

# Neutrino physics

5-6 May 2025

**NDM 2025**

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# Lecture 2: Current picture and open questions

## Aims

Apply neutrino oscillations formalism to exp and discuss the current knowledge of neutrinos and the open questions

## Outline

Neutrino oscillations in matter

- constant density case

- varying density

- solar neutrino oscillations and the MSW effect

Brief review of current and past neutrino oscillation experiments

Open questions

- Nature of neutrinos

- Neutrino masses

- CP violation

# RECAP

## Neutrino mixing

Mixing is described by the *Pontecorvo-Maki-Nakagawa-Sakata* matrix:  $\nu_{\alpha} = \sum_i U_{\alpha i} \nu_i$

Flavour field  $\nu_{\alpha}$  ← Mass field  $\nu_i$

which enters in the CC interactions

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{k\alpha} (U_{\alpha k}^* \bar{\nu}_{kL} \gamma^{\rho} l_{\alpha L} W_{\rho} + \text{h.c.})$$

This implies that in an interaction with an electron, the corresponding (anti-)neutrino will be produced, as a superposition of different mass eigenstates.

# Implications of the existence of neutrino oscillations

The oscillation probability is given by

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| \sum_i U_{\alpha i}^* U_{\beta i} e^{-i \frac{\Delta m_{i1}^2}{2E} L} \right|^2$$

- **neutrinos have mass** (as the different components of the initial state need to propagate with different phases)
- **neutrinos mix** (as U needs not be the identity. If they do not mix the flavour eigenstates are also eigenstates of the propagation Hamiltonian and they do not evolve)

## 3-neutrino oscillations

They depend on two mass squared-differences

$$\Delta m_{21}^2 \ll \Delta m_{31}^2$$

In general the formula is quite complex

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} e^{-i \frac{\Delta m_{21}^2}{2E} L} + U_{\alpha 3}^* U_{\beta 3} e^{-i \frac{\Delta m_{31}^2}{2E} L} \right|^2$$

### 2-neutrino limits relevant for experiments

For a given L, the neutrino energy determines the impact of a mass squared difference. Various limits are of interest in concrete experimental situations. Using the current values

$$\Delta m_{21}^2 = 7.5 \times 10^{-5} \text{eV}^2 \quad \Delta m_{31}^2 = 2.5 \times 10^{-3} \text{eV}^2$$

Type of neutrinos	Average E	(Typical) L	$\frac{\Delta m_{21}^2}{4E} L$	$\frac{\Delta m_{31}^2}{4E} L$
Atmospheric neutrinos	(0.1-100 GeV) 10 GeV	(10-12000 km) 6000 km	0.06	1.9
Reactor neutrinos I	3 MeV	1 km	0.03	1.1
Reactor neutrinos II	3 MeV	100 km	3.2	106
Accelerator (T2K)	0.7 GeV	300 km	0.04	1.36
Accelerator (DUNE)	3 GeV	1300 km	0.04	1.37
SBL	0.8 GeV	500 m	0.00006	0.001

# *Neutrinos oscillations in matter*

- When neutrinos travel through a medium, they interact with the background of electron, proton and neutrons and acquire an effective mass.
- This modifies the mixing between flavour states and propagation states and the eigenvalues of the Hamiltonian, leading to a different oscillation probability w.r.t. vacuum.
- Typically the background is CP and CPT violating, e.g. the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations are CP and CPT violating.

## Effective potentials

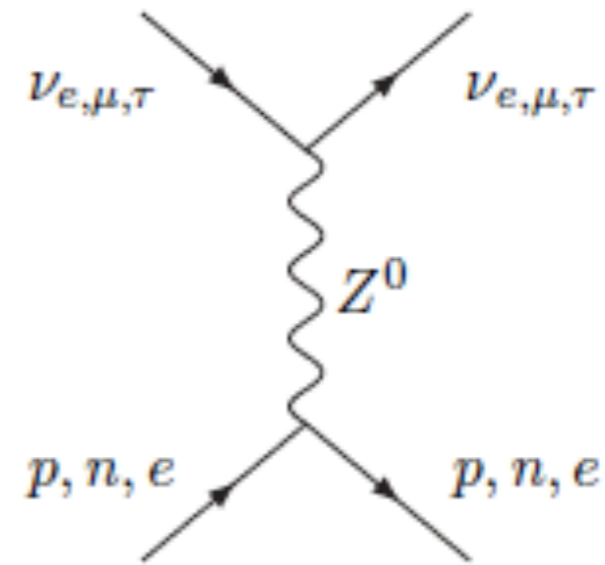
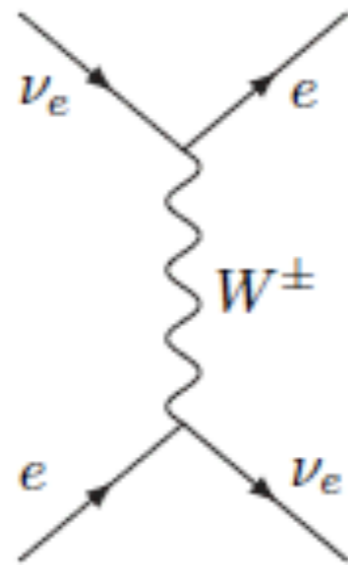
Inelastic scattering and absorption processes go as  $G_F^2$  and are typically negligible. An order of magnitude estimate for the interaction cross section is

$$\sigma_\nu \sim G_F^2 EM \sim 10^{-38} \text{ cm}^2 \frac{EM}{\text{GeV}^2},$$

For the Earth density, the mean free path of a 1 GeV neutrino is  $10^{14}$  cm. The Earth becomes opaque to neutrinos only above 100 TeV.

Neutrinos undergo also **forward elastic scattering**, in which they do not change momentum. [L. Wolfenstein, Phys. Rev. D 17, 2369 (1978); ibid. D 20, 2634 (1979), S. P. Mikheyev, A. Yu Smirnov, Sov. J. Nucl. Phys. 42 (1986) 913.]

Electron neutrinos have CC and NC interactions, while muon and tau neutrinos only the latter.



For a useful discussion, see E. Akhmedov, hep-ph/0001264; A. de Gouvea, hep-ph/0411274.

The effective Hamiltonian density for CC interactions is

$$H_{CC} = 2\sqrt{2}G_F[\bar{e}\gamma_\mu P_L\nu_e][\bar{\nu}_e\gamma^\mu P_L e] = 2\sqrt{2}G_F[\bar{\nu}_e\gamma^\mu P_L\nu_e][\bar{e}\gamma_\mu P_L e]$$

Fierz transformation

We treat the electrons as a background, averaging over it and we take into account that neutrinos see only the left-handed component of the electrons.

$$\langle \bar{e}\gamma_0 e \rangle = N_e \quad \langle \bar{e}\vec{\gamma}e \rangle = \langle \vec{v}_e \rangle \quad \langle \bar{e}\gamma_0\gamma_5 e \rangle = \left\langle \frac{\vec{\sigma}_e \cdot \vec{p}_e}{E_e} \right\rangle \quad \langle \bar{e}\vec{\gamma}\gamma_5 e \rangle = \langle \vec{\sigma}_e \rangle$$

For an unpolarised at rest background, the only term is the first one.  $N_e$  is the electron density.

The neutrino dispersion relation can be found by solving the Dirac eq with plane waves, in the ultrarelativistic limit

$$E \simeq p \pm \sqrt{2}G_F N_e \mp \sqrt{2}G_F N_n / 2$$

## The Hamiltonian

Let's start with the vacuum Hamiltonian for 2-neutrinos

$$i \frac{d}{dt} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

Recalling that  $|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle$ , one can go into the flavour basis

$$\begin{aligned} i \frac{d}{dt} \begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} &= U \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} U^\dagger \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} \\ &= \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} \end{aligned}$$

We have neglected common terms on the diagonal as they amount to an overall phase in the evolution.

The **full Hamiltonian in matter** can then be obtained by adding the potential terms, diagonal in the flavour basis.

$$H^m = H^0 + \text{diag}(V_e, V_\mu, V_\tau)$$

For electron and muon neutrinos

$$i \frac{d}{dt} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

For antineutrinos the potential has the opposite sign.

In general the evolution is a complex problem but there are few cases in which analytical or semi-analytical results can be obtained.

## 2-neutrino case in constant density

$$i \frac{d}{dt} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2} G_F N_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

If the electron density is constant (a good approximation for oscillations in the Earth crust), it is easy to solve. We need to diagonalise the Hamiltonian.

- Eigenvalues:

$$E_A - E_B = \sqrt{\left( \frac{\Delta m^2}{2E} \cos(2\theta) - \sqrt{2} G_F N_e \right)^2 + \left( \frac{\Delta m^2}{2E} \sin(2\theta) \right)^2}$$

- The diagonal basis and the flavour basis are related by a unitary matrix with **angle in matter**

Exercise  
Derive

$$\tan(2\theta_m) = \frac{\frac{\Delta m^2}{2E} \sin(2\theta)}{\frac{\Delta m^2}{2E} \cos(2\theta) - \sqrt{2} G_F N_e}$$

• If  $\sqrt{2}G_F N_e \ll \frac{\Delta m^2}{2E} \cos 2\theta$ , we recover the vacuum case and  $\theta_m \simeq \theta$

• If  $\sqrt{2}G_F N_e \gg \frac{\Delta m^2}{2E} \cos(2\theta)$ , matter effects dominate and oscillations are suppressed.

• If  $\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E} \cos 2\theta$ : resonance and maximal mixing

$$\theta_m = \pi/4$$

• The resonance condition can be satisfied for

- neutrinos if  $\Delta m^2 > 0$
- antineutrinos if  $\Delta m^2 < 0$

$$P(\nu_e \rightarrow \nu_\mu; t) = \sin^2(2\theta_m) \sin^2 \frac{(E_A - E_B)L}{2}$$

Matter effects modify the oscillation probability in LBL experiments.

$$P_{\nu_{\mu} \rightarrow \nu_e} = \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta m_{13}^m L}{2}$$

The probability enhancement happens for

- neutrinos if  $\Delta m^2 > 0$
- antineutrinos if  $\Delta m^2 < 0$

The impact of matter effects is stronger at higher energies and at longer baselines.

## 2-neutrino oscillations with varying density

Let's consider the case in which  $N_e$  depends on time. This happens, e.g., if a beam of neutrinos is produced and then propagates through a medium of varying density (e.g. Sun, supernovae).

$$i \frac{d}{dt} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2} G_F N_e(t) & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

At a given instant of time  $t$ , the Hamiltonian can be diagonalised by a unitary transformation as before. We find the **instantaneous matter basis and the instantaneous values of the energy**. The expressions are exactly as before but with the angle which depends on time,  $\theta(t)$ .

We have

$$|\nu_\alpha\rangle = U(t)|\nu_I\rangle, \quad U^\dagger(t)H_{m,fl}U(t) = \text{diag}(E_A(t), E_B(t))$$

Starting from the Schroedinger equation, we can express it in the instantaneous basis

$$i\frac{d}{dt}U_m(t) \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2}G_F N_e(t) & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} U_m(t) \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

$$i\frac{d}{dt} \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} E_A(t) & -i\dot{\theta}(t) \\ i\dot{\theta}(t) & E_B(t) \end{pmatrix} \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

The evolution of  $\nu_A$  and  $\nu_B$  are not decoupled. In general, it is very difficult to find an analytical solution to this problem.

## Adiabatic case

In the adiabatic case, each component evolves independently. In the non adiabatic one, the state can “jump” from one to the other.

If the evolution is sufficiently slow (adiabatic case):

$$|\dot{\theta}(t)| \ll |E_A - E_B|$$

we can follow the evolution of each component independently.

## Adiabaticity condition

$$\gamma^{-1} \equiv \frac{2|\dot{\theta}|}{|E_A - E_B|} = \frac{\sin(2\theta) \frac{\Delta m^2}{2E}}{|E_A - E_B|^3} |\dot{V}_{CC}| \ll 1$$

In the Sun, typically we have

$$\gamma \sim \frac{\Delta m^2}{10^{-9} \text{eV}^2} \frac{\text{MeV}}{E_\nu}$$

## Solar neutrinos: MSW effect

The oscillations in matter were first discussed by L. Wolfenstein, S. P. Mikheyev, A. Yu Smirnov.

- Production in the centre of the Sun: matter effects dominate at high energy, negligible at low energy.

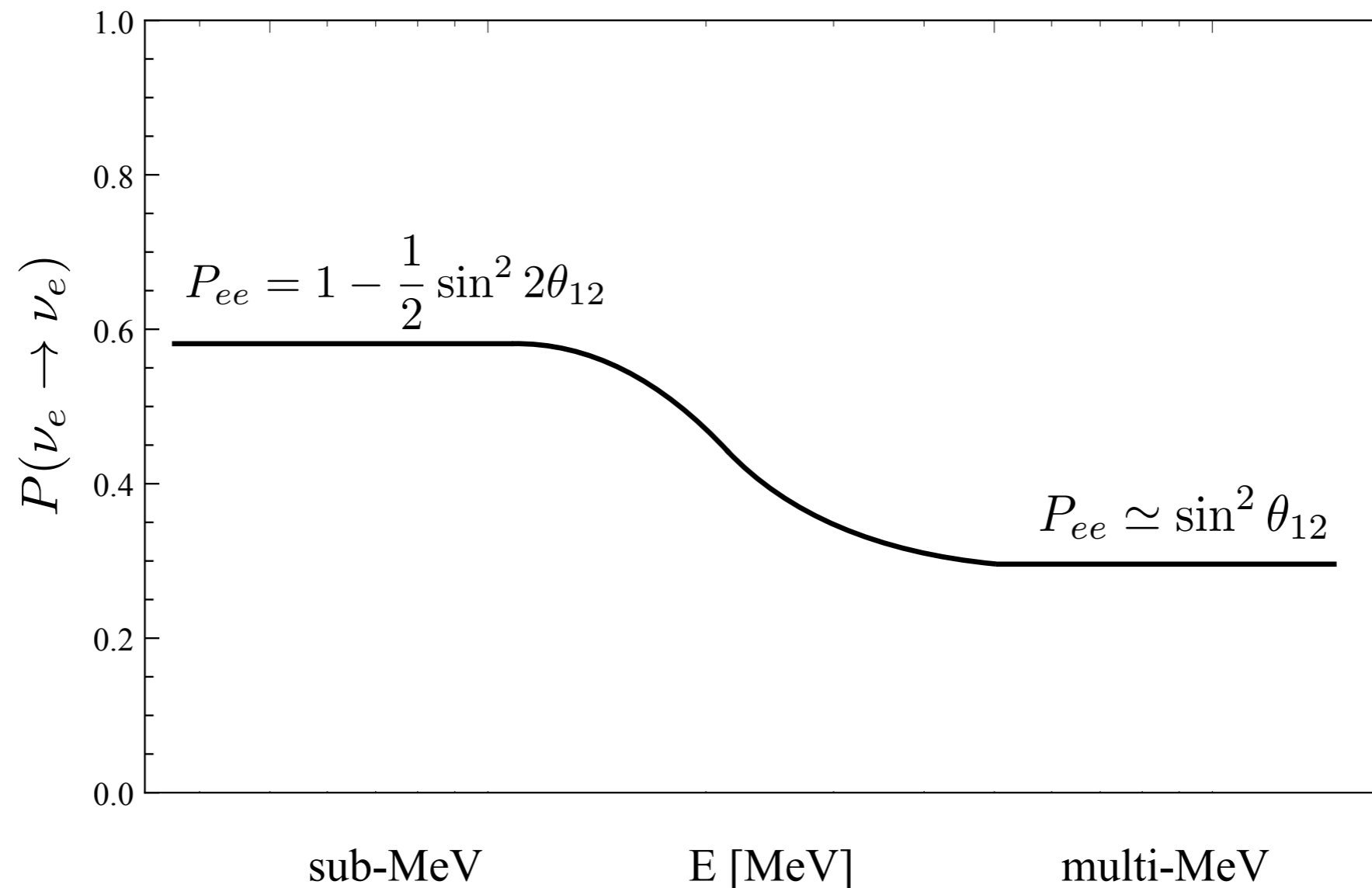
The probability of  $\nu_e$  to be  $\nu_A$  is  $\cos^2 \theta_m$   
 $\nu_B$  is  $\sin^2 \theta_m$

If matter effects dominate,  $\sin^2 \theta_m \simeq 1$  corresponding to the heavier state. If the evolution is adiabatic, the state will continue to be the heavier one.

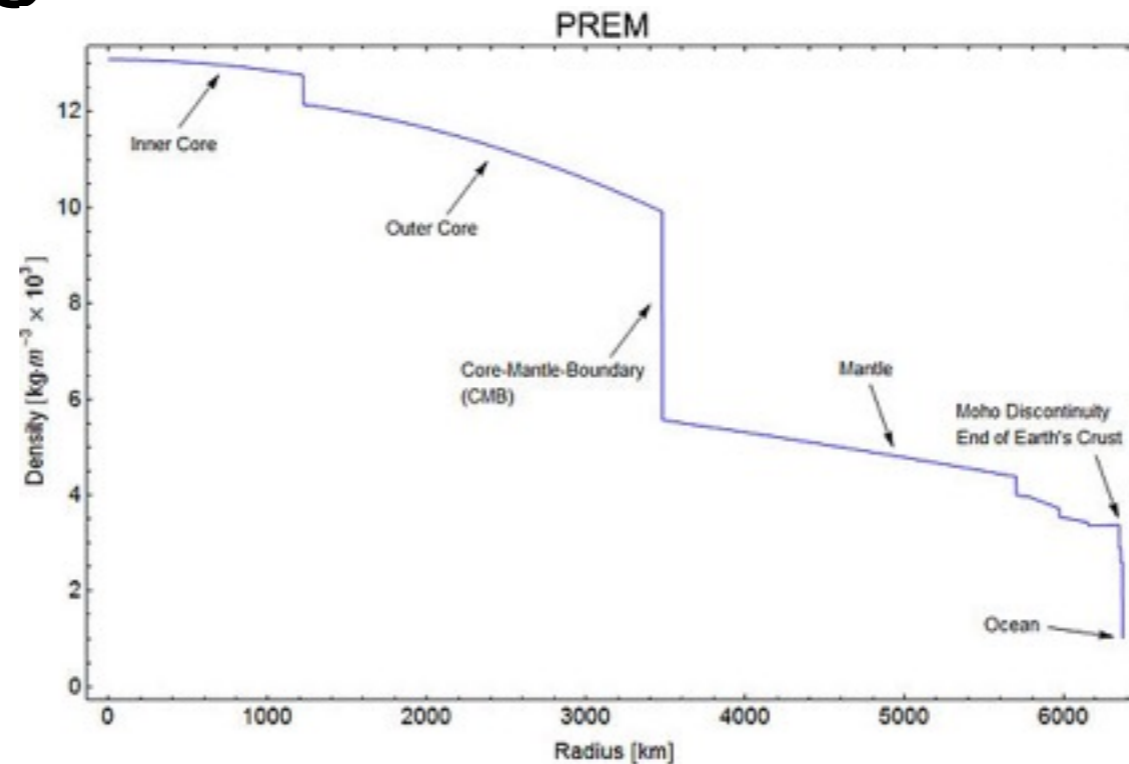
At the Sun surface, the state will be a mass state  $\nu_2$ . To find the oscillation probability

$$P(\nu_e \rightarrow \nu_e) = |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2 \theta$$

- $P(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2} \sin^2(2\theta)$  (averaged vacuum oscillations), when matter effects are negligible (low energies)
- $P(\nu_e \rightarrow \nu_e) = \sin^2 \theta$  (dominant matter effects and adiabaticity) (high energies)



**Neutrino oscillations in Earth:** Matter effects are relevant also for atmospheric neutrinos. The profile of the Earth induces interesting effects that can be exploited to gain information on the neutrino mass ordering.



Wikipedia

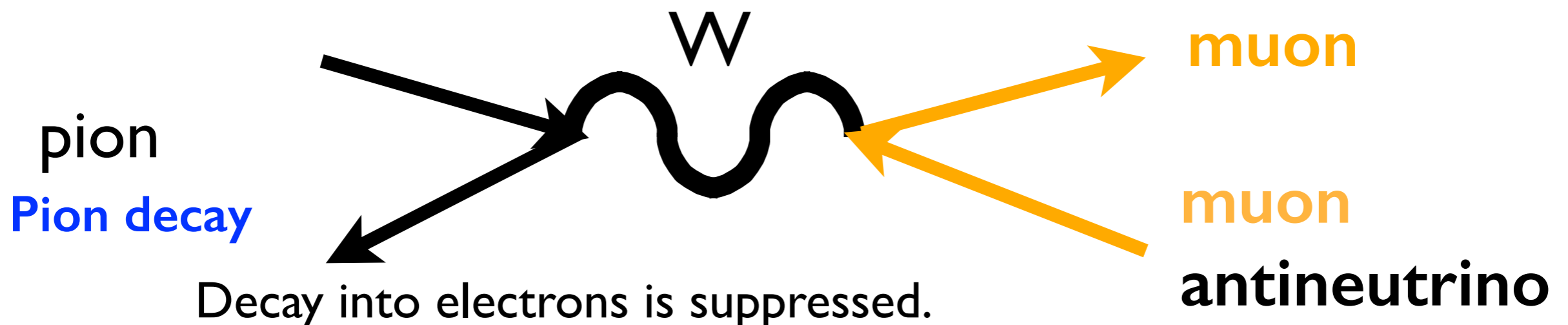
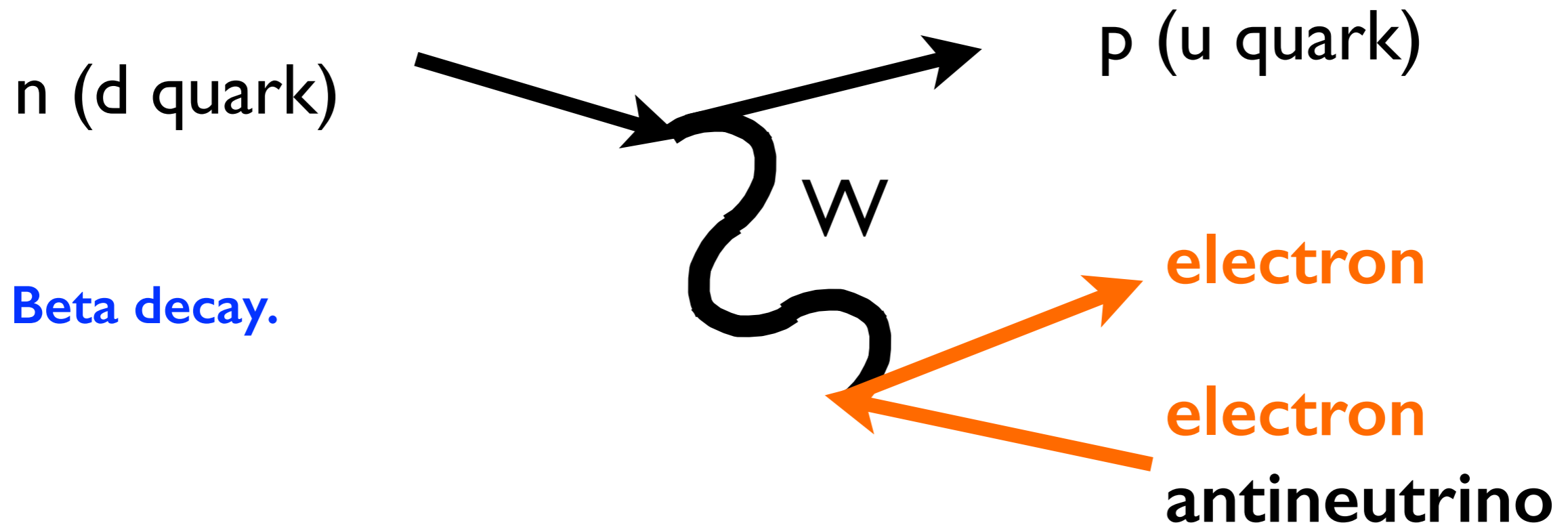
**Neutrino oscillations in supernovae:** In a SN explosion  $\sim 99\%$  of energy is emitted in neutrinos in  $\sim 10$  s. These neutrinos undergo complex oscillations and carry information about the dynamics of the explosion.

See G. Raffelt's lectures

# Neutrinos oscillations in experiments

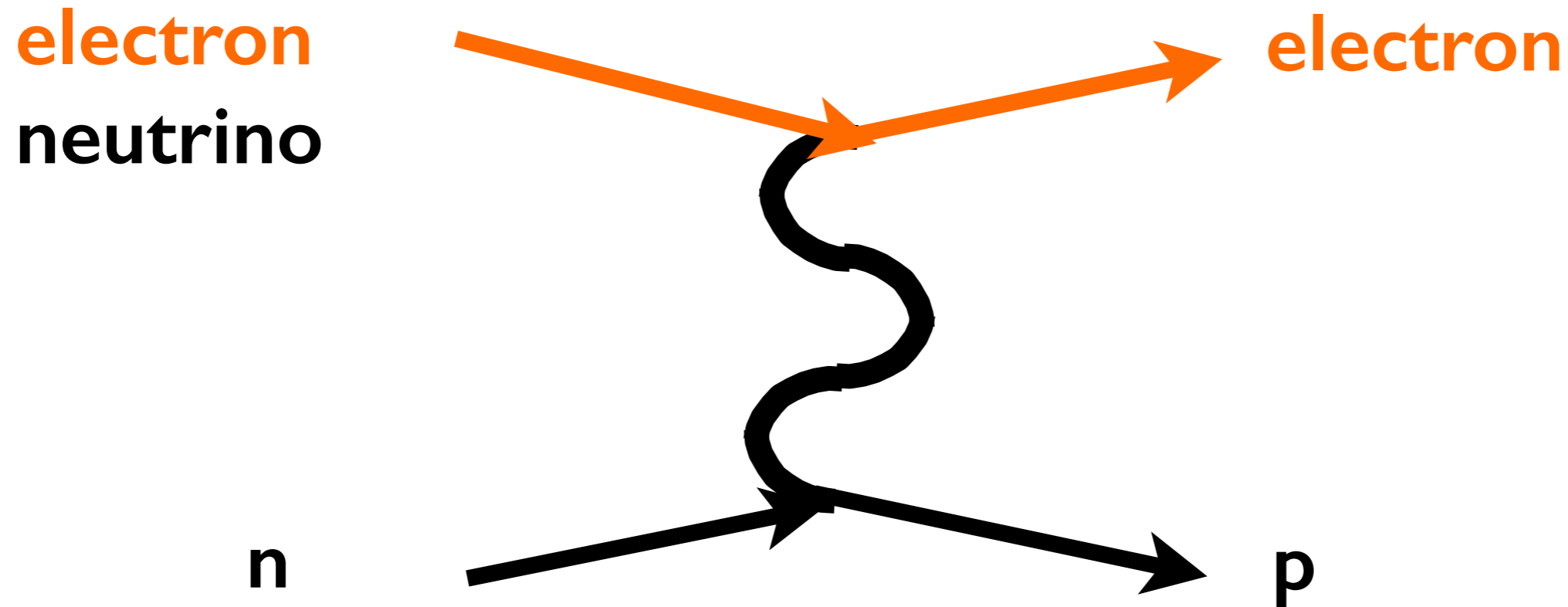
## Neutrino production

In CC (NC) SU(2) interactions, the W boson (Z boson) will be exchanged leading to the production of neutrinos.



## Neutrino detection

Neutrino detection proceeds via CC (and NC) SU(2) interactions. Example:

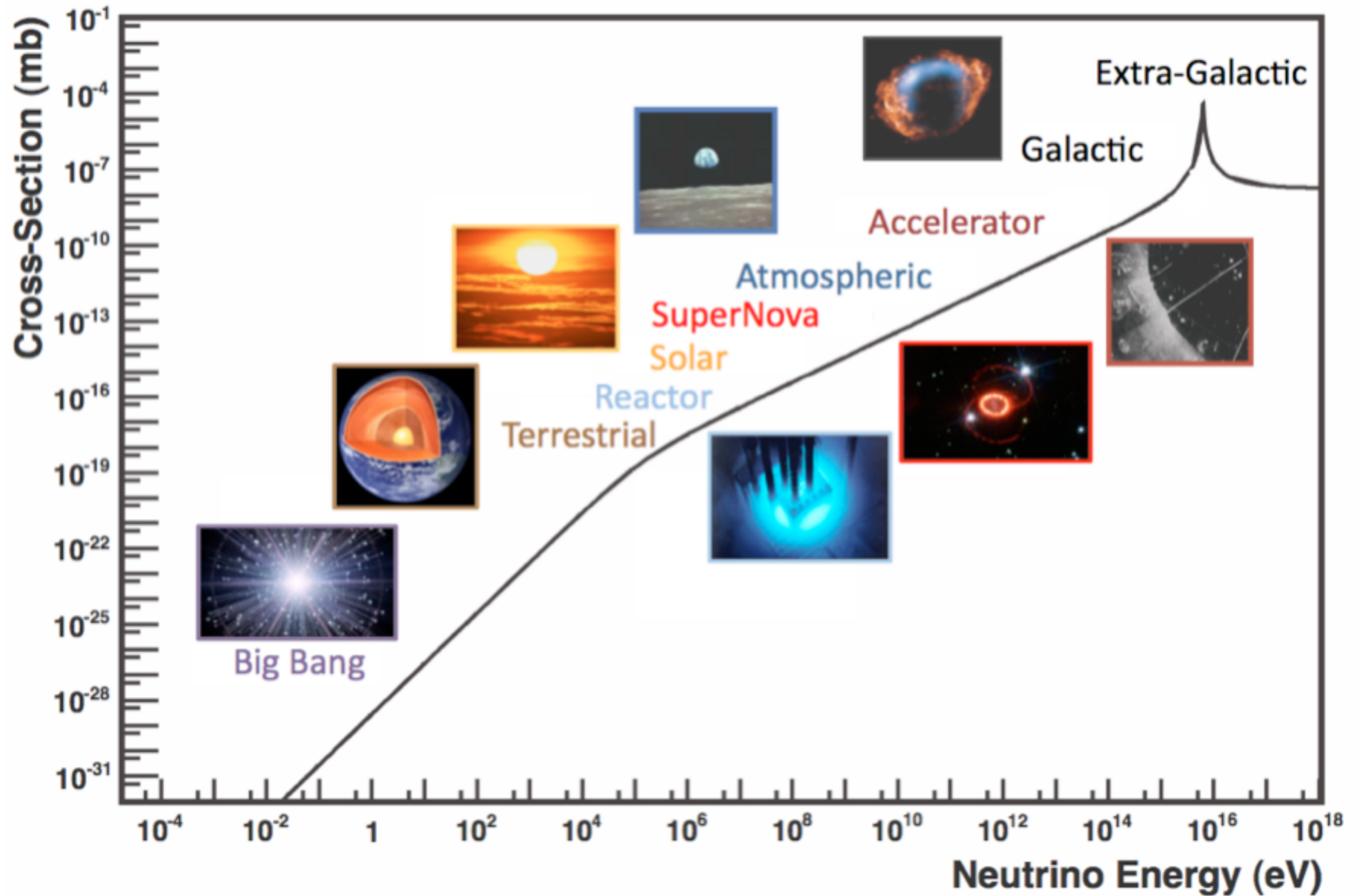


Notice that the leptons have different masses:

$$m_e = 0.5 \text{ MeV} < m_{\mu} = 105 \text{ MeV} < m_{\tau} = 1700 \text{ MeV}$$

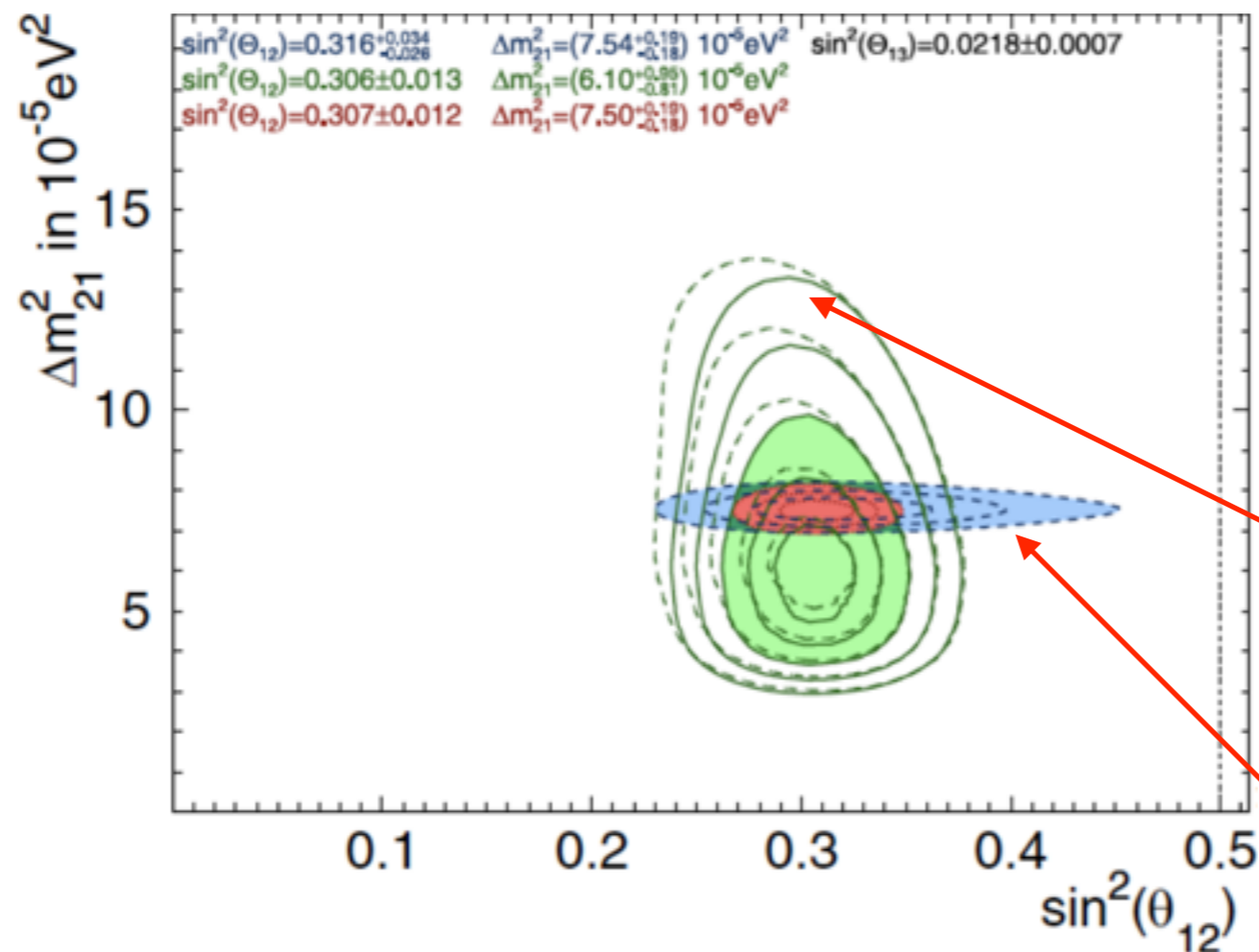
A certain lepton will be produced in a CC only if the neutrino has sufficient energy.

# Neutrino sources



J. Formaggio and S. Zeller, I305.75 I3

# Solar and KamLAND neutrinos



J. Maneira, solar neutrino review Neutrino 2024

MSW for solar neutrinos, SuperK and SNO+ data

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; t) \simeq c_{13}^4 \left( 1 - \sin^2(2\theta_{12}) \sin^2 \left( \frac{\Delta m_{21}^2 L}{4E} \right) \right) + s_{13}^4$$

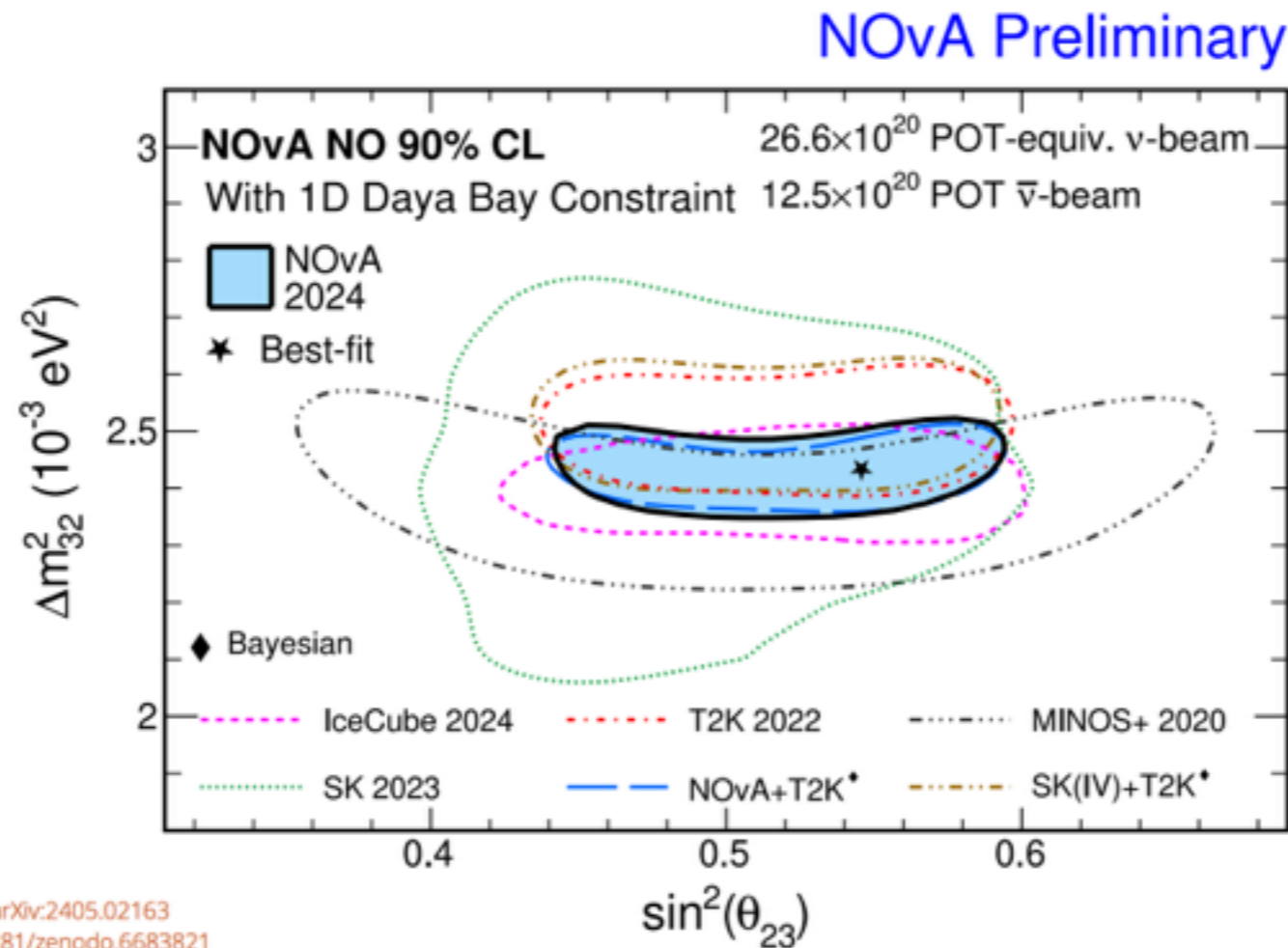
Solar experiments best constrain the “solar mixing” angle of  $\theta_{12}$  to be large (but non-maximal). The mass squared difference is around  $7 \times 10^{-5} \text{ eV}^2$ .

# Atmospheric, reactor and accelerator neutrinos

The muon neutrino disappearance probability:

$$P(\nu_\mu \rightarrow \nu_\mu) \simeq \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

Sensitivity to  $\theta_{23}$  and  $\Delta m_{31}^2$ .



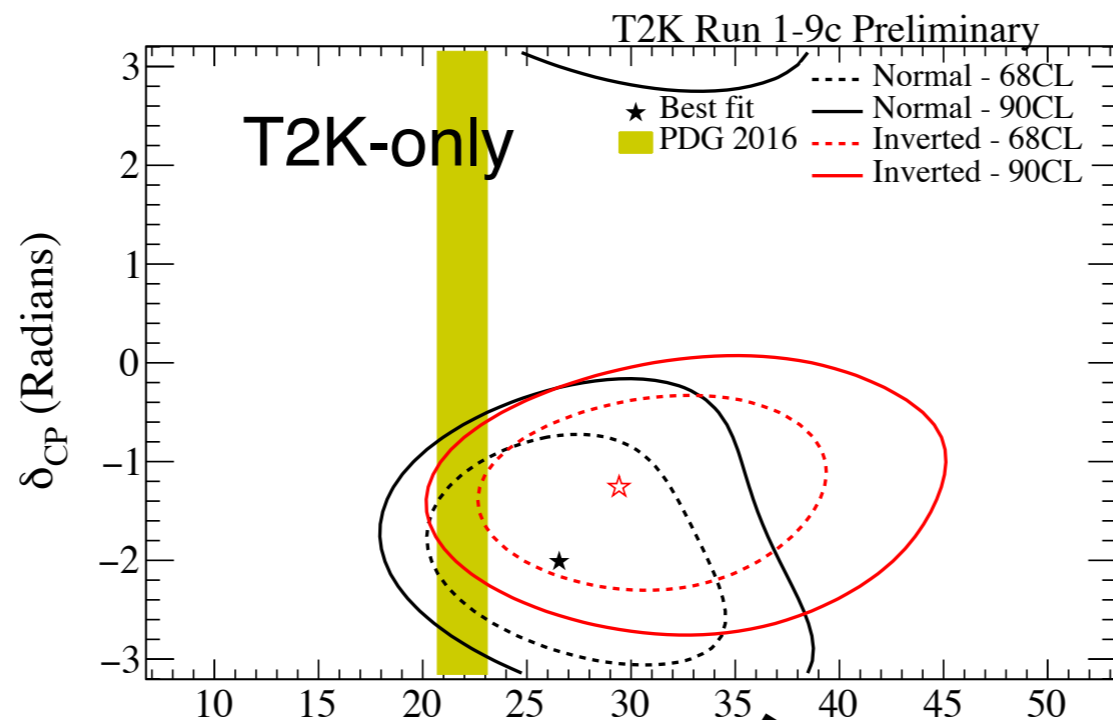
J. Wolcott, for NOvA, Neutrino 2024

# The reactor electron anti-nu disappearance probability:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; t) = 1 - \sin^2(2\theta_{13}) \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

$$P(\nu_\mu \rightarrow \nu_e) \sim s_{23}^2 \sin^2 \theta_{13} \sin^2 \frac{\Delta m_{31}^2 L}{4E} + \text{matter effect} + \text{CPV}$$

