Neutrino physics

5-6 May 2025

NDM 2025

Silvia Pascoli

HIDDeν
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos
Horizon2020
ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA
INFN
Istituto Nazionale di Fisica Nucleare

Lecture 3: Neutrino masses BSM

Aims

Explore the origin of neutrino masses in models BSM.

Outline

Neutrino mass terms

Neutrino masses BSM:

- Dirac masses
- Weinberg operator
- See-saw Type I and leptogenesis
- other models

The problem of flavour

Current status of neutrino parameters

NuFIT 6.0 (2024)

	Normal Ordering ($\Delta\chi^2 = 0.6$)		Inverted Ordering (best fit)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$
$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
$\sin^2\theta_{23}$	$0.561^{+0.012}_{-0.015}$	$0.430 \to 0.596$	$0.562^{+0.012}_{-0.015}$	$0.437 \to 0.597$
$\theta_{23}/^\circ$	$48.5^{+0.7}_{-0.9}$	$41.0 \to 50.5$	$48.6^{+0.7}_{-0.9}$	$41.4 \to 50.6$
$\sin^2\theta_{13}$	$0.02195^{+0.00054}_{-0.00058}$	$0.02023 \to 0.02376$	$0.02224^{+0.00056}_{-0.00057}$	$0.02053 \to 0.02397$
$\theta_{13}/^\circ$	$8.52^{+0.11}_{-0.11}$	$8.18 \to 8.87$	$8.58^{+0.11}_{-0.11}$	$8.24 \to 8.91$
$\delta_{\rm CP}/^\circ$	177^{+19}_{-20}	$96 \to 422$	285^{+25}_{-28}	$201 \to 348$
$\frac{\Delta m^2_{21}}{10^{-5}\ \rm eV^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \rm eV^2}$	$+2.534^{+0.025}_{-0.023}$	$+2.463 \to +2.606$	$-2.510^{+0.024}_{-0.025}$	$-2.584 \to -2.438$

nufit.org, M. C. Gonzalez-Garcia et al., 2410.05380

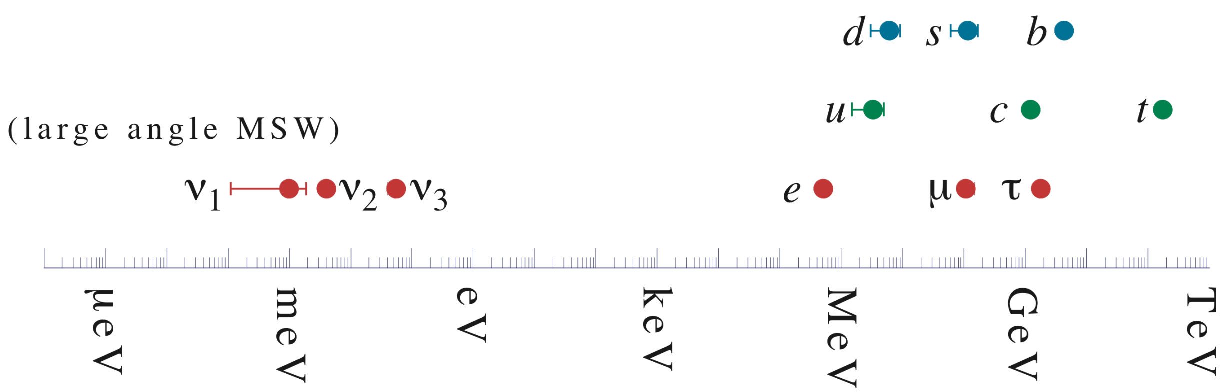
Current knowledge of neutrino properties:

- 2 mass squared differences
- 3 sizable mixing angles,
- hints of CPV?
- mild indications in favour of NO

Open window on Physics beyond the SM

Neutrinos give a new perspective on physics BSM.

1. Origin of masses



Why neutrinos have mass?
and why are they so much lighter?
and why their hierarchy is at most mild?

2. Problem of flavour

$$\begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

$$\begin{pmatrix} 0.8 & 0.5 & 0.16 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

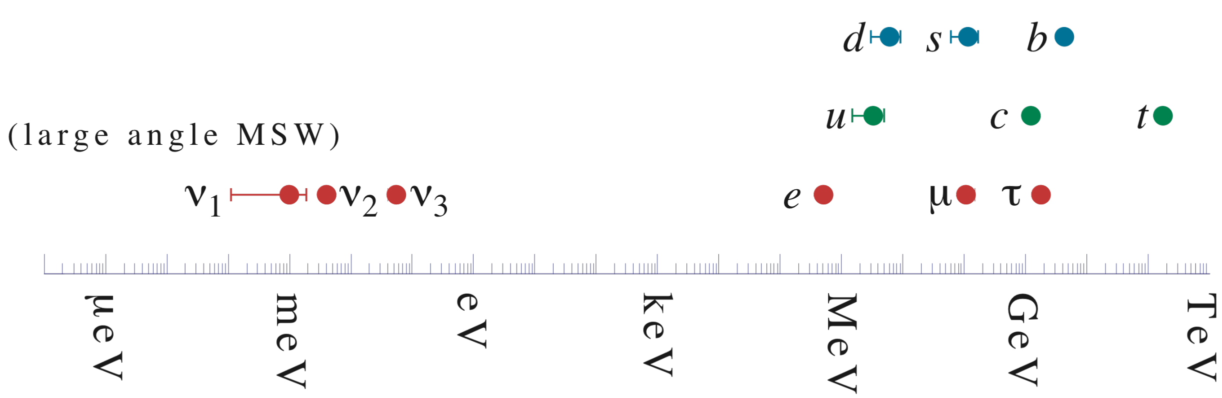
Why leptonic mixing is so different from quark mixing?

This information is **complementary** with the one from flavour physics experiments and from colliders.

Open window on Physics beyond the SM

Neutrinos give a new perspective on physics BSM.

1. Origin of masses



Why neutrinos have mass?
and why are they so much lighter?
and why their hierarchy is at most mild?

2. Problem of flavour

$$\begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

$$\begin{pmatrix} 0.8 & 0.5 & 0.16 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

Why leptonic mixing is so different from quark mixing?

This information is **complementary** with the one from flavour physics experiments and from colliders.

First step:
We need to augment the SM Lagrangian with terms which describe neutrino masses.

What kind of masses can neutrinos have?

Neutrino masses in the nuSM lagrangian

A mass term for a fermion connects a left-handed field with a right-handed one. For example the "usual" Dirac mass

$$m_\psi(\bar{\psi}_R \psi_L + \mathrm{h.c.}) = m_\psi \bar{\psi}\psi$$

Exercise
check this formula

Dirac masses

This is the simplest case. We assume that we have two independent Weyl fields: ν_L , ν_R
and we can write down the term as above.

$$\mathcal{L}_{mD} = -m_\nu(\bar{\nu}_R \nu_L + \mathrm{h.c.})$$

Does it conserve lepton number?

$$\nu_L \to e^{i(+1)\alpha}\nu_L$$
$$\nu_R \to e^{i(?)\alpha}\nu_R$$

Neutrino masses in the nuSM lagrangian

A mass term for a fermion connects a left-handed field with a right-handed one. For example the "usual" Dirac mass

$$m_\psi(\bar{\psi}_R \psi_L + \text{h.c.}) = m_\psi \bar{\psi}\psi$$

Exercise
check this formula

Dirac masses

This is the simplest case. We assume that we have two independent Weyl fields: ν_L , ν_R
and we can write down the term as above.

$$\mathcal{L}_{mD} = -m_\nu(\bar{\nu}_R \nu_L + \text{h.c.})$$

This conserves lepton number!

$$\nu_L \to e^{i\alpha}\nu_L \qquad \mathcal{L}_{mD} \to \mathcal{L}_{mD}$$
$$\nu_R \to e^{i\alpha}\nu_R$$

Diagonalize a Dirac mass term

If there are several fields, there will be a Dirac mass matrix.

$$\mathcal{L}_{mD} = -\bar{\nu}_{Ra} \, (m_D)_{ab} \, \nu_{Lb} + \text{h.c.}$$

This requires two unitary mixing matrices to diagonalise it

$$m_D = V m_{\text{diag}} U^\dagger$$

and the massive states are

$$n_L = U^\dagger \nu_L \quad n_R = V^\dagger \nu_R$$

This is the mixing matrix which enters in neutrino oscillations. So the form of the mass matrix determines the mixing pattern.

Majorana masses

If we have only the left-handed field, we can still write down a mass term, called Majorana mass term. We use the fact that

$$(\psi_L)^c = (\psi^c)_R$$

then the mass term is

$$\mathcal{L}_{mM} \propto -M_M \bar{\nu_L^c} \nu_L + \text{h.c.} = M_M \nu_L^T C^{-1} \nu_L$$

Exercise
Show that these two formulations are equivalent.

Hint:

$$\begin{aligned} \bar{\nu_L^c}\nu_L &= (C\bar{\nu}_L^T)^\dagger \gamma^0 \nu_L = \bar{\nu}_L^* C^\dagger \gamma^0 \nu_L \\ &= \nu_L^T \gamma^{0*} C^\dagger \gamma^0 \nu_L = -\nu_L^T C^{-1} \nu_L \end{aligned}$$

This breaks lepton number!

$$\nu_L \to e^{i\alpha} \nu_L \qquad \mathcal{L}_{mM} \to e^{2i\alpha} \mathcal{L}_{mM}$$

Diagonalize a Majorana mass term

If there are several fields, there will be a Majorana mass matrix. We can show that it is symmetric.

$$\boxed{M_M = M_M^T}$$

In fact:

$$\nu_L^T M_M C^{-1} \nu_L = (\nu_L^T M_M C^{-1} \nu_L)^T$$
$$= -\nu_L^T M_M^T C^{-1,T} \nu_L = \nu_L^T M_M^T C^{-1} \nu_L$$

This implies that only one unitary mixing matrix is required to diagonalise it

$$\boxed{M_M = (U^\dagger)^T m_{\rm diag} U^\dagger}$$

The massive fields are related to the flavour ones as

$$\boxed{n_L = U^\dagger \nu_L}$$

and the Lagrangian can be rewritten in terms of a Majorana field

$$\mathcal{L}_M = -\frac{1}{2}\overline{n_L^c} m_{\rm diag} n_L - \frac{1}{2}\overline{n_L} m_{\rm diag} n_L^c = -\frac{1}{2}\bar{\chi} m_{\rm diag} \chi$$

with $$\boxed{\chi \equiv n_L + n_L^c \Rightarrow \chi = \chi^c}$$

A Majorana mass term (breaks L) leads to Majorana neutrinos (breaks L).

