$\mathscr{I}$ and the black hole horizon

- new conserved quantities from a geometric duality -

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Mathematical Physics of Gravity and Symmetry Workshop
Institut de Mathématiques de Bourgogne (IMB)
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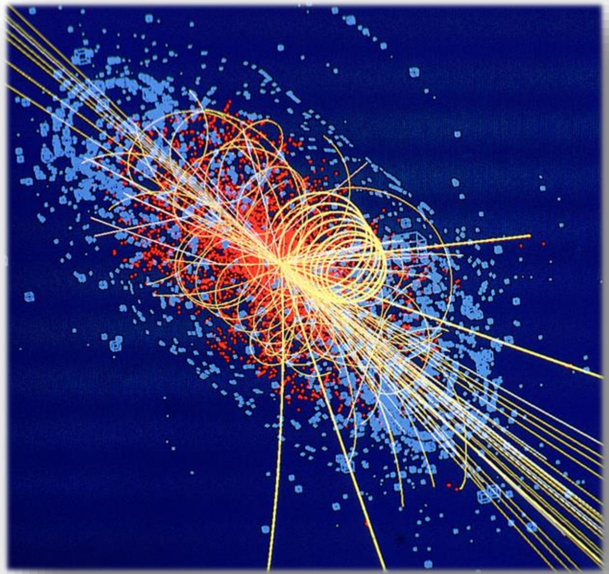
SISSA

Motivation

Quantum gravity in 4d asymptotically flat spacetimes → vanishing cosmological constant

$$\Lambda = 0$$

These spacetimes are relevant



from collider physics …

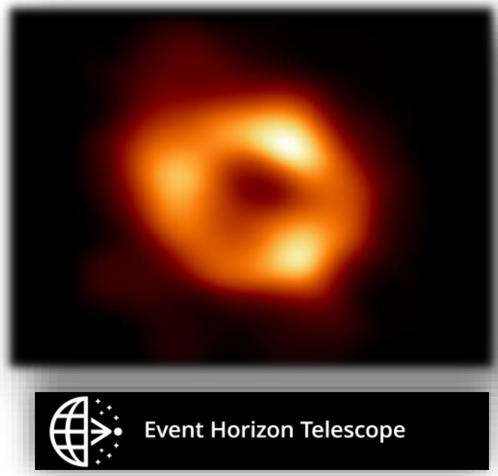
… to astrophysics
(< cosmological scales)

Motivation

Quantum gravity in 4d asymptotically flat spacetimes

Black holes

Our understanding of quantum properties of black holes goes *hand-in-hand* with the **spectacular advances** of the **holographic** or **AdS/CFT correspondence**.



$$\boxed{S_{BH} = \frac{Ac^3}{4G\hbar}}$$

→ 'Primordial holographic relationship'

[Bekenstein][Hawking]

Problem: we do not live in Anti-de Sitter spacetime

→ need to develop a **holographic correspondence** for **flat spacetimes**

Motivation

The holographic principle

Quantum gravity is *encoded* in a different theory that lives in a *lower-dimensional* spacetime.

['t Hooft '93; Susskind '94; Maldacena '97]

$\rightarrow$ **How general is it?**

Anti-de Sitter

$$\Lambda < 0$$

CFT

Flat

$$\boxed{\Lambda = 0}$$

??

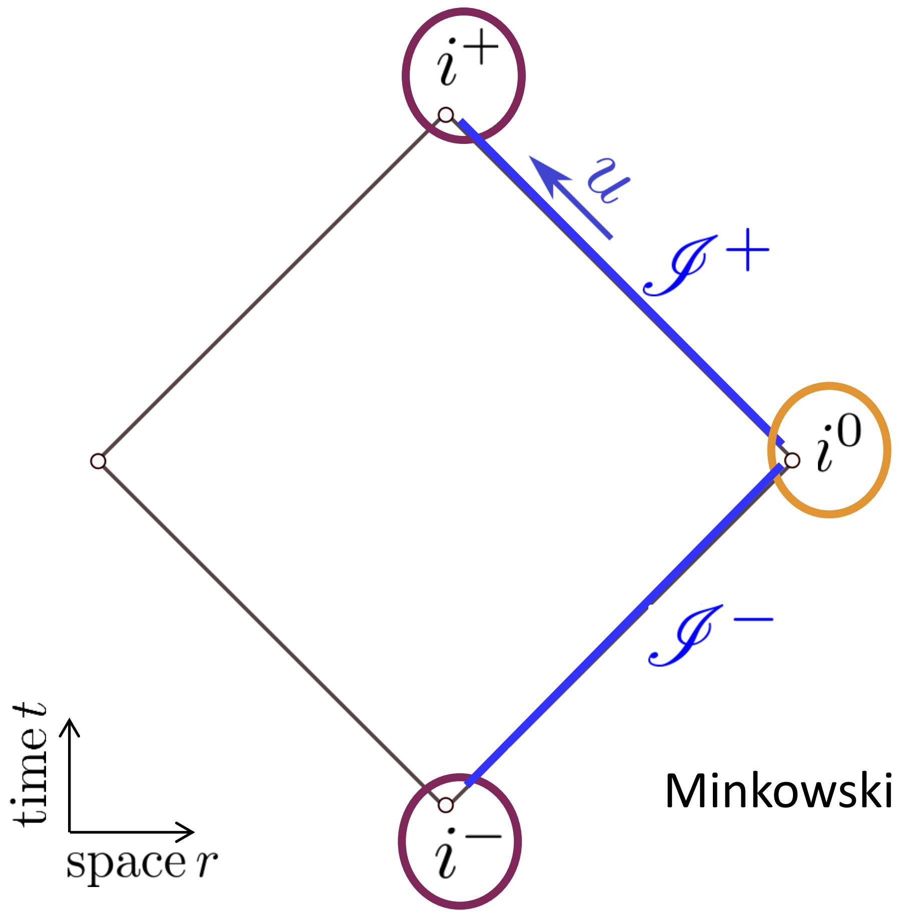
Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Main obstructions/difficulties:

1 The conformal **boundary** includes

future/past timelike infinity
future/past null infinity
spatial infinity



Minkowski

Flat space holography

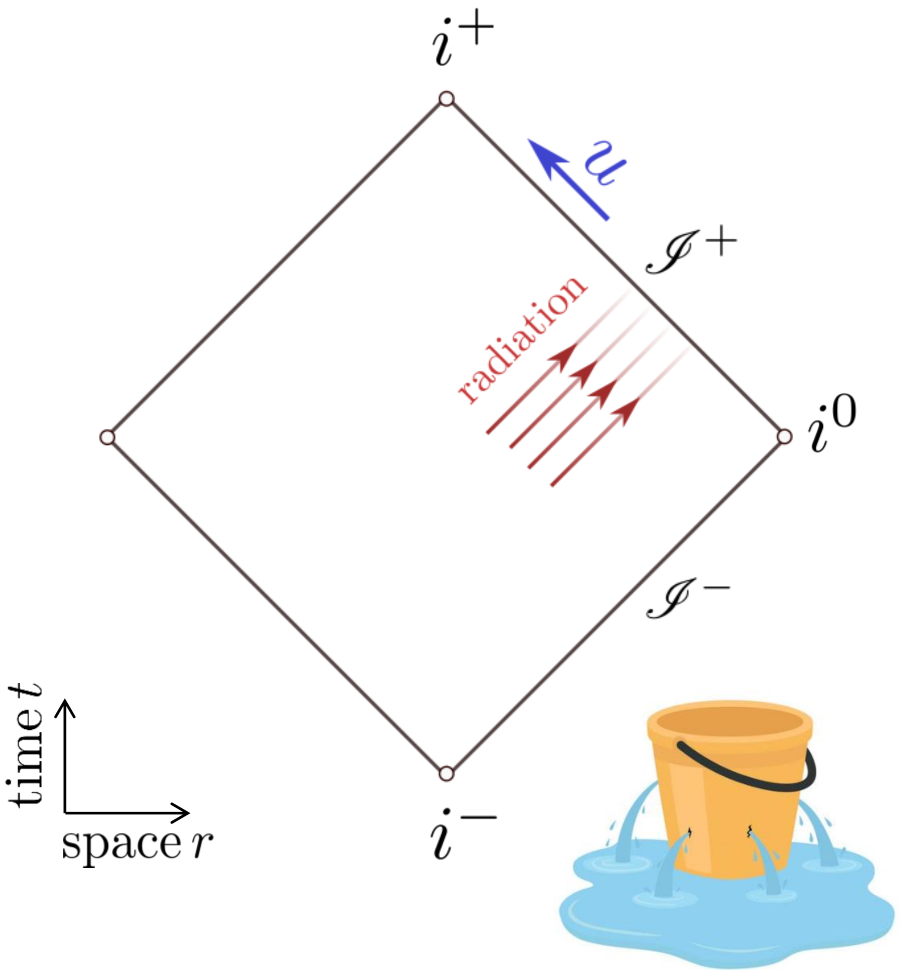
Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Main obstructions/difficulties:

1. The conformal **boundary** includes

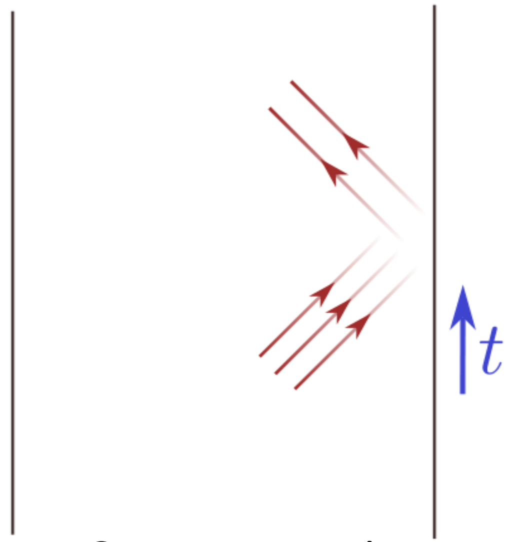
   future/past timelike infinity
   future/past null infinity
   spatial infinity

2. There are **fluxes** leaking out the boundary



vs

AdS



Quantum gravity 'in a **box**'

Flat space holography

- -> Road map: symmetries

- The phenomenon of **symmetry enhancement** is a key feature of asymptotically flat spacetimes, due to the presence of gravitational radiation

4 Poincaré translations

↓ *Symmetry enhancement*

∞ BMS supertranslations

what was expected

Poincaré

what was found

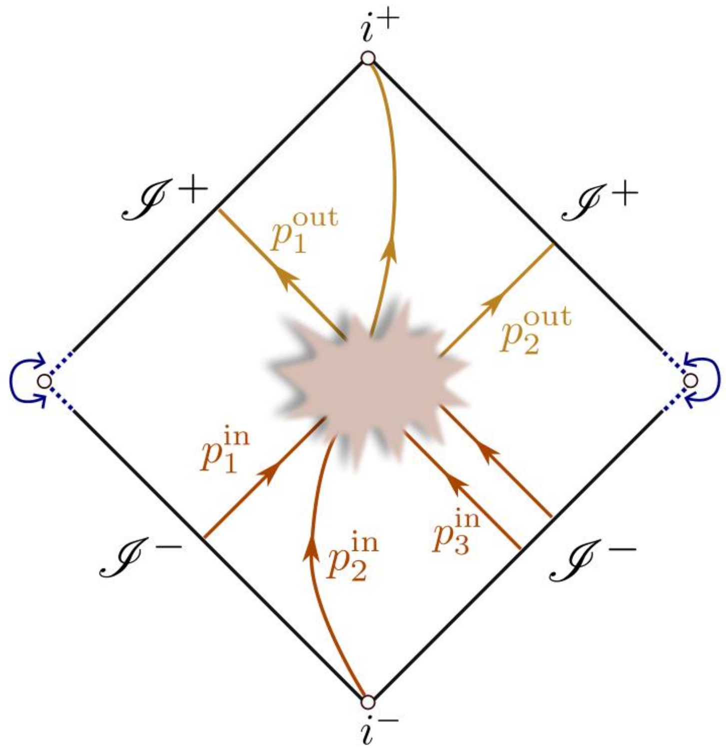
Bondi-Metzner-Sachs (BMS) ('62)

+ van der Burg

Flat space holography

While BMS symmetries were originally *disregarded*, it was realized (50 years later!) that they

- constrain the gravitational **S-matrix** [Strominger ‘13]



$$\langle \text{out}|S|\text{in}\rangle$$

Flat space holography

While BMS symmetries were originally *disregarded*, it was realized (50 years later!) that they

- constrain the gravitational **S-matrix** [Strominger ‘13]
- have associated low-energy **observables** (memory effects) [Ashtekar ‘14][Strominger, Zhiboedov ‘14]

Donnay, Goncharov, Harms, *Phys. Rev. Lett.* 2024

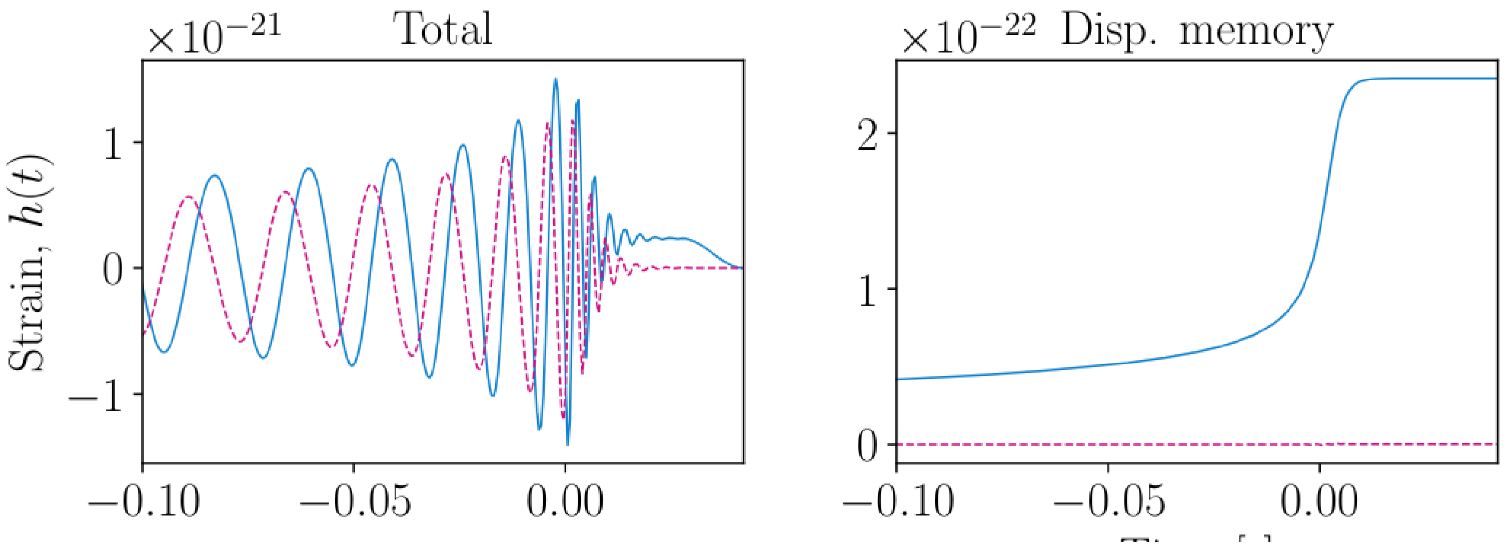


FIG. 4: Demonstration of the GW memory contribution to strain from a merger of two non-spinning BBHs in the extended BMS scenario, $(m_1, m_2, \theta_{jn}, z) = (30\ M_\odot, 30\ M_\odot, \pi/3, 0.06)$. Solid lines show h_+, dashed lines show $h_\times$.

Flat space holography

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- constrain the gravitational **S-matrix** [Strominger '13]
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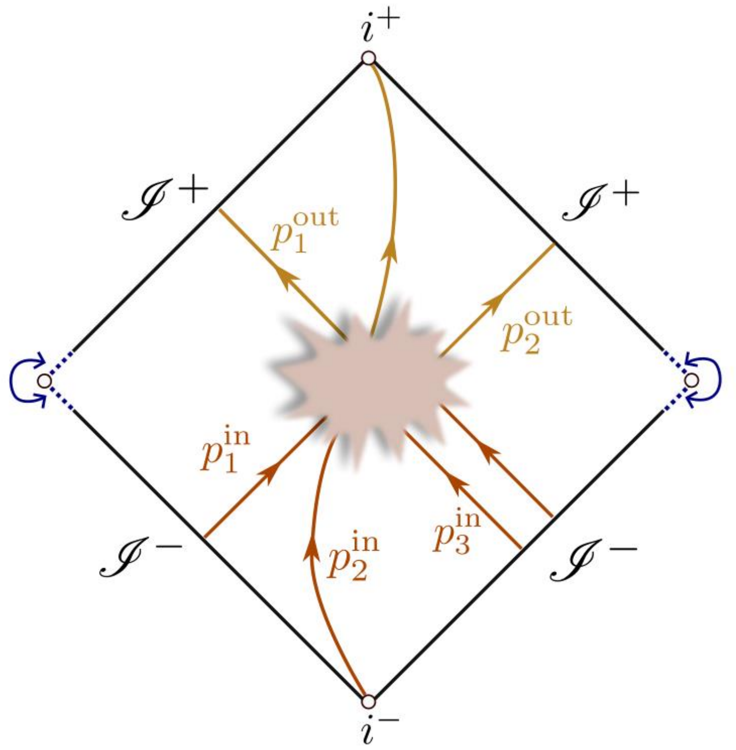
- allow further extensions, including the local **conformal** group [Barnich, Troessaert '09]



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Flat space holography

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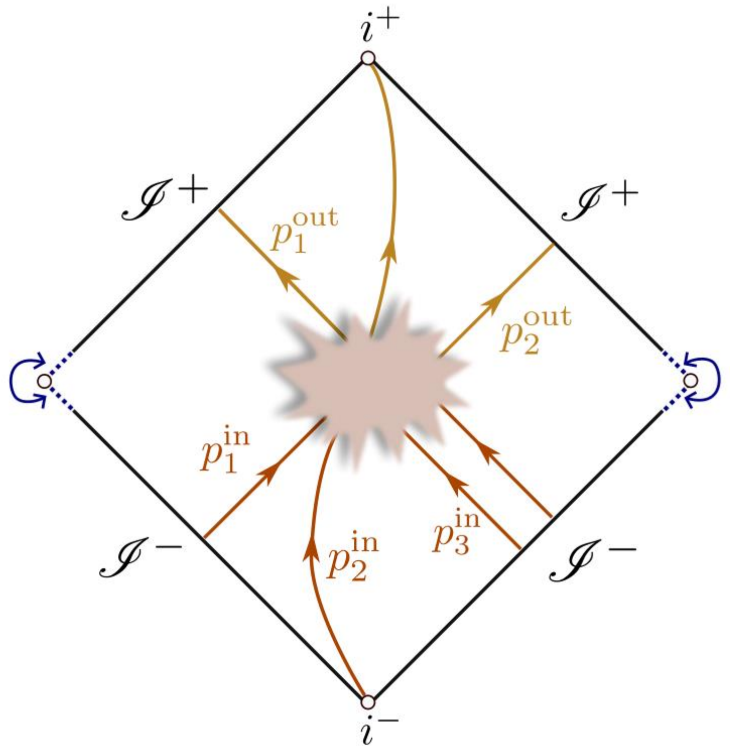
↓

revival of **flat holography**

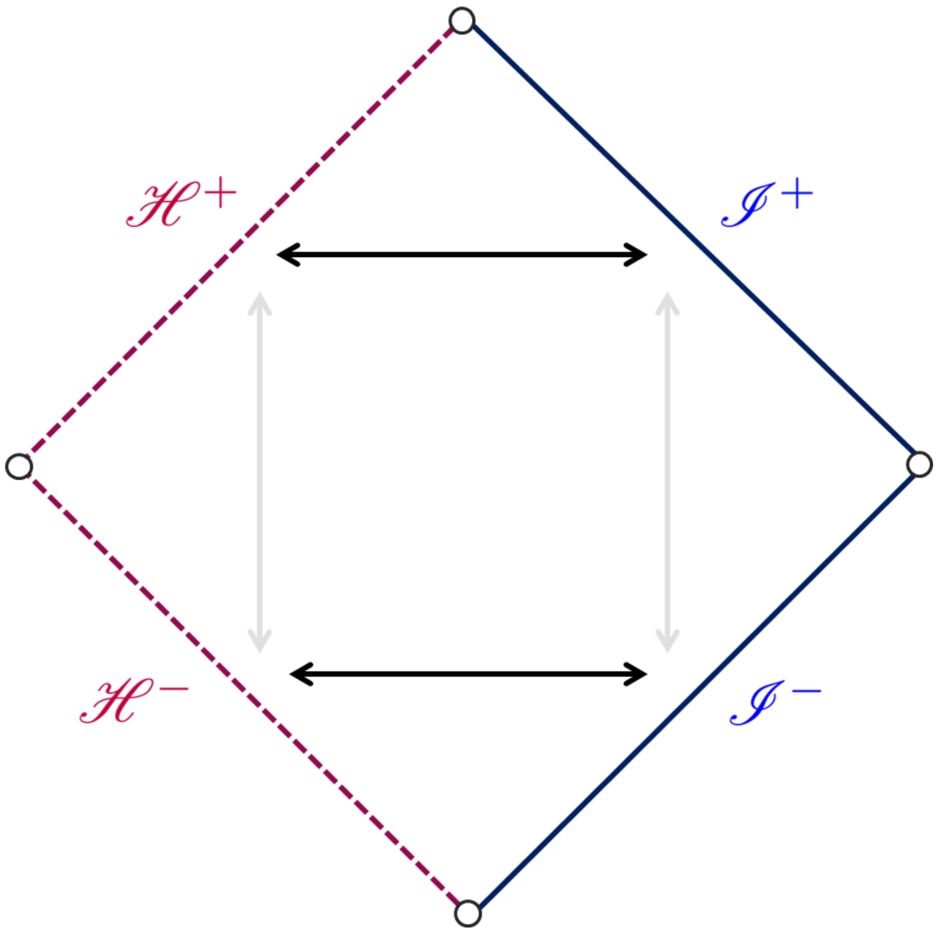


$$\langle \text{out}|S|\text{in}\rangle$$

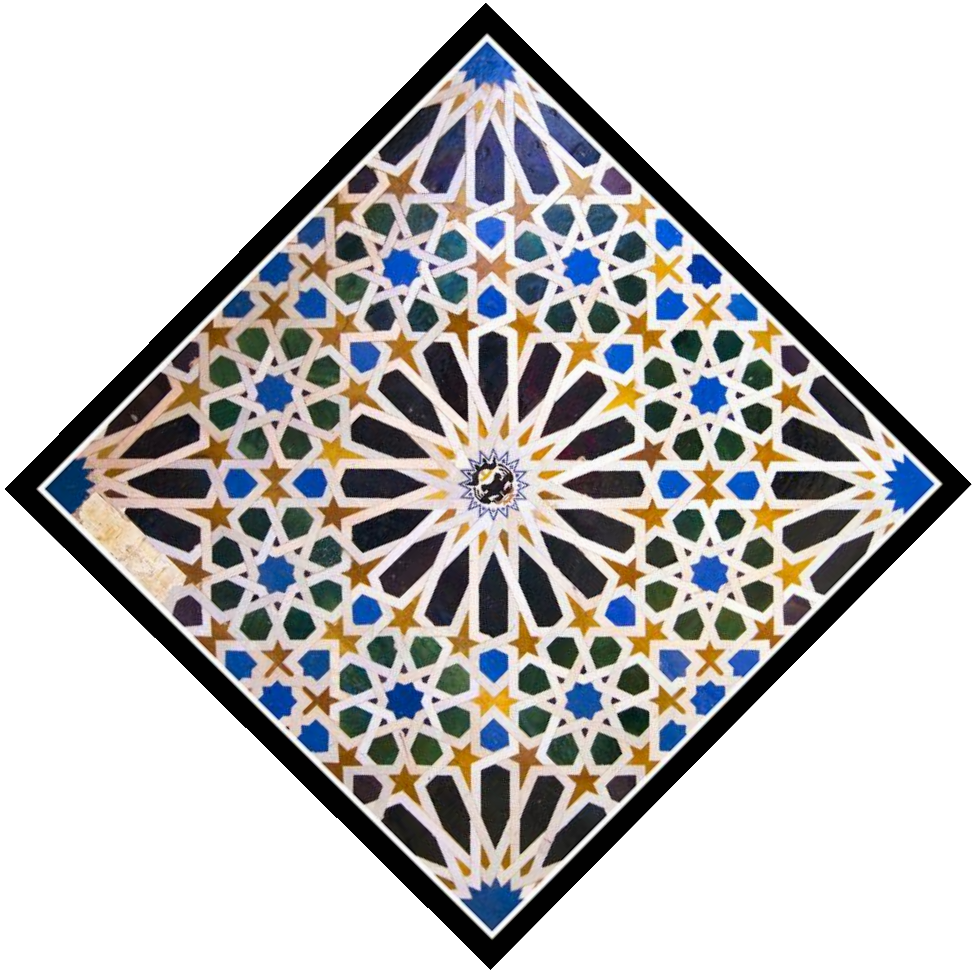
$$\langle \text{out}|S|\text{in}\rangle = \langle \mathcal{O}_{\Delta_1,J_1} \cdots \mathcal{O}_{\Delta_m,J_m}\rangle_{CCFT_2}$$

review L. Donnay, *Celestial holography: an asymptotic symmetry perspective*, Phys. Rept. 1073 (2024)

Flat space holography with horizons?



What is the global form of conservation laws?



Alhambra الحَمْراء tile (13^{th} century)

OUTLINE

① BLACK HOLE HORIZON VS NULL INFINITY

A) GEOMETRY

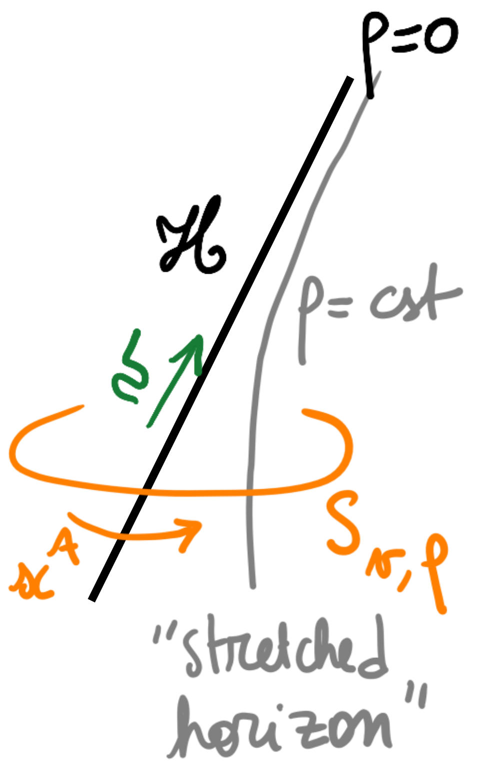
B) INFINITE SYMMETRIES

② $\mathscr{H}$ / $\mathscr{I}$ « DUALITY »

③ MAP OF ∞ CONSERVED QUANTITES $\mathscr{H}^{\pm} \leftrightarrow \mathscr{I}^{\pm}$

Geometry of a black hole horizon

- - -> see the review [E. Gourgoulhon, J.L. Jaramillo '05]



Coordinates : (v, ρ, x^A)

advanced null time

radial coordinate

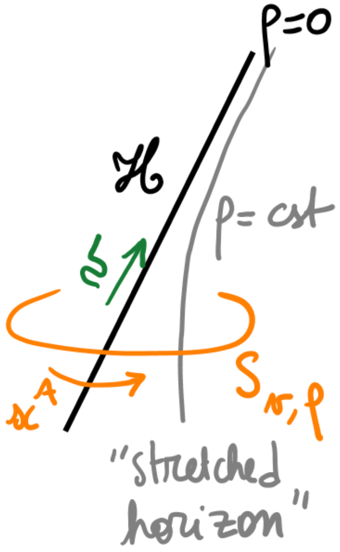
(d-2) sphere coordinates

- The near-horizon geometry is (expanding in small ρ):

$$ds^2 = -2\rho\kappa\, dv^2 + 2\, dv d\rho + 2\rho\, \theta_A\, dv\, dx^A + (\Omega_{AB} + \rho\lambda_{AB})\, dx^A dx^B + \underbrace{\dots}_{\text{subleading terms}}$$

κ, θ_A, Ω_{AB} : functions of (v, x^A)

Geometry of a black hole horizon



$$ds^2 = -2\rho\kappa\, dv^2 + 2\, dv\, d\rho + 2\rho\,\theta_A\, dv\, dx^A + (\Omega_{AB} + \rho\lambda_{AB})\, dx^A dx^B + \dots$$

The horizon extrinsic geometry is given by

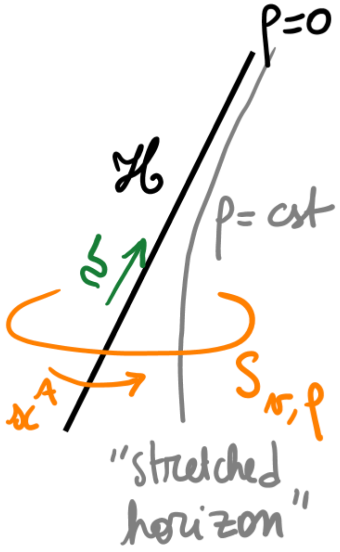
$$\Sigma_{AB} = \frac{1}{2}\partial_v \Omega_{AB}$$

trace → $\Theta = \Omega^{AB}\Sigma_{AB}$ « expansion »

traceless part → $\sigma_{AB} = \frac{1}{2}\partial_v \Omega_{AB} - \frac{\Theta}{d-2}\Omega_{AB}$ « shear »

$$\omega_A = -\frac{1}{2}\theta_A$$ « twist »

Geometry of a black hole horizon



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The horizon extrinsic geometry is given by

$$\Sigma_{AB} = \frac{1}{2}\partial_v \Omega_{AB} \xrightarrow{\text{trace}} \Theta = \Omega^{AB}\Sigma_{AB}$$

"expansion"

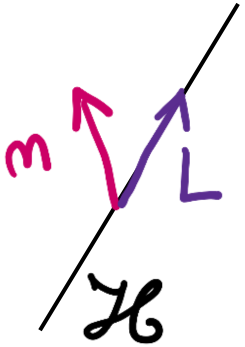
$$\Sigma_{AB} \xrightarrow{\text{traceless part}} \sigma_{AB} = \frac{1}{2}\partial_v \Omega_{AB} - \frac{\Theta}{d-2}\Omega_{AB}$$

"shear"

$$\omega_A = -\frac{1}{2}\theta_A$$ "twist"

κ : surface gravity $\left(L^b D_b L^a \hat{=} \kappa L^a\right)$

↑ normal to $\mathcal{H}$



$$\Xi_{AB} = -\frac{1}{2}\lambda_{AB} \xrightarrow{\text{trace}} \Theta^{(m)}$$

$$\Xi_{AB} \xrightarrow{\text{traceless}} \sigma^{(m)}_{AB}$$: transversal shear

Horizon dynamics

- **Null Raychaudhuri** equation: $(\partial_v - \kappa)\,\Theta + \frac{1}{d-2}\Theta^2 + \sigma_{AB}\sigma^{AB} = 0$

--> describes how the **expansion** evolves along the null geodesic congruence
Key in the proof of singularity theorems (+ energy conditions)

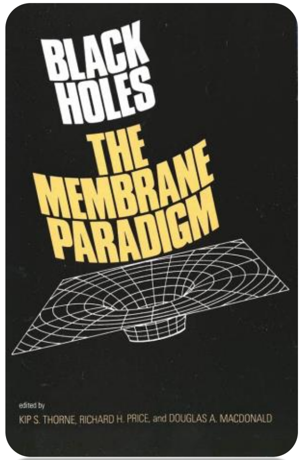
Horizon dynamics

- **Null Raychaudhuri** equation: $$(\partial_v - \kappa)\,\Theta + \frac{1}{d-2}\Theta^2 + \sigma_{AB}\sigma^{AB} = 0$$

- **Damour** equation: $$(\partial_v + \Theta)\,\theta_A + 2D_A\left(\kappa + \frac{d-3}{d-2}\Theta\right) - 2D^B\sigma_{AB} = 0$$

  --> originally thought of as the Navier-Stokes equation for a viscous **fluid** [Damour '79][Price,Thorne '86]

  In fact, it is a conservation equation of a ***Carrollian*** (not a Galilean) fluid [LD, Marteau '19]



Horizon dynamics

EINSTEIN EQUATIONS

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- **Transverse shear** evolution equation:

$$R_{AB}\left[\Omega\right] - (\partial_v + \kappa)\,\lambda_{AB} - 2D_{(A}\omega_{B)} - 2\omega_A\omega_B$$
$$+ 2\sigma^C{}_{(A}\left[\lambda_{B)C} - \frac{1}{4}\Omega_{B)C}\lambda^D{}_D\right] - \frac{d-6}{2\,(d-2)}\Theta\left[\lambda_{AB} + \frac{1}{d-6}\Omega_{AB}\lambda^C{}_C\right] = 0$$

Horizon dynamics

EINSTEIN EQUATIONS

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- **Tidal force** equation:

$$(\partial_v - \kappa)\,\sigma_{AB} - \sigma_{AC}\sigma_B{}^C - \frac{1}{d-2}\Omega_{AB}\sigma_{CD}\sigma^{CD} = 0$$

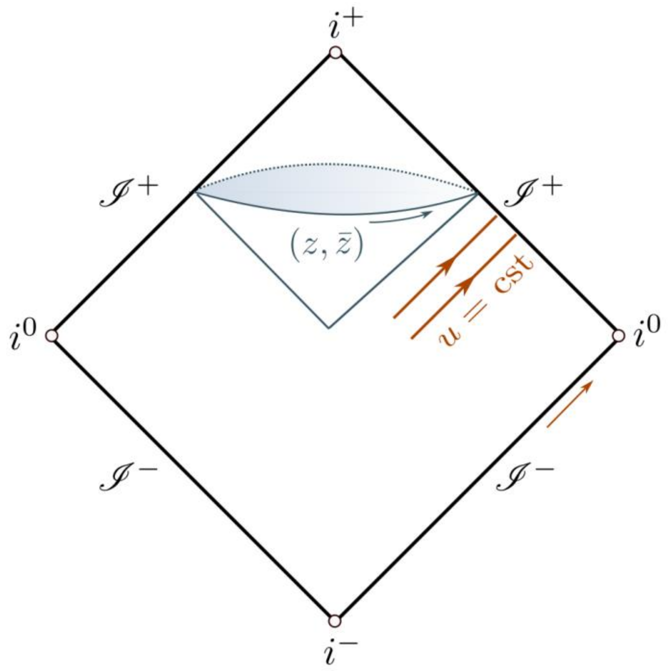
Geometry of null infinity

- **Asymptotically flat** spacetime (in Newman-Unti gauge) [Bondi, van der Burg, Metzner ’62] [Sachs ’62] [Newman, Unti ‘62]

$$r \to \infty$$

$$\left(u, r, x^A\right), \quad x^A = (z, \bar{z})$$

$$ds^2_{\mathscr{I}^+} = -F du^2 - 2\, du dr + r^2 \mathcal{H}_{AB} \left(dx^A - \frac{U^A}{r^2} du\right)\left(dx^B - \frac{U^B}{r^2} du\right)$$

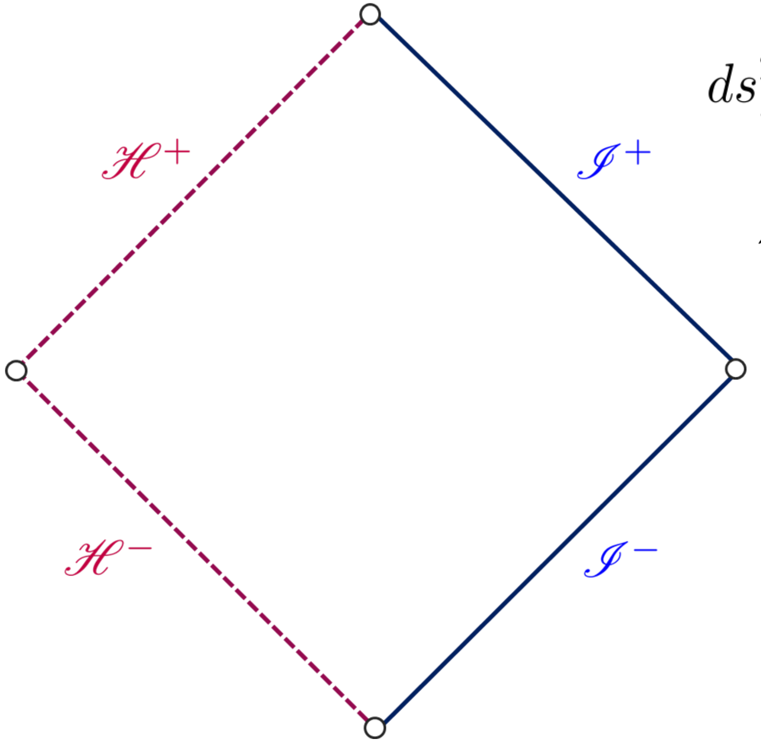


$$\mathcal{H}_{AB}\left(u, r, x^C\right) = q_{AB}\left(x^C\right) + \frac{1}{r} C_{AB}\left(u, x^C\right) + o\left(r^{-1}\right),$$

$$F\left(u, r, x^A\right) = \frac{R[q]}{(d-2)(d-3)} - \frac{1}{d-2}\partial_u C_A{}^A - \frac{2G m_{\mathrm{B}}}{r} + o\left(r^{-1}\right),$$

$$U^A\left(u, r, x^B\right) = \frac{1}{2(d-3)}\left(D_B C^{AB} - D^A C_B{}^B\right) + \frac{2}{3r}\left[N^A - \frac{1}{2} C^{AB} D^C C_{BC}\right] + o\left(r^{-1}\right)$$

$\mathscr{I}$ vs $\mathscr{H}$



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$$ds^2_{\mathscr{H}^+} = -\rho \mathcal{F}\, dv^2 + 2\, dv d\rho + g_{AB}\left(dx^A + \rho\, \vartheta^A dv\right)\left(dx^B + \rho\, \vartheta^B dv\right) ,$$

$$\mathcal{F}\left(v, \rho, x^A\right) = 2\kappa\left(v, x^A\right) + \rho \mathcal{F}_0\left(v, x^A\right) + o(\rho) ,$$

$$\vartheta^A\left(v, \rho, x^B\right) = \theta^A\left(v, x^B\right) + o\left(\rho^{+0}\right) ,$$

$$g_{AB}\left(v, \rho, x^C\right) = \Omega_{AB}\left(v, x^C\right) + \rho\, \lambda_{AB}\left(v, x^C\right) + o(\rho) .$$

Infinite-dimensional symmetries

[Newman, Unti '62][Barnich, Lambert '13]

Newman - Unti gauge

$$\mathcal{L}_\xi g_{rr} = 0 \qquad \xi^u = f(u, x^A)$$

$$\mathcal{L}_\xi g_{rr} = 0 \quad \Rightarrow \quad \xi^r = -r\partial_u f + J$$

$$\mathcal{L}_\xi g_{rA} = 0 \qquad \xi^A = Y^A(u, x^A) + I^A$$

+ asymptotic flatness

$$\mathcal{L}_\xi g_{uA} = O(r^0), \quad \mathcal{L}_\xi g_{AB} = O(r)$$

$$\mathcal{L}_\xi g_{uu} = O(r^{-1}) \quad \Rightarrow \quad \begin{cases} \partial_u Y^A = 0 & \mathcal{L}_Y q_{AB} = \frac{2}{d-2} q_{AB} D_C Y^C \\ \partial_u f = \frac{1}{d-2} D_C Y^C \end{cases}$$

BMS

Infinite-dimensional symