Neutrino physics

5-6 May 2025

NDM 2025

Silvia Pascoli

HIDDeν
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

Horizon2020

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

INFN
Istituto Nazionale di Fisica Nucleare

Lecture 1: Neutrino in history and the SM; Neutrino mixing and oscillations

Aims

Introduce neutrinos and briefly recap the anomalies that led to the discovery of neutrino oscillations (see A. McDonald's lectures).

Develop the formalism of neutrino mixing.

Discuss theoretical aspects of neutrino oscillations.

Outline

1. Neutrino all around us
2. Keys steps in the history of neutrino physics
3. Neutrino mixing
4. Neutrino oscillation formalism
    - general case
    - 2-neutrino oscillations in vacuum
    - 3-neutrino oscillations in vacuum

Why study neutrinos?

- Neutrino masses imply new physics BSM.
Their origin is a necessary ingredient for the newSM.

- The least know of all SM fermions (a window on the BSM?).

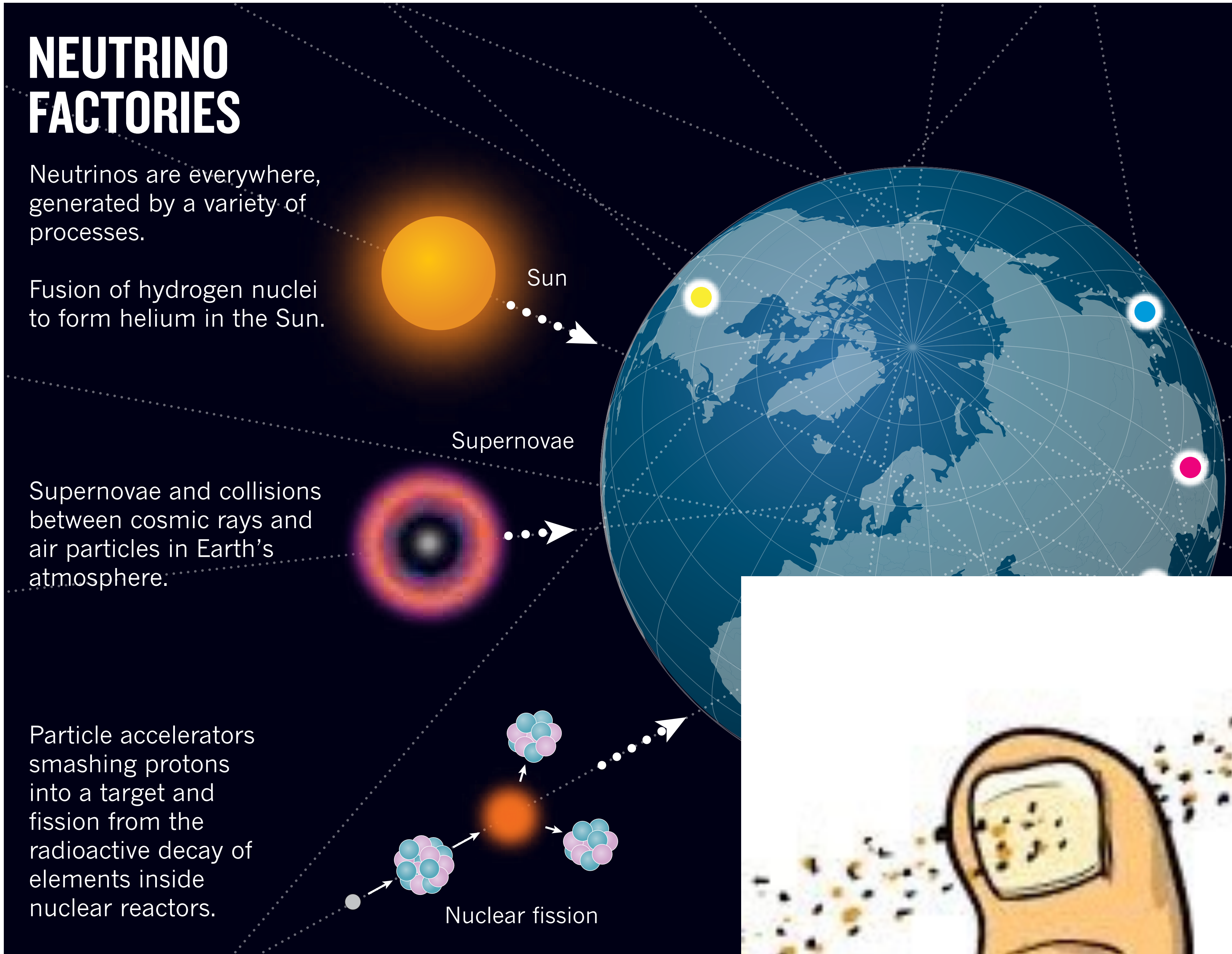
- Their nature (and mass) is related to the fundamental symmetries of nature (lepton number, link with proton decay).

- The most abundant of all fermions in the Universe with strong impact of its evolution.

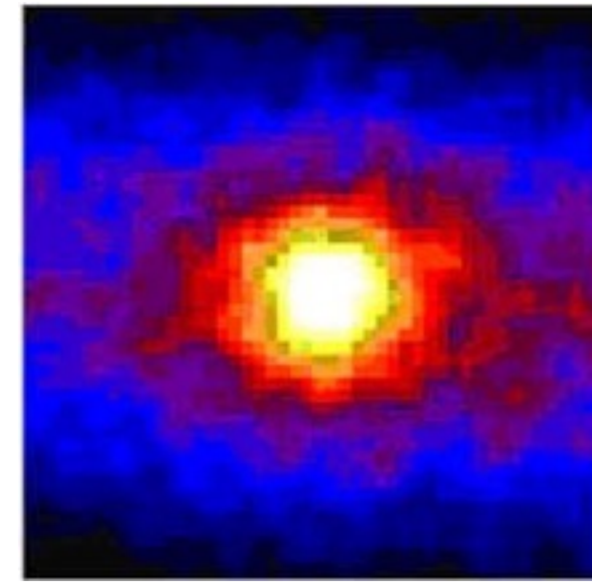
- Neutrino mass models can explain the baryon asymmetry of the Universe.

- A complementary window on the problem of flavour.

Neutrinos are all around and through us in large amounts.



@Nature, 2015



@Super-Kamiokande

Neutrinoscope

Neutrinoscope is a free App for iPhone and iPad developed by Cambridge Consultants and Durham University. It allows to visualise the neutrinos as they are around us.

Useful references

- C. Giunti, C.W. Kim, Fundamentals of Neutrino Physics and Astrophysics, Oxford University Press, USA (May 17, 2007)

- M. Fukugita, T. Yanagida, Physics of Neutrinos and applications to astrophysics, Springer 2003

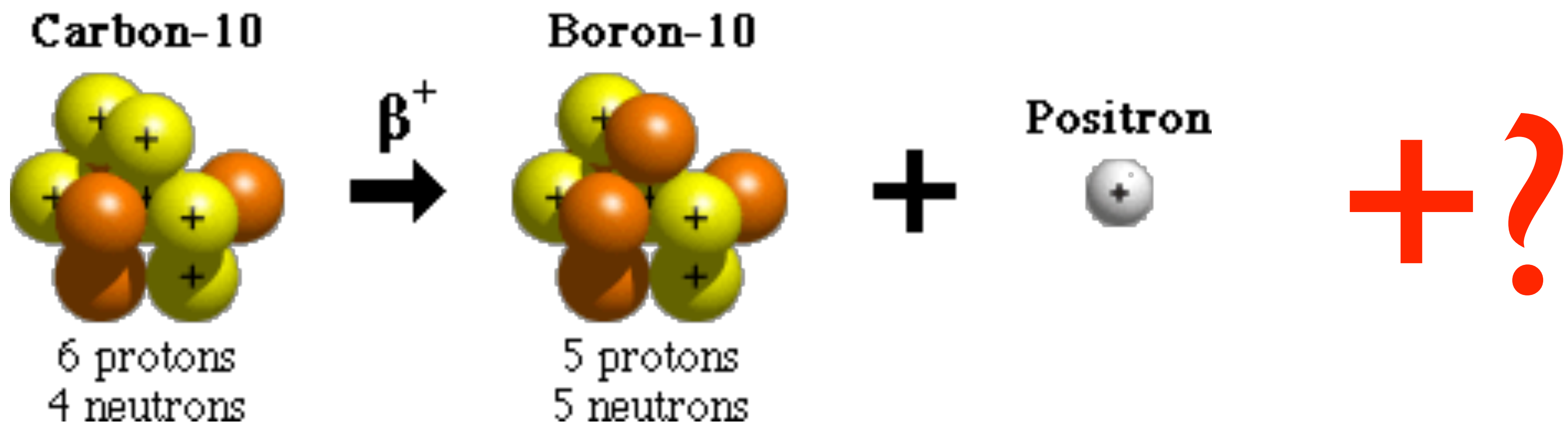
- A. De Gouvea, TASI lectures, hep-ph/0411274