## Sensitivity to $\theta_{13}$ and $\delta$ .



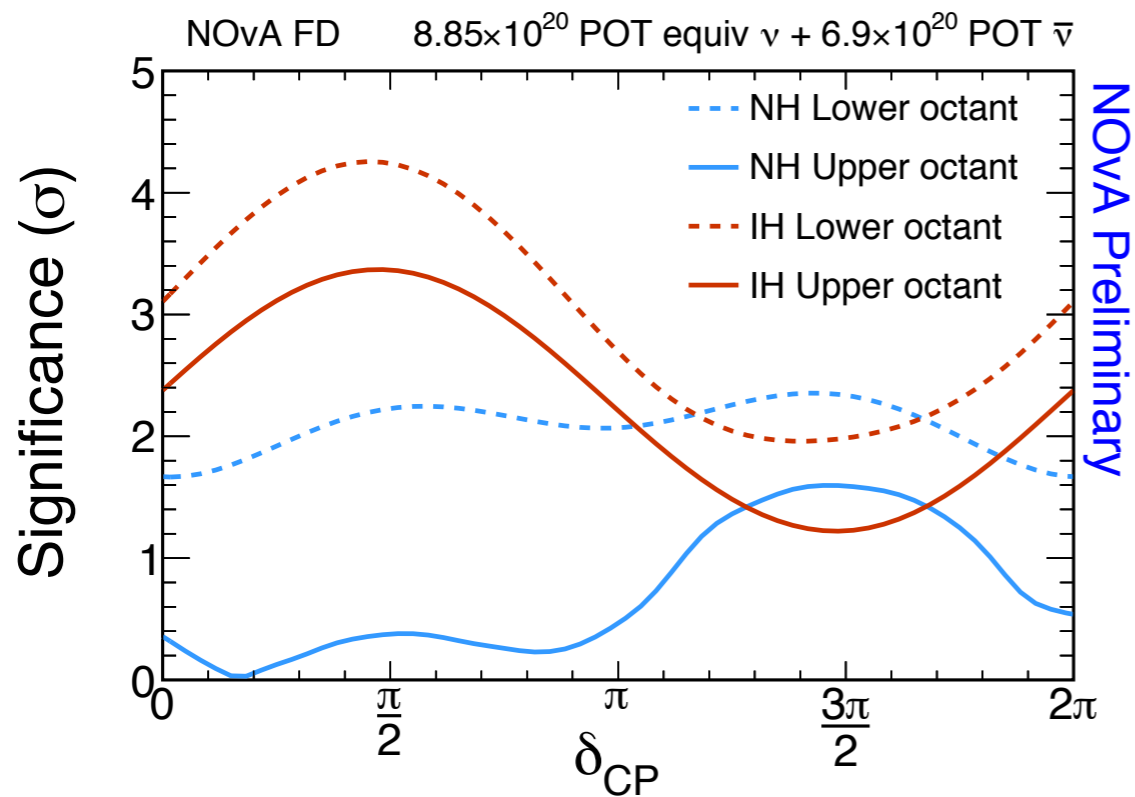
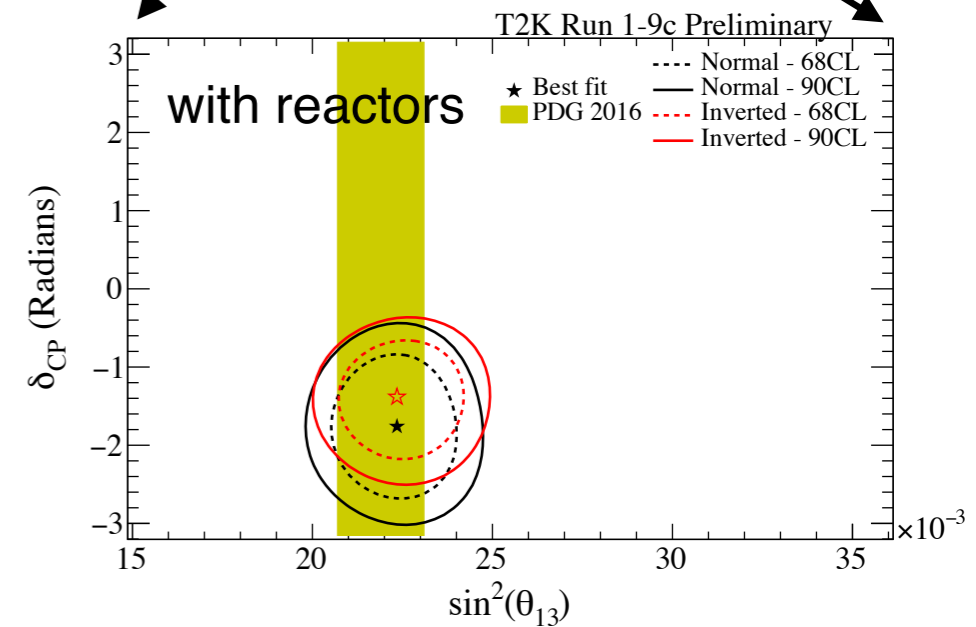
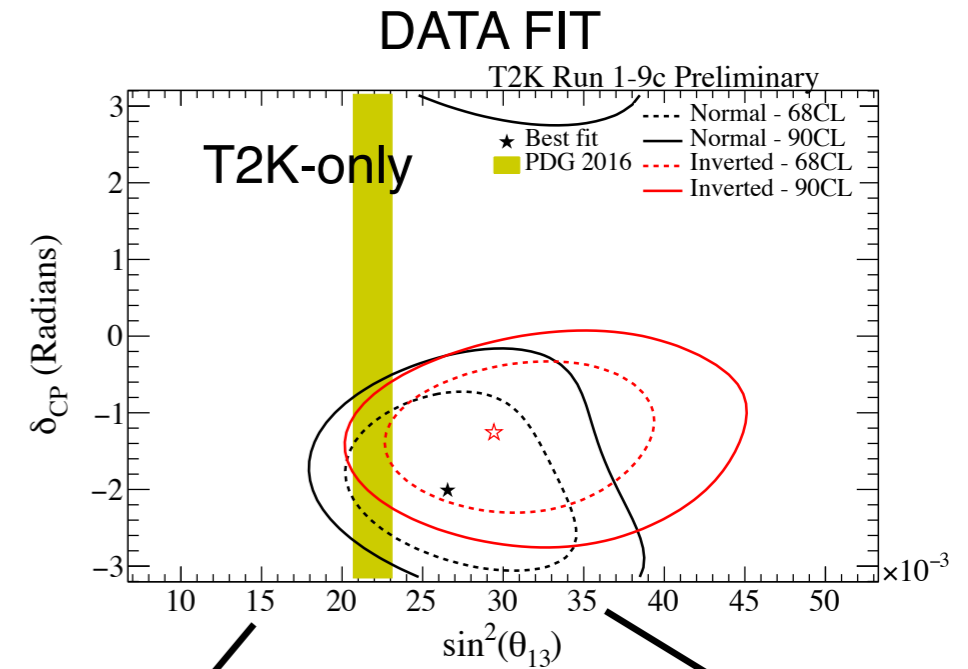
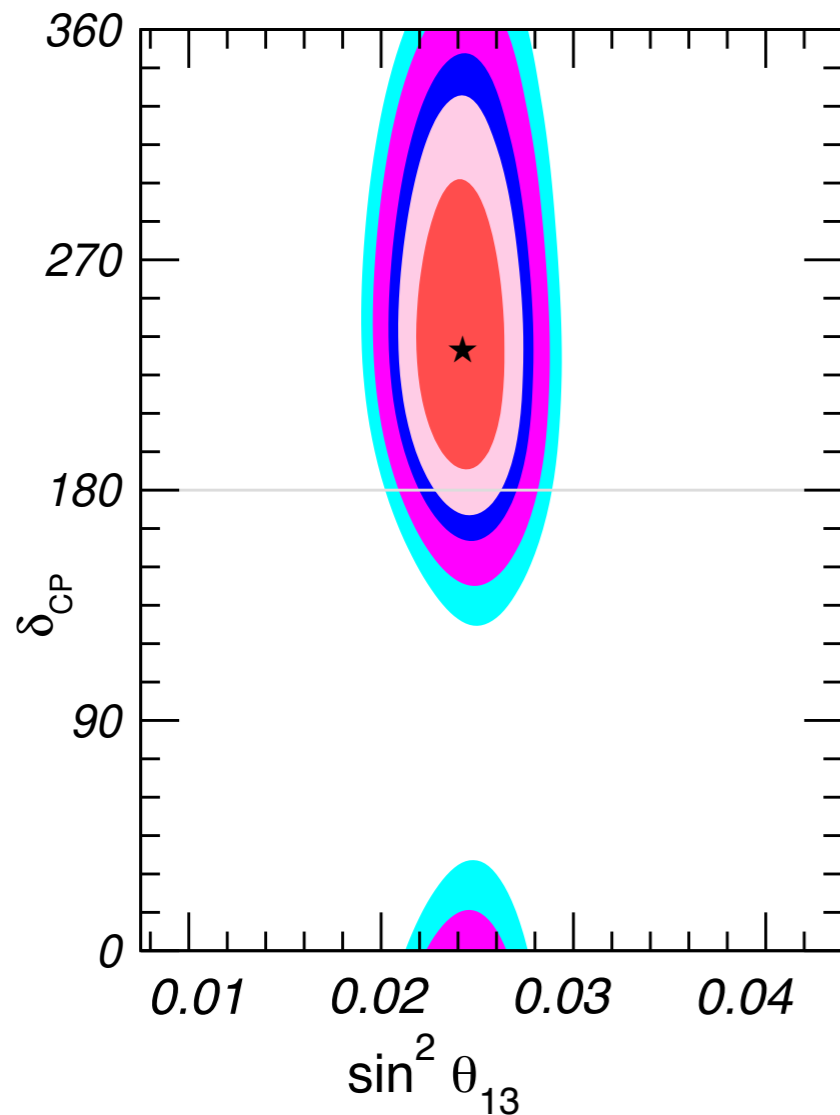
M. Wasko,  
for T2K,  
Neutrino  
2018

# Current Hints for CP violation?

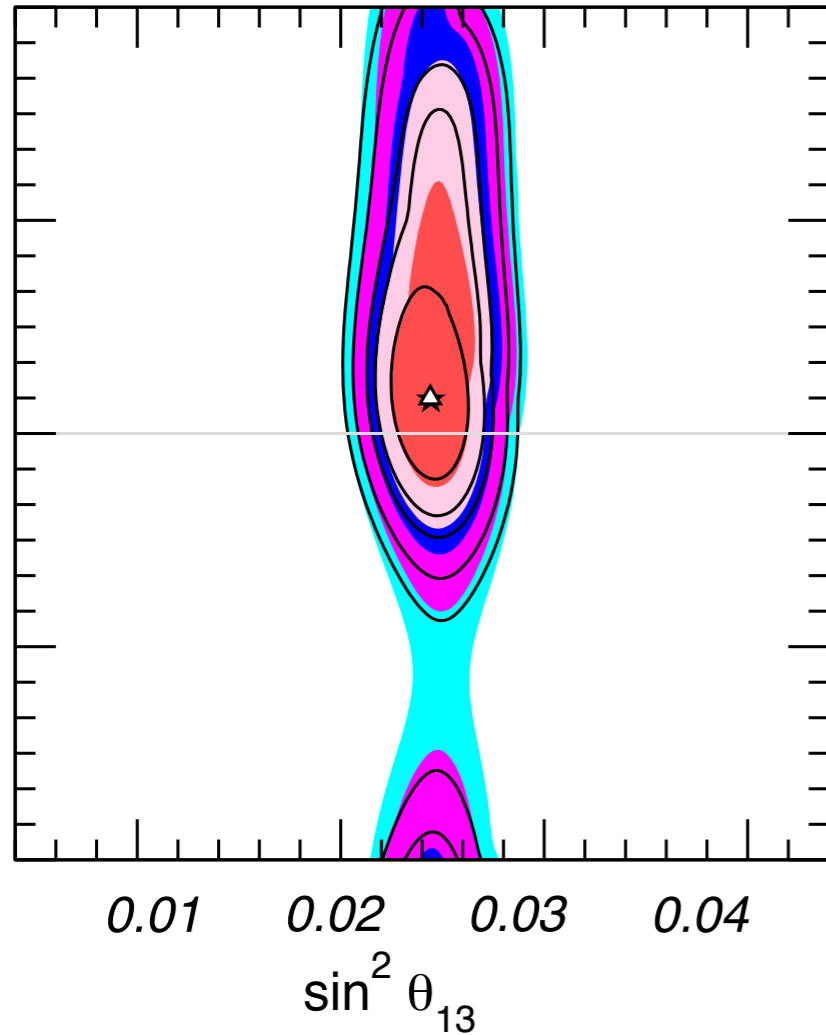
M. C. Gonzalez-Garcia  
et al., NuFit,  
1611.01514  
Pre-Neutrino 2018

M. Wasko, for  
T2K, Neutrino  
2018

M. Sanchez, for  
NOvA, Neutrino  
2018



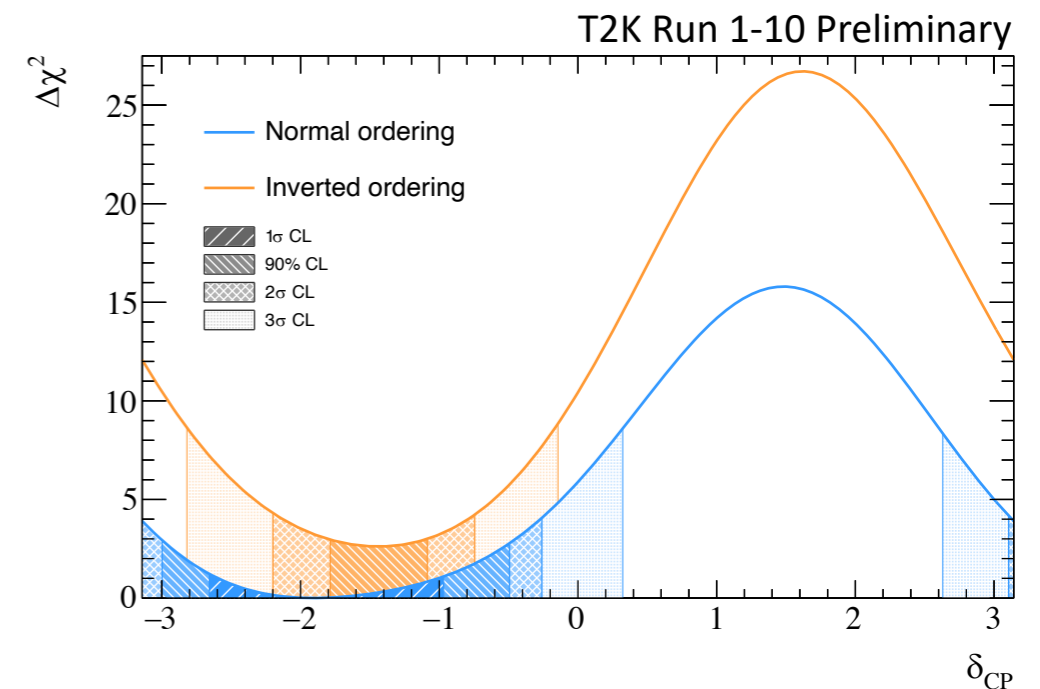
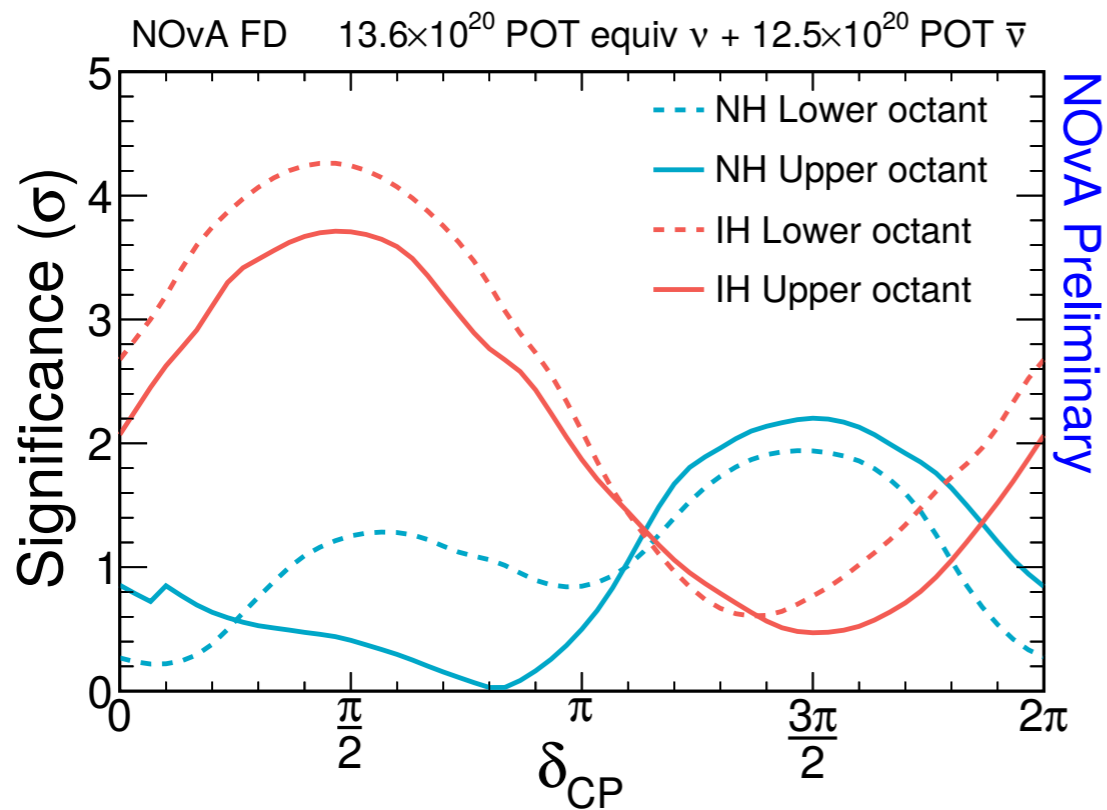
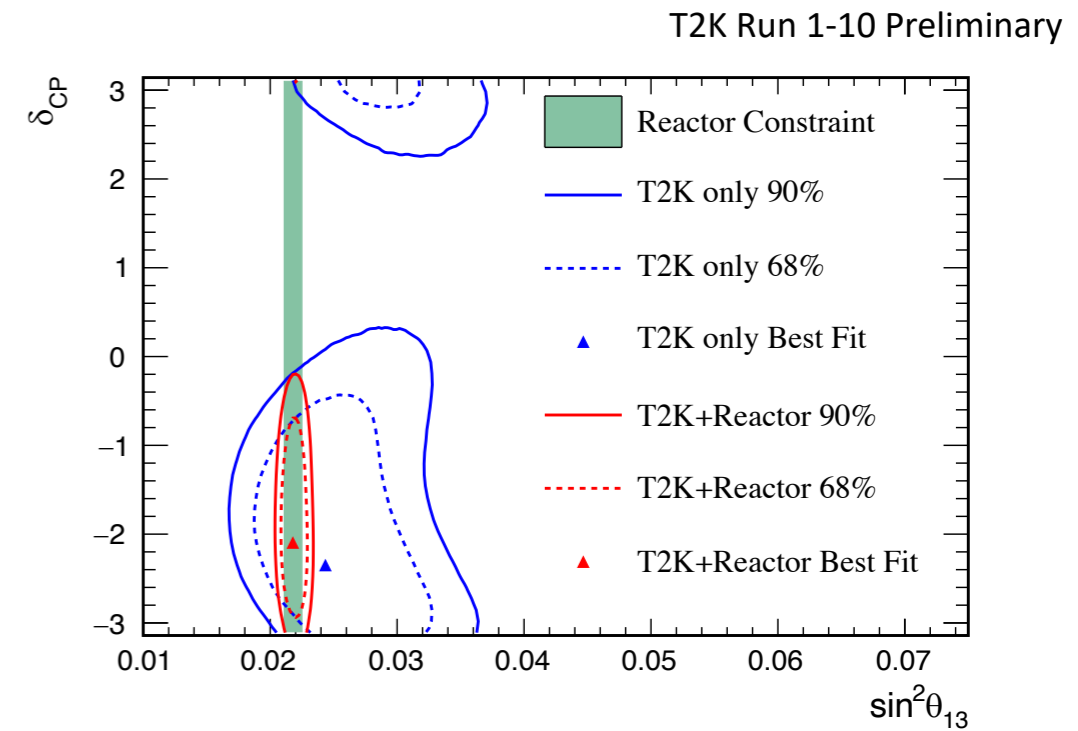
# Current Hints for CP violation?



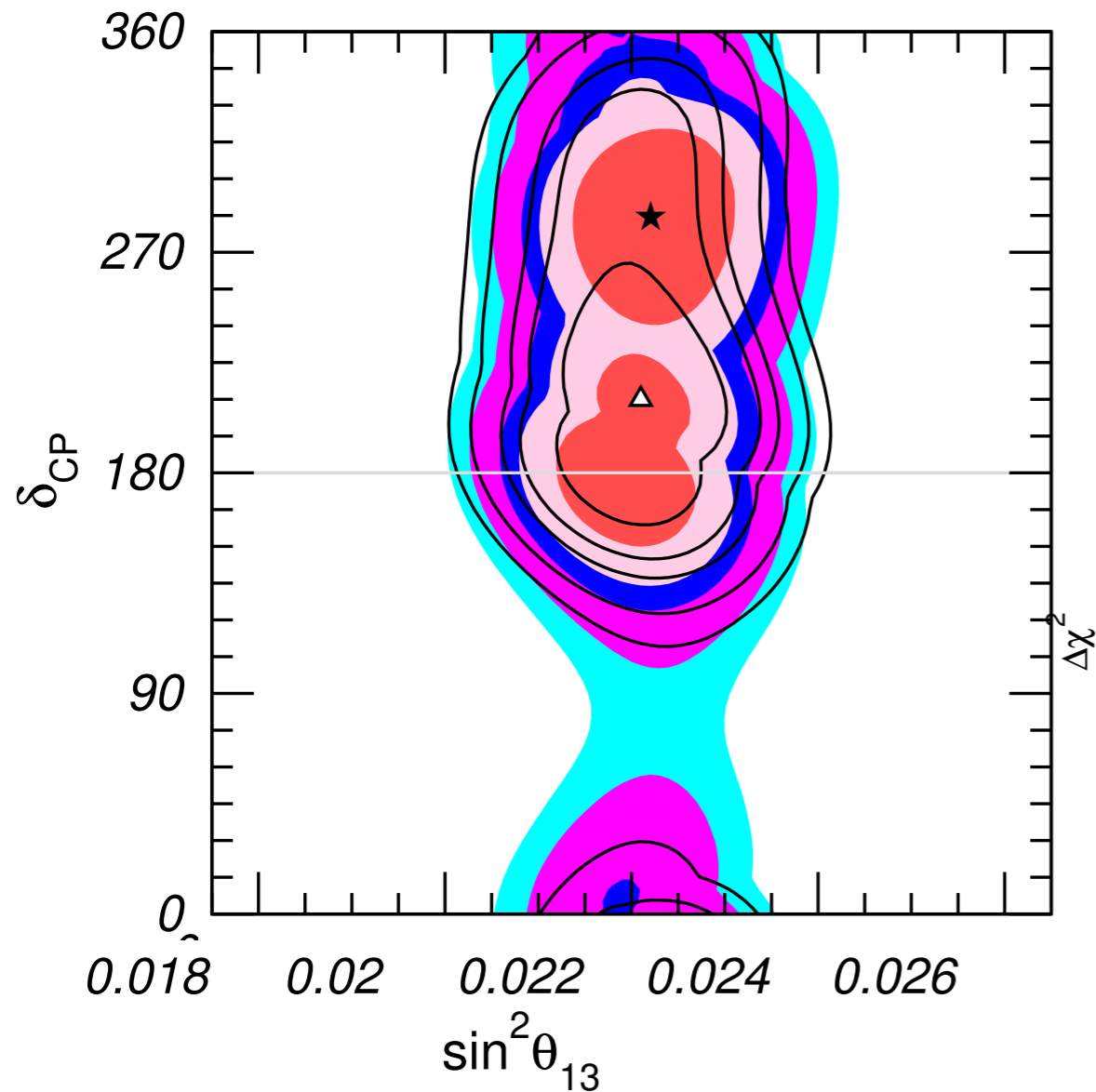
M. C. Gonzalez-Garcia  
et al., NuFit,  
2007.14792

