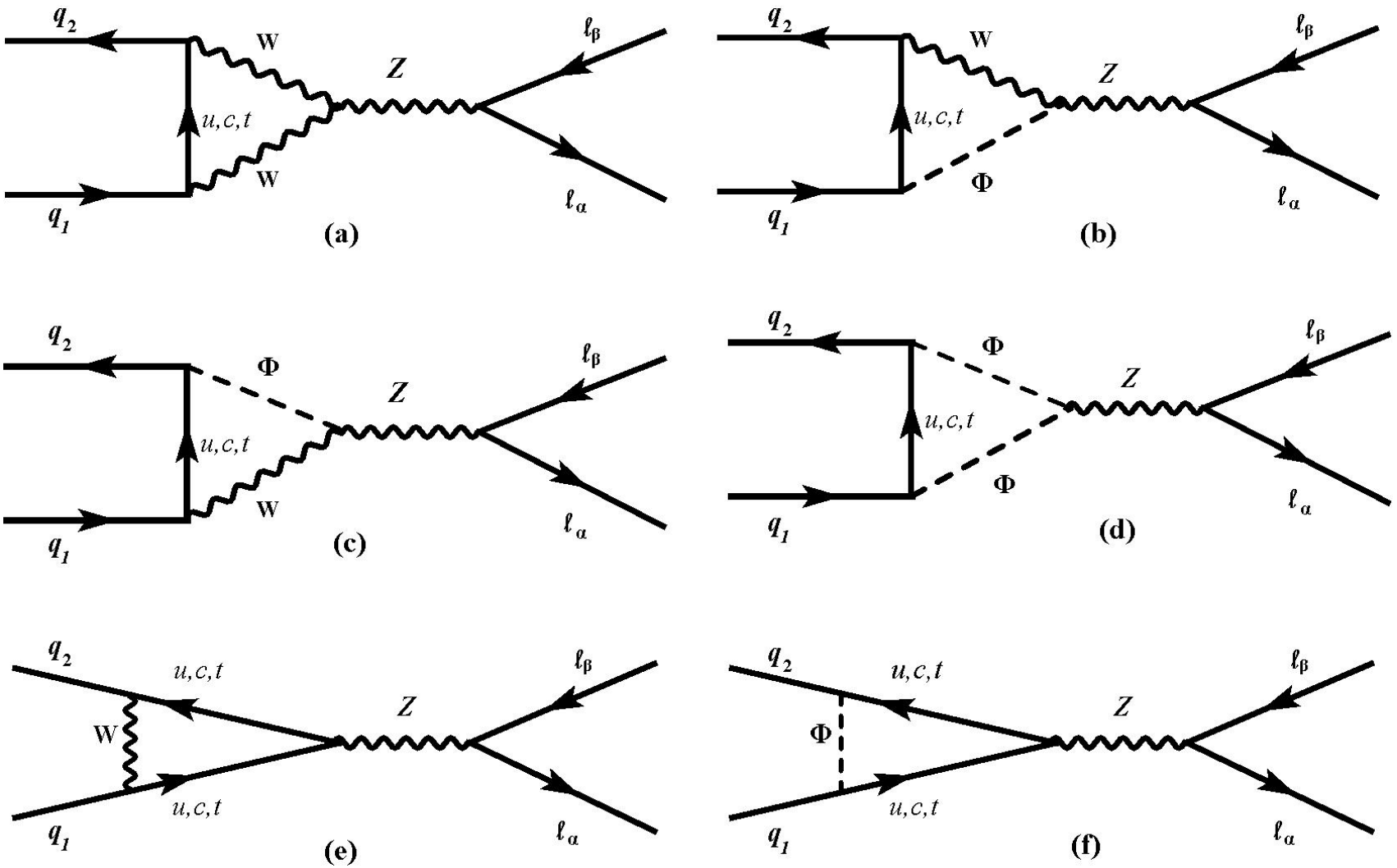
$$\mathbf{U} = \begin{pmatrix} (1+\eta)U_\nu & R \\ -R^\dagger U_\nu & 1+\eta' \end{pmatrix}, \quad R = M_D^* M_\Sigma^{-1}, \eta = -\frac{1}{2} R R^\dagger, \eta' = -\frac{1}{2} R^\dagger R$$

$$\mathbf{U_L} = \begin{pmatrix} 1+2\eta & \sqrt{2}R \\ -\sqrt{2}R^\dagger & 1+2\eta' \end{pmatrix} \quad \mathbf{U_R} = \begin{pmatrix} 1 & \sqrt{2} m_\ell R M_\Sigma^{-1} \\ -\sqrt{2} M_\Sigma^{-1} R^\dagger m_\ell & 1 \end{pmatrix}$$