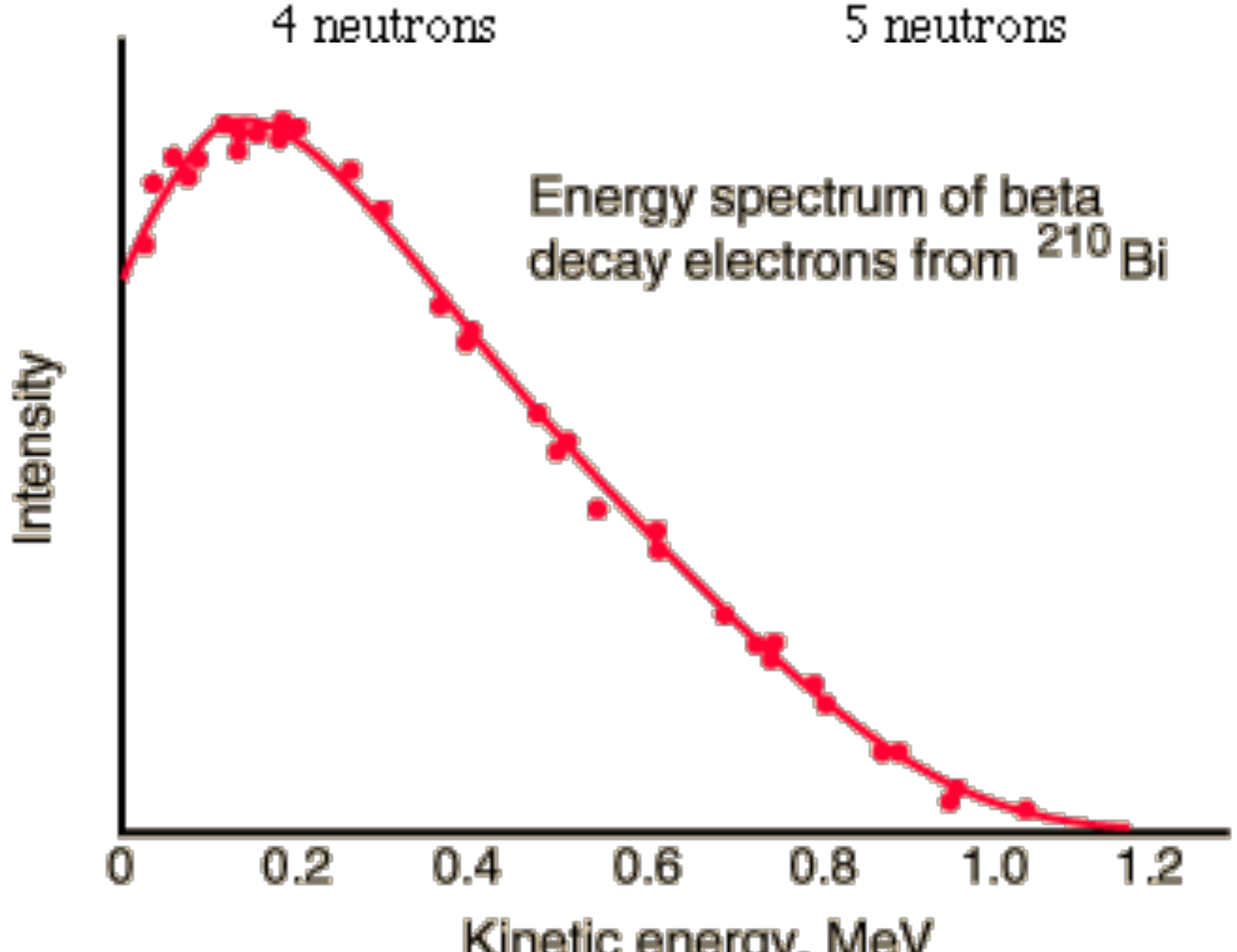
- A. Strumia and F. Vissani, hep-ph/0606054.

- Talks at the Neutrino 2024 conference

A brief history of our knowledge of neutrinos

Despite being the most abundant fermion in the Universe, we did not even realise they existed till the '30. The idea came about in the study of beta decays and a puzzle which troubled physicists for several decades.





The experimental results were puzzling. The positron (= anti-electron) carries away different amounts of energy in each individual observed decay.

G. J. Neary, Roy. Phys. Soc. (London), A175, 71 (1940).

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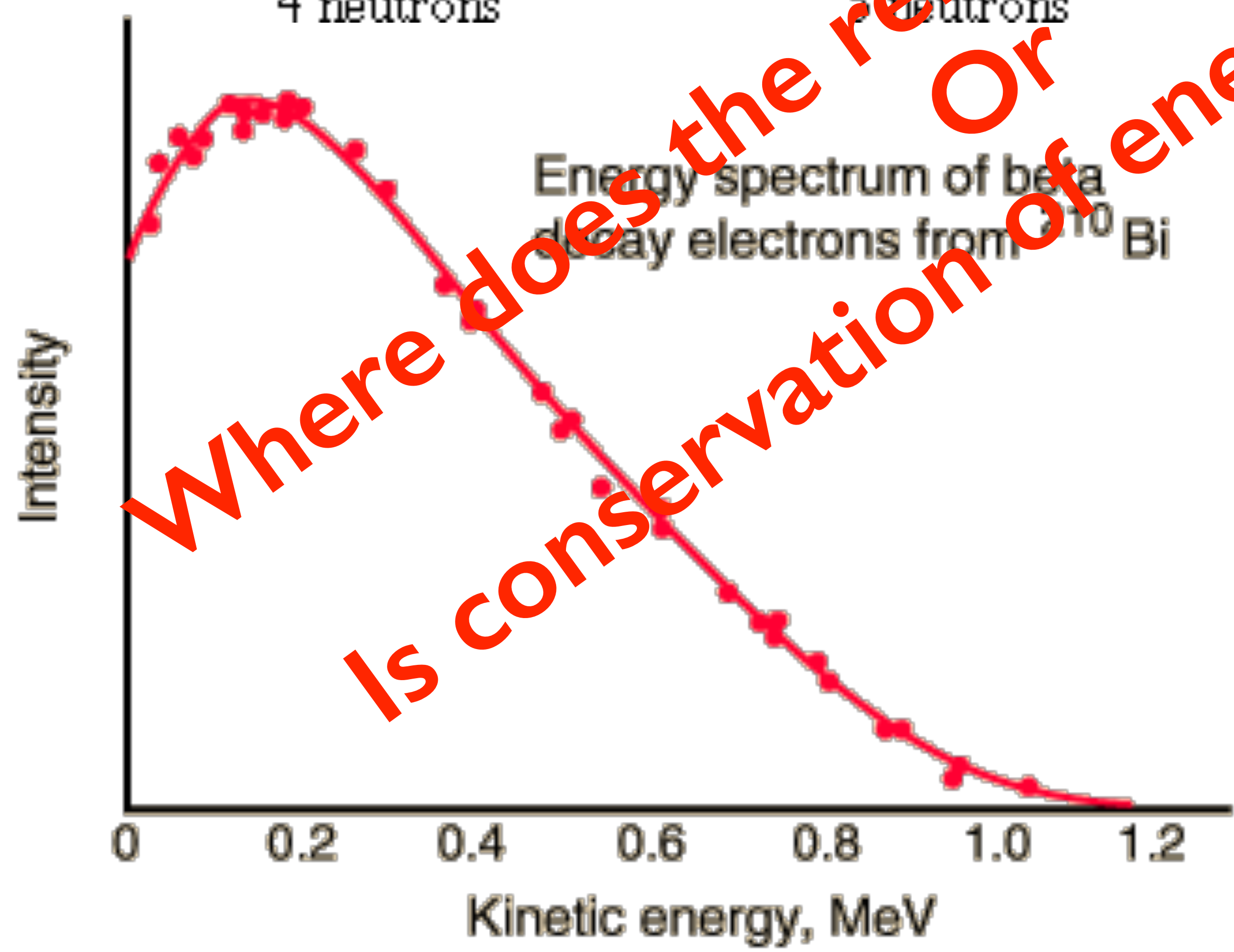

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The proposal of the “neutrino” was put forward by W. Pauli in 1930. [Pauli Letter Collection, CERN]

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

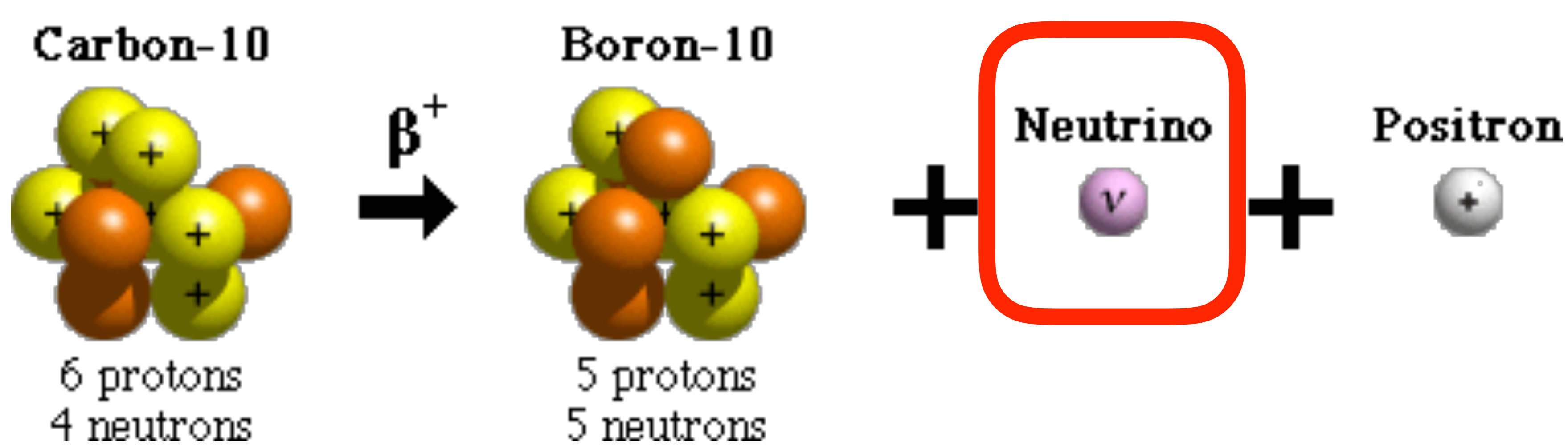
Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

Dear radioactive ladies and gentlemen,

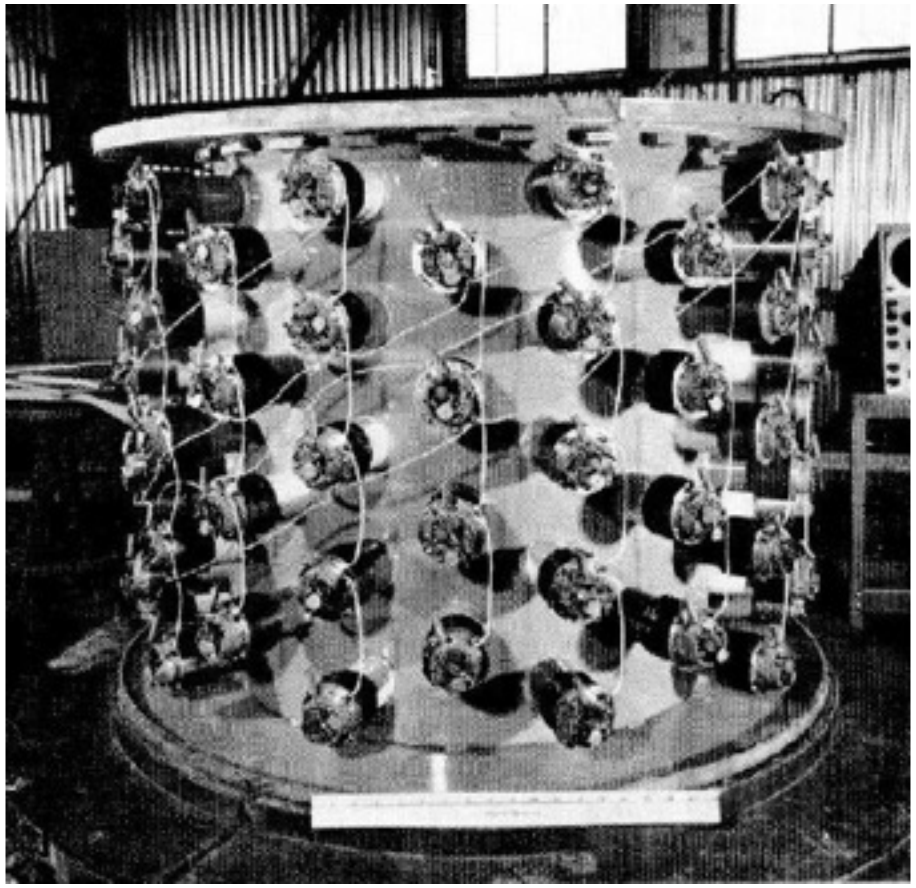
…I have hit upon a desperate remedy to save the … energy theorem. Namely the possibility that there could exist in the nuclei electrically neutral particles that I wish to call neutrons, which have spin 1/2 … The mass of the neutron must be … not larger than 0.01 proton mass. …in β decay a neutron is emitted together with the electron, in such a way that the sum of the energies of neutron and electron is constant.