P. Dunne, for  
T2K, Neutrino  
2020

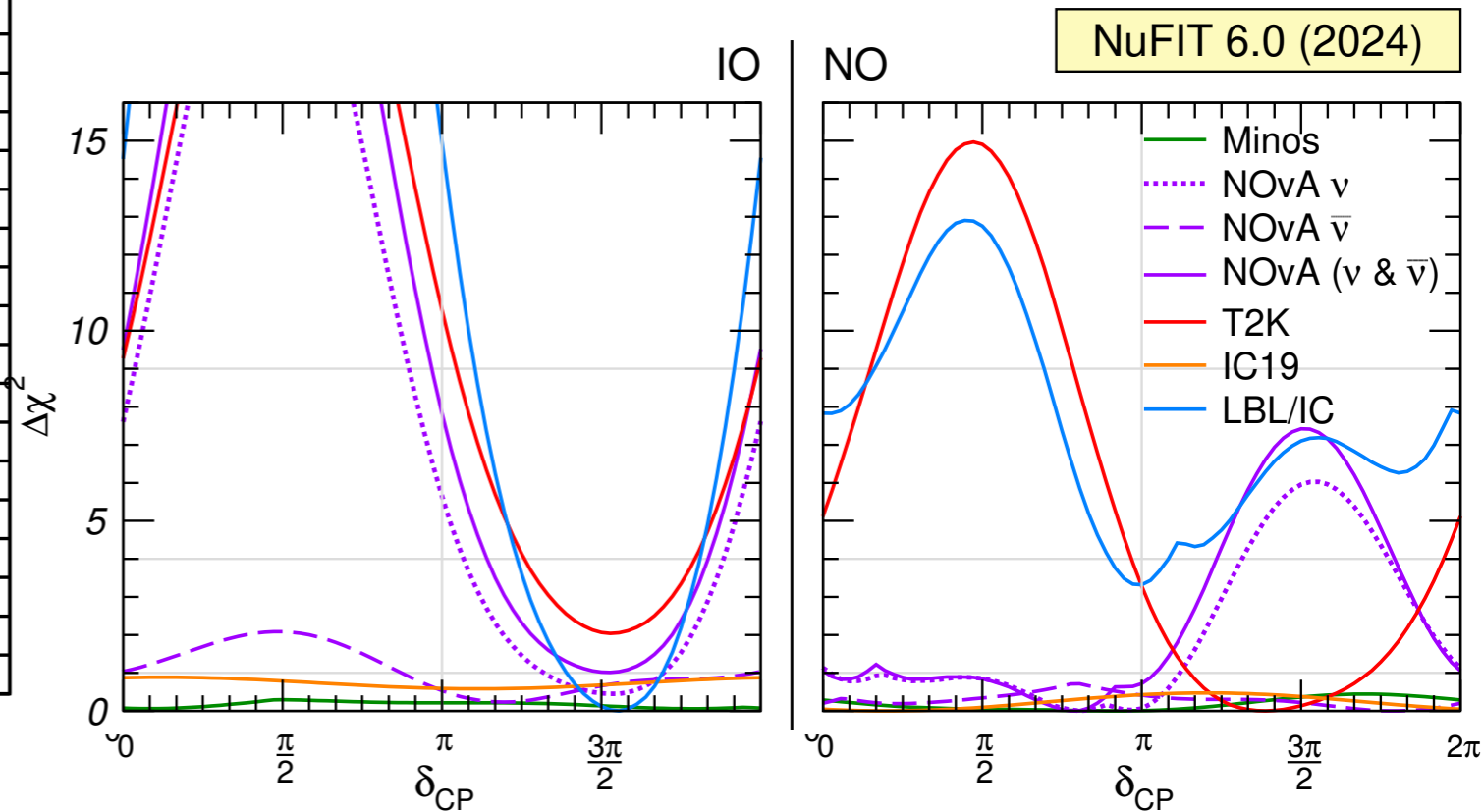
A. Himmel, for  
NOvA, Neutrino  
2020



# Current Hints for CP violation?



M. C. Gonzalez-Garcia et al., 2410.05380



Some **mild preference for CP-violation**, but “tension” between NOvA and T2K remains. Puzzling.

T2K and NOvA appearance data individually favor NO, but together IO. Disappearance data prefer NO. All together there is no preference (unless one adds SK data).

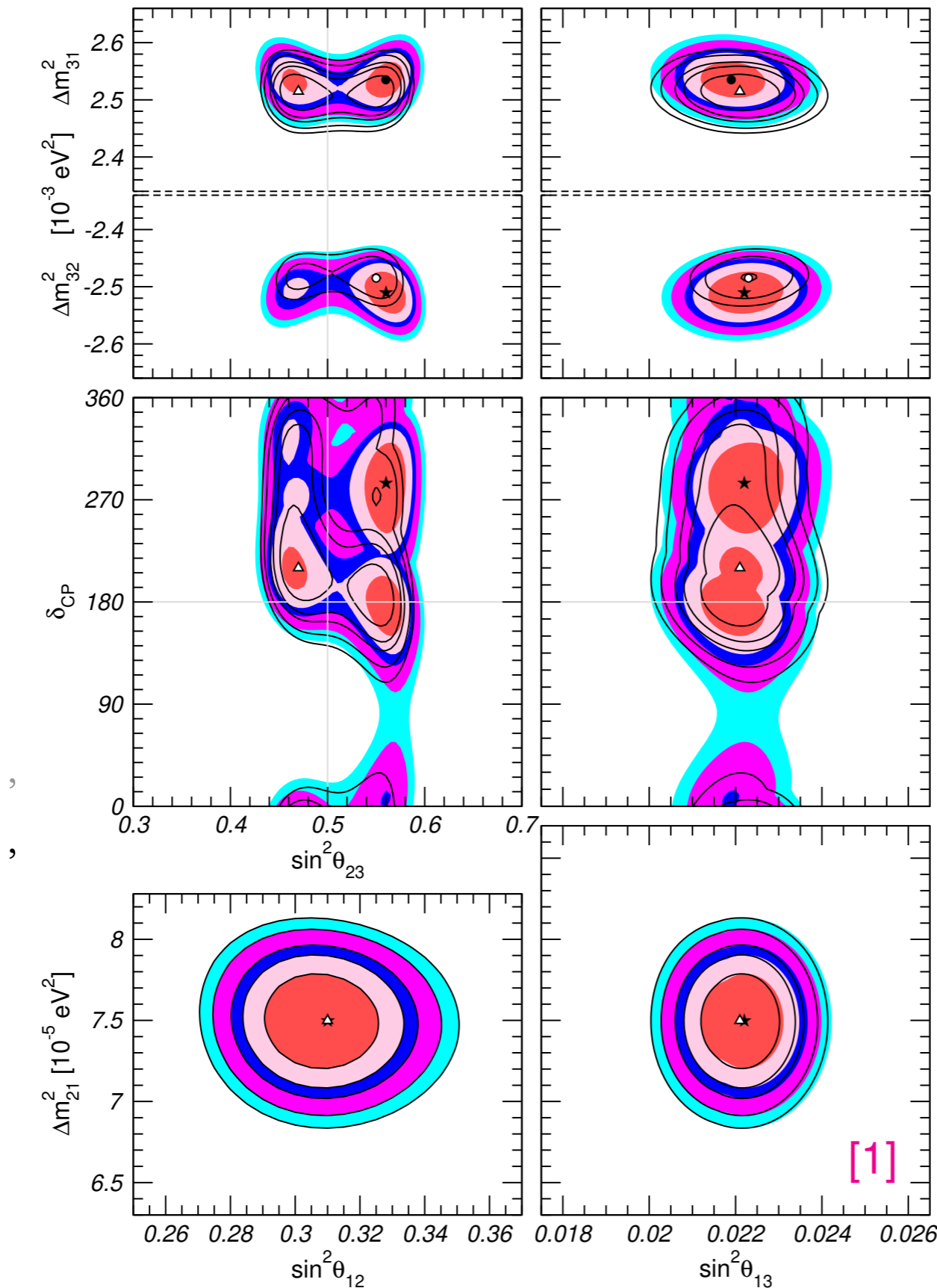
*Current knowledge  
of neutrino  
properties*

# Current status of neutrino parameters

NuFIT 6.0 (2024)

	Normal Ordering ( $\Delta\chi^2 = 0.6$ )		Inverted Ordering (best fit)	
	bf $\pm 1\sigma$	$3\sigma$ range	bf $\pm 1\sigma$	$3\sigma$ range
$\sin^2 \theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.345$
$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \rightarrow 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \rightarrow 35.95$
$\sin^2 \theta_{23}$	$0.561^{+0.012}_{-0.015}$	$0.430 \rightarrow 0.596$	$0.562^{+0.012}_{-0.015}$	$0.437 \rightarrow 0.597$
$\theta_{23}/^\circ$	$48.5^{+0.7}_{-0.9}$	$41.0 \rightarrow 50.5$	$48.6^{+0.7}_{-0.9}$	$41.4 \rightarrow 50.6$
$\sin^2 \theta_{13}$	$0.02195^{+0.00054}_{-0.00058}$	$0.02023 \rightarrow 0.02376$	$0.02224^{+0.00056}_{-0.00057}$	$0.02053 \rightarrow 0.02397$
$\theta_{13}/^\circ$	$8.52^{+0.11}_{-0.11}$	$8.18 \rightarrow 8.87$	$8.58^{+0.11}_{-0.11}$	$8.24 \rightarrow 8.91$
$\delta_{\text{CP}}/^\circ$	$177^{+19}_{-20}$	$96 \rightarrow 422$	$285^{+25}_{-28}$	$201 \rightarrow 348$
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \rightarrow 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \rightarrow 8.05$
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.534^{+0.025}_{-0.023}$	$+2.463 \rightarrow +2.606$	$-2.510^{+0.024}_{-0.025}$	$-2.584 \rightarrow -2.438$

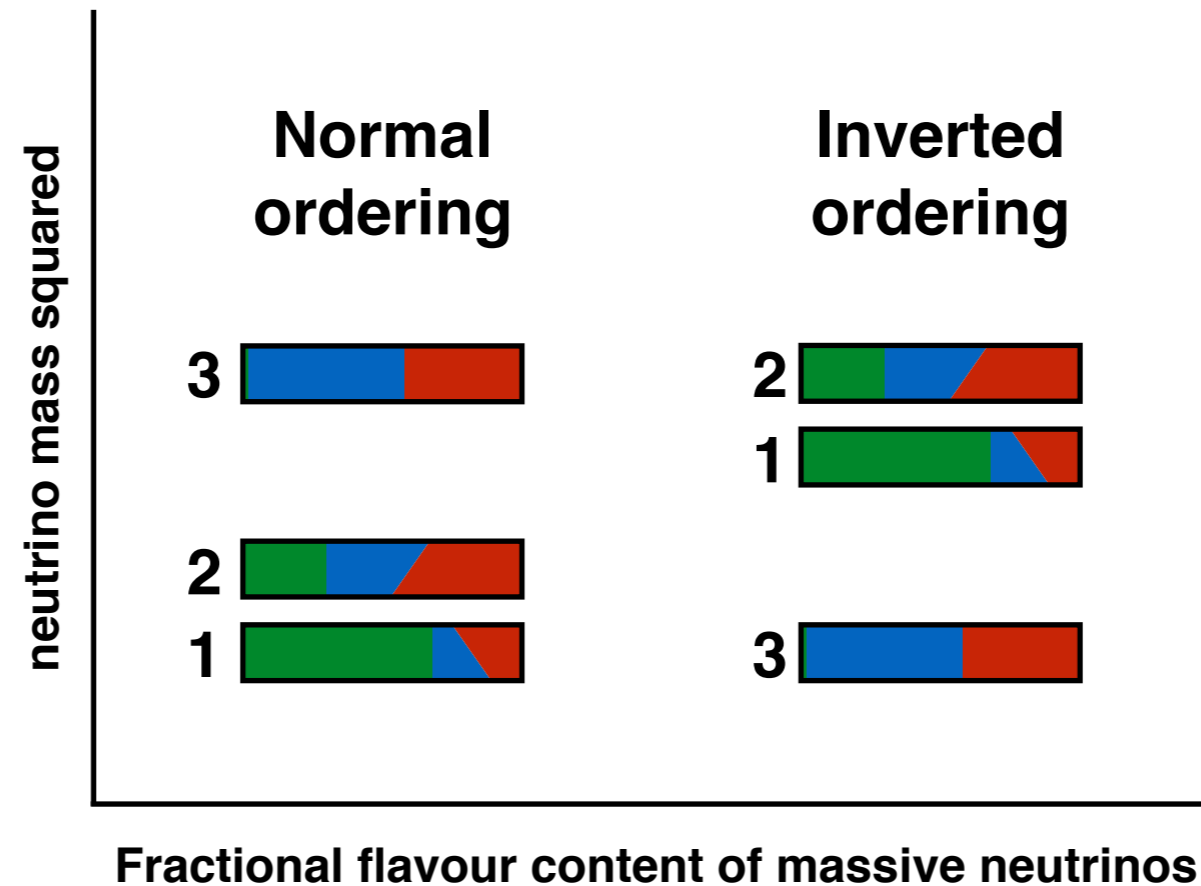
[nufit.org](http://nufit.org), M. C. Gonzalez-Garcia et al., 2410.05380



- Current knowledge of neutrino properties:
- 2 mass squared differences
  - 3 sizable mixing angles,
  - some mild preference for CPV
  - mild indications in favour of NO

# Neutrino masses

$\Delta m_s^2 \ll \Delta m_A^2$  implies at least 3 massive neutrinos.



Using

$$m_2 = \sqrt{m_2^2} = \sqrt{m_2^2 \pm m_1^2} = \sqrt{m_1^2 + \Delta m_{21}^2}$$

we can express the masses in terms of MO and  $m_{\text{MIN}}$ :

$$m_1 = m_{\text{min}}$$

$$m_2 = \sqrt{m_{\text{min}}^2 + \Delta m_{\text{sol}}^2}$$

$$m_3 = \sqrt{m_{\text{min}}^2 + \Delta m_A^2 + \Delta m_{\text{sol}}^2/2}$$

$$m_3 = m_{\text{min}}$$

$$m_1 = \sqrt{m_{\text{min}}^2 + |\Delta m_A^2| - \Delta m_{\text{sol}}^2/2}$$

$$m_1 = \sqrt{m_{\text{min}}^2 + |\Delta m_A^2| + \Delta m_{\text{sol}}^2/2}$$

There are three limiting cases:

- **normal hierarchical spectrum** (NH): requires NO and  $m_1 \sim 0$ .

$$m_1 \ll m_2 \simeq \sqrt{\Delta m_{21}^2} \ll m_3 \simeq \sqrt{\Delta m_{31}^2}$$

- **inverted hierarchical spectrum** (IH): requires IO and  $m_3 \sim 0$ .

$$m_3 \ll m_1 \simeq m_2 \simeq \sqrt{\Delta m_{32}^2}$$

- **quasi-degenerate spectrum** (QD):  
for  $m_1 > 0.1$  eV.

$$m_1 \simeq m_2 \simeq m_3 \gg \sqrt{\Delta m_{32}^2}$$

Measuring the masses requires:

- the mass scale:  $m_{\min}$
- the mass ordering. Currently there is a slight hint in favour of NO based mainly on atmospheric data.

# Mixing and CP-violation

## The Pontecorvo-Maki-Nakagawa-Sakata matrix

CPV?

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

- $\theta_{23}$  maximal or close to maximal
- $\theta_{12}$  significantly different from maximal
- $\theta_{13}$  quite large: challenge to flavour models
- Mixings very different from quark sector
- Possibly, large CPV. CPV is a **fundamental question, possibly related to** the origin of the baryon asymmetry and to the origin of the flavour structure

# Open questions

- **What is the nature of neutrinos? Dirac vs Majorana?**  
Neutrinoless  $\delta\beta$  decay
- **What are the values of the masses? Absolute scale**  
(KATRIN, ...?) and the ordering.  
LBL: T2K, NOvA, DUNE, T2HK, ESSnuSB, Daedalus, nuFACT..., PINGU, ORCA, INO, JUNO
- **Is there CP-violation? Its discovery**  
in the next generation of LBL depends on the value of  $\delta$ .
- **What are the precise values**  
of mixing angles? Do they suggest an underlying pattern?  
reactor SBL and MBL, atm, LBL, ...
- **Is the standard picture correct? Are there NSI? Sterile neutrinos? Other effects?** MINOS+, MicroBooNE, SoLid, ...

# What do we still need to know?

- **What is the nature of neutrinos? Dirac vs Majorana?**
- **What are the values of the masses? Absolute scale (KATRIN, ...?) and the ordering.**
- Is there CP-violation? Its discovery in the next generation of LBL depends on the value of delta.
- What are the precise values of mixing angles? Do they suggest an underlying pattern?
- Is the standard picture correct? Are there NSI? Sterile neutrinos? Other effects?

# Nature of neutrinos: Dirac vs Majorana

Neutrinos can be **Majorana** or **Dirac particles**. In the SM only neutrinos can be Majorana because they are **neutral**.

**Majorana** particles are indistinguishable from antiparticles.

**Dirac** neutrinos are labelled by the lepton number.

The **nature** of neutrinos is linked to the **conservation** of the **Lepton number (L)**. This information is crucial in understanding the Physics BSM: with or without L-conservation? and it can be linked to the existence of matter in the Universe.

# Charge conjugation

This operation changes a field in its charge-conjugate (opposite quantum numbers):

$$\psi^c = C\bar{\psi}^T = i\gamma^2\psi^*$$

Properties:  $C\gamma^{\alpha T}C^\dagger = -\gamma^\alpha$  ,  $CC^\dagger = 1$  ,  $C^T = -C$

In Weyl representation:  $C = i\gamma^2\gamma^0$

Let's apply it to a left-handed field

$$(\psi_L)^c = i\gamma^2\psi_L^* = i \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \eta^* \end{pmatrix} = \begin{pmatrix} i\sigma^2\eta^* \\ 0 \end{pmatrix}$$

We find that it behaves as a right-handed field!

$$(\psi_L)^c = (\psi^c)_R$$

**Exercise**  
using the properties of C,  
show that this equation is true  
independently of the gamma  
representation.

# Majorana fields

A Majorana field satisfies the Majorana condition

$$\psi = \psi^c$$

Majorana fields satisfy the Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

It can be written as

$$\psi = \psi_L + \psi_L^c,$$

It has 2 (and not 4) degrees of freedom.

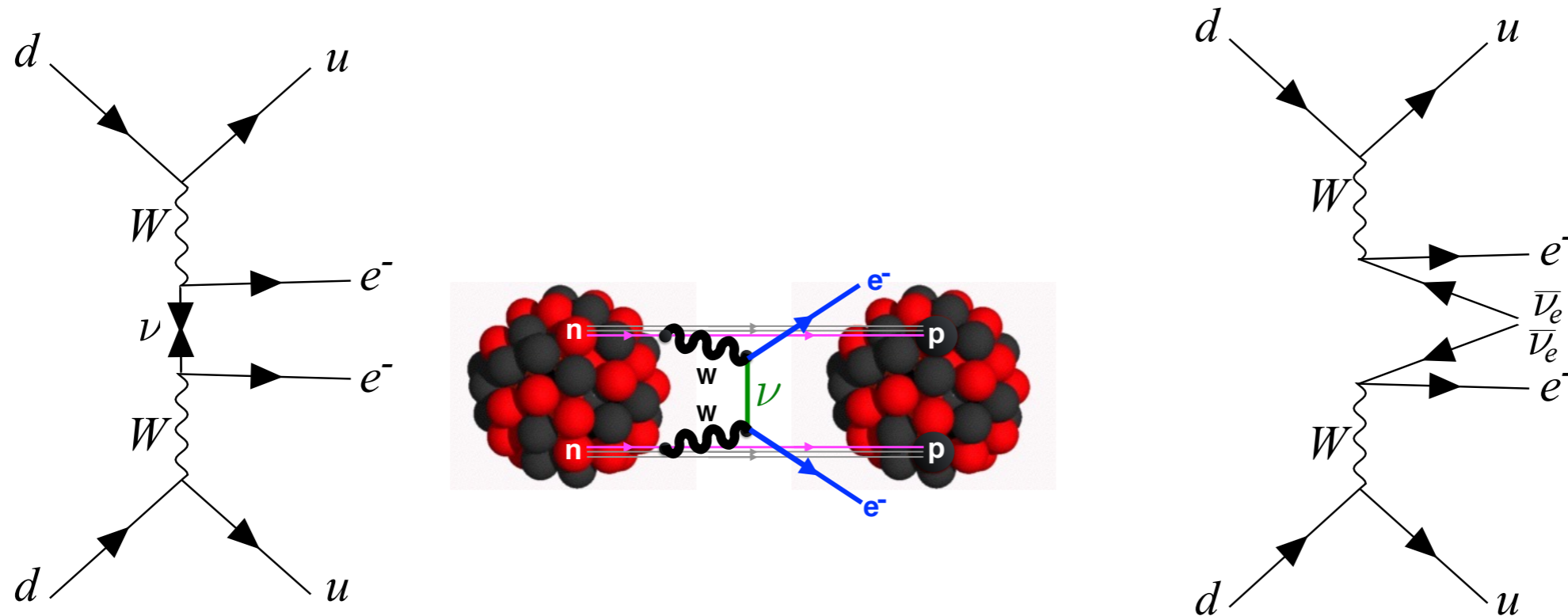
They cannot carry any U(1) charge. In particular, Majorana neutrinos break lepton number

$$e_L \xrightarrow{U_{\text{Lepton}}} e^{i\alpha} e_L$$
$$\nu_L \xrightarrow{U_{\text{Lepton}}} e^{i\alpha} \nu_L$$

$$\psi = \psi^c \xrightarrow{U_{\text{Lepton}}} \psi e^{i\alpha} = \psi^c e^{-i\alpha}$$

# Neutrinoless double beta decay

Neutrinoless double beta decay,  $(A, Z) \rightarrow (A, Z+2) + 2 e^-$ , will test the nature of neutrinos.