Dirac + Majorana masses

If we have both the left-handed and right-handed fields, we can write down three mass terms:

- a Dirac mass term
- a Majorana mass term for the left-handed field and
- a Majorana mass term for the right-handed field.

$$\mathcal{L}_{mD+M} = -m_\nu \bar{\nu}_R \nu_L - \frac{1}{2}\nu_L^T M_{M,L} C^{-1} \nu_L - \frac{1}{2}\nu_R^T M_{M,R} C^{-1} \nu_R + \text{h.c.}$$

What do we expect the massive neutrinos to be? Dirac, Majorana, both?

Dirac + Majorana masses

If we have both the left-handed and right-handed fields, we can write down three mass terms:

- a Dirac mass term
- a Majorana mass term for the left-handed field and
- a Majorana mass term for the right-handed field.

$$\mathcal{L}_{mD+M} = -m_\nu \bar{\nu}_R \nu_L - \frac{1}{2}\nu_L^T M_{M,L} C^{-1} \nu_L - \frac{1}{2}\nu_R^T M_{M,R} C^{-1} \nu_R + \mathrm{h.c.}$$

This breaks lepton number, in both the Majorana mass terms.

The expectation is that, as lepton number is not conserved, neutrinos will be Majorana particles. Let's prove it.

We start by rewriting $\mathcal{L}_{mD+M} = -\frac{1}{2}\bar{\psi^c_L}\mathcal{M}\psi_L + \text{h.c.}$

with $\psi_L \equiv \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$ and $\mathcal{M} \equiv \begin{pmatrix} M_{M,L} & m_D^T \\ m_D & M_{M,R} \end{pmatrix}$

In fact

$$\mathcal{L}_{mD+M} = -\frac{1}{2}\bar{\nu^c_L} M_{M,L}\nu_L - \frac{1}{2}\bar{\nu_R} M_{M,R}\nu_R - \bar{\nu}_R m_D \nu_L + \text{h.c.}$$

and one can use $\bar{\nu^c_L} m_D^T \nu_R^c = \bar{\nu}_R m_D \nu_L$

Exercise
Show that these two formulations are equivalent.

Then, we need to diagonalise the full mass matrix, and we find the Majorana massive states, in analogy to what we have done for the Majorana mass case.

$$\chi \equiv n_L + n_L^c \Rightarrow \chi = \chi^c$$

The difference is that $n_L = U_j \nu_L + U_k \nu_R^c$

Not unitary

Mixing between mass states and sterile neutrinos

Summary of neutrino mass terms

Dirac masses

$$\mathcal{L}_{mD} = -m_\nu(\bar{\nu}_R \nu_L + \text{h.c.})$$

This term conserves lepton number.

Majorana masses

$$\mathcal{L}_{mM} \propto -M_M \bar{\nu_L^c} \nu_L + \text{h.c.} = M_M \nu_L^T C^{-1} \nu_L$$

This term breaks lepton number.

Dirac + Majorana masses

$$\mathcal{L}_{mD+M} = -m_\nu \bar{\nu}_R \nu_L - \frac{1}{2} \nu_L^T M_{M,L} C^{-1} \nu_L - \frac{1}{2} \nu_R^T M_{M,R} C^{-1} \nu_R + \text{h.c.}$$

Lepton number is broken -> Majorana neutrinos.

Can neutrino masses arise in the SM? and if not, how can we extend the SM to generate them?

Ingredients:

- gauge and Lorentz invariance
- particle content
- renormalisability

This leads to the SM, as we know it, with no neutrino masses. We need to relax some of these conditions.

Ingredients:

- gauge and Lorentz invariance
- particle content
- renormalisability

Neutrino masses in the SM and beyond

In the SM, neutrinos do not acquire mass and mixing:

- like the other fermions as there are no right-handed neutrinos.

$$m_e \bar{e}_L e_R \qquad\qquad m_\nu \bar{\nu}_L \cancel{\nu_R}$$

Solution: Introduce ν_R for Dirac masses

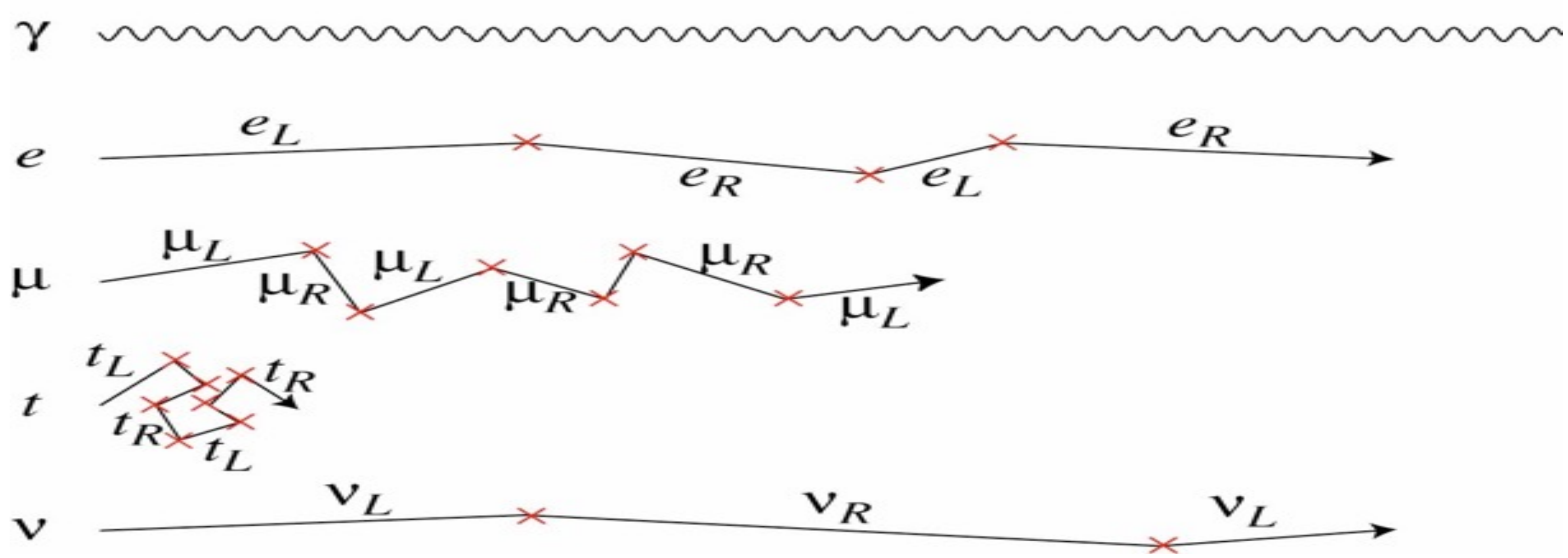
- they do not have a Majorana mass term

$$M \nu_L^T C \nu_L$$

as this term breaks the SU(2) gauge symmetry.
Solution: Introduce an SU(2) scalar triplet or gauge invariant non-renormalisable terms (D>4). This term breaks Lepton Number.

Dirac Masses

If we introduce a right-handed neutrino, then a lepton-number conserving interaction with the Higgs boson emerges.



Thanks to
H. Murayama

$$\mathcal{L} = -y_\nu \bar{L} \cdot \tilde{H} \nu_R + \text{h.c.}$$

with $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ and $\tilde{H} = \begin{pmatrix} H^{0,*} \\ -H^- \end{pmatrix}$

This term is
- SU(2) invariant and
- respects lepton number

When the neutral component of the Higgs field gets a vev, a Dirac mass term for neutrinos is generated.

$$
\begin{aligned}
\mathcal{L}_{\nu H} &= -y_\nu (\bar{\nu}_L, \bar{\ell}_L) \cdot \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix} \nu_R + \mathrm{h.c.} \\
&= -y_\nu (\bar{\nu}_L H^{0*} - \bar{\ell}_L H^-) \nu_R + \mathrm{h.c.} \\
&= -y_\nu \frac{v_H}{\sqrt{2}} \bar{\nu}_L \nu_R + \mathrm{h.c.} + \ldots
\end{aligned}
$$

$H^0 \to \frac{v_H}{\sqrt{2}} + h^0$

It follows that

$$
y_\nu \sim \frac{\sqrt{2} m_\nu}{v_H} \sim \frac{0.2 \text{ eV}}{200 \text{ GeV}} \sim 10^{-12}
$$

Tiny couplings!

Many theorists consider this explanation of neutrino masses not satisfactory. We would expect this Yukawa couplings to be similar to the ones in the quark sector:

1. why the coupling is so small????
2. why the mixings are large? (instead of small as in the quark sector)
3. why neutrino masses have at most a mild hierarchy if they are not quasi-degenerate? instead of what happens to quarks?

Dirac masses are strictly linked to lepton number conservation. But this is an accidental global symmetry. Should it be conserved at high scales?

Majorana Masses

In order to have an SU(2) invariant mass term for neutrinos, it is necessary to introduce a Dimension 5 operator (or to allow for new scalar fields, e.g. a scalar triplet):

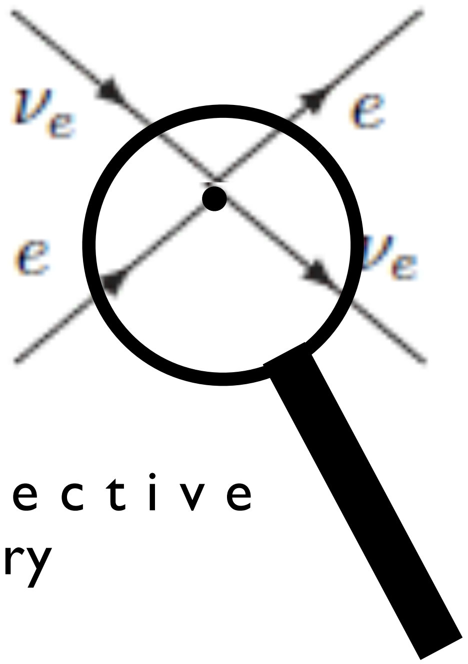
D=5 term

$$-\mathcal{L} = \lambda \frac{\bar{L}^c \ i\sigma_2 H \ H^T i\sigma_2 L}{M} = \frac{\lambda v_H^2}{M} \nu_L^T C^\dagger \nu_L$$

Weinberg operator, PRL 43

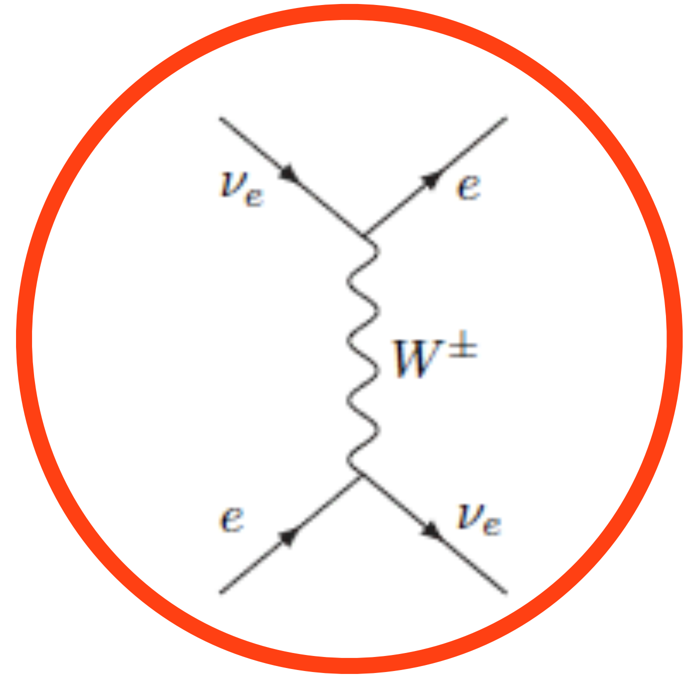
Lepton number violation!

If neutrino are Majorana particles, a Majorana mass can arise as the low energy realisation of a higher energy theory (new mass scale!).



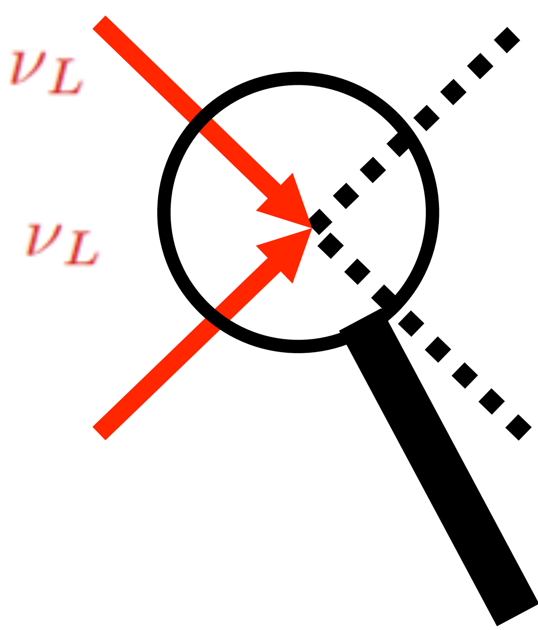
effective theory

$$\mathcal{L} \propto G_F(\bar{e}_L\gamma_\mu\nu_L)(\bar{\nu}_L\gamma^\mu e_L)$$



Standard Model: W exchange

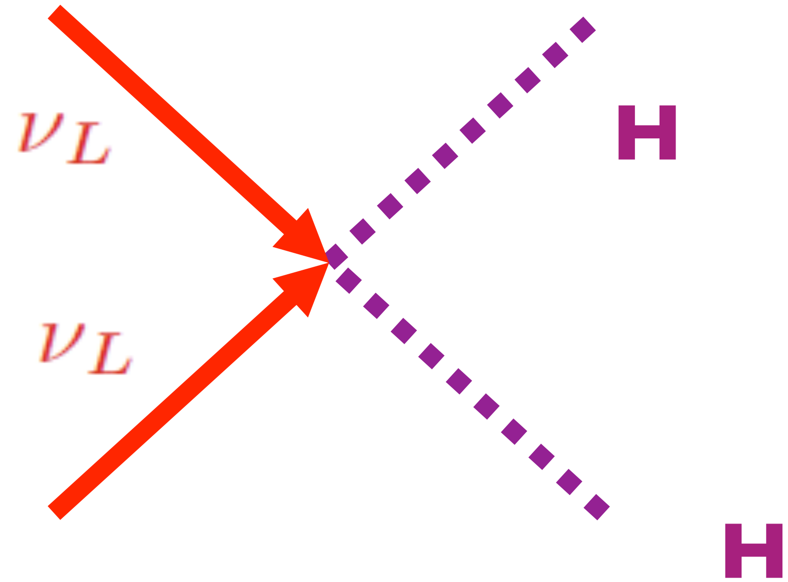
$$\mathcal{L}_{SM} \propto g\bar{\nu}_L\gamma^\mu e_L W_\mu \Rightarrow G_F \propto \frac{g^2}{m_W^2}$$



Neutrino mass

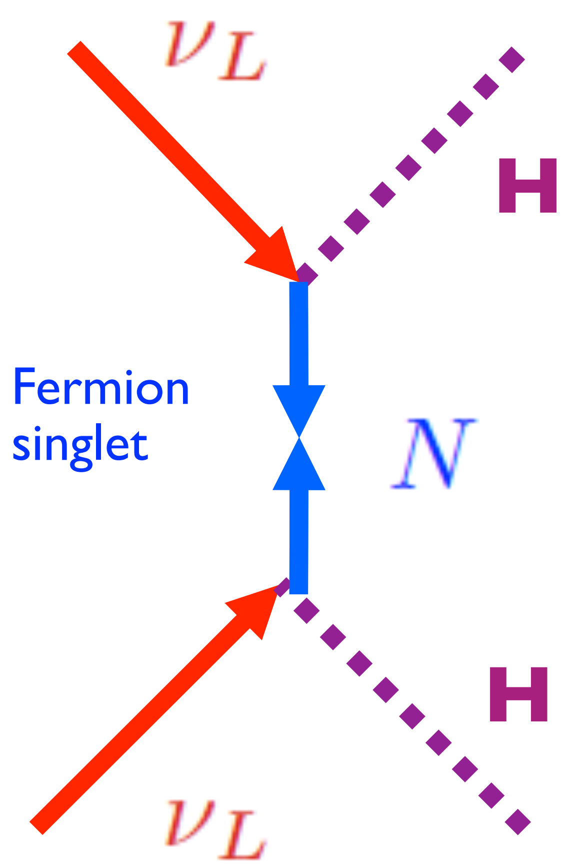
$$-\mathcal{L} = \lambda\frac{L\cdot H L\cdot H}{M}$$

New theory: new particle exchange with mass M



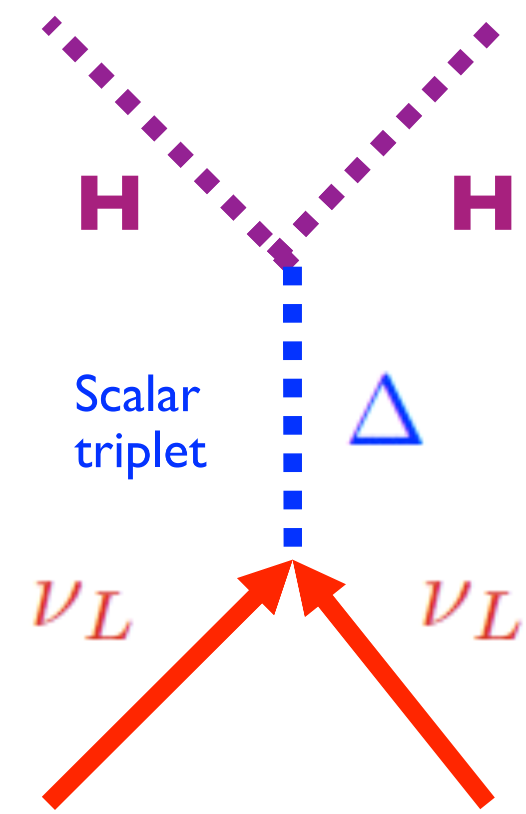
$$-\mathcal{L} = \lambda \frac{L \cdot H L \cdot H}{M}$$

See-saw Type I



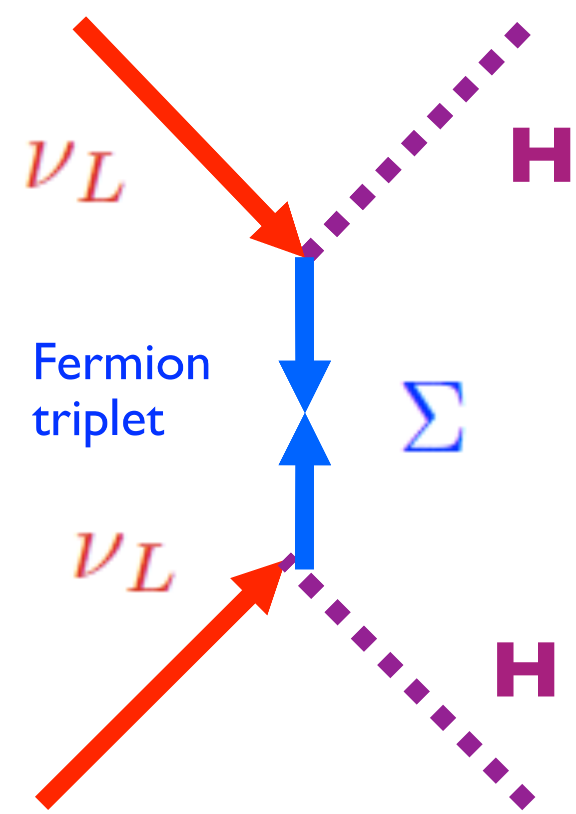
Minkowski, Yanagida, Glashow, Gell-Mann, Ramond, Slansky, Mohapatra, Senjanovic