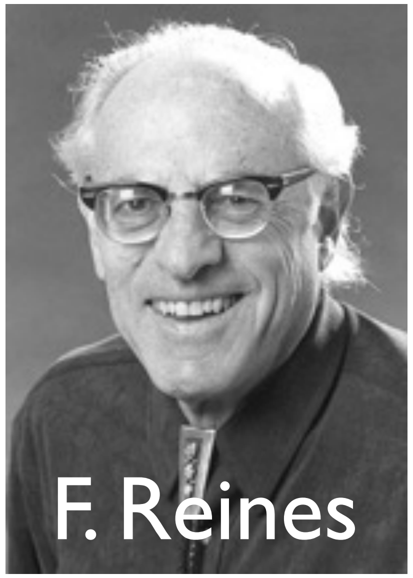
- Since the neutron was discovered in 1932 by J. Chadwick, Fermi, following E. Amaldi, used the name "neutrino" (little neutron) and later proposed the Fermi theory of beta decay.

E. Fermi

- Reines and Cowan discovered neutrinos in 1956 using inverse beta decay.

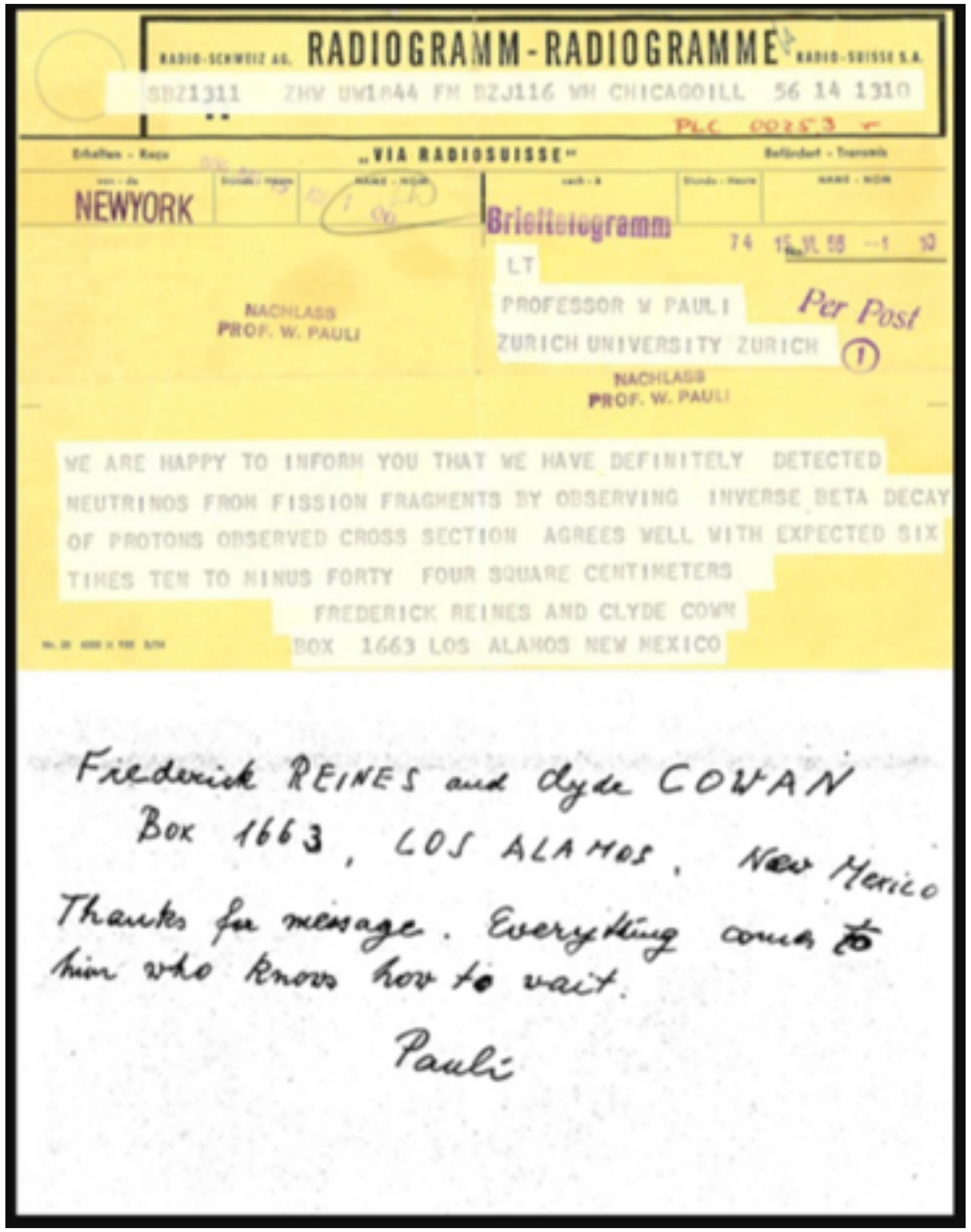


Savannah River experiment



F. Reines

The Nobel Prize in Physics 1995



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PROFESSOR W PAULI

ZURICH UNIVERSITY ZURICH

Per Post

NACHLASS PROF. W. PAULI

WE ARE HAPPY TO INFORM YOU THAT WE HAVE DEFINITELY DETECTED NEUTRINOS FROM FISSION FRAGMENTS BY OBSERVING INVERSE BETA DECAY OF PROTONS OBSERVED CROSS SECTION AGREES WELL WITH EXPECTED SIX TIMES TEN TO MINUS FORTY FOUR SQUARE CENTIMETERS

FREDERICK REINES AND CLYDE COWN

BOX 1663 LOS ALAMOS NEW MEXICO

Frederick REINES and Clyde COWAN

Box 1663, LOS ALAMOS, New Mexico

Thanks for message. Everything comes to him who knows how to wait.

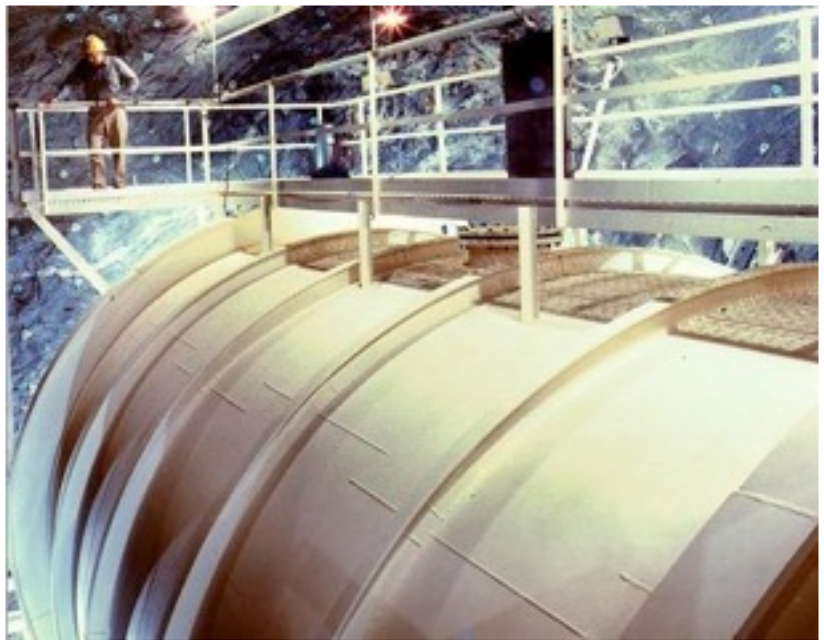
Pauli

- Madame Wu in 1956 demonstrated that the parity symmetry is violated in weak interactions. Neutrinos come only as left-handed (spin opposite to momentum) differently from all other fermions.

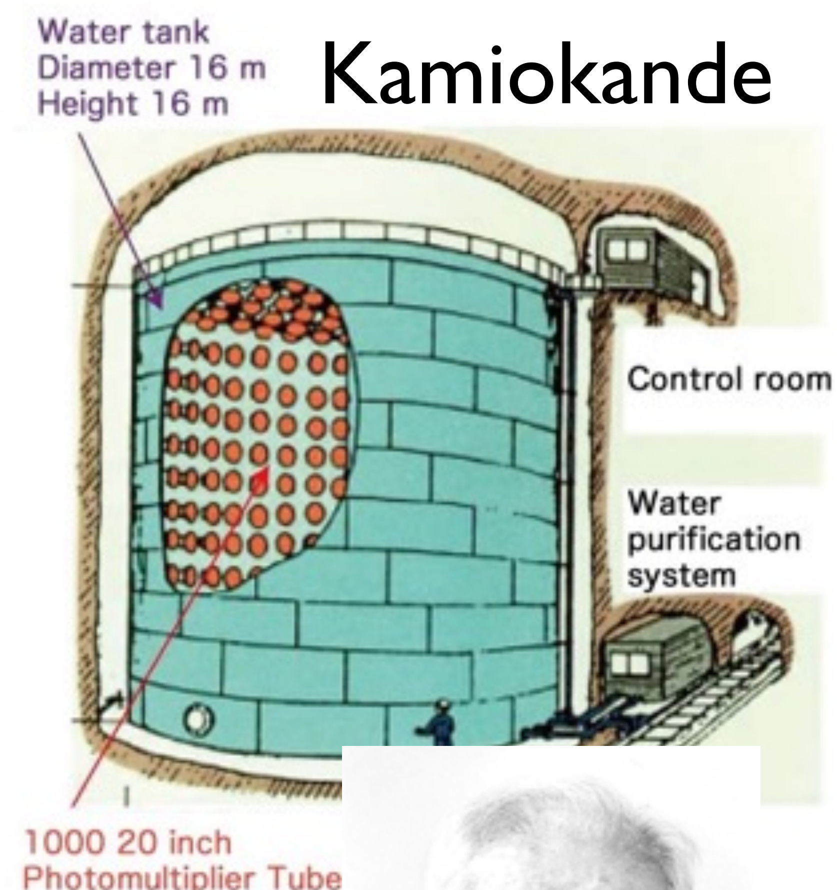
- Muon neutrinos were discovered in 1962 by L. Lederman, M. Schwartz and J. Steinberger.