Massive Majorana neutrinos mediate this process.

It has a special role in the study of neutrino properties as it probes **lepton number violation** and the nature of neutrinos and can provide information on neutrino masses and (possibly) on CP-violation.

The decay rate depends on  $T_{1/2}^{-1} \simeq \frac{G_{0\nu}}{m_e} |m_{\beta\beta}|^2 M_{\text{NUCL}}^2$   
 via the effective Majorana mass parameter:

$$|m_{\beta\beta}| \equiv \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i(\alpha_{31} - 2\delta)} \right|$$

The predictions for  $m_{\beta\beta}$  depend on the neutrino masses

- **NH** ( $m_1 \ll m_2 \ll m_3$ ):

$$m_1 \simeq 0$$

$$m_2 \simeq \sqrt{\Delta m_{21}^2}$$

$$m_3 \simeq \sqrt{\Delta m_{31}^2}$$

$$|U_{e1}| = \cos \theta_{13} \cos \theta_{12}$$

$$|U_{e2}| = \cos \theta_{13} \sin \theta_{12}$$

$$|U_{e3}| = \sin \theta_{13}$$

**Exercise**

Choose one of the neutrino mass spectra and compute  $|\langle m \rangle|$ , using the latest values of the oscillation parameters

- **NH** ( $m_1 \ll m_2 \ll m_3$ ):  $|\langle m \rangle| \sim 1\text{-}5 \text{ meV}$

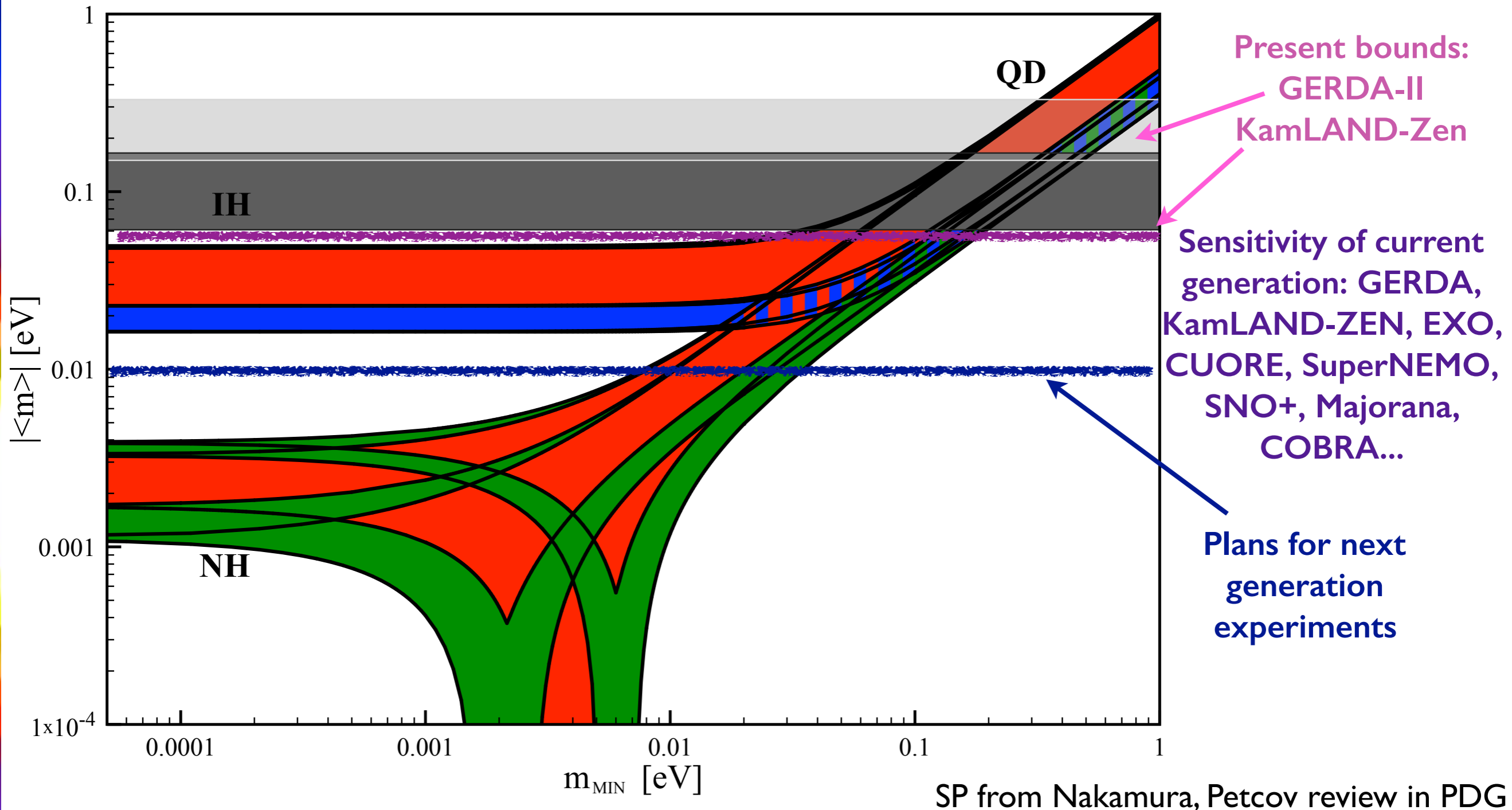
$$|\langle m \rangle| \simeq \left| \sqrt{\Delta m_{21}^2} \cos^2 \theta_{13} \sin^2 \theta_{12} + \sqrt{\Delta m_{31}^2} \sin^2 \theta_{13} e^{i(\alpha_{32} + 2\delta)} \right|$$

- **IH** ( $m_3 \ll m_1 \sim m_2$ ):  $15 \text{ meV} < |\langle m \rangle| < 50 \text{ meV}$

$$\sqrt{\Delta m_{31}^2} \cos 2\theta_{12} \leq |\langle m \rangle| \simeq \sqrt{(1 - \sin^2 2\theta_{12} \sin^2 \frac{\alpha_{21}}{2}) \Delta m_{31}^2} \leq \sqrt{\Delta m_{31}^2}$$

- **QD** ( $m_1 \sim m_2 \sim m_3$ ):  $44 \text{ meV} < |\langle m \rangle| < m_1$

$$|\langle m \rangle| \simeq m_0 \left| (\cos^2 \theta_{12} + \sin^2 \theta_{12} e^{i\alpha_{21}}) \cos^2 \theta_{13} + \sin^2 \theta_{13} e^{i\alpha_{31}} \right|$$

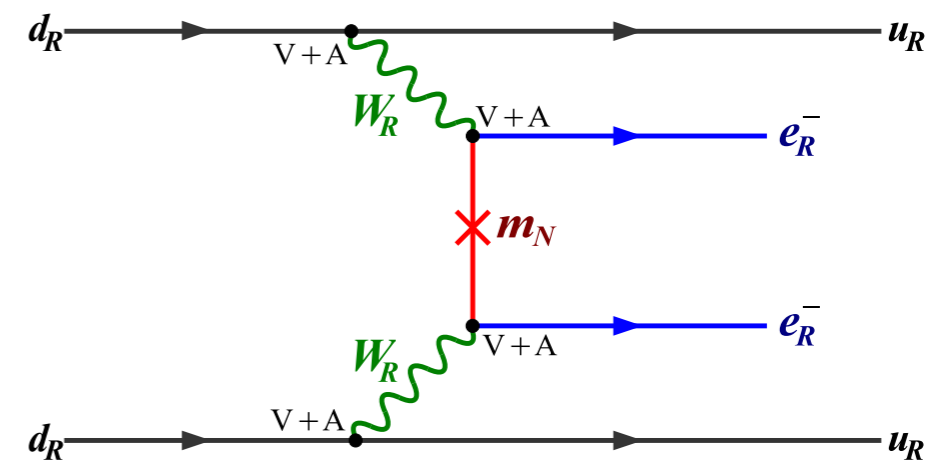
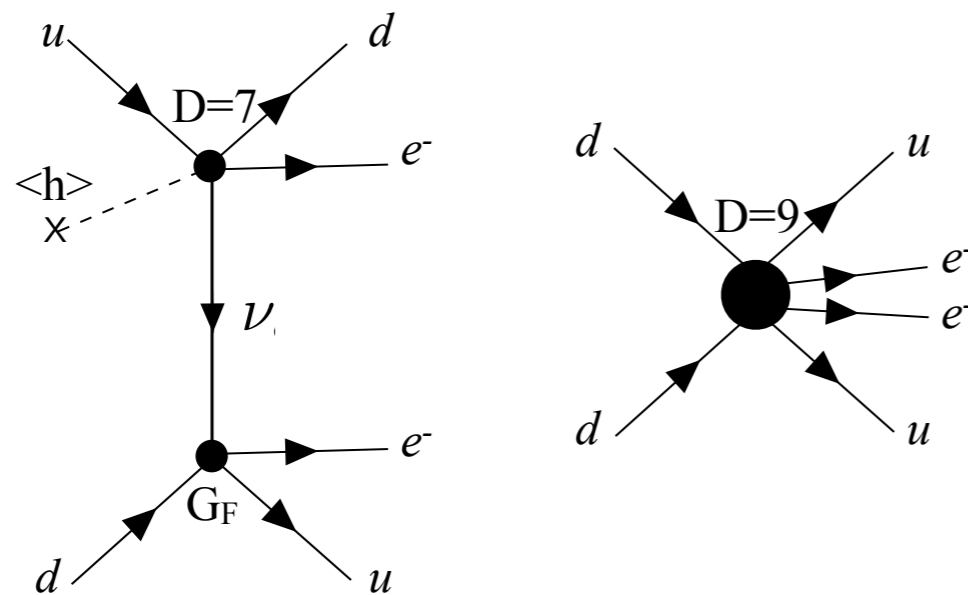
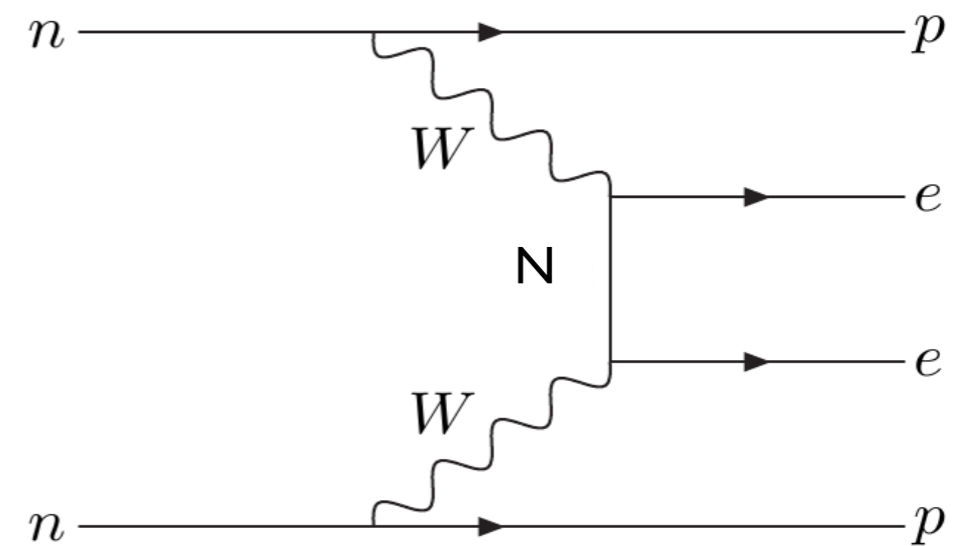


SP from Nakamura, Petcov review in PDG

**Wide experimental program** for the future: **a positive signal would indicate that L is violated!**

**Other LNV mechanisms**, possibly advocated for the origin of neutrino masses and/or leptogenesis, would also mediate neutrinoless double beta decay at some level.

- Light sterile neutrinos
- Heavy sterile neutrinos
- R-parity violating SUSY
- Extra dimensional models
- Left-Right models

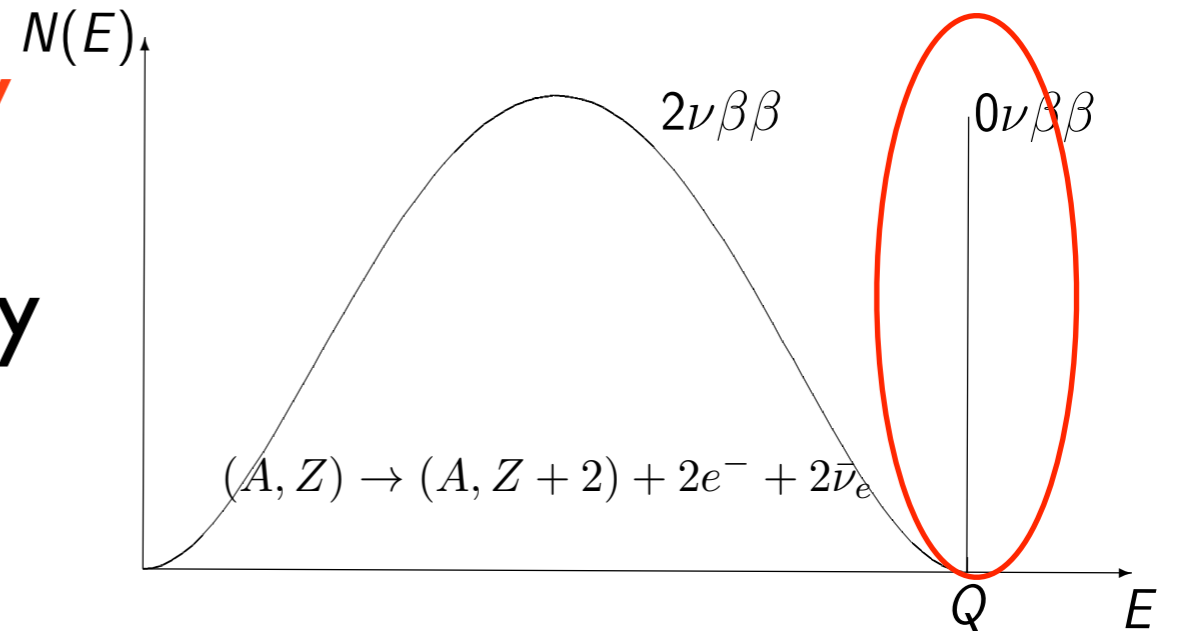


Deppisch, Hirsch, Pas, 1208.0727

# Experimental searches of betabeta decay

Neutrinoless double beta decay can be tested in nuclei in which single beta decay is kinematically forbidden ( $^{76}\text{Ge}$ ,  $^{100}\text{Mo}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$ ...).

It is a very rare process:



$$T_{0\nu} \propto \sqrt{\frac{M t}{B \Delta E}}$$

Detector mass  
< energy resolution  
< backgrounds



News

## Roman ingots to shield particle detector

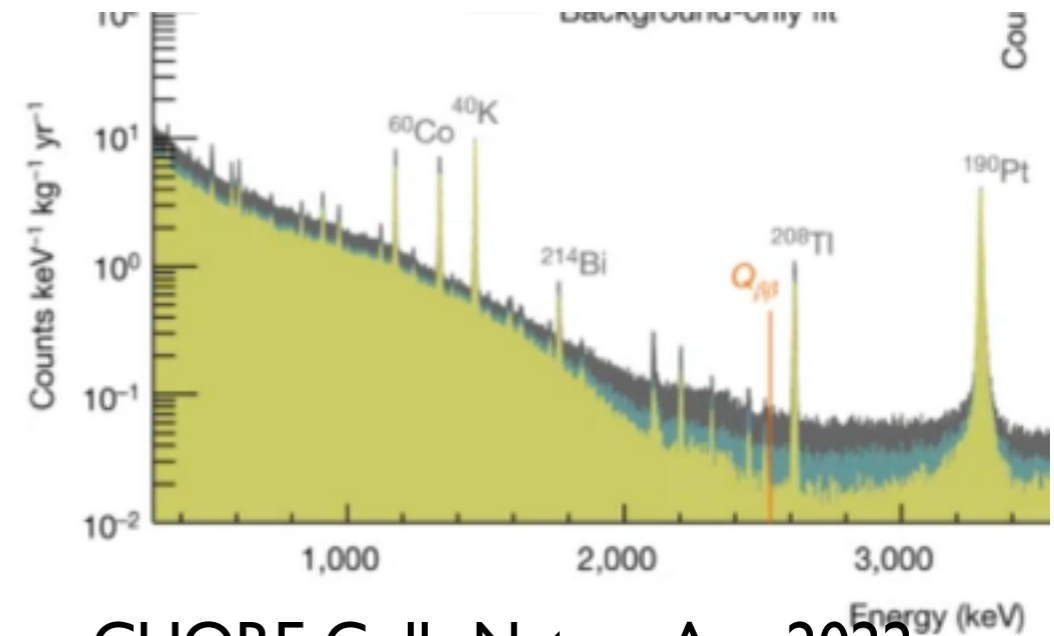
Lead from ancient shipwreck will line Italian neutrino experiment.

Large detectors, in deep underground labs, and with precise control of backg.ds

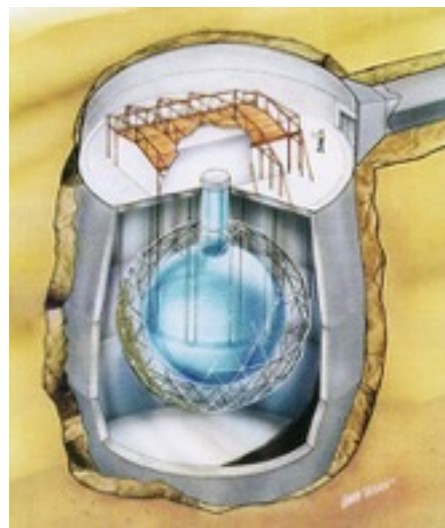
**KamLAND-Zen** Loaded LSc with 380 kg  $^{136}\text{Xe}$ ,  
 $T_{1/2} > 1.07 \times 10^{26}$  yrs (90% C.L.),  $m_{bb} < 61-165$  meV

**CUORE**  $^{130}\text{Te}$ , ~206 kg,  $T_{1/2} > 2.9 \times 10^{25}$  yrs

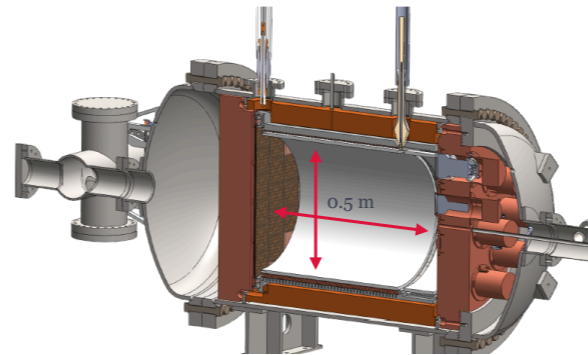
Also, **EXO-200**, **GERDA**, **MAJORANA**



CUORE Coll., Nature Apr 2022

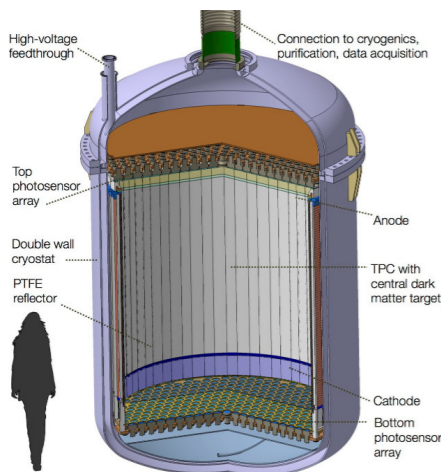


**SNO+**



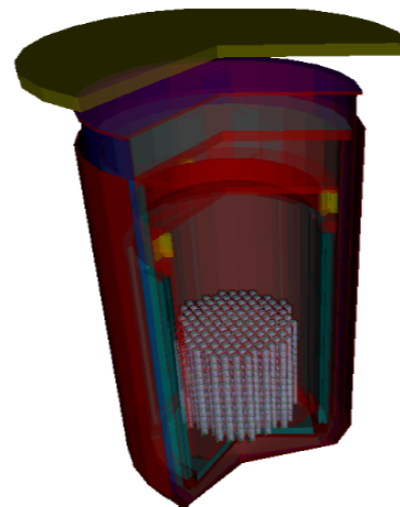
**NEXT-HD** **nEXO**

40 t liquid Xe TPC  
 (with 8.9%  $^{136}\text{Xe}$  → 3.6 t of  $^{136}\text{Xe}$ )



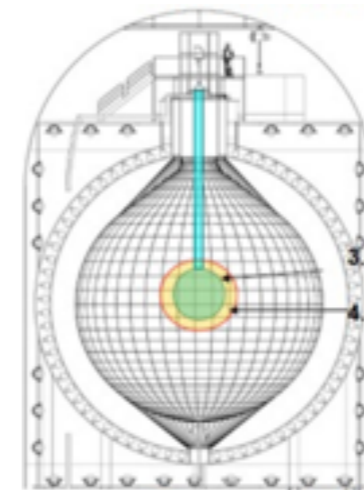
**LEGEND**

**LEGEND**  
**DARWIN**



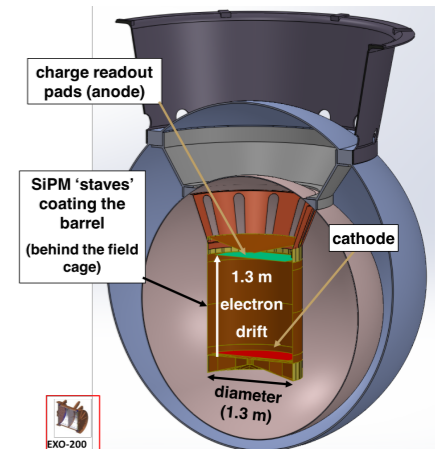
**CUPID**

Large Enriched Germanium Experiment for Neutrinoless  $\beta\beta$  Decay



**KamLAND2-Zen**

HPXe: PANDAX-III  
 Bolometers: AMoRE



5 ton LSc Xe

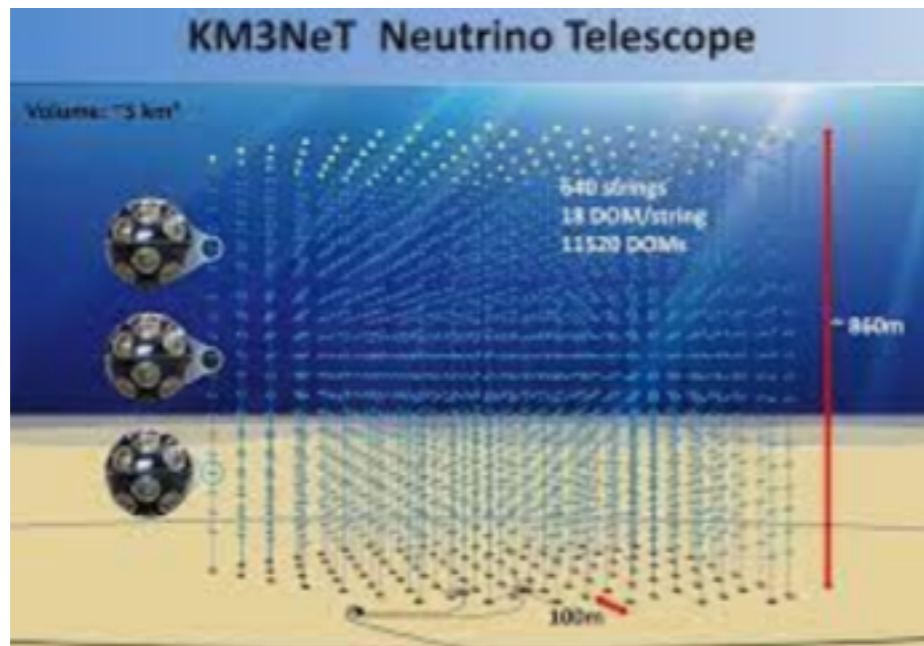
The ultimate goal of next generation is  $m_{bb} \sim 15-20$  meV.