- Unlike type I see-saw, type-III seesaw contains flavour changing neutral currents (FCNCs) in the charged lepton sector.

$$\mathcal{L}^\ell_{FCNC} = -\frac{g}{2\cos\theta_W} \left(R_{\alpha k} R^\dagger_{k\beta} \right) \bar{\ell}_\alpha \gamma_\mu (1-\gamma_5) \ell_\beta Z^\mu + \text{ h.c.}$$

CLFV Meson decay in Type-III



Effective Hamiltonian

- The effective Hamiltonian for all diagrams in Type-III seesaw:

$$\mathcal{H}_{eff} = f_{Type-III}\left[\overline{q_2}\gamma^{\mu}(1-\gamma_5)q_1\right]\underbrace{\left[\bar{\ell}_{\alpha}\gamma^{\mu}(1-\gamma_5)\ell_{\beta}\right]}_{\text{V-A}}$$

- All the six triangle diagrams in Type-III have comparable contributions.

$$C_{9\ Type-III} \simeq x_t\left[R_{e1}R^{*}_{\mu 1}\right]\left[\mathcal{I}_{\rm tot}(x_t\ ,x_1)\right]$$

Seesaw Models

Seesaw	Additional Fields	Mass Matrix	Mixing
Type-I $\bar{L}\, Y_D\, \widetilde{H}\, N_R$ $\frac{1}{2}\overline{N_R^c}\, M_R\, N_R$	Fermion $N(1,0)$	$\begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}$ $m_\nu \approx -M_D M_R^{-1} M_D^T$	$\begin{pmatrix} (\mathbb{I} - \frac{1}{2} RR^\dagger)\, U_\nu & RU_N \\ -R^\dagger U_\nu & (\mathbb{I} - \frac{1}{2} R^\dagger R)\, U_N \end{pmatrix}$ $R = M_D M_R^{-1} + \mathcal{O}(M_D^3 (M_R^{-1})^3)$
Type-II $Y_\Delta \ell^T C \Delta \ell + \text{h.c.}$ $\mu \widetilde{H}^T \Delta \widetilde{H} + \text{h.c.}$ $+M_\Delta^2\, \text{Tr}\left(\Delta^\dagger \Delta\right)$	Scalar $\Delta(3,1)$ $\begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+\sqrt{2} \end{pmatrix}$	$m_\nu^{II} = 2 Y_\Delta u_\Delta$ with $u_\Delta = \frac{\mu v^2}{M_\Delta^2}$, $\left\langle \Delta^0 \right\rangle = u_\Delta$	$(m_\nu^{II})_{\ell_i \ell_j} = (U^*\, dia(m_1, m_2, m_3) U^\dagger)_{\ell_i \ell_j}$ $(Y_\Delta)_{\ell_i \ell_j} = \frac{1}{2u_\Delta} (m_\nu^{II})_{\ell_i \ell_j}$
Type-III $Y_\Sigma \bar{\ell} \Sigma \tilde{H} +$ $\frac{1}{2} M_\Sigma\, \text{Tr}\left(\overline{\Sigma^c} \Sigma\right)$	Fermion $\Sigma(3,0)$ $\begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0\sqrt{2} \end{pmatrix}$	$\begin{pmatrix} 0 & M_D \\ M_D^T & M_\Sigma \end{pmatrix}$ $m_\nu^{III} = -Y_\Sigma^T \frac{v^2}{M_\Sigma} Y_\Sigma$	$\begin{pmatrix} (\mathbb{I} - \frac{1}{2}\varepsilon)\, U_\nu & M_D M_\Sigma^{-1} U_N \\ -(M_D M_\Sigma^{-1})^\dagger U_\nu & (\mathbb{I} - \frac{1}{2}\varepsilon^\dagger)\, U_N \end{pmatrix}$ $\varepsilon = M_D M_\Sigma^{-1} (M_D M_\Sigma^{-1})^\dagger$