The Nobel Prize in Physics 1988

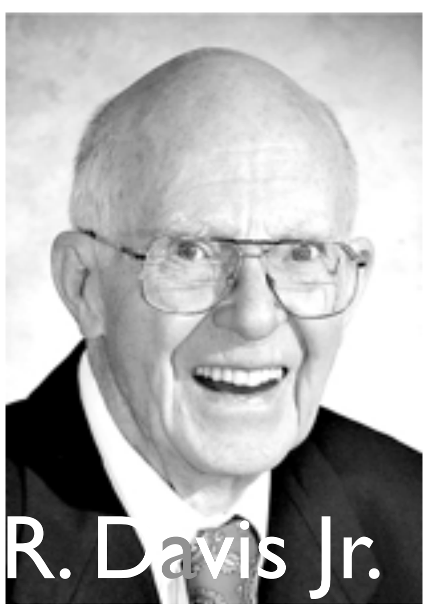
After their discovery by Cowan and Reines, searches were performed for astrophysical neutrinos, produced in the Sun, Supernova (just by chance) and in the atmosphere.



The Homes take experiment.



Kamiokande



Nobel prize in 2002

The first atmospheric neutrinos were observed in 1965 by the Kolar Gold Field (KGF) and Reines' experiments.

The first idea of neutrino oscillations was put forward by B. Pontecorvo in 1957.

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1950: Hunt for missing atomic scientist

The British intelligence service MI5 has been brought into the hunt for the missing atomic scientist Bruno Pontecorvo who has not been seen for about seven weeks.

Professor Pontecorvo and his family arrived in Finland at the beginning of September but they have since disappeared. There is speculation the family may have gone to the Soviet Union.

The professor had recently left his post as a principal scientific officer at Harwell atomic research station in Oxfordshire and was due to begin a new job at Liverpool University in January.

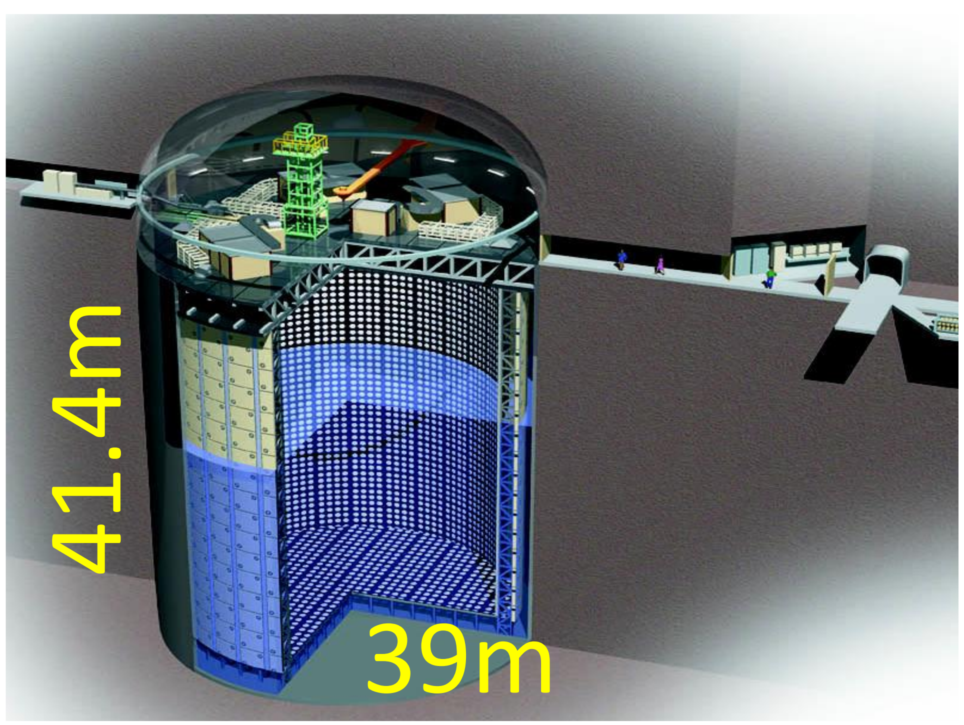
Professor Pontecorvo was last seen seven weeks ago in Finland

In Context

Bruno Pontecorvo's post as Professor of Experimental Physics at the University of Liverpool was cancelled.

Neutrinos seemed to fit well in the picture of the SM which was forming. But soon some anomalies started to appear.

- First indications of ν oscillations came from solar ν: less electron neutrinos were observed than expected. Where did the others go?

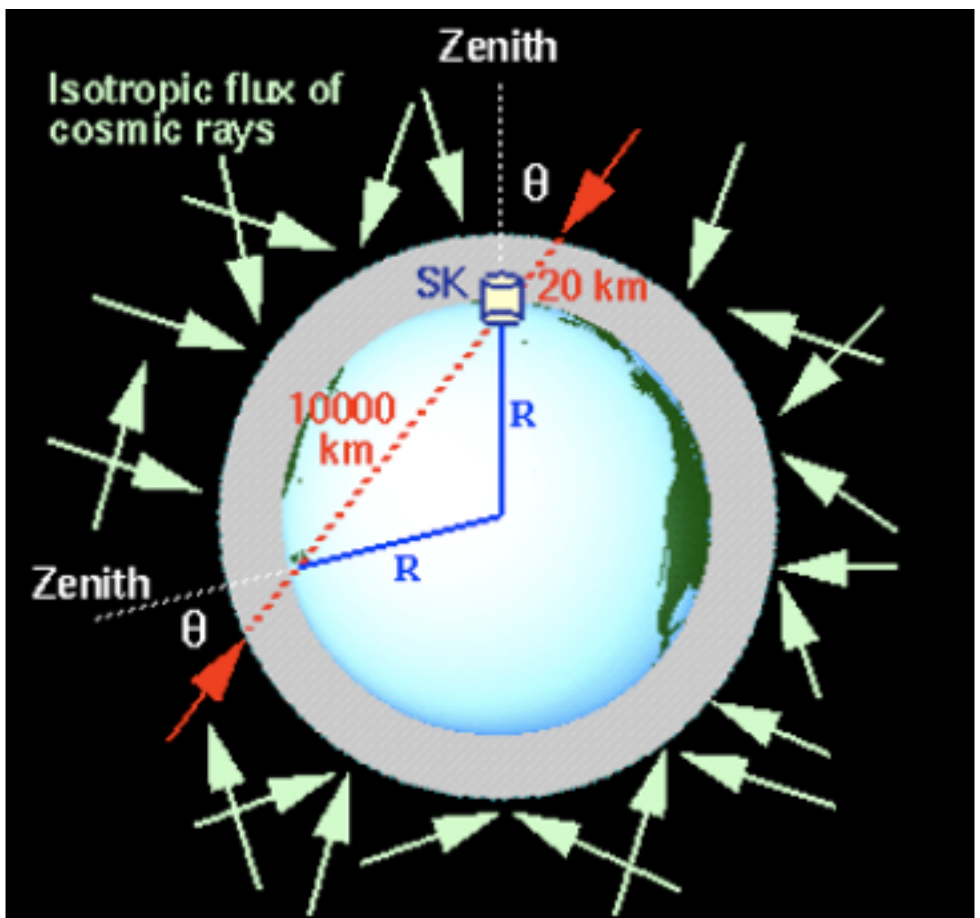


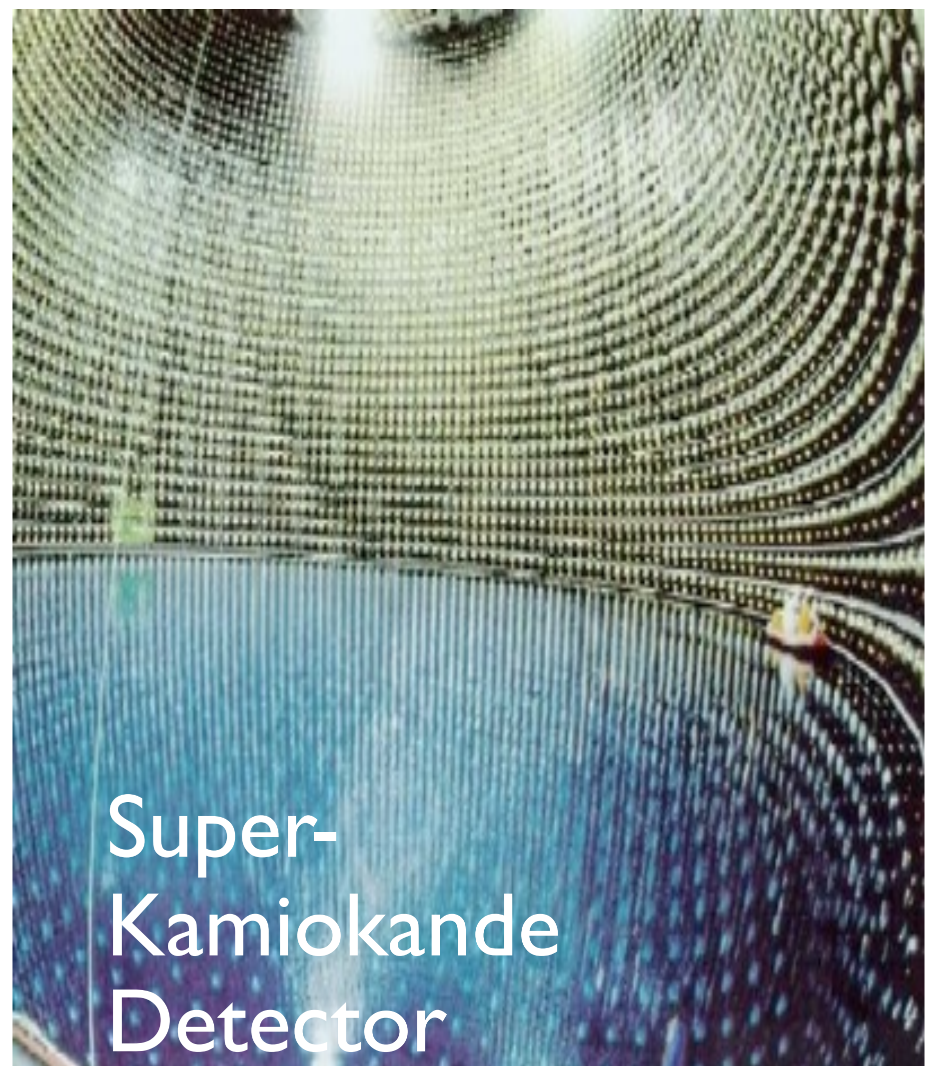
Super-Kamiokande

The SNO Detector

Nobel Prize in Physics 2015

- Indications of an anomaly in atmospheric neutrinos was presented in 1988, subsequently confirmed by MACRO.
- More muon neutrinos were seen going down than coming up from the other side of the Earth.
- Discovery was presented in 1998 by SuperKamiokande.





Nobel Prize in Physics 2015

What was going on?

These experiments showed that neutrinos oscillate, i.e. that can change flavour (going into flavours that some detectors cannot see).

Neutrinos are **chameleon** particles.

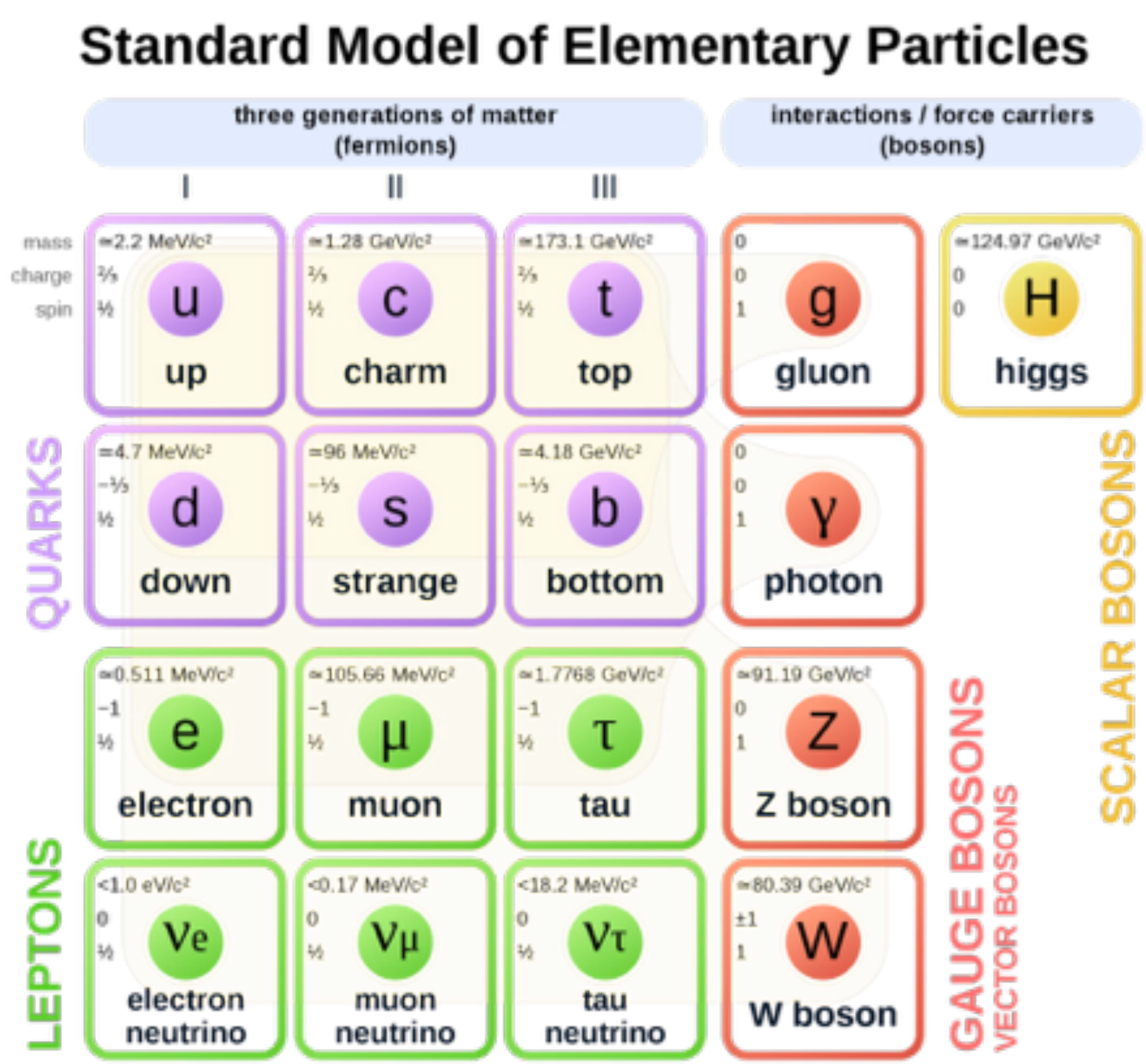
Neutrinos in the SM

The Standard Model

SM gauge group and EW symmetry breaking

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

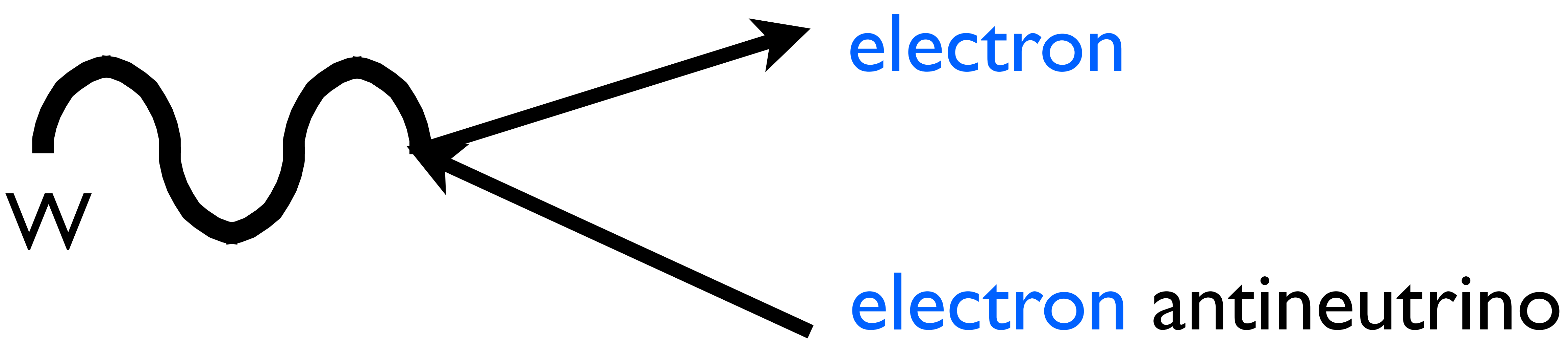
Particle content



- There are 3 generations of quarks and leptons.
- The theory is chiral as left and right components behave differently.

- Neutrino flavours correspond to each of the charged leptons.

Particles	$SU(3)$	$SU(2)_L$	$U(1)_Y$
Leptons			
$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L, \quad \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L, \quad \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L$	**1**	**2**	$-1/2$
$e_R\,, \quad \mu_R\,, \quad \tau_R$	**1**	**1**	-1
Quarks			
$\begin{pmatrix}u \\ d\end{pmatrix}_L, \quad \begin{pmatrix}c \\ s\end{pmatrix}_L, \quad \begin{pmatrix}t \\ b\end{pmatrix}_L$	**3**	**2**	$1/6$
$u_R\,, \quad c_R\,, \quad t_R$	**3**	**1**	$2/3$
$d_R\,, \quad s_R\,, \quad b_R$	**3**	**1**	$-1/3$



- They have hypercharge -1/2 and charge 0.

- Neutrinos in the SM are described by Weyl spinors with left-handed chirality ($P_L=1-\gamma_5/2$).

- They are massless as nuR are not present (no Yukawa coupling with the Higgs) and Majorana mass term (of left-handed neutrinos) is not gauge invariant.

Note: For massless neutrinos, helicity and chirality correspond as the chiral and helicity operators are the same up to m/E.

Note: LH neutrino states are accompanied by RH antineutrino ones as required by CPT.

Note: Natural units. We will assume c=1, hbar=1.

They have charge current (CC) and neutral current (NC) interactions

$$\mathcal{L}_{\rm SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{\alpha L}\gamma^{\mu}\ell_{\alpha L}W_{\mu} - \frac{g}{2\cos\theta_W} \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{\alpha L}\gamma^{\mu}\nu_{\alpha L}Z_{\mu} + \text{h.c.}$$

Number of active neutrinos

The invisible width of the Z (measured precisely at LEP) restricts the number of active neutrinos to

$$N_\nu = \frac{\Gamma_{inv}}{\Gamma_{\bar{\nu}\nu}} = 2.984 \pm 0.008$$

Note: Additional neutrinos can be present but they cannot partake of the SM interactions and are called sterile neutrinos (See later).

Neutrino mixing

Neutrino mixing

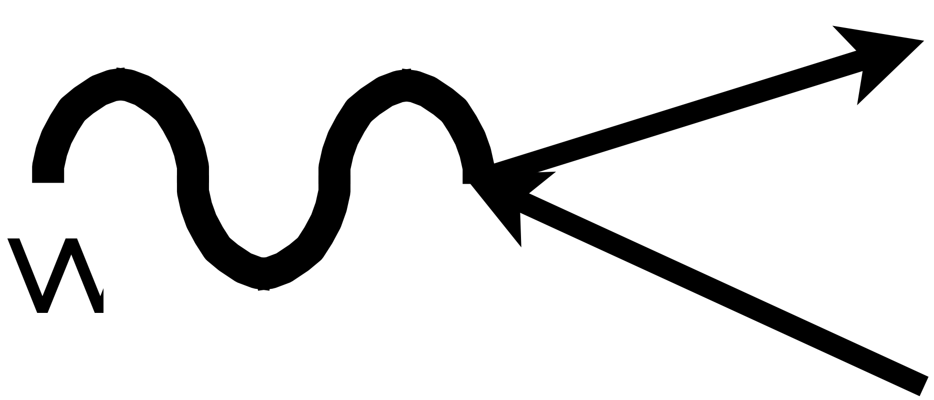
Mixing is described by the *Pontecorvo-Maki-Nakagawa-Sakata* matrix: $\nu_{\alpha} = \sum_i U_{\alpha i} \nu_i$ ⟵ Mass field

↑ Flavour field

which enters in the CC interactions

$$\mathcal{L}_{CC} \quad = \quad -\frac{g}{\sqrt{2}} \sum_{k\alpha} (U^*_{\alpha k} \bar{\nu}_{kL} \gamma^{\rho} l_{\alpha L} W_{\rho} + \text{h.c.})$$

This implies that in an interaction with an electron, the corresponding (anti-)neutrino will be produced, as a superposition of different mass eigenstates.



electron neutrino $= \sum_i U_{ei} \nu_i$

Neutrino mixing

Mixing is described by the *Pontecorvo-Maki-Nakagawa-Sakata* matrix: $\nu_\alpha = \sum_i U_{\alpha i} \nu_i$ ← Mass field

↑ Flavour field

which enters in the CC interactions

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{k\alpha} \left(U^*_{\alpha k} \bar{\nu}_{kL} \gamma^\rho l_{\alpha L} W_\rho + \text{h.c.} \right)$$

Note: this holds in the basis under which the charged lepton mass matrix is diagonal.

Note: A flavour neutrino is produced in a CC interaction as a superposition of mass eigenstates under the assumption that the mass differences can be ignored (= coherence of the final state).

• 2-neutrino mixing matrix depends on 1 angle only. The phases get absorbed in a redefinition of the leptonic fields (a part from 1 Majorana phase).

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

• 3-neutrino mixing matrix has 3 angles and 1(+2) CPV phases.

$$\begin{pmatrix} \bar{\nu}_1 & \bar{\nu}_2 & \bar{\nu}_3 \end{pmatrix} e^{i\psi} \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \text{CKM-type} \end{pmatrix} \begin{pmatrix} e^{i\rho_e} & 0 & 0 \\ 0 & e^{i\rho_\mu} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

One can always rephrase a field as:

$$\begin{aligned} e &\rightarrow e^{-i(\rho_e+\psi)} e \\ \mu &\rightarrow e^{-i(\rho_\mu+\psi)} \mu \\ \tau &\rightarrow e^{-i\psi} \tau \end{aligned}$$

The kinetic, NC and mass terms are not modified. One can choose a rephrasing to eliminate 3 phases from the U mixing matrix in the full Lagrangian. These phases are therefore unphysical.

$$\begin{pmatrix} \bar{\nu}_1 & \bar{\nu}_2 & \bar{\nu}_3 \end{pmatrix} e^{i\psi} \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \text{CKM-} \\ \text{type} \end{pmatrix} \begin{pmatrix} e^{i\rho_e} & 0 & 0 \\ 0 & e^{i\rho_\mu} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

For Dirac neutrinos, a similar rephasing can be done eliminating the two phases on the left. Only one phase remains physical.

For Majorana neutrinos, the Majorana condition forbids such rephasing: 2 physical CP-violating phases.

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix}$$

$$\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

For antineutrinos, $U \rightarrow U^*$

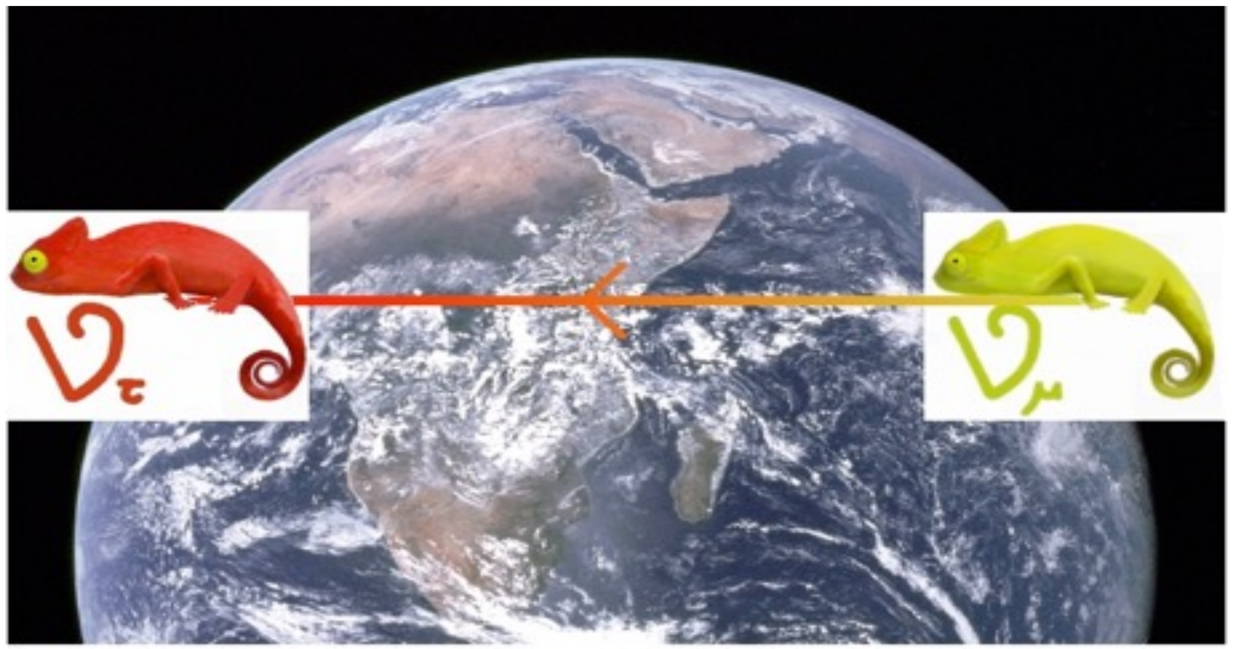
CP-conservation requires U is real $\Rightarrow \delta = 0, \pi$

It is useful to express the CP violating effects in a rephrasing invariant manner (Jarlskog invariant):

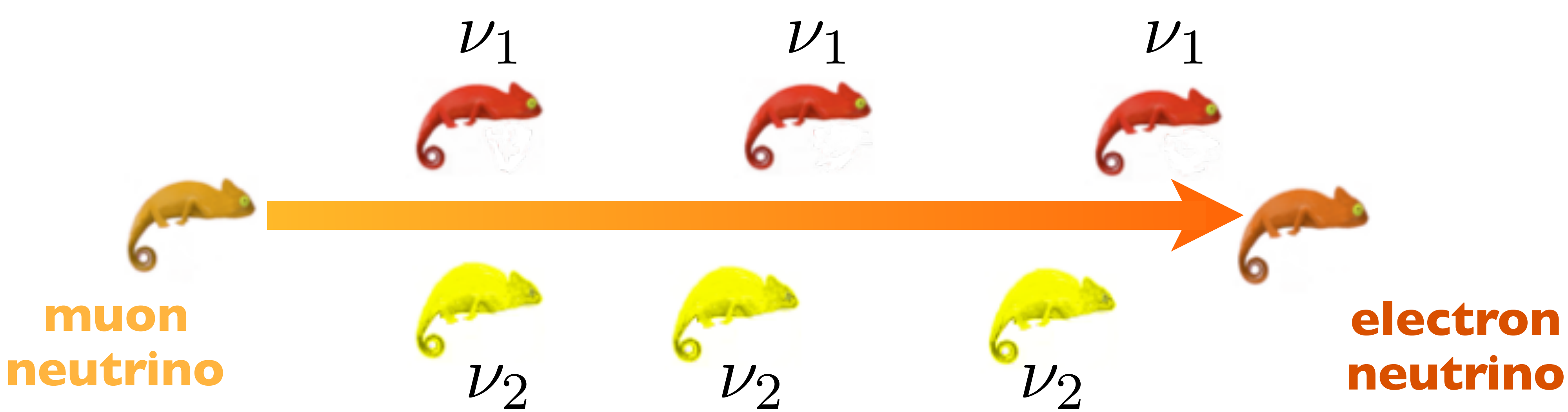
$$J \equiv \Im[U_{\mu 3} U_{e2} U^*_{\mu 2} U^*_{e3}] = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

Neutrino oscillations: theory

Neutrinos oscillations: the basic picture

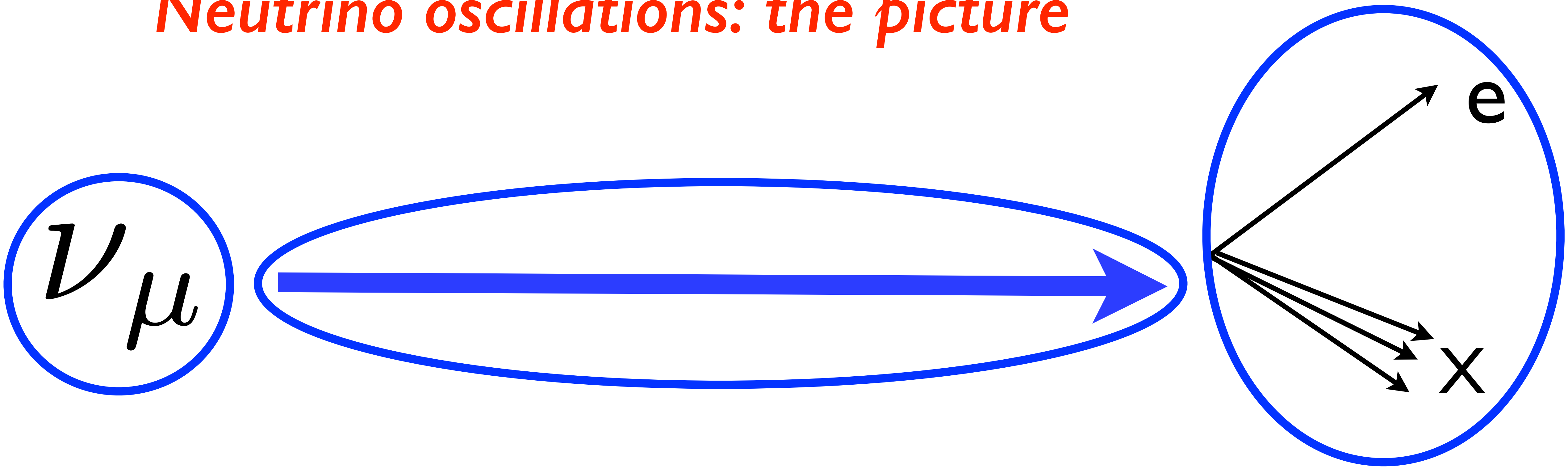


Contrary to what expected in the SM, neutrinos oscillate: after being produced, they can **change their flavour**.



Neutrino oscillations imply that neutrinos have mass and they mix.
First evidence of physics beyond the SM.

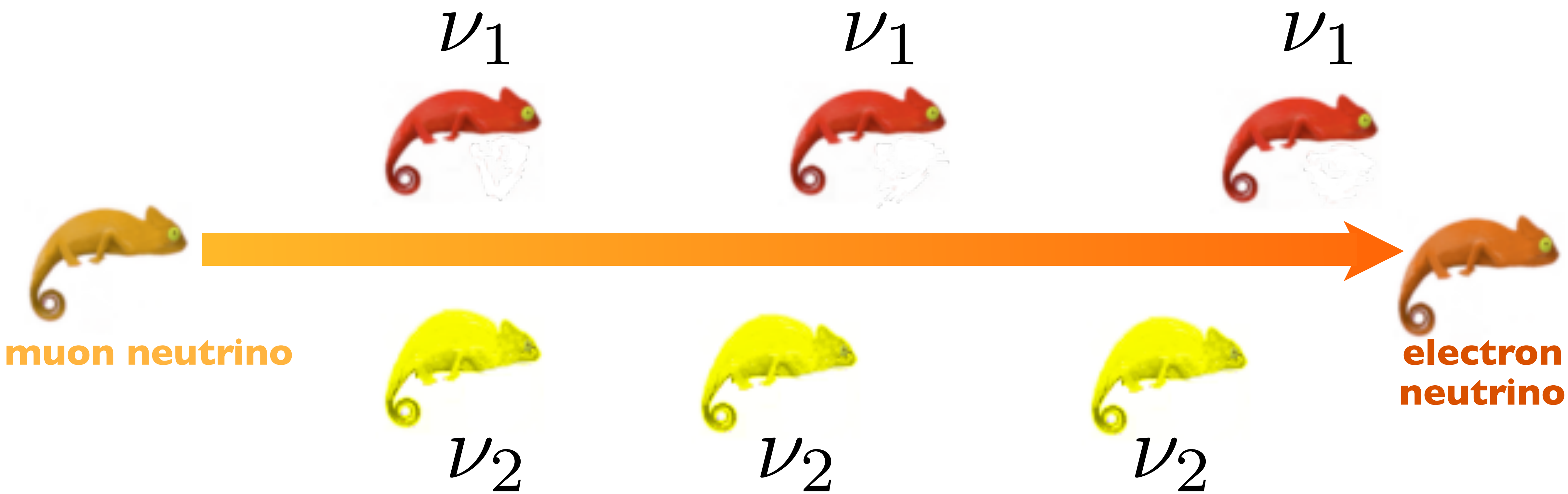
Neutrino oscillations: the picture



Production	Propagation	Detection
Flavour states	Massive states (eigenstates of the Hamiltonian)	Flavour states

At production, coherent superposition of massive states:

$$|\nu_\mu\rangle = U^*_{\mu 1}|\nu_1\rangle + U^*_{\mu 2}|\nu_2\rangle + U^*_{\mu 3}|\nu_3\rangle$$



Production

$$|\nu_\mu\rangle = \sum_i U^*_{\mu i} |\nu_i\rangle$$

Propagation

$$\nu_1 : \quad e^{-iE_1 t}$$
$$\nu_2 : \quad e^{-iE_2 t}$$
$$\nu_3 : \quad e^{-iE_3 t}$$

Detection: projection over

$$\langle \nu_e |$$

As the propagation phases are different, the state evolves with time and can change to other flavours.