# What do we still need to know?

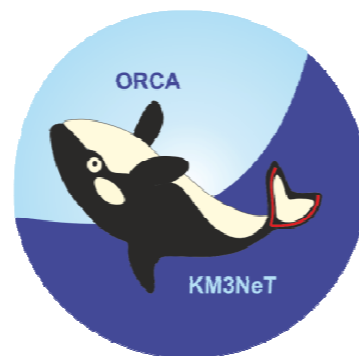
- What is the nature of neutrinos? Dirac vs Majorana?
- **What are the values of the masses? Absolute scale (KATRIN, ...?) and the ordering.**
- Is there CP-violation? Its discovery in the next generation of LBL depends on the value of delta.
- What are the precise values of mixing angles? Do they suggest an underlying pattern?
- Is the standard picture correct? Are there NSI? Sterile neutrinos? Other effects?

- **Mass ordering** via **neutrino oscillation** in **matter** or in **vacuum** (JUNO). Discovery expected within 6-7 years thanks to relatively large  $\theta_{13}$ .

## Atm neutrinos



Exploit matter effects in large detectors.



Long baseline neutrino oscillation experiments

## JUNO



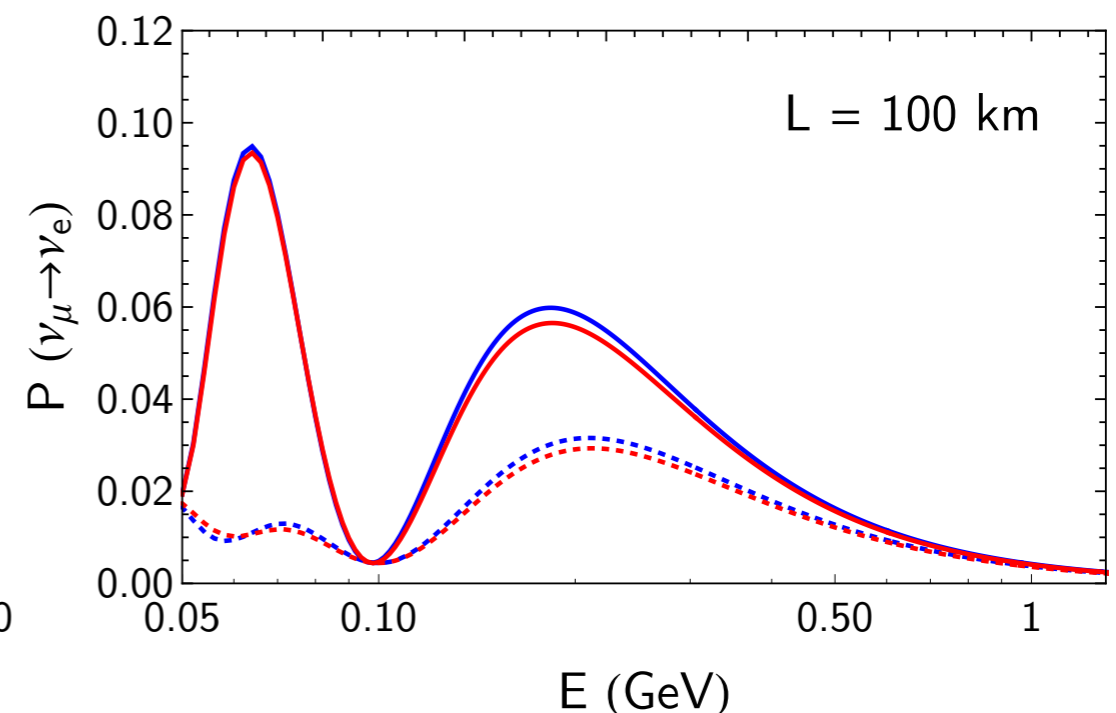
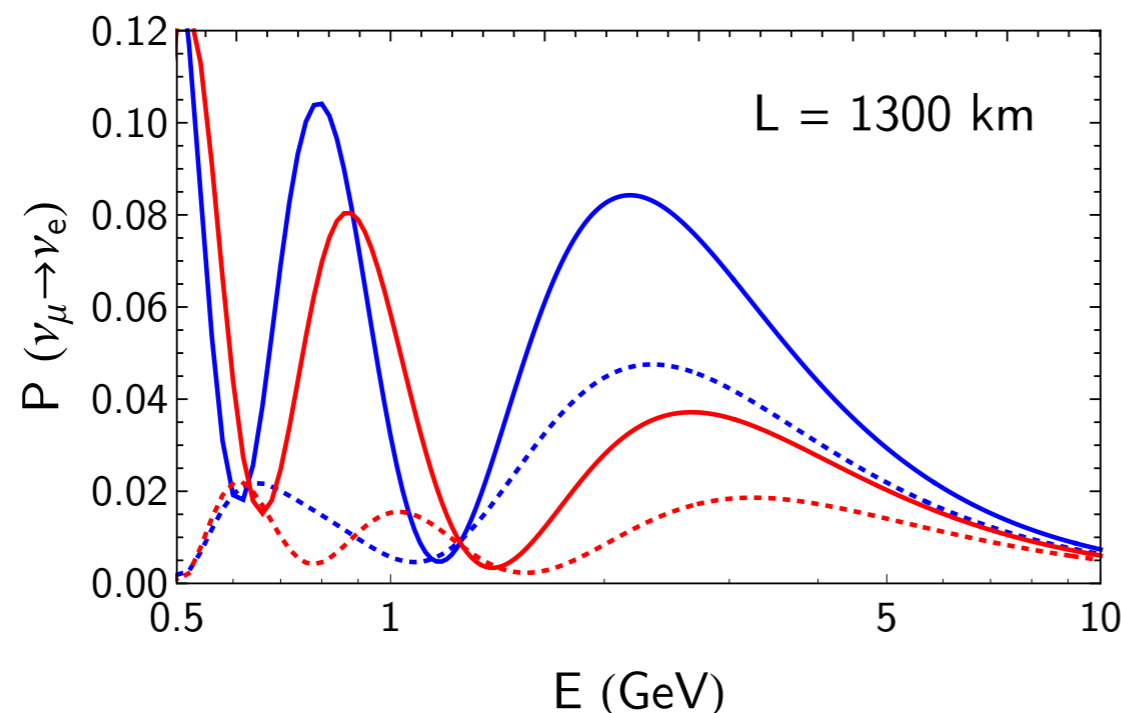
JUNO Coll., 2104.02565

JUNO uses a 20kton LSc detector to detect reactor nus at a baseline of  $\sim 60$  km. Excellent energy resolution is needed. Started.

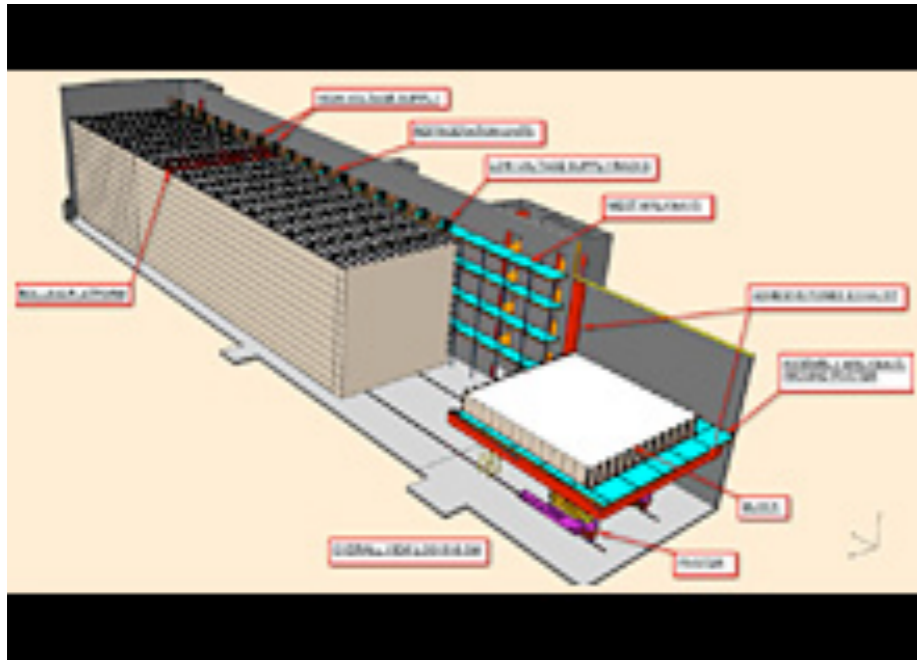
# Long baseline oscillations and the ordering

Long baseline neutrino oscillation experiments (T2K, NOvA, DUNE, T2HK) study the subdominant channels

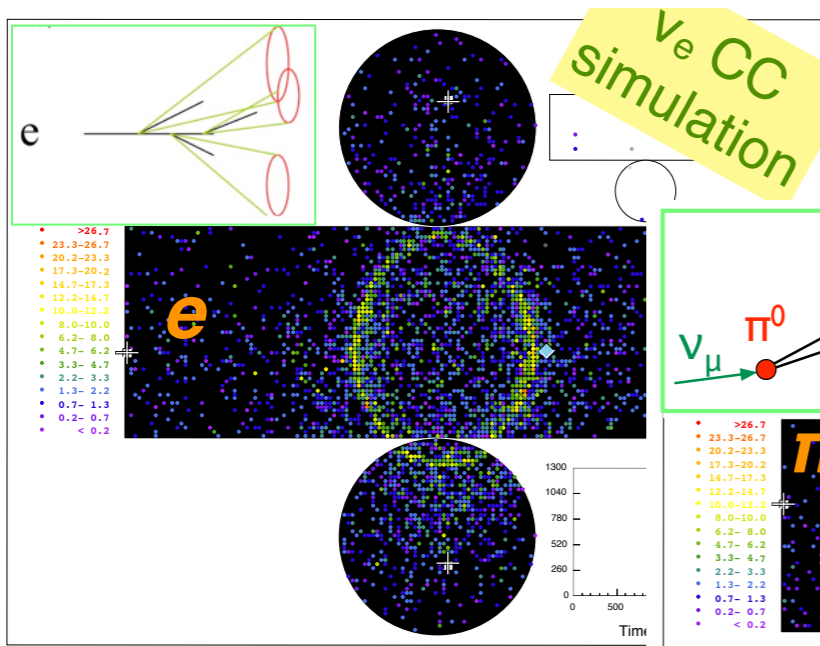
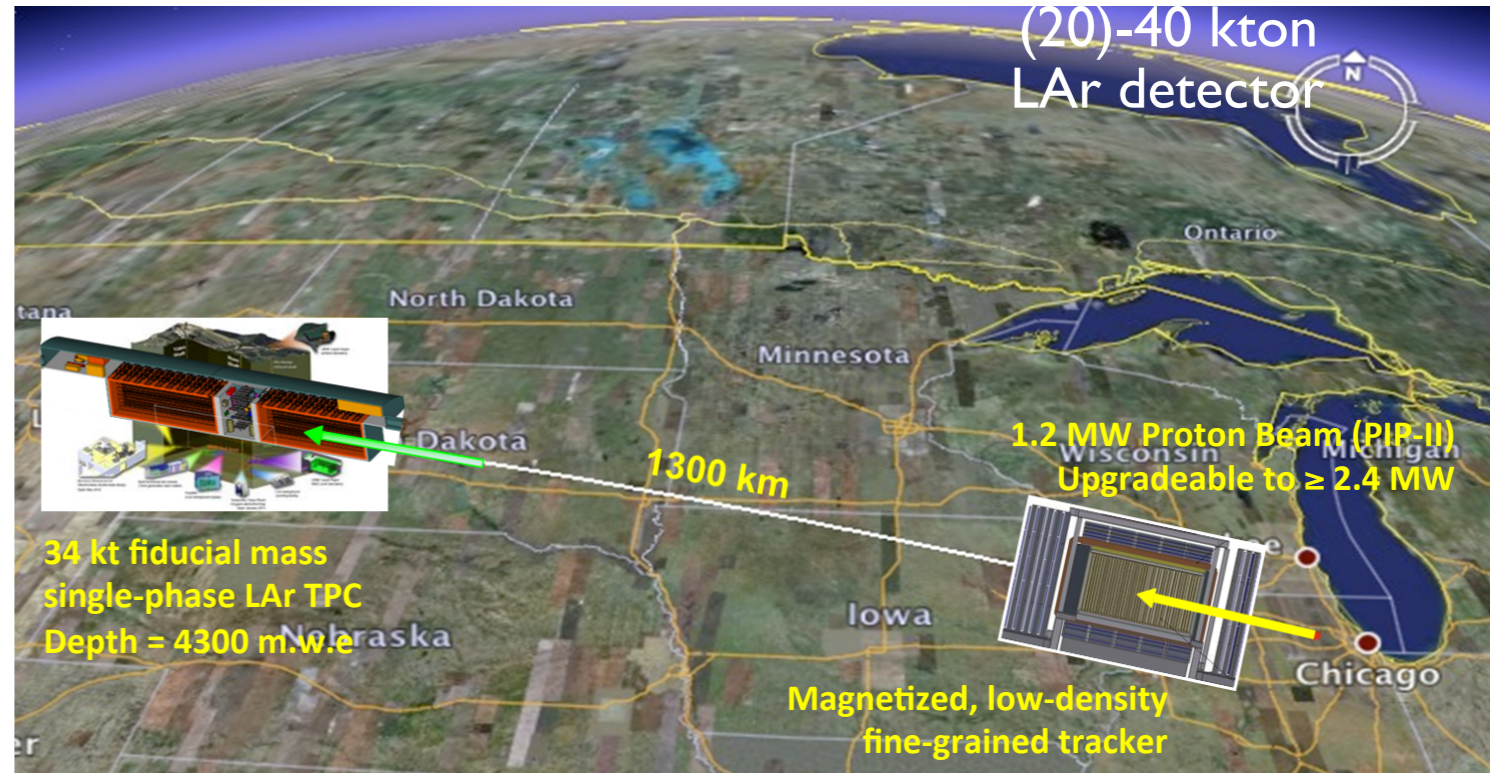
$$\begin{aligned}
 P_{\mu e} \simeq & 4c_{23}^2 s_{13}^2 \frac{1}{(1 - r_A)^2} \sin^2 \frac{(1 - r_A)\Delta_{31}L}{4E} && \text{A. Cervera et al., hep-ph/0002108;} \\
 & + \sin 2\theta_{12} \sin 2\theta_{23} s_{13} \frac{\Delta_{21}L}{2E} \sin \frac{(1 - r_A)\Delta_{31}L}{4E} \cos \left( \delta - \frac{\Delta_{31}L}{4E} \right) && \text{K. Asano, H. Minakata, I 103.4387;} \\
 & + s_{23}^2 \sin^2 2\theta_{12} \frac{\Delta_{21}^2 L^2}{16E^2} - 4c_{23}^2 s_{13}^4 \sin^2 \frac{(1 - r_A)\Delta_{31}L}{4E} && \text{S. K. Agarwalla et al., I 302.6773...}
 \end{aligned}$$



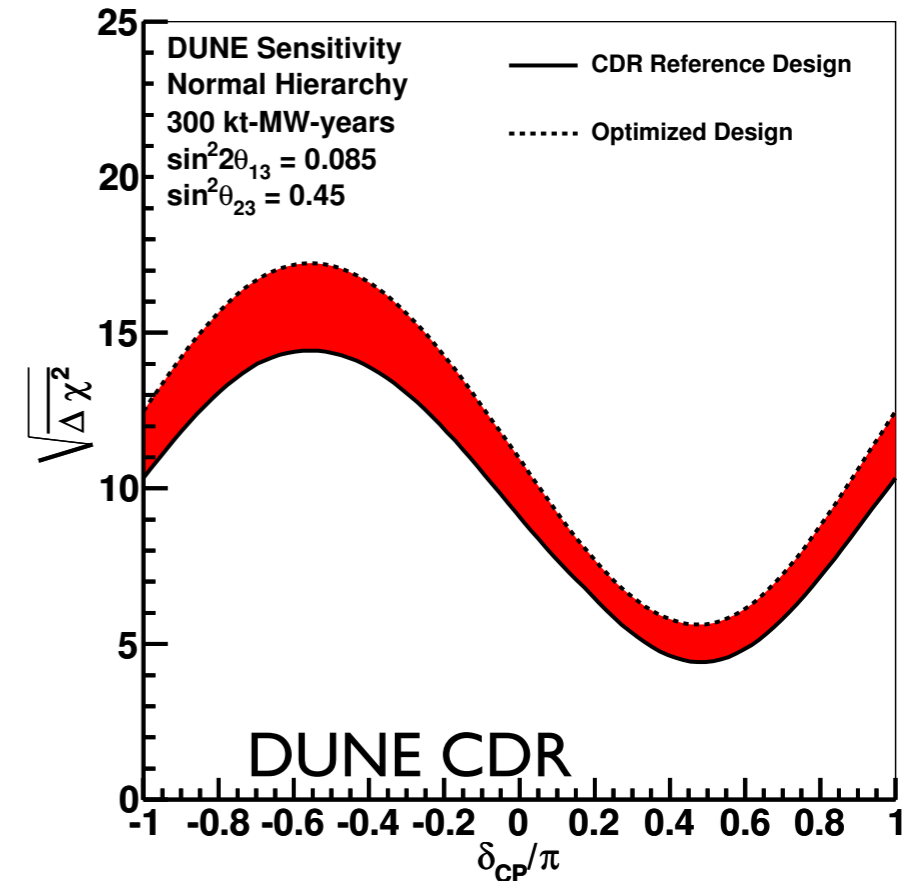
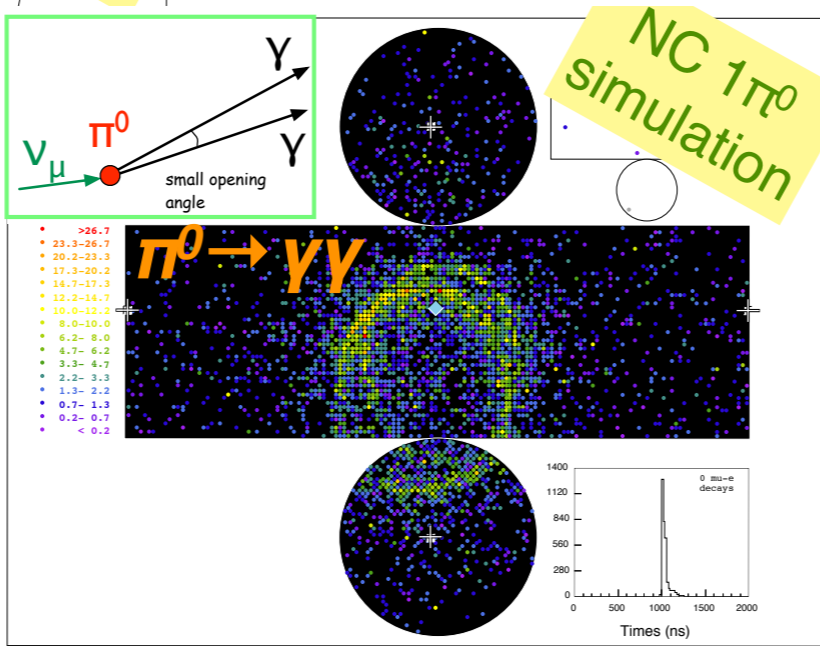
# Present/Future LBL exp **DUNE**: 1300 km on-axis (20)-40 kton LAr detector



**NOVA**: 810 km off-axis  
 ~14 kton plastic scintillator detector  
**T2K**: 295 km off-axis  
 ~22.5 kton WC detector



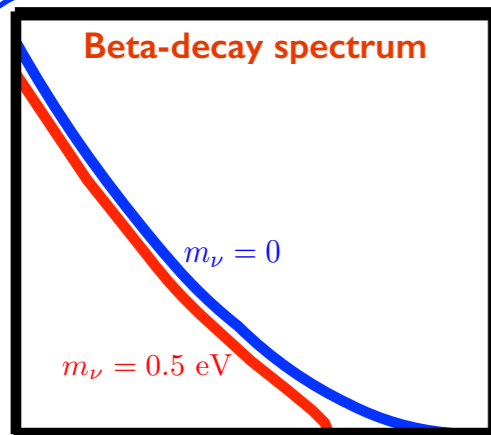
**T2HK**: 295 km off-axis  
 ~1 Mton WC detector



M. Shiozawa, for T2HK coll., NuPhys 2014

# Measuring neutrino masses

- Absolute mass scale.

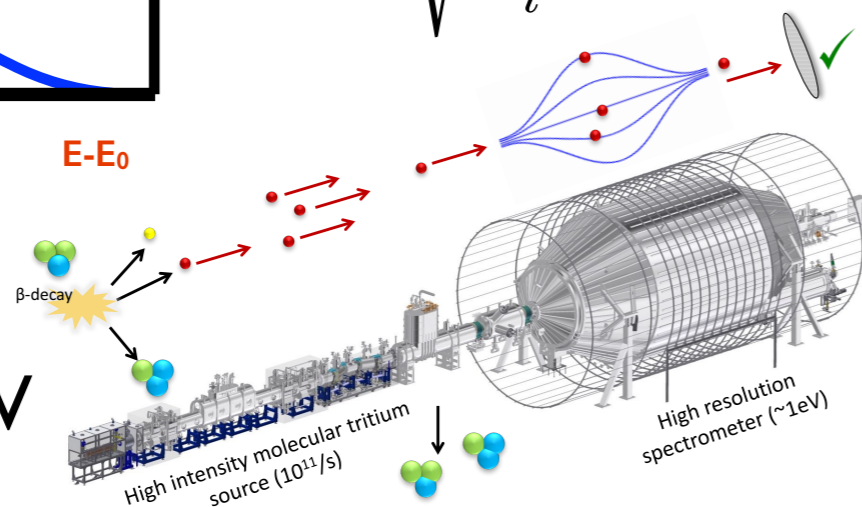


## Beta decay

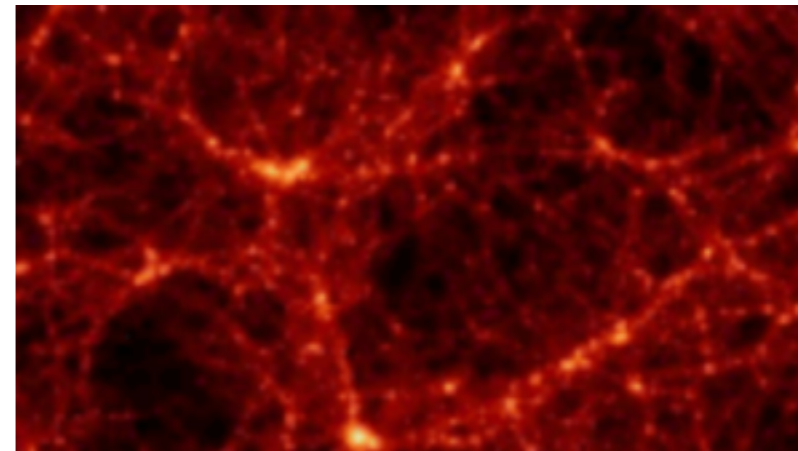
$$m_\beta \equiv \sqrt{\sum_i U_{ei}^2 m_i^2}$$

KATRIN

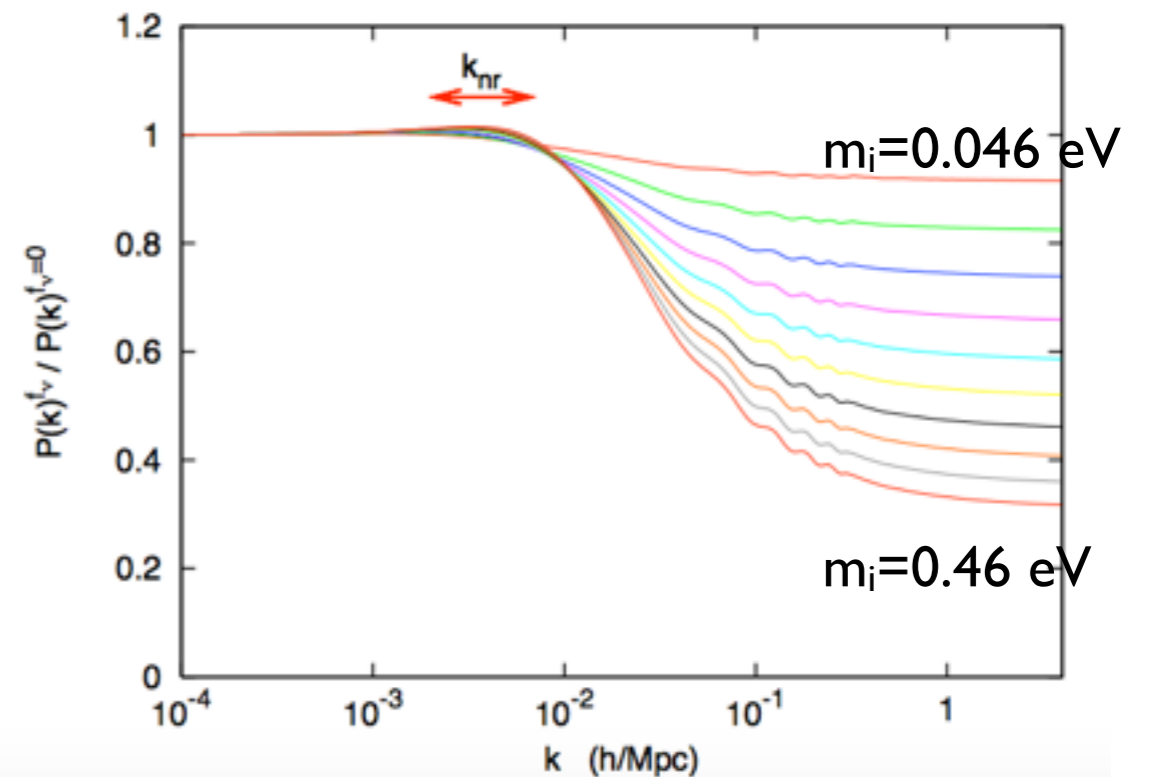
$m < 0.45 \text{ eV}$



## Cosmology

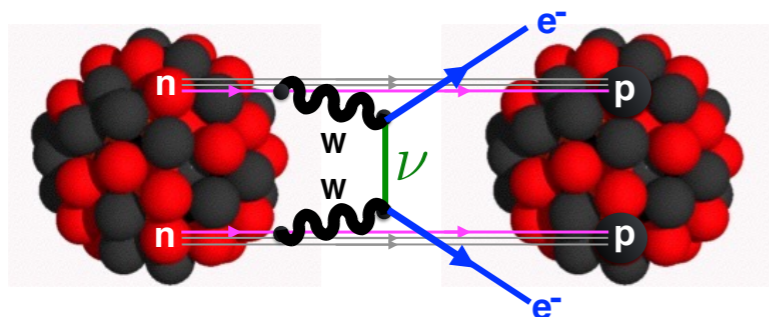


$$\sum_i m_i$$



J. Lesgourgues and S. Pastor, Phys. Rep. 429

## Neutrinoless dbeta decay



$$m_{\beta\beta} \equiv f(m_i, \alpha_{21}, \alpha_{31}, \delta)$$

# Direct mass measurement

The electron spectrum in beta decays depends on neutrino masses as

$$\frac{d\Gamma}{dE_e} = \sum_i |U_{ei}|^2 \frac{d\Gamma_i}{dE_e}(m_i) \quad \text{with}$$

$$\frac{d\Gamma_i}{dE_e} = \mathcal{C} |M|^2 p_e (E_e + m_e) (E_e - E_0) \sqrt{(E_e - E_0)^2 - m_i^2} F(E_e)$$

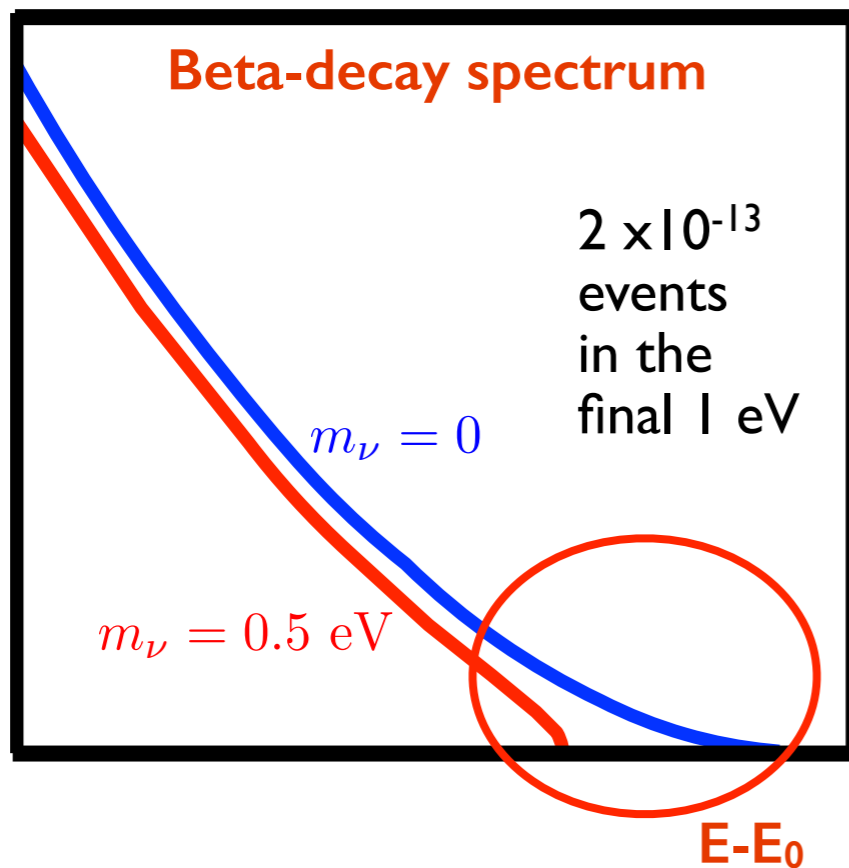
for QD:  $m_\beta \sim \sqrt{|U_{ei}|^2 m_i^2} \sim m_0$

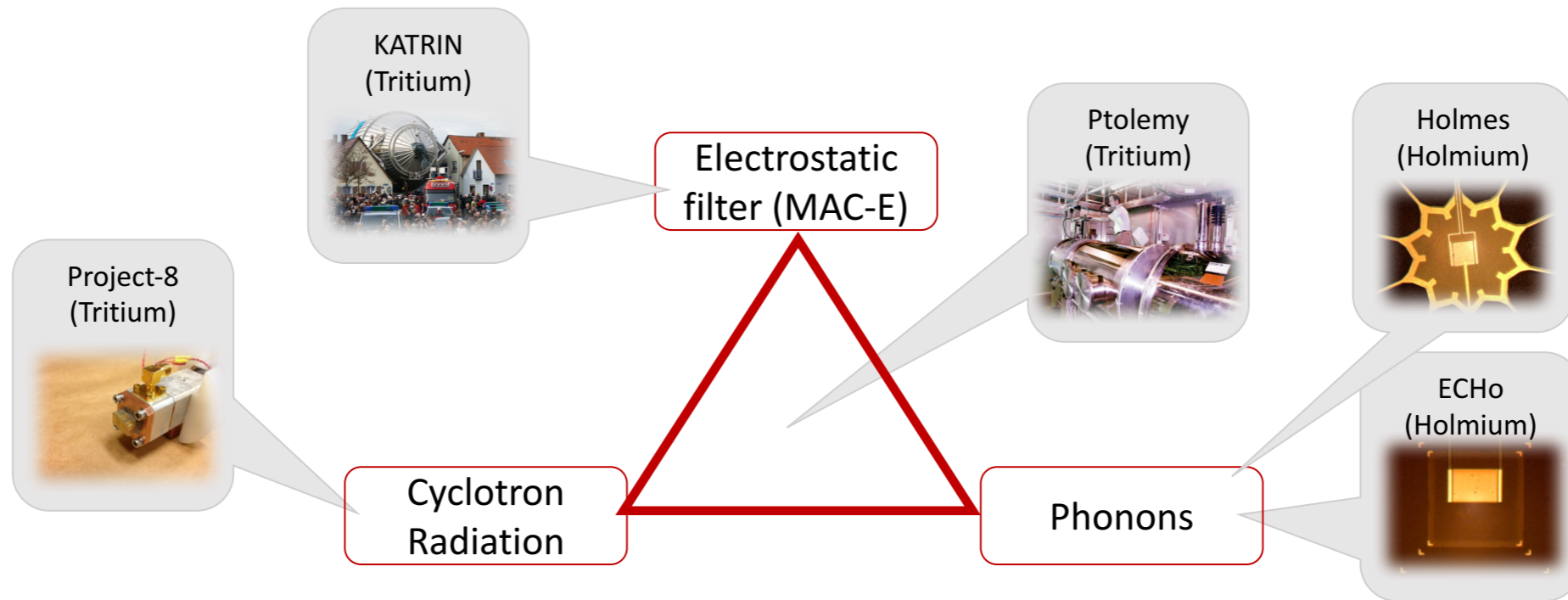
for NH:  $\frac{d\Gamma}{dE_e} \simeq \frac{d\Gamma(m_i = 0)}{dE_e}$

for IH:  $\frac{d\Gamma(\sqrt{|\Delta m_{23}^2|})}{dE_e}$

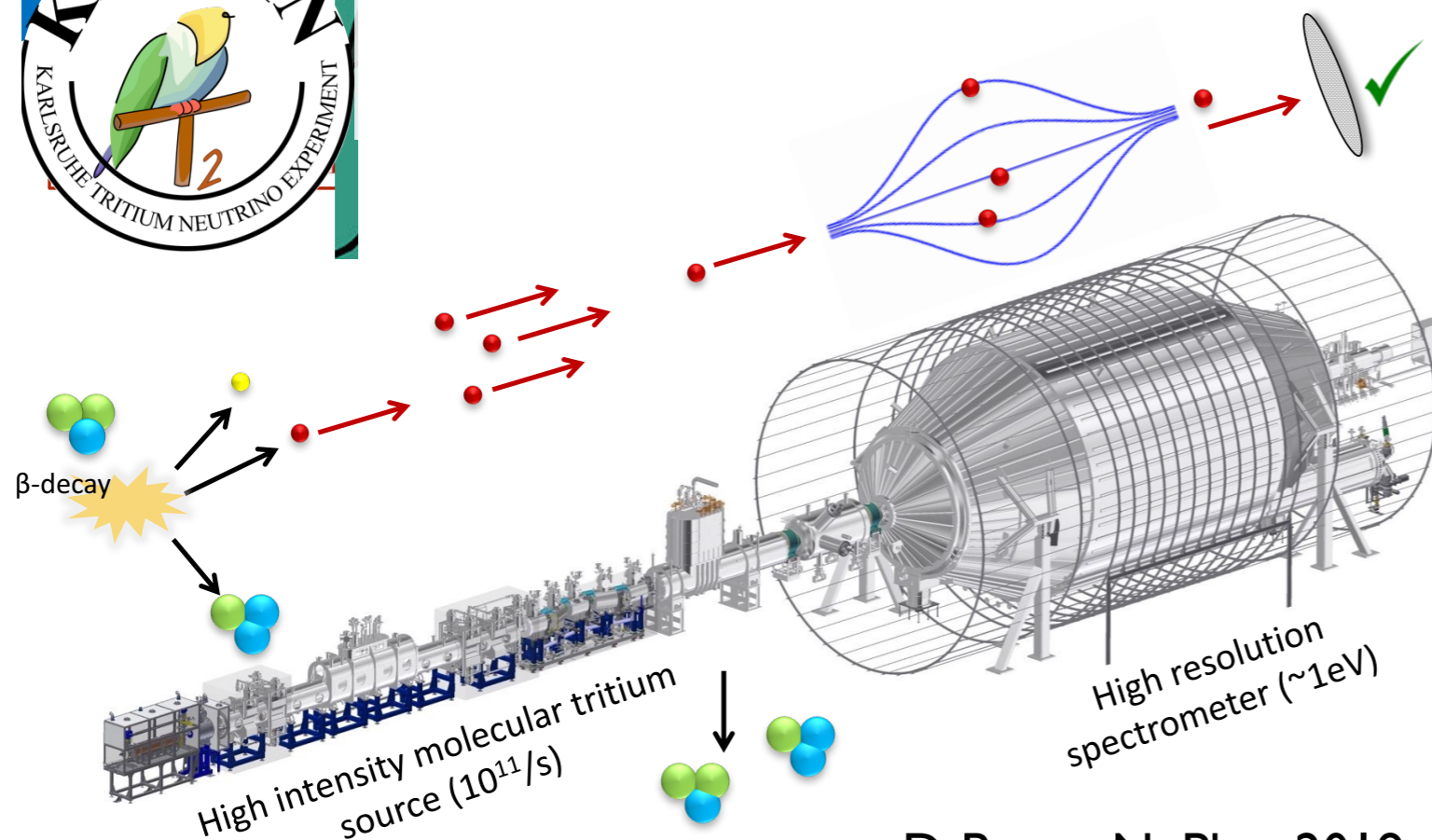
for IO, NO with partial hierarchy:

$$\frac{d\Gamma}{dE_e} = \sum_i |U_{ei}|^2 \frac{d\Gamma(m_i)}{dE_e}$$





S. Mertens, Granada Open Symposium ESPP



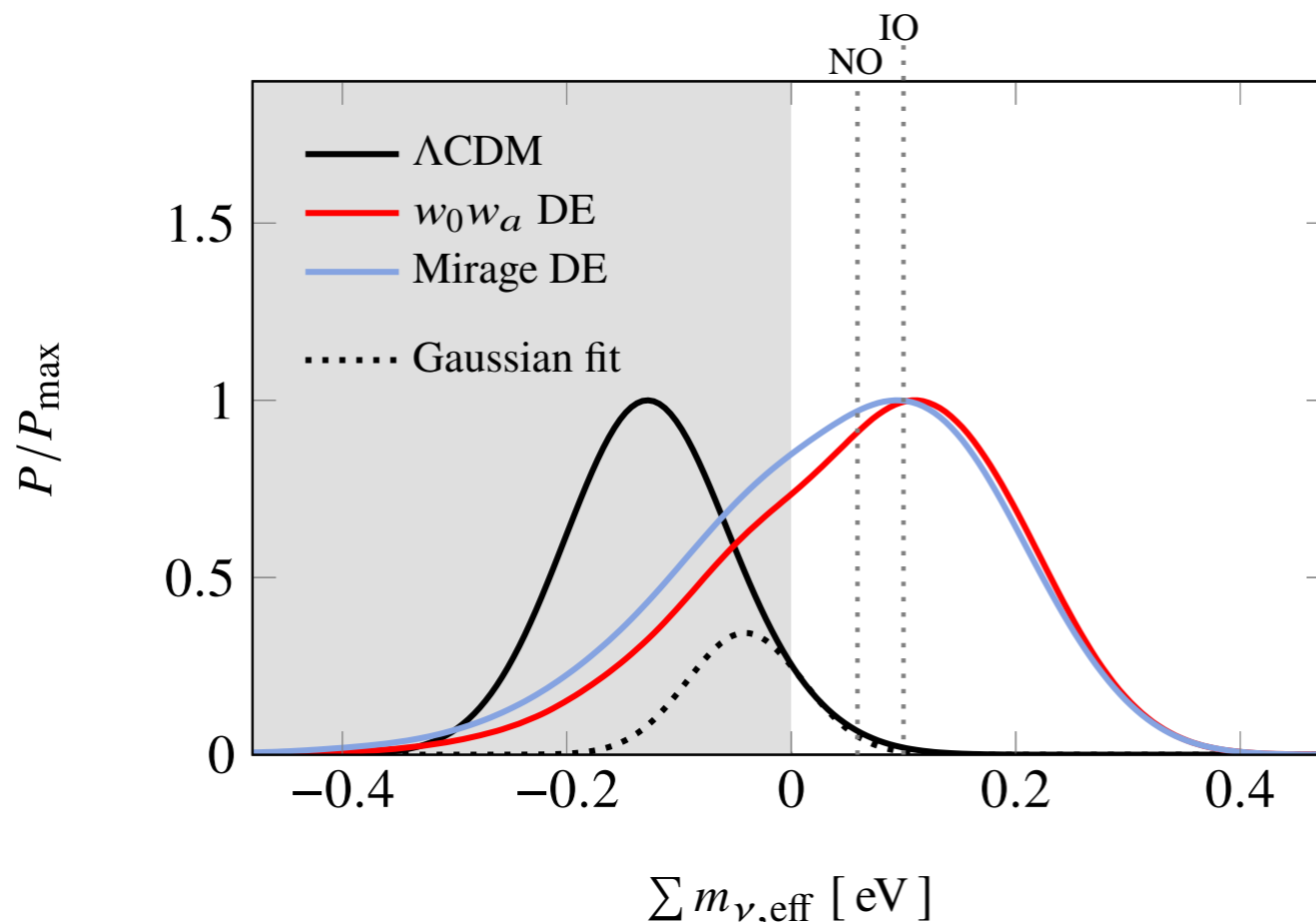
D. Parno, NuPhys 2019

**KATRIN** has started data taking.  
 Latest results:  $m < 0.45$  eV.  
 The ultimate sensitivity is to  $m_b < 0.2$  eV.

# Neutrinos in cosmology

Neutrinos contribute to the radiation density early on and to the matter density when they become non relativistic.

$$\Omega_\nu(a) = \sum_{i=1}^{N_\nu} \frac{8GT_\nu^4}{3\pi H_0^2} \int_0^\infty \frac{x^2 \sqrt{x^2 + a^2 m_i^2 / T_\nu^2}}{1 + e^x} dx.$$



Cosmology favours **NOT** negative neutrino masses but a component that has an effect opposite to that of neutrinos.

see Elbers et al., 2407.10695

Most precise determination of masses in future. Problem of underlying cosmological model and systematic errors.

# What do we still need to know?

- What is the nature of neutrinos? Dirac vs Majorana?
- What are the values of the masses? Absolute scale (KATRIN, ...?) and the ordering.
- **Is there CP-violation? Its discovery in the next generation of LBL depends on the value of delta.**
- **What are the precise values of mixing angles? Do they suggest an underlying pattern?**
- Is the standard picture correct? Are there NSI? Sterile neutrinos? Other effects?