See-saw Type II



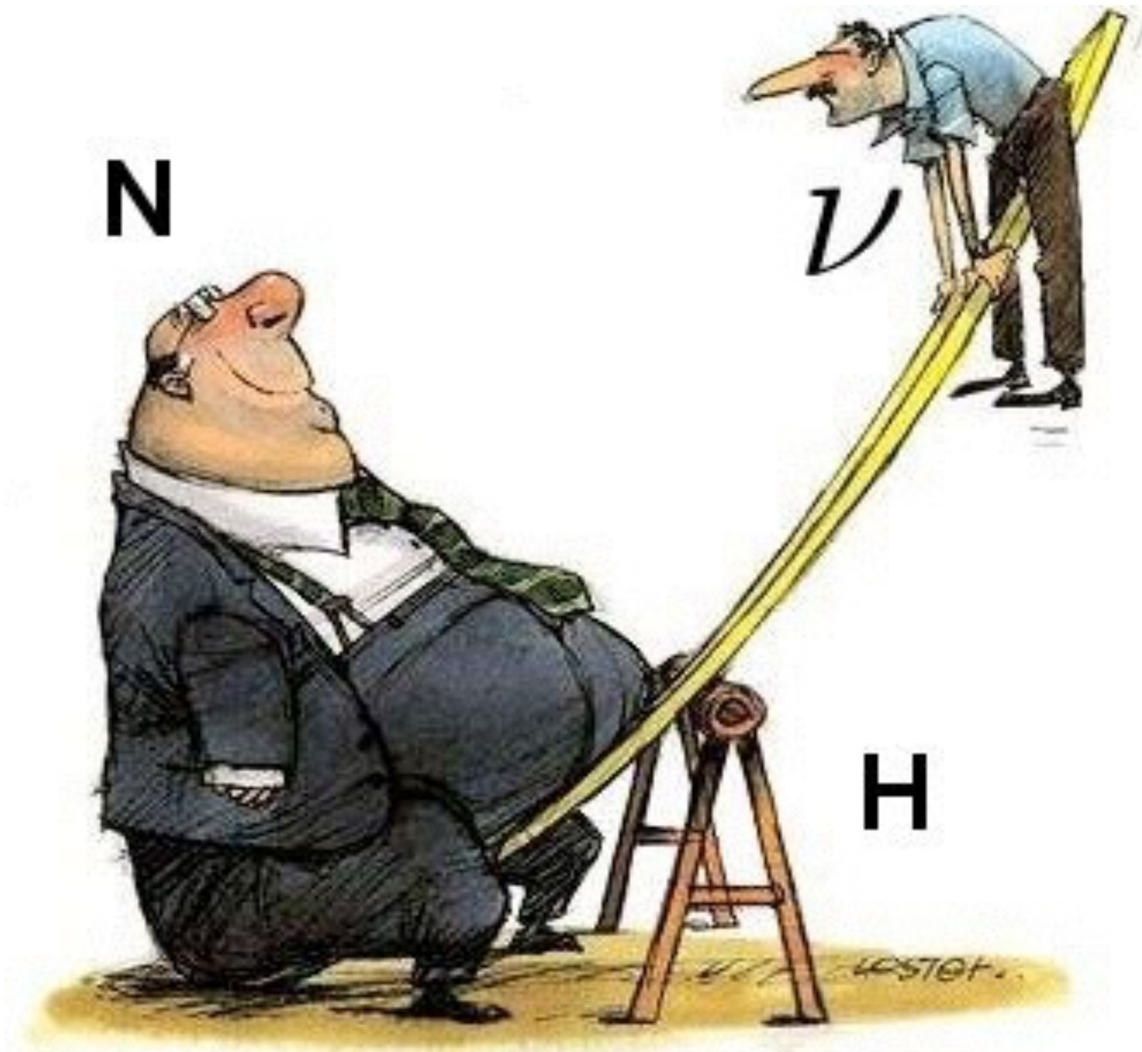
Magg, Wetterich, Lazarides, Shafi. Mohapatra, Senjanovic, Schecter, Valle

See-saw Type III



Ma, Roy, Senjanovic, Hambye

Models of neutrino masses BSM



See-saw type I

- Introduce a right handed neutrino **N (sterile neutrino)**
- Couple it to the Higgs and left handed neutrinos

breaks lepton number

The Lagrangian is

$$\mathcal{L} = -Y_\nu \bar{N} L \cdot H - 1/2 \bar{N}^c M_R N$$

When the Higgs boson gets a vev, Dirac masses will be generated. The mass matrix will be (for one generation)

$$\mathcal{L} = \left(\nu_L^T N^T\right) \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N \end{pmatrix}$$

This is of the Dirac+Majorana type we discussed earlier. So we know that the massive states are found by diagonalising the mass matrix and the massive states will be Majorana neutrinos.

$$\begin{vmatrix} -\lambda & m_D \\ m_D & M - \lambda \end{vmatrix} = 0$$

$$\lambda^2 - M\lambda - m_D^2 = 0$$

$$\lambda_{1,2} = \frac{M \pm \sqrt{M^2 + 4m_D^2}}{2} \simeq \begin{cases} M \\ \frac{M-M}{2} - \frac{4m_D^2}{4M} = -\frac{m_D^2}{M} \end{cases}$$

One massive state remains very heavy, the light neutrino masses acquires a tiny mass!

$$m_\nu \simeq \frac{m_D^2}{M} \sim \frac{1\,\mathrm{GeV}^2}{10^{10}\,\mathrm{GeV}} \sim 0.1\,\mathrm{eV}$$

Mixing between active neutrinos and heavy neutrinos will emerge but it will be typically very small

$$\tan 2\theta = \frac{2m_D}{M}$$

and can be related to neutrino masses $m_\nu \simeq \frac{m_D^2}{M} \simeq \sin^2\theta M$

Pros and cons of type I see-saw models

Pros:

- they explain "naturally" the smallness of neutrino masses.
- can be embedded in GUT theories!
- neutrino masses are a indirect test of GUT theories
- have several phenomenological consequences (depending on the mass scale), e.g. leptogenesis, LFV

Cons:

- the new particles are typically too heavy to be produced at colliders (but TeV scale see-saws)
- the mixing with the new states are tiny
- in general: difficult to test

Leptogenesis in see-saw models

There is evidence of the baryon asymmetry:

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.18 \pm 0.06) \times 10^{-10}$$

Planck, 1502.01589, AA 594

In order to generate it dynamically in the Early Universe, the Sakharov's conditions need to be satisfied:

- B (or L) violation;

$$X^c \to \bar{q}q \qquad X \to \bar{q}q$$
$$X \to \ell q \qquad X^c \to \bar{\ell}\bar{q}$$
$$X \to \bar{q}q \qquad X \to \ell q$$

- C, CP violation;

- departure from thermal equilibrium.

- At T>M, N are in equilibrium:

$$N \leftrightarrow \ell H \quad N \leftrightarrow \ell H \quad N \leftrightarrow \ell H$$
$$N \leftrightarrow \ell H \quad N \leftrightarrow \ell H$$

- At T<M, N drops out of equilibrium:

$$N \rightarrow \ell H \quad N \rightarrow \ell^c H^c \quad N \rightarrow \ell H$$
$$N \rightarrow \ell^c H^c \quad N \rightarrow \ell^c H^c$$

- A lepton asymmetry can be generated if

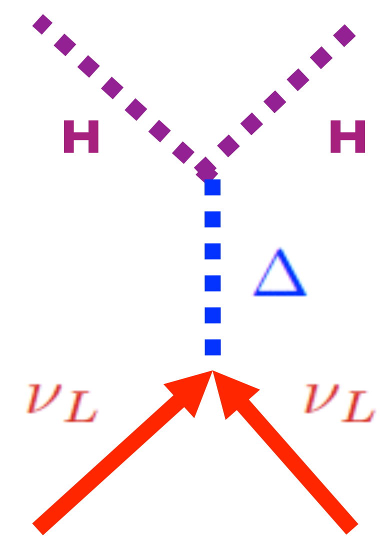
$$\Gamma(N \rightarrow \ell H) \neq \Gamma(N \rightarrow \ell^c H^c)$$

- $$\Delta L \xrightarrow{sphalerons} \Delta B$$

The observation of L violation and of CPV in the lepton sector would be a strong indication (even if not a proof) of leptogenesis as the origin of the baryon asymmetry.

See-saw type II

We introduce a Higgs triplet which couples to the Higgs and left handed neutrinos. It has hypercharge 2.



$$\mathcal{L}_\Delta \propto y_\Delta L^T C^{-1} \sigma_i \Delta_i L + \text{h.c.}$$

with $$\Delta_i = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix}$$

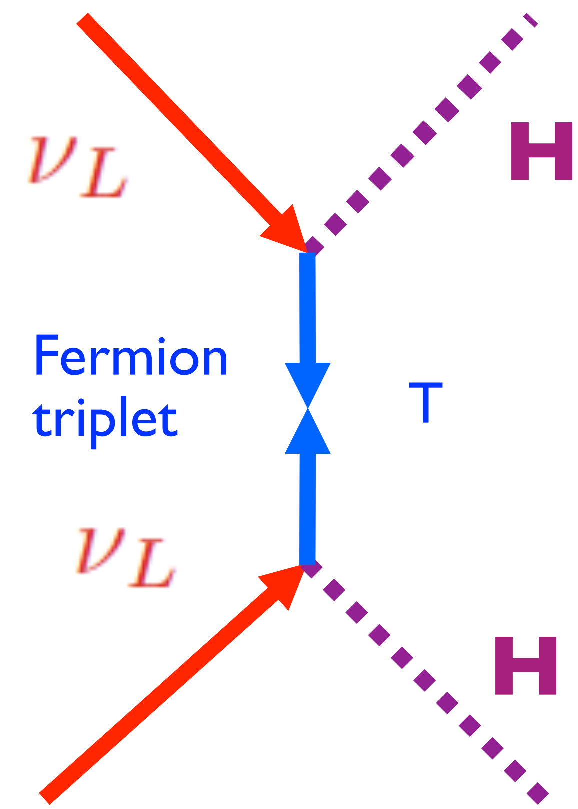
Once the Higgs triplet gets a vev, Majorana neutrino masses arise:

$$\boxed{m_\nu \sim y_\Delta v_\Delta}$$

Cons: why the vev is very small?
Pros: the component of the Higgs triplet could tested directly at the LHC.

See-saw type III

We introduce a fermionic triplet which has hypercharge 0.



$$\mathcal{L}_T \propto y_T \bar{L} \sigma H \cdot T + \text{h.c.}$$

with $T = \begin{pmatrix} T^0 & T^+ \\ T^- & -T^0 \end{pmatrix}$

Majorana neutrino masses are generated as in see-saw type I:

$$\boxed{m_\nu \simeq -y_T^T M_T^{-1} y_T v_H^2}$$

Pros: the component of the fermionic triplet have gauge interactions and can be produced at the LHC
Cons: why the mass of T is very large?

Extensions of the see saw mechanism

Models in which it is possible to lower the mass scale (e.g. TeV or below), keeping large Yukawa couplings have been studied. Examples: inverse and extended see-saw.

Let's introduce two right-handed singlet neutrinos.

$$\mathcal{L} = Y\bar{L}\cdot HN_1 + Y_2\bar{L}\cdot HN_2^c + \Lambda\bar{N}_1 N_2 + \mu' N_1^T C N_1 + \mu N_2^T C N_2$$

$$\begin{pmatrix} 0 & Yv & Y_2 v \\ Yv & \mu' & \Lambda \\ Y_2 v & \Lambda & \mu \end{pmatrix}$$

$$m_{tree} \simeq -m_D^T M^{-1} m_D \simeq \frac{v^2}{2(\Lambda^2 - \mu'\mu)} \left(\mu Y_1^T Y_1 + \epsilon^2 \mu' Y_2^T Y_2 - \Lambda\epsilon (Y_2^T Y_1 + Y_1^T Y_2)\right)$$

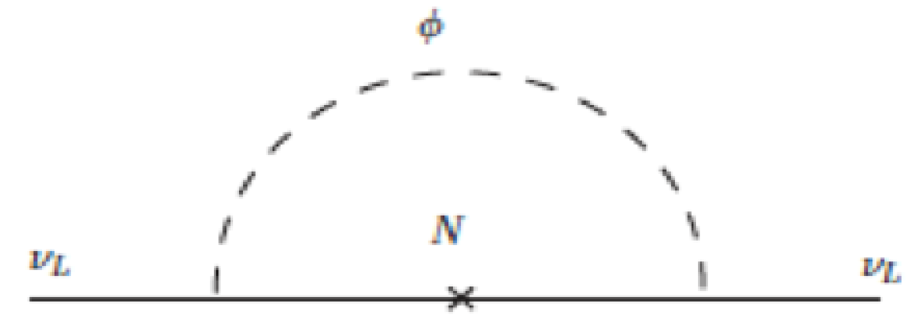
Small neutrino masses emerge due to cancellations between the contributions of the two sterile neutrinos (typically associated to small breaking of some L).