Neutrinos oscillations in vacuum: the theory

Let's assume that at t=0 a muon neutrino is produced

$$|\nu, t=0\rangle = |\nu_\mu\rangle = \sum_i U^*_{\mu i} |\nu_i\rangle$$

The time-evolution is given by the solution of the Schroedinger equation with free Hamiltonian:

$$|\nu, t\rangle = \sum_i U^*_{\mu i} e^{-iE_i t} |\nu_i\rangle$$

In the same-momentum approximation:

$$E_1 = \sqrt{p^2 + m_1^2} \quad E_2 = \sqrt{p^2 + m_2^2} \quad E_3 = \sqrt{p^2 + m_3^2}$$

Note: other derivations are also valid (same E formalism, etc).

At detection one projects over the flavour state as these are the states which are involved in the interactions. The probability of oscillation is

$$
\begin{aligned}
P(\nu_\mu \to \nu_\tau) &= |\langle \nu_\tau | \nu, t\rangle|^2 \\
&= \left| \sum_{ij} U^*_{\mu i} U_{\tau j} e^{-iE_i t} \langle \nu_j | \nu_i \rangle \right|^2 \\
&= \left| \sum_{i} U^*_{\mu i} U_{\tau i} e^{-iE_i t} \right|^2
\end{aligned}
$$

Typically, neutrinos are very relativistic: $E_i \simeq p + \frac{m_i^2}{2p}$

$$
\begin{aligned}
&= \left| \sum_{i} U^*_{\mu i} U_{\tau i} e^{-i\frac{m_i^2}{2E} t} \right|^2 \\
&= \left| \sum_{i} U^*_{\mu i} U_{\tau i} e^{-i\frac{m_i^2 - m_1^2}{2E} t} \right|^2
\end{aligned}
$$

Δm_{i1}^2

Exercise
Derive

Implications of the existence of neutrino oscillations

The oscillation probability implies that

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_i U^*_{\alpha 1} U_{\beta 1} e^{-i \frac{\Delta m^2_{i1}}{2E} L} \right|^2$$

- **neutrinos have mass** (as the different components of the initial state need to propagate with different phases)

- **neutrinos mix** (as U needs not be the identity. If they do not mix the flavour eigenstates are also eigenstates of the propagation Hamiltonian and they do not evolve)

General properties of neutrino oscillations

- Neutrino oscillations conserve the total lepton number: a neutrino is produced and evolves with times

- They violate the flavour lepton number as expected due to mixing.

- Neutrino oscillations do not depend on the overall mass scale and on the Majorana phases.

- CPT invariance: $$P(\nu_\alpha \to \nu_\beta) \quad = \quad P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$$

- CP-violation:

$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ requires $U \neq U^*(\delta \neq 0, \pi)$

- Appearance channel:

$$\nu_\alpha \rightarrow \nu_\beta \qquad \alpha \neq \beta$$

- Disappearance channel:

$$\nu_\alpha \rightarrow \nu_\alpha$$

- The total probability of oscillation is 1:

$$\sum_\beta P(\nu_\alpha \rightarrow \nu_\beta, t) = 1$$

- The argument of the sin can be expressed as:

$$\frac{\Delta m^2_{i1}}{4E} L = 1.27 \frac{\Delta m^2_{i1}[\text{eV}^2]}{E[\text{GeV}]} L[\text{km}]$$

Exercise
Derive

Further theoretical issues on neutrino oscillations

Energy-momentum conservation

Let's consider for simplicity a 2-body decay: $\pi \rightarrow \mu \; \bar{\nu}_\mu$.

Energy-momentum conservation seems to require:

$$E_\pi = E_\mu + E_1 \quad \text{with } E_1 = \sqrt{p^2 + m_1^2}$$
$$E_\pi = E_\mu + E_2 \quad \text{with } E_2 = \sqrt{p^2 + m_2^2}$$

?

How can the picture be consistent?

Further theoretical issues on neutrino oscillations

Energy-momentum conservation

Let's consider for simplicity a 2-body decay: $\pi \to \mu \, \bar{\nu}_\mu$.

Energy-momentum conservation seems to require:

$$E_\pi = E_\mu + E_1 \quad \text{with } E_1 = \sqrt{p^2 + m_1^2}$$
$$E_\pi = E_\mu + E_2 \quad \text{with } E_2 = \sqrt{p^2 + m_2^2}$$

These two requirements seems to be incompatible. Intrinsic quantum uncertainty, localisation of the initial pion lead to an uncertainty in the energy-momentum and allow coherence of the initial neutrino state.

The need for wavepackets

- In deriving the oscillation formulas we have implicitly assumed that neutrinos can be described by plane-waves, with definite momentum.

- However, production and detection are well localised and very distant from each other. This leads to a momentum spread which can be described by a wave-packet formalism.

Typical sizes:

- e.g. production in decay: the relevant timescale is the pion lifetime (or the time travelled in the decay pipe),

$$\Delta t \sim \tau_\pi \Rightarrow \Delta E \Rightarrow \Delta p \quad \Delta x$$

For details see, Akhmedov, Smirnov, 1008.2077; Giunti and Kim, Neutrino Physics and Astrophysics.

Decoherence and the size of a wave-packet

- The different components of the wavepacket, $\nu 1$, $\nu 2$ and $\nu 3$, travel with slightly different velocities (as their mass is different).

- If the neutrinos travel extremely long distances, these components stop to overlap, destroying coherence and oscillations.

- In terrestrial experimental situation this is not relevant. But this can happen for example for supernovae neutrinos.

2-neutrino case

Let's recall that the mixing is

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

We compute the probability of oscillation

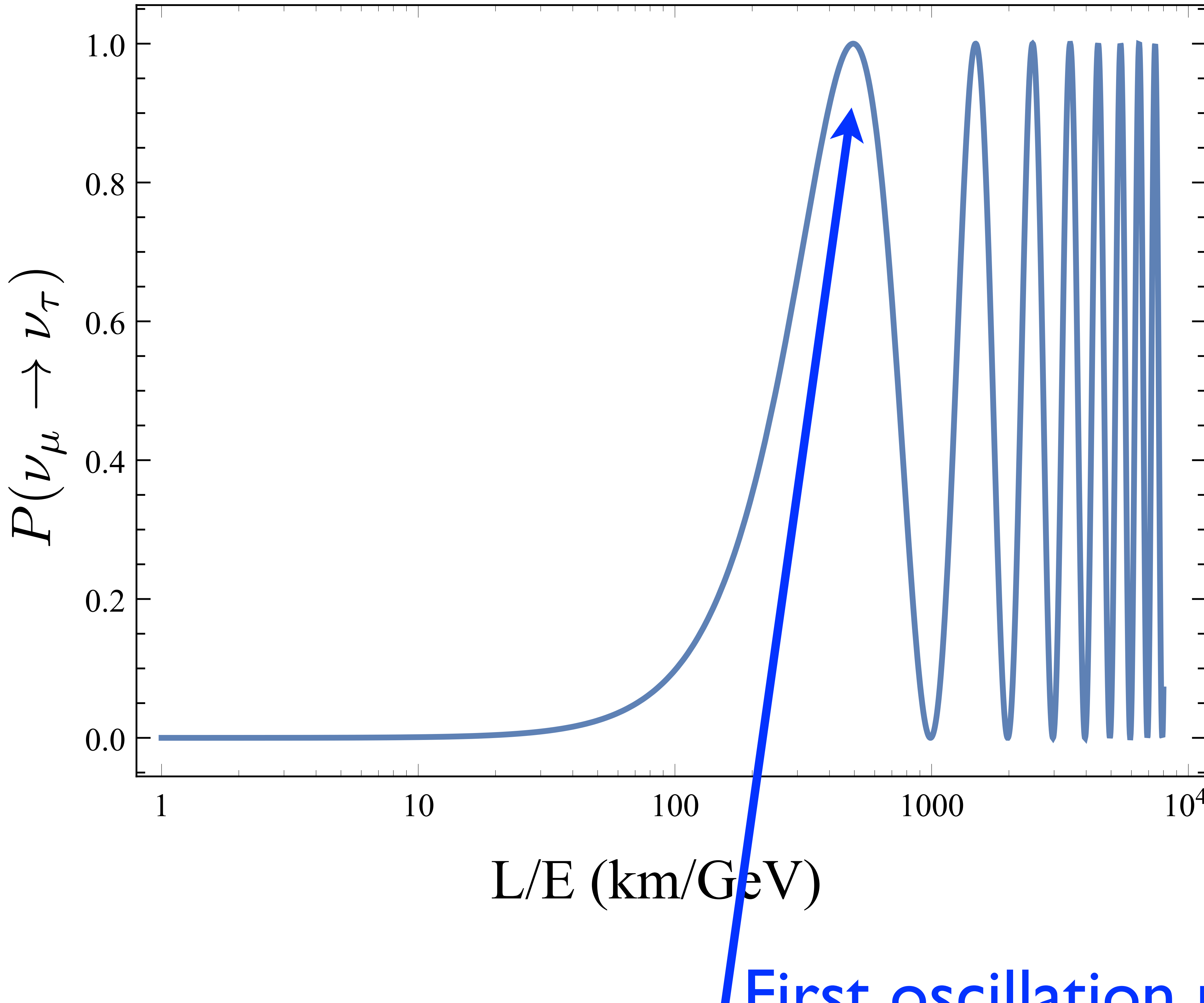
$$P(\nu_\alpha \to \nu_\beta) = \left| U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2} e^{-i\frac{\Delta m^2_{21}}{2E}L} \right|^2$$

$$= \left| \cos\theta \sin\theta - \cos\theta \sin\theta e^{-i\frac{\Delta m^2_{21}}{2E}L} \right|^2$$

$$= \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2_{21}}{4E}L\right)$$

$$P(\nu_\alpha \rightarrow \nu_\beta) \simeq 0$$

$$P(\nu_\alpha \rightarrow \nu_\beta) \simeq \frac{1}{2}\sin^2(2\theta)$$



First oscillation maximum

Properties of 2-neutrino oscillations

- Appearance probability:

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta)\sin^2(\frac{\Delta m^2_{21}}{4E}L)$$

- Disappearance probability:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2(2\theta)\sin^2(\frac{\Delta m^2_{21}}{4E}L)$$

- No CP-violation as there is no Dirac phase in the mixing matrix

$$P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

- Consequently, no T-violation (using CPT):

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$$

3-neutrino oscillations

They depend on two mass squared-differences

$$\Delta m^2_{21} \ll \Delta m^2_{31}$$

In general the formula is quite complex

$$P(\nu_\alpha \to \nu_\beta) = \left| U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2} e^{-i\frac{\Delta m^2_{21}}{2E}L} + U^*_{\alpha 3} U_{\beta 3} e^{-i\frac{\Delta m^2_{31}}{2E}L} \right|^2$$

2-neutrino limits relevant for experiments

For a given L, the neutrino energy determines the impact of a mass squared difference. Various limits are of interest in concrete experimental situations. Using the current values

$$\Delta m^2_{21} = 7.5 \times 10^{-5} \mathrm{eV}^2 \qquad \Delta m^2_{31} = 2.5 \times 10^{-3} \mathrm{eV}^2$$

Type of neutrinos	Average E	(Typical) L	$\frac{\Delta m_{21}^2}{4E} L$	$\frac{\Delta m_{31}^2}{4E} L$
Atmospheric neutrinos	(0.1-100 GeV) 10 GeV	(10-12000 km) 6000 km	0.06	1.9
Reactor neutrinos I	3 MeV	1 km	0.03	1.1
Reactor neutrinos II	3 MeV	100 km	3.2	106
Accelerator (T2K)	0.7 GeV	300 km	0.04	1.36
Accelerator (DUNE)	3 GeV	1300 km	0.04	1.37
SBL	0.8 GeV	500 m	0.00006	0.001

"Atmospheric regime"

$$\frac{\Delta m^2_{21}}{4E} L \ll 1$$

The first limit applies to atmospheric, reactor with short baseline, accelerator neutrinos (K2K, MINOS, T2K, NOvA, DUNE, T2HK) at leading order.

$$P(\nu_\alpha \to \nu_\beta) = \left| U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2} e^{-i \frac{\Delta m^2_{21}}{2E} L} + U^*_{\alpha 3} U_{\beta 3} e^{-i \frac{\Delta m^2_{31}}{2E} L} \right|^2$$

$$P(\nu_\alpha \to \nu_\beta) = \left| U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2} e^{-i \frac{\Delta m^2_{21}}{2E} L} + U^*_{\alpha 3} U_{\beta 3} e^{-i \frac{\Delta m^2_{31}}{2E} L} \right|^2$$

$$P(\nu_\alpha \to \nu_\beta) = \left| \underline{U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2}} + U_{\alpha 3} U^*_{\beta 3} e^{-i \frac{\Delta m^2_{31}}{2E} L} \right|^2$$

We use the fact that $U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} + U_{\alpha 3} U^*_{\beta 3} = \delta_{\alpha\beta}$

$$P(\nu_\alpha \to \nu_\beta) = \left| -U_{\alpha 3}U^*_{\beta 3} + U_{\alpha 3}U^*_{\beta 3} e^{-i\frac{\Delta m^2_{31}}{2E}L} \right|^2$$

$$= \left|U_{\alpha 3}U^*_{\beta 3}\right|^2 \left| -1 + e^{-i\frac{\Delta m^2_{31}}{2E}L} \right|^2$$

The same we have encountered in the 2-neutrino case

$$= 2\left|U_{\alpha 3}U_{\beta 3}\right|^2 \sin^2\left(\frac{\Delta m^2_{31}}{4E}L\right)$$

Exercise
Derive

Exp relevant probabilities:

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - 4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2)\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) \simeq 1 - \sin^2 2\theta_{23}\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) + \mathcal{O}(s^2_{13})$$

$$P(\nu_\mu \to \nu_e) \simeq 4|U_{e3}|^2|U_{\mu 3}|^2\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) \simeq s^2_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

$$P(\overline{\nu}_e \to \overline{\nu}_e) \simeq 1 - \sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

Exercise
Derive

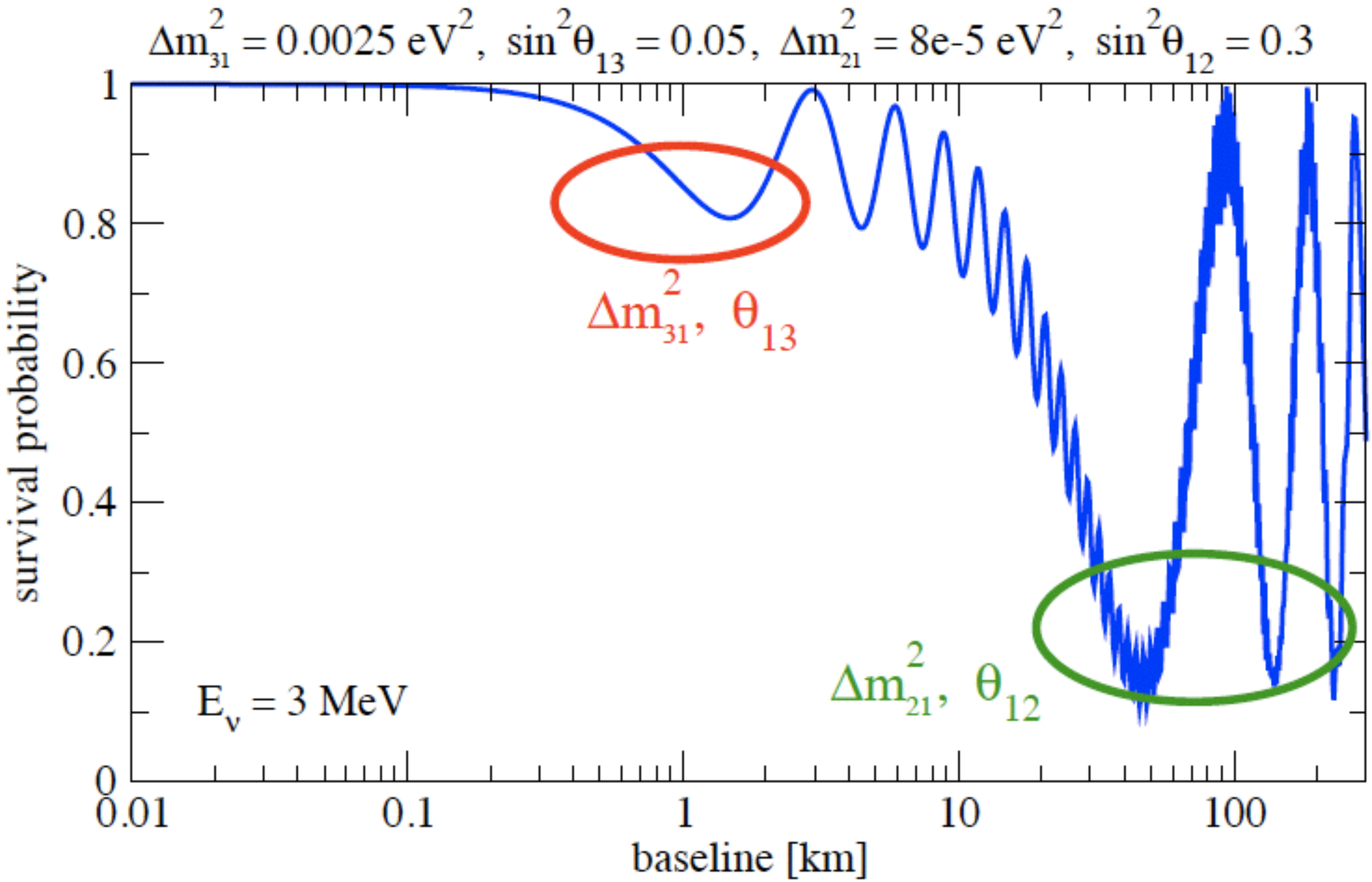
“Small Dm2 regime”

$$\frac{\Delta m^2_{31}}{4E} L \gg 1$$

The oscillations due to the atmospheric mass squared differences get averaged out.

Derivation of oscillation probability

$$P(\bar{\nu}_e \to \bar{\nu}_e; t) \quad \simeq \quad c^4_{13} \left(1 - \sin^2(2\theta_{12}) \sin^2 \frac{\Delta m^2_{21} L}{4E}\right) + s^4_{13}$$



Thanks to T. Schwetz

"CPV effects"

The oscillations due to the atmospheric mass squared differences get averaged out.

CP-violation will manifest itself in neutrino oscillations, due to the delta phase. Let's consider the CP-asymmetry:

$$P(\nu_\alpha \to \nu_\beta; t) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta; t) =$$

$$= \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i \frac{\Delta m^2_{21} L}{2E}} + U_{\alpha 3} U^*_{\beta 3} e^{-i \frac{\Delta m^2_{31} L}{2E}} \right|^2 - (U \to U^*)$$

$$= U_{\alpha 1} U^*_{\beta 1} U^*_{\alpha 2} U_{\beta 2} e^{i \frac{\Delta m^2_{21} L}{2E}} + U^*_{\alpha 1} U_{\beta 1} U_{\alpha 2} U^*_{\beta 2} e^{-i \frac{\Delta m^2_{21} L}{2E}} - (U \to U^*) + \cdots$$

$$= 4 s_{12} c_{12} s_{13} c^2_{13} s_{23} c_{23} \sin\delta \left[\sin\left(\frac{\Delta m^2_{21} L}{2E}\right) + \left(\frac{\Delta m^2_{23} L}{2E}\right) + \left(\frac{\Delta m^2_{31} L}{2E}\right) \right]$$

Exercise**
Derive

Notice:

- CP-violation requires all angles to be nonzero. It is a genuinely 3-mixing effect.

- It is proportional to the sine of the delta phase. It can be expressed in terms of the Jarlskog invariant.

- If one can neglect Δm^2_{21}, the asymmetry goes to zero as we have seen that effective 2-neutrino probabilities are CP-symmetric.

This implies that searching for leptonic CPV requires P which are sensitive to both Dm^2.