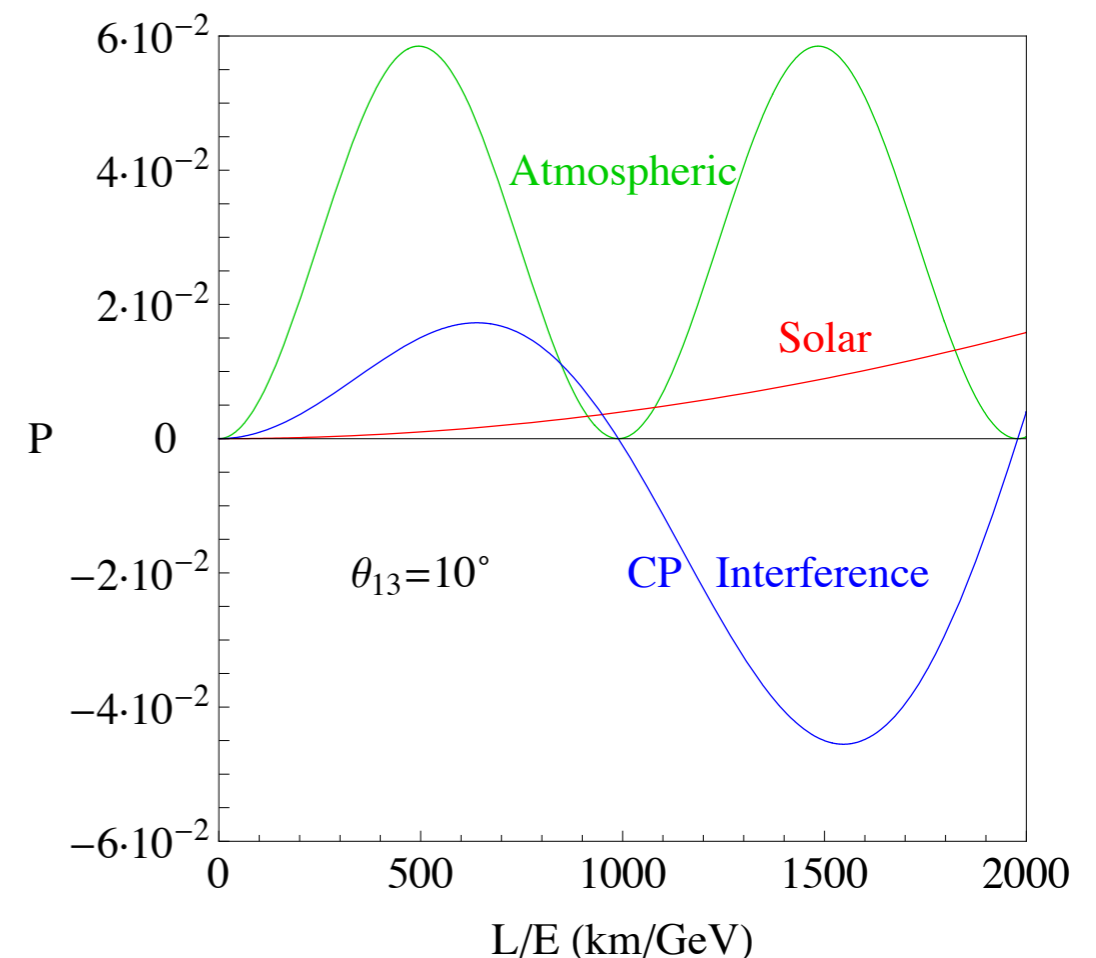
$$P_{\mu e} \simeq 4c_{23}^2 s_{13}^2 \frac{1}{(1-r_A)^2} \sin^2 \frac{(1-r_A)\Delta_{31}L}{4E}$$

$$+ \sin 2\theta_{12} \sin 2\theta_{23} s_{13} \frac{\Delta_{21}L}{2E} \sin \frac{(1-r_A)\Delta_{31}L}{4E} \cos \left( \delta - \frac{\Delta_{31}L}{4E} \right)$$

$$+ s_{23}^2 \sin^2 2\theta_{12} \frac{\Delta_{21}^2 L^2}{16E^2} - 4c_{23}^2 s_{13}^4 \sin^2 \frac{(1-r_A)\Delta_{31}L}{4E}$$

A. Cervera et al., hep-ph/0002108;  
K. Asano, H. Minakata, I 103.4387;  
S. K. Agarwalla et al., I 302.6773...

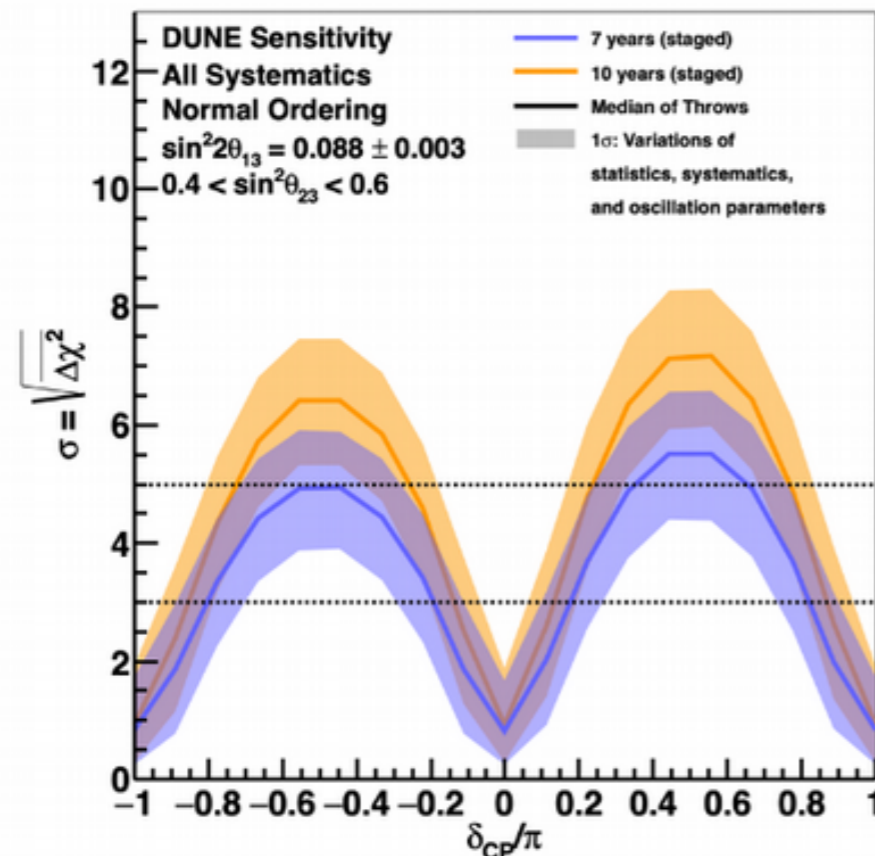
- Large  $\theta_{13}$  makes its searches possible but not ideal.
- Degeneracies with the mass hierarchy and  $\theta_{23}$ .
- CPV effects are more pronounced at low energy.



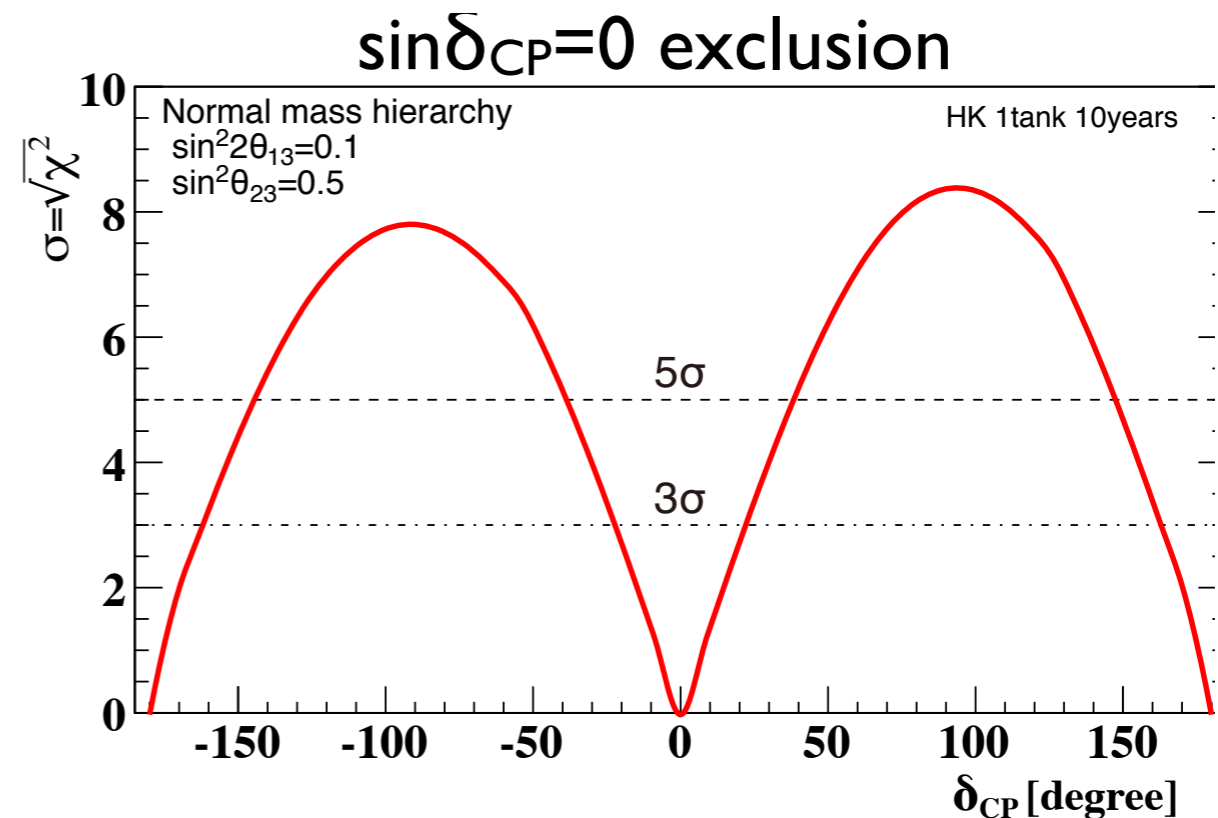
P. Coloma, E. Fernandez-Martinez, JHEP1204

- If delta is not too close to 0 or pi, CP violation can be discovered in the next decade.

### True Normal Ordering



M. Mooney, for DUNE, Neutrino 2020



M. Shiozawa, for HK, Neutrino 2018

- Majorana CPV is even more challenging and would require an exceedingly good measurement in neutrino less double beta decay and neutrino masses.

# Precision measurements of the oscillation parameters in LBL experiments

The precision measurement of the oscillation parameters is a primary physics goal of future LBL and reactor exp.

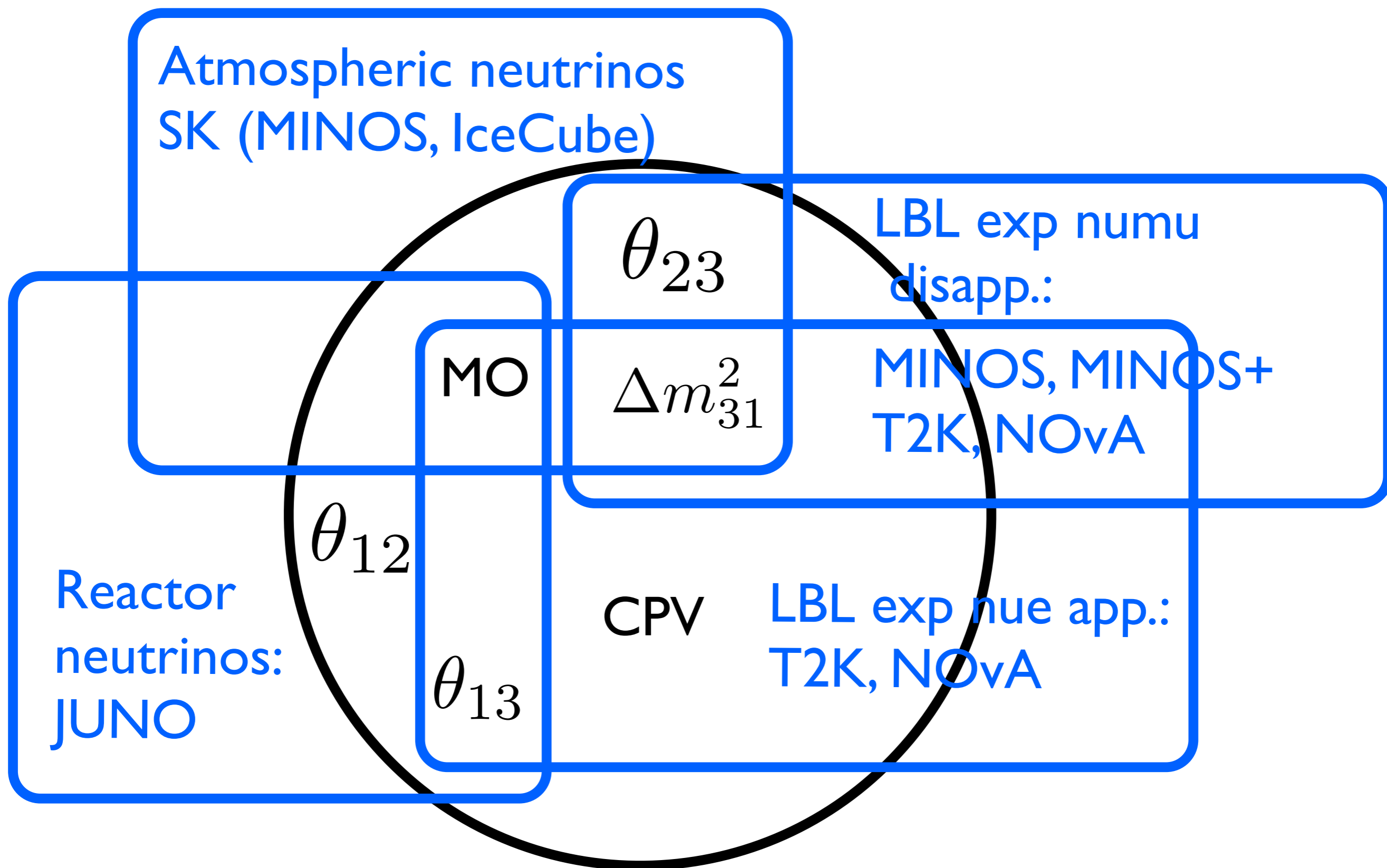
- The values of the mixing angles seem to indicate an underlying symmetry:  $\theta_{23} \sim 45^\circ$ ,  $\theta_{13}$  not too far from 0.
- Predictions for the CPV phase delta and relations among parameters in flavour models (e.g. sum rules). Example:

$$a = \sigma r \cos \delta \quad \sigma = 1, -1/2$$

with  $\sin \theta_{12} = \frac{1+s}{\sqrt{3}}$ ,  $\sin \theta_{13} = \frac{r}{\sqrt{2}}$ ,  $\sin \theta_{23} = \frac{1+a}{\sqrt{2}}$  King, 0710.0530

Crucial information in order to discriminate between different flavour models.

# Measurement of oscillation parameters



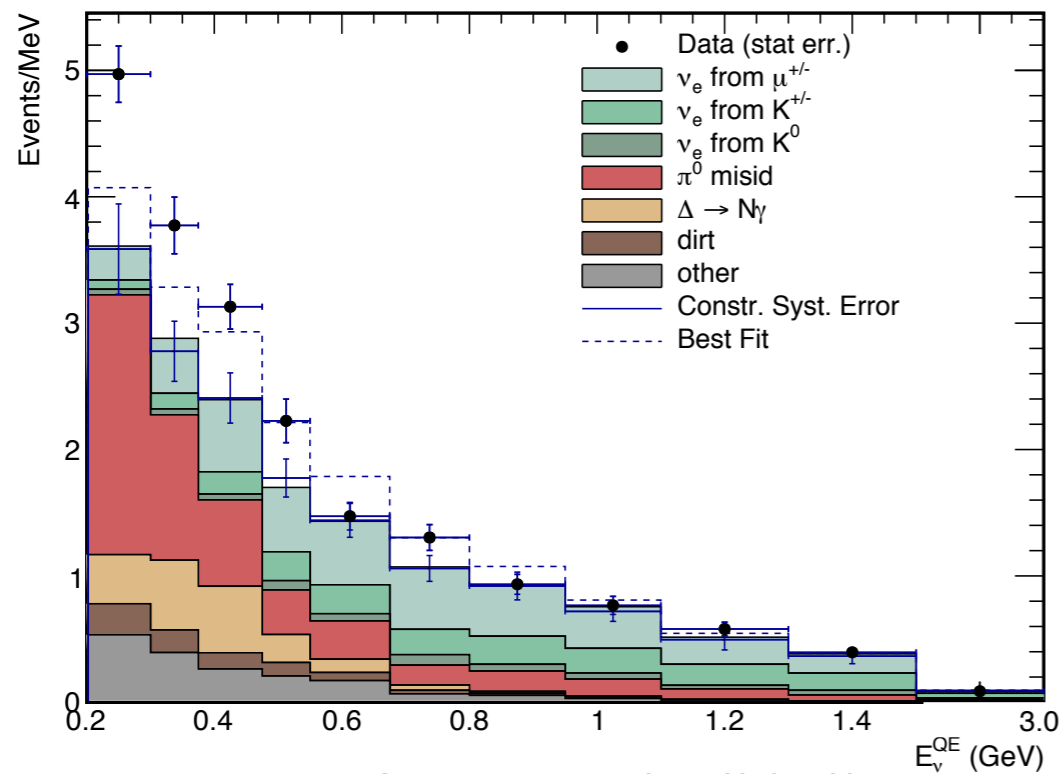
Also: Tests of standard neutrino paradigm

# What do we still need to know?

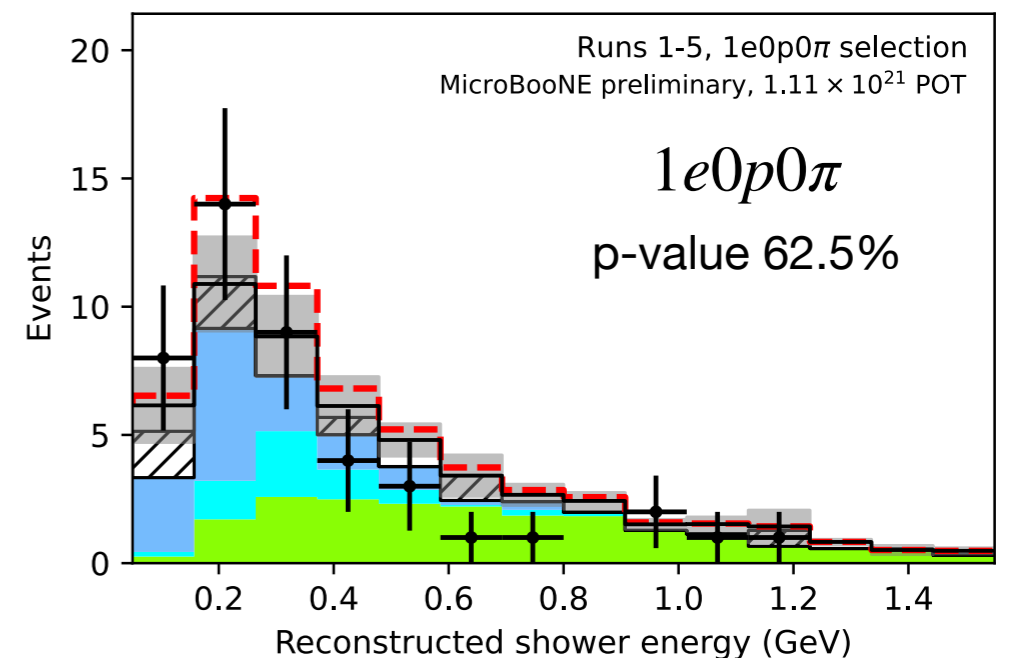
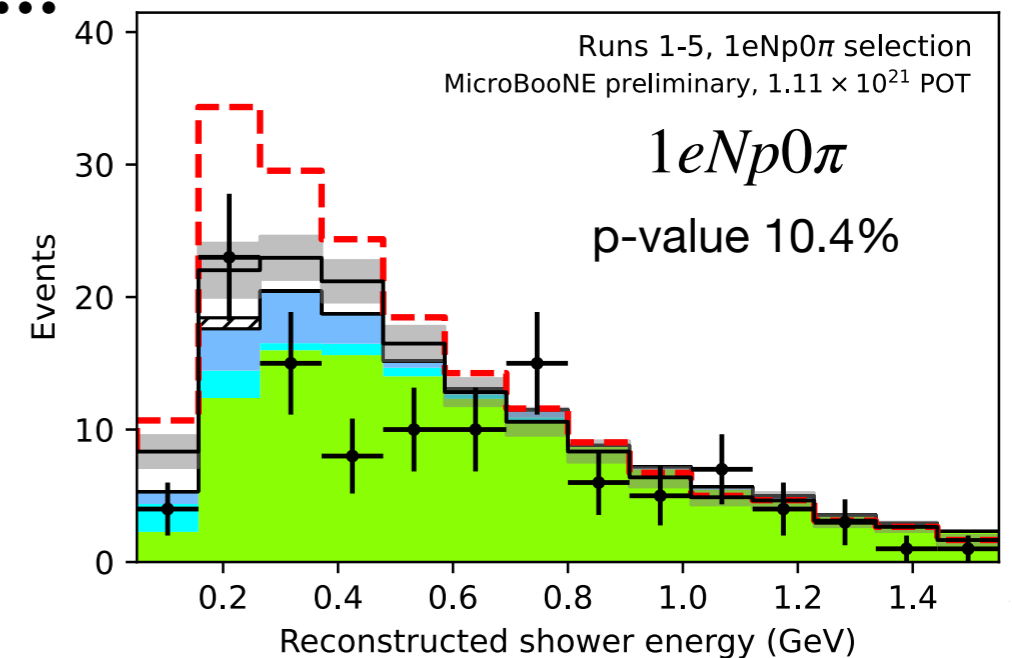
- What is the nature of neutrinos? Dirac vs Majorana?
- What are the values of the masses? Absolute scale (KATRIN, ...?) and the ordering.
- Is there CP-violation? Its discovery in the next generation of LBL depends on the value of delta.
- What are the precise values of mixing angles? Do they suggest an underlying pattern?
- **Is the standard picture correct? Are there NSI? Sterile neutrinos? Other effects?**

# Tests of the standard 3-neutrino paradigm

- Sterile neutrinos (as suggested or not by current hints). Synergy with SBN.
- New interactions: NSI, light mediators, trident...
- Decoherence, Lorentz violation...



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A deviation from the standard picture would have a groundbreaking impact.

**The ultimate goal is to understand**

- where do neutrino masses come from?**
- why there is leptonic mixing? and what is at the origin of the observed structure?**