Other models of neutrino masses

Radiative masses

If neutrino masses emerge via loops, in models in which Dirac masses are forbidden, there is an additional suppression.

Some of these models have also dark matter candidates.



$$m_\nu \propto \frac{g^2}{16\pi^2} f(M, \mu_\phi^2)$$

See Ma, PRL81; also e.g. Boehm et al., PRD77; Zee-Babu model …

R-parity violating SUSY

In the MSSM, there are no neutrino masses. But it is possible to introduce terms which violate R (and L).

$$V = \ldots - \mu H_1 H_2 + \epsilon_i \tilde{L}_i H_2 + \lambda'_{ijk} \tilde{L}_i \tilde{L}_j \tilde{E}_k + \ldots$$

See e.g. Aulakh, Mohapatra, PLB119; Hall, Suzuki, NPB231; Ross, Valle, PLB151; Ellis et al., NPB261; Dawson, PRD57, ...

The bilinear term induces mixing between neutrinos and higgsino, the trilinear term masses at loop-level.

What is the new physics?

sub-eV eV keV MeV GeV TeV GUT scale

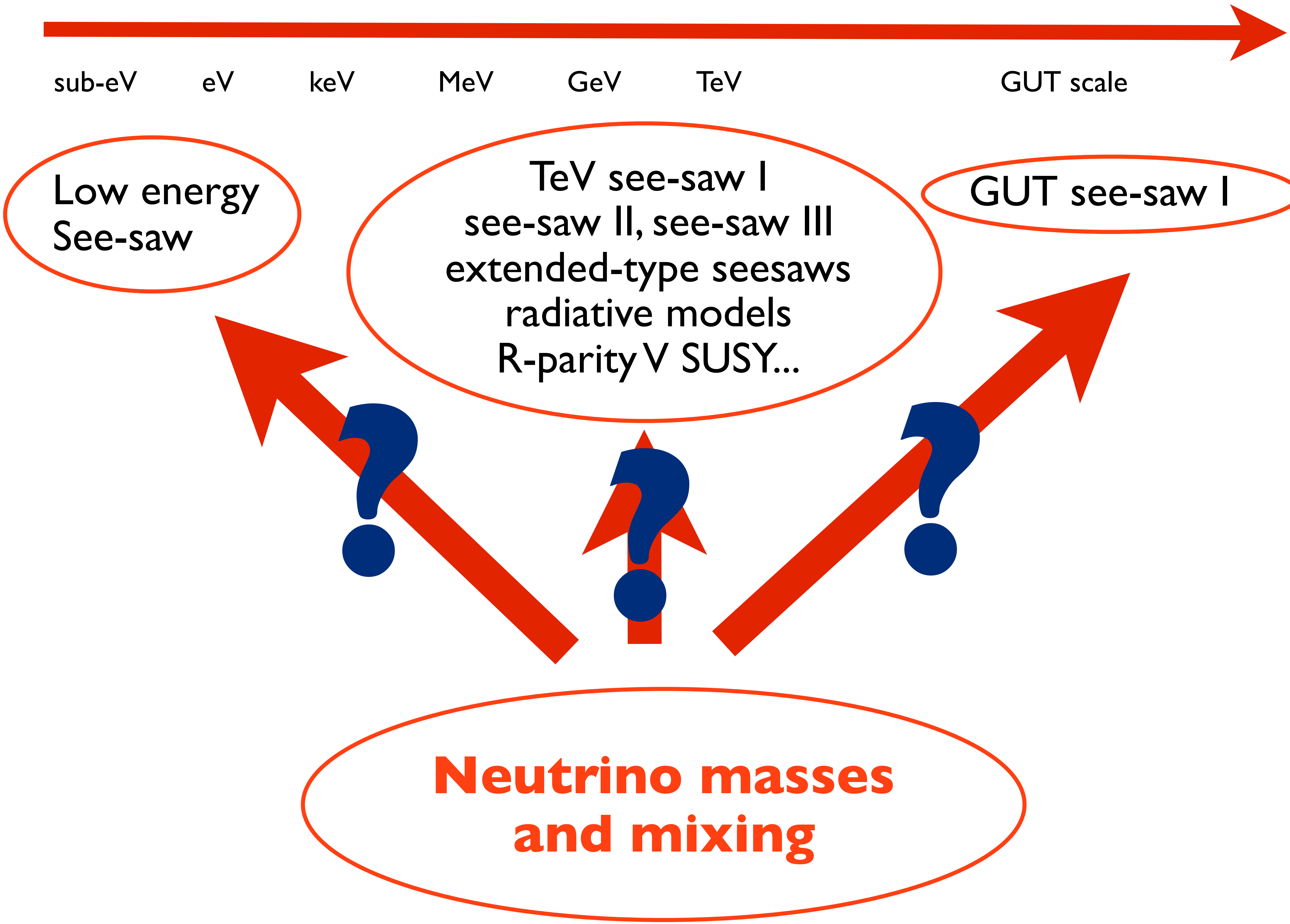
Low energy
See-saw

TeV see-saw I
see-saw II, see-saw III
extended-type seesaws
radiative models
R-parity V SUSY...

GUT see-saw I

Neutrino masses and mixing

What is the new physics?



Complementarity with other searches

There are many (direct and indirect) signatures of these extensions of the SM.

Neutrino masses

Signatures

Indirect signals (proton decay)

Peak searches

Direct signals in colliders

Charged lepton flavour violation

Kinks in beta decay

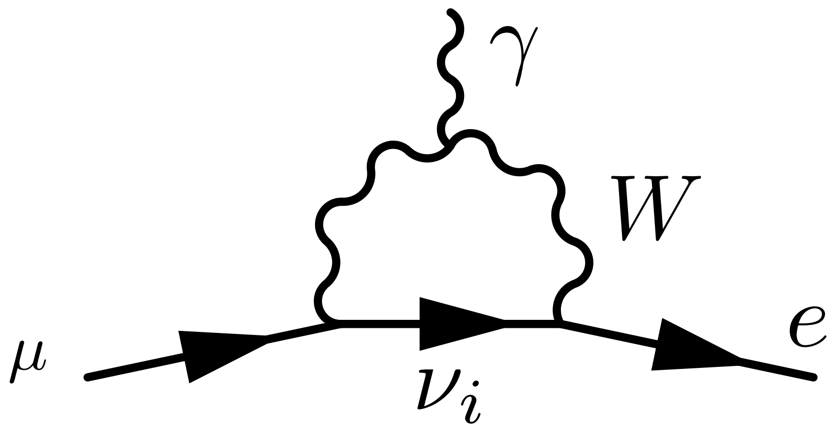
Nuless 2beta decay

Leptogenesis

Establishing the origin of neutrino masses requires to have as much information as possible about the masses and to combine it with other signatures of the models.

Charged lepton flavour violation

Neutrino masses induce very suppressed LFV processes.



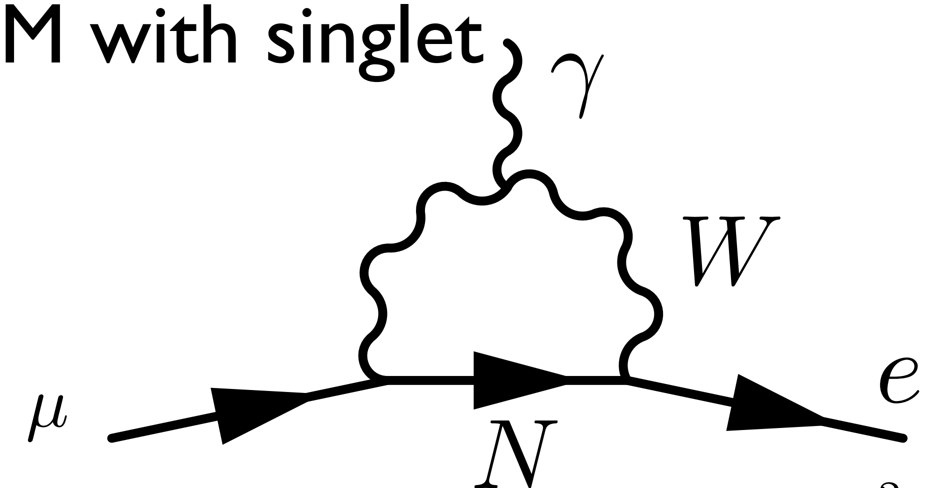
$$Br(\mu \to e\gamma)$$

$$\sim \frac{3\alpha}{32\pi}\left(\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta^2 m_{i1}}{m_W^2}\right)^2 \sim 10^{-53}$$

S. Petcov, SJNP 25 (1977)

Any observation of CLFV would show new physics BSM and provide clues on the origin of neutrino masses.

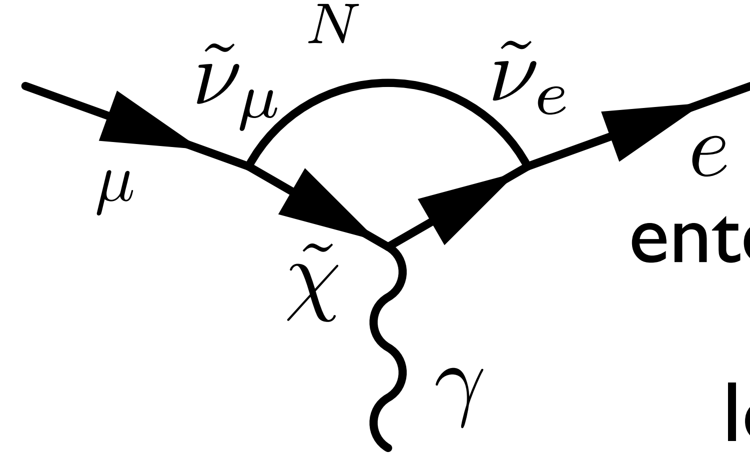
Example: extension of the SM with singlet N



$$Br(\mu \to e\gamma) \sim \frac{3\alpha}{8\pi}\left(\sum_j U^*_{\mu j} U_{ej}\, g\!\left(\frac{M_N^2}{m_W^2}\right)\right)^2$$

Example: SUSY see-saw

$$Br \propto \left|\sum_N Y^*_{N\mu} Y_{Ne} \ln(m_0/m_N)\right|^2$$



The same parameters enter in LFV, nu masses and leptogenesis.

Borzumati, Masiero, PRL 57

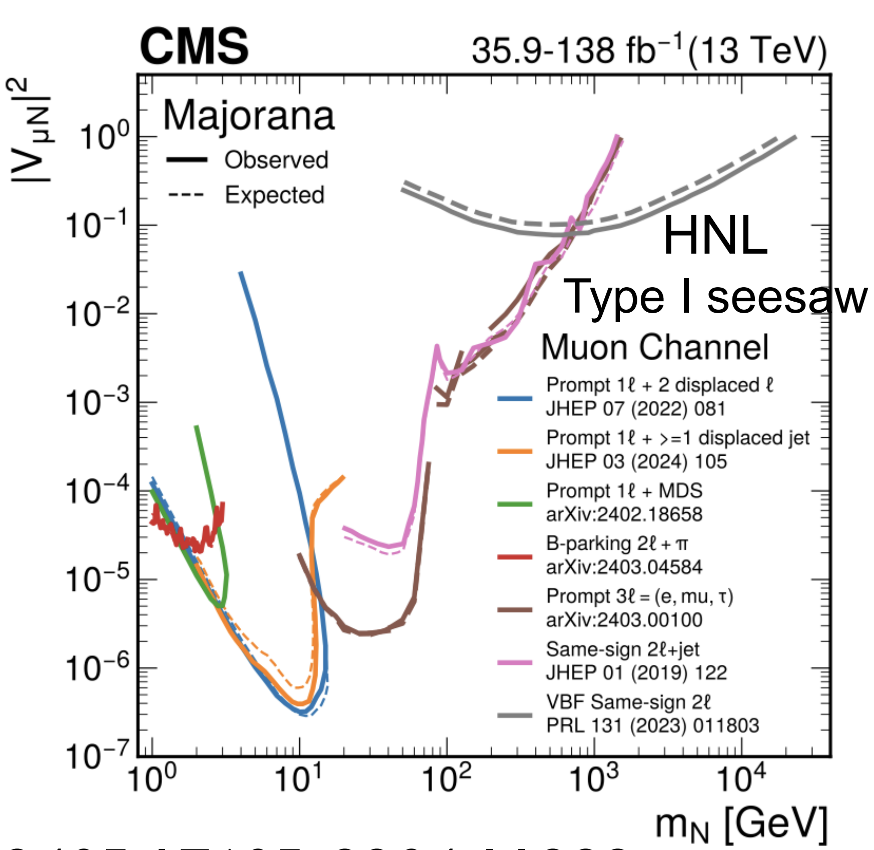
New physics scale? Going to high energy

eV keV MeV GeV **TeV** **Intermediate scale** **GUT scale**

TeV see-saw I, II, III, extended seesaws, radiative models, extra-D, R-parity V SUSY...

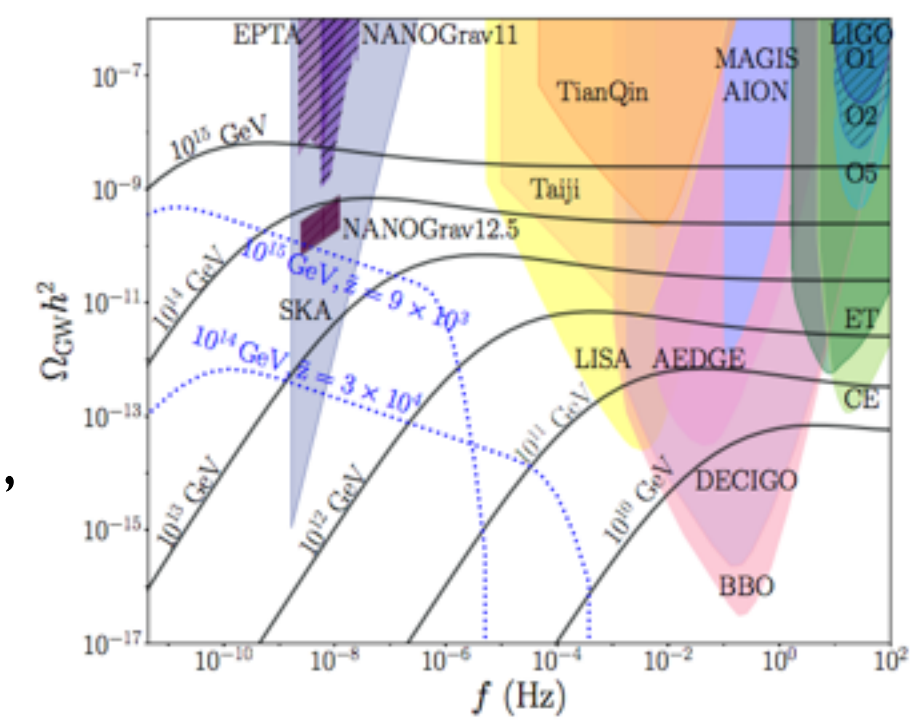
At the TeV scale, link with searches at colliders and meson/ tau decays.

Atre et al., 0901.3589



2405.17605, 2204.11988

S. King et al., PRL 126 (2021)

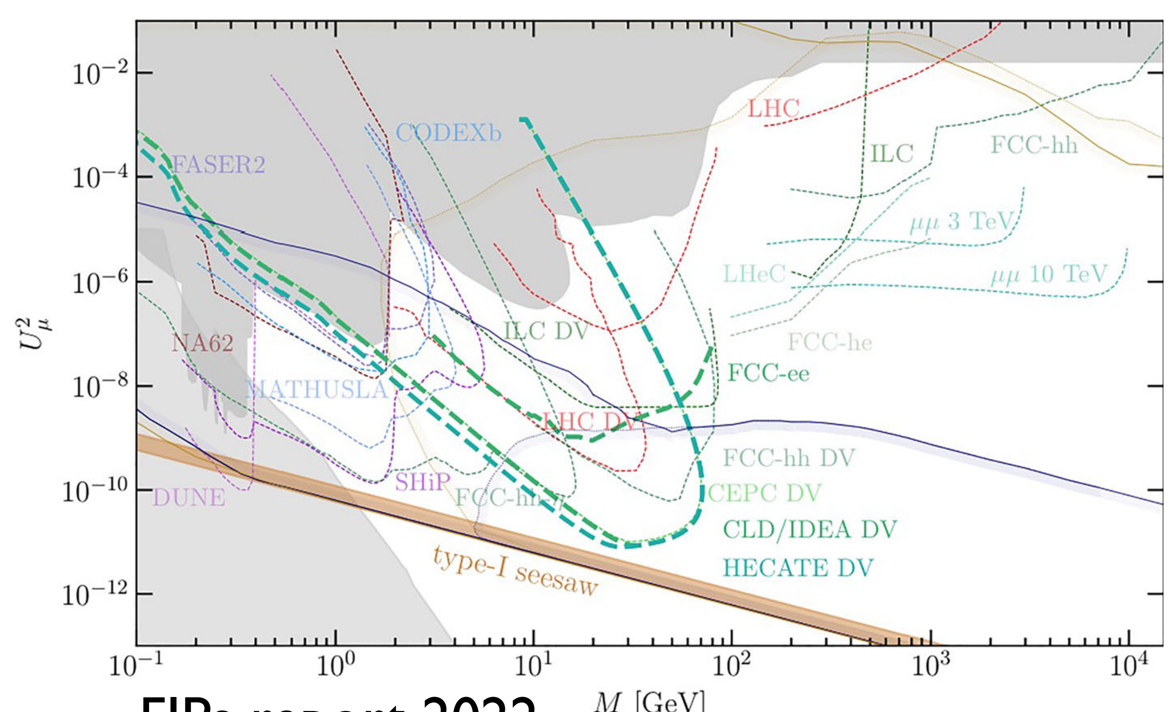


GUT theories lead to proton decay and to GW (due to cosmic strings from U(1) breaking).

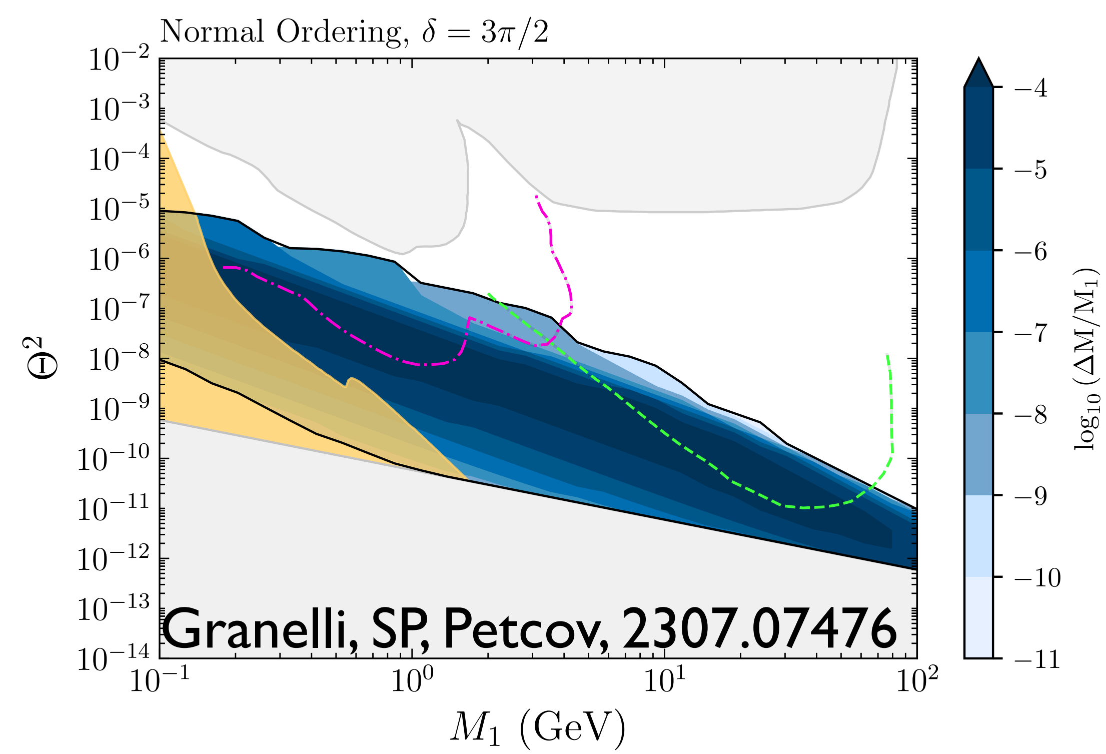
Going low in energy: Dark sectors

Dark sector (feebly interacting with SM below EW scale) have gained major attention in recent years.

E. K. Akhmedov, V. A. Rubakov, and A. Y. Smirnov; T. Asaka and M. Shaposhnikov; P. Hernandez, et al.; M. Drewes, et al; A. Granelli, K. Moffat, and S. T. Petcov,; J. Klaric, et al.,….



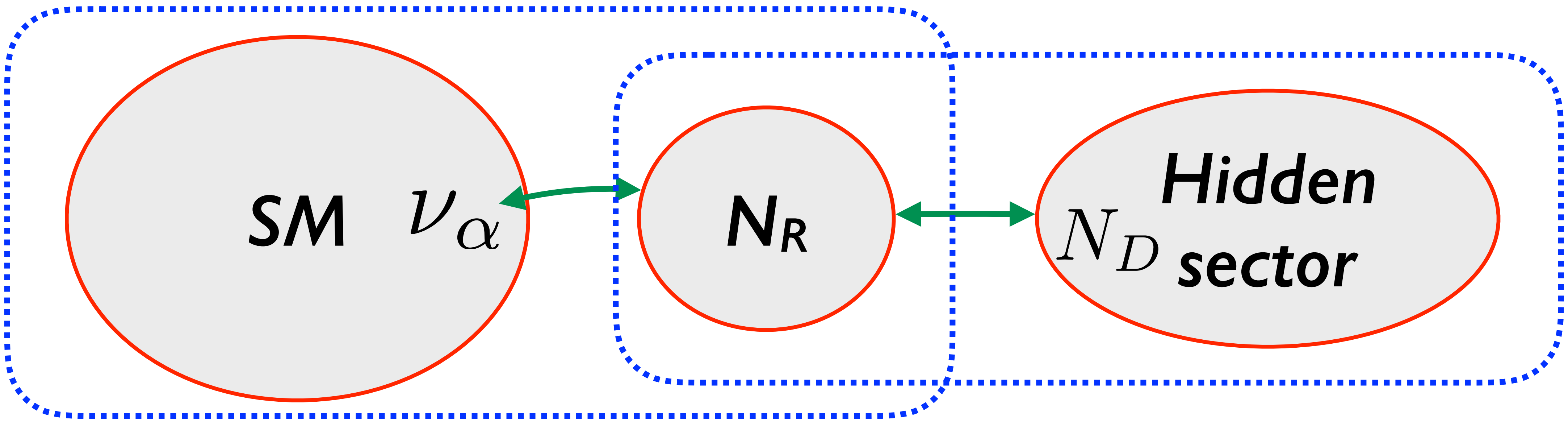
FIPs report 2022



How does the dark sector "talk" to the SM?

- mixing/Yukawa couplings
- new gauge interactions
- kinetic mixing
- new scalars via the Higgs portal
- EFTs

Neutrino portal $\bar{L} \cdot H N_R \quad (+ ... \overline{N_R} N_S)$

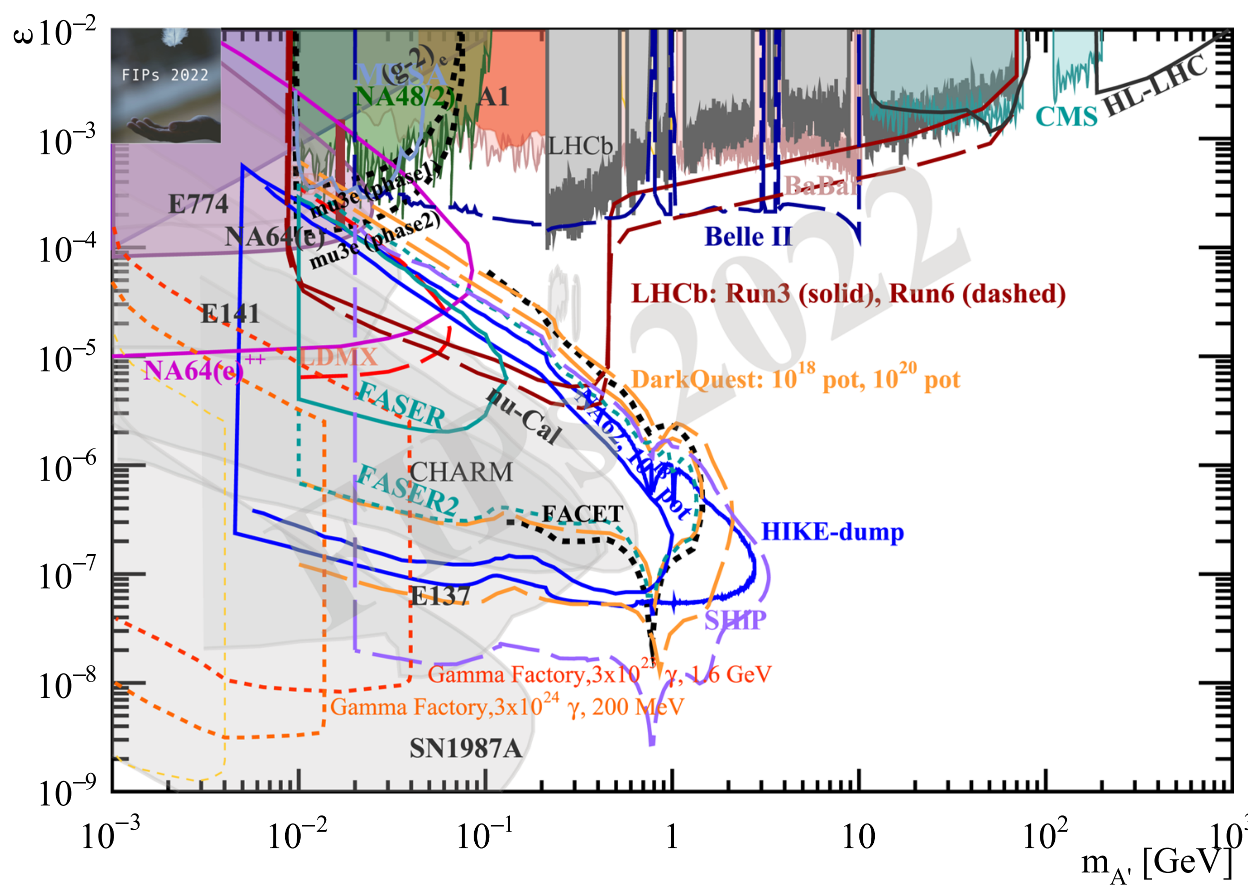


Higgs portal $\lambda_{\phi H} \phi^\dagger \phi H^\dagger H$

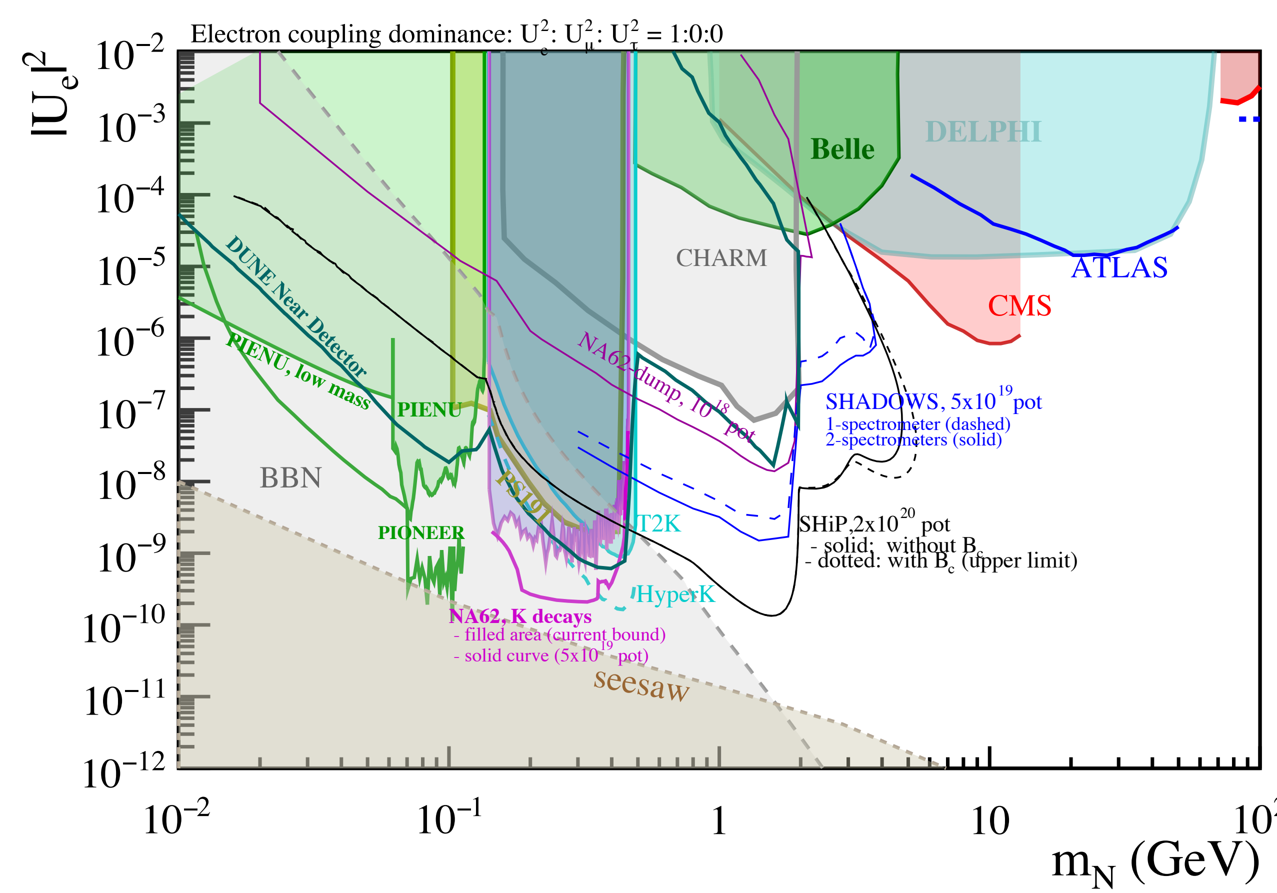
Vector portal $\frac{\sin \chi}{2} X_{\mu\nu} B^{\mu\nu}$

Very strong bounds have already been obtained and more sensitive experiments are ongoing/planned.

For very comprehensive review:
see FIPS reports 2020 and 2022.



FIPs report 2022



Adbullahi et al., 2203.08039

A rich dark sector

Non-minimality is strongly motivated: the SM is highly non-minimal and it exists.

E.g. P. Ballett, M. Hostert, SP, PRD 101 (2020)

U(1)'

$$\lambda_{\phi H} \phi^\dagger \phi \, H^\dagger H$$

$$\frac{\sin\chi}{2} X_{\mu\nu} B^{\mu\nu}$$

$$\bar{L} \cdot H N_R \quad (+ \ldots \overline{N_R} N_S)$$

Scalar sector
dark scalar needed to break U(1)'

Gauge sector
Z' (dark photon, A')

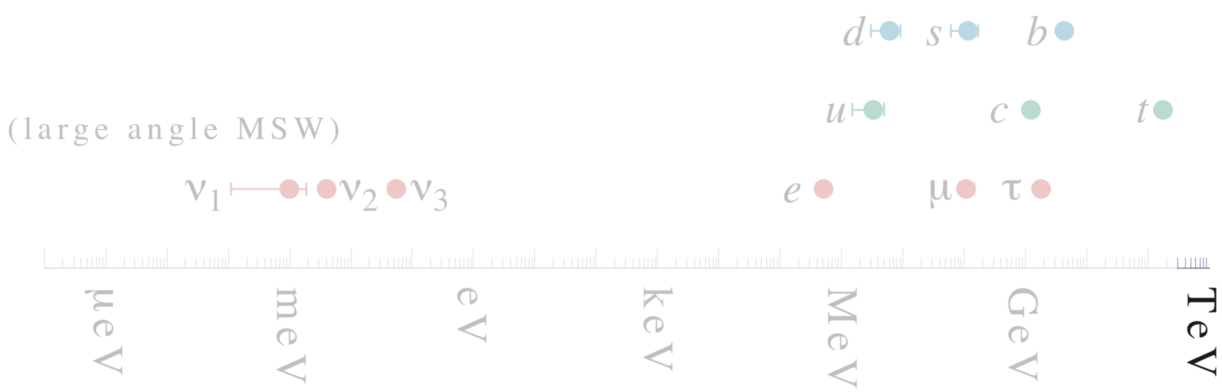
Fermion sector
dark neutrinos charged under the U(1)'

This type of structure is typical of rich dark sectors, that contain multiple particles and interactions.

Open window on Physics beyond the SM

Neutrinos give a new perspective on physics BSM.

1. Origin of masses



Why neutrinos have mass? and why are they so much lighter? and why their hierarchy is at most mild?

2. Problem of flavour

$$\begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

$$\begin{pmatrix} 0.8 & 0.5 & 0.16 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

Why leptonic mixing is so different from quark mixing?

This information is **complementary** with the one from **flavour physics experiments and from colliders**.

Why do we observe a specific pattern of mixing angles? Is there an underlying principle?

Masses and mixing from the mass matrix

Neutrino masses and the mixing matrix arises from the diagonalisation of the mass matrix

$$\boxed{M_M = (U^\dagger)^T m_{\rm diag} U^\dagger} \Rightarrow \boxed{n_L = U^\dagger \nu_L}$$

Theory

Experiments

Example. In the diagonal basis for the charged leptons

$$\mathcal{M}_\nu = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

the angle is $\tan 2\theta = \dfrac{2b}{a-c} \gg 1 \quad \text{for} \quad a \sim c \text{ and, or } a, c \ll b$

and masses $m_{1,2} \simeq \dfrac{a + c \pm 2b}{2}$

In a model of flavour, both the mass matrix for leptons and neutrinos will be predicted and need to be diagonalised:

$$(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) \mathcal{M}_\ell \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad (\bar{\nu}^c_{eL}, \bar{\nu}^c_{\mu L}, \bar{\nu}^c_{\tau L}) \mathcal{M}_\nu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

$$(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) V_L V_L^\dagger \mathcal{M}_\ell V_R V_R^\dagger \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad (\bar{\nu}^c_{eL}, \bar{\nu}^c_{\mu L}, \bar{\nu}^c_{\tau L}) U_\nu^* U_\nu^T \mathcal{M}_\nu U_\nu U_\nu^\dagger \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

$$(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \mathcal{M}_{\rm diag} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \qquad (\bar{\nu}^c_{1L}, \bar{\nu}^c_{2L}, \bar{\nu}^c_{3L}) \mathcal{M}_{{\rm diag},\nu} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

In a model of flavour, both the mass matrix for leptons and neutrinos will be predicted and need to be diagonalised:

$$(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L)\mathcal{M}_\ell \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad (\bar{\nu}^c_{eL}, \bar{\nu}^c_{\mu L}, \bar{\nu}^c_{\tau L})\mathcal{M}_\nu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

$$(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L)V_L V_L^\dagger \mathcal{M}_\ell V_R V_R^\dagger \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad (\bar{\nu}^c_{eL}, \bar{\nu}^c_{\mu L}, \bar{\nu}^c_{\tau L})U_\nu^* U_\nu^T \mathcal{M}_\nu U_\nu U_\nu^\dagger \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

$$(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L)\mathcal{M}_{\mathrm{diag}} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \qquad (\bar{\nu}^c_{1L}, \bar{\nu}^c_{2L}, \bar{\nu}^c_{3L})\mathcal{M}_{\mathrm{diag},\nu} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

in the CC interactions (and oscillations):

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L)\gamma^\mu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} W_\mu \Rightarrow \frac{g}{\sqrt{2}}(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L)\gamma^\mu U_{\mathrm{osc}} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} W_\mu$$

$$\boxed{U_{\mathrm{osc}} = V_L^\dagger U_\nu}$$

Phenomenological approaches

Various strategies and ideas can be employed to understand the observed pattern (many many models!).

- Mixing related to mass ratios

$$\theta_{12,23,13} = \text{function}(\underbrace{\frac{m_e}{m_\mu}}_{\text{too small}}, \ldots, \frac{m_1}{m_2})$$

- Flavour symmetries

- Complementarity between quarks and leptons

$$\theta_{12} + \theta_C \simeq 45^o$$

- Anarchy (all elements of the matrix of the same order).

Example I: mu-tau symmetry

Large theta23 motivates to consider the mu-tau symmetry.

$$\mathcal{M}_\nu = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

The mixing is given by $\tan 2\theta = \frac{2b}{0} = \infty \Rightarrow \theta_{23} = 45^o$

For 3 generations, this mass matrix respects the symmetry

$$\mathcal{M}_\nu = \sqrt{\Delta m_A^2} \begin{pmatrix} \sim 0 & a\epsilon & a\epsilon \\ a\epsilon & 1+\epsilon & 1 \\ a\epsilon & 1 & 1+\epsilon \end{pmatrix}$$

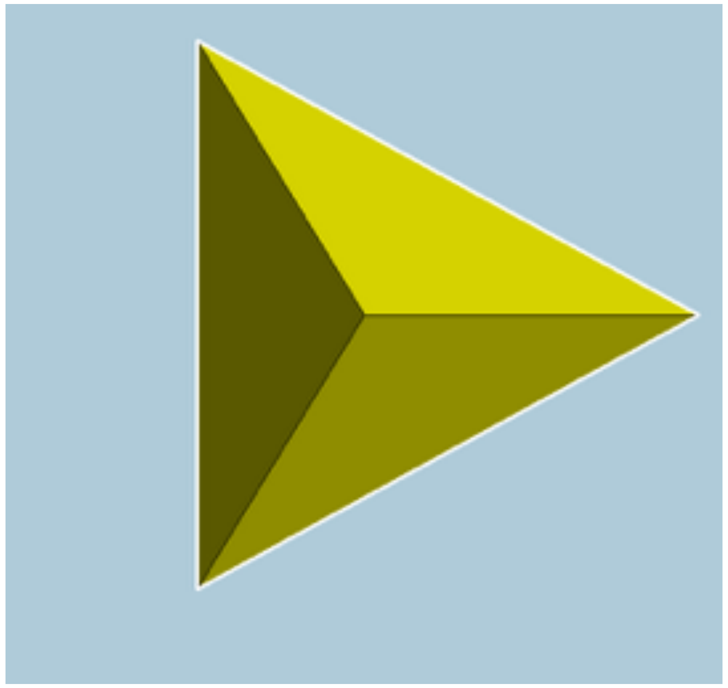
leading to $\theta_{23} = \frac{\pi}{4} - \frac{\Delta m_\odot^2}{\Delta m_A^2}$ $\qquad \theta_{13} \sim \epsilon^2 \sim \frac{\Delta m_\odot^2}{\Delta m_A^2} \sim 0.04$

The large value of theta13 needs more corrections.

Example 2: a discrete symmetry A4

An example of discrete symmetry: Z2 (reflections).

A4 is the group of even permutations of (1234).
This is a very studied example of discrete symmetry.
It is the invariant group of a tetrahedron.



There are 12 elements and it has the following representations: 1, 1', 1", and 3.

We need to assign fermions to the representations:

$$L \to 3$$
$$e_R \to 1$$
$$\mu_R \to 1'$$
$$\tau_R \to 1''$$

As usual, masses require the "product" of two fermions:

$$1' \times 1' = 1''$$
$$1'' \times 1'' = 1'$$
$$1' \times 1'' = 1$$
$$3 \times 3 = 1 + 1' + 1'' + 3 + 3$$

In order to break the symmetry, scalars (called 'flavons') are needed: $\phi(3), \quad \phi'(3), \quad \xi(1)$

Requiring that the Lagrangian is invariant w.r.t. the flavour symmetry, the allowed interactions are fixed:

$$\mathcal{L} = y_e \bar{e}_R (\phi L) \frac{H_d}{\Lambda} + y_\mu \bar{\mu}_R (\phi L) \frac{H_d}{\Lambda} + y_\tau \bar{\tau}_R (\phi L) \frac{H_d}{\Lambda} + j_a \xi (LL) \frac{H_u H_u}{\Lambda^2} + j_b (\phi' LL) \frac{H_u H_u}{\Lambda^2}$$

1 $(33)_1$ 1' $(33)_{1''}$ 1'' $(33)_{1'}$ 1 $(33)_1$ $(333)_1$

The flavons get a vev

$$\langle \phi \rangle = (v, v, v) \quad \langle \phi' \rangle = (v', 0, 0) \quad \langle \xi \rangle = u$$

and the resulting mass matrices are

$$M_l = v \frac{v_{Hd}}{\Lambda} \begin{pmatrix} y_e & y_e & y_e \\ y_\mu & y_\mu e^{i4\pi/3} & y_\mu e^{i2\pi/3} \\ y_\tau & y_\tau e^{i2\pi/3} & y_\tau e^{i4\pi/3} \end{pmatrix} \qquad M_\nu = \frac{v_u^2}{\Lambda^2} \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}$$

Finally, the two matrices can be diagonalised and the resulting mixing matrix is found.

Summary and Conclusions

Neutrino oscillations imply that they have mass and mix.

Since their discovery, neutrino oscillations have provided us with a precise picture of neutrino properties.

Key open questions remain open and a wide experimental programme is ongoing/planned to answer them: neutrino nature, leptonic CPV, neutrino masses, precise measurement of mixing parameters, tests of standard 3-neutrino mixing paradigm.

The ultimate goal is to understand the physics BSM at the origin of neutrino masses and leptonic flavour, uncovering a New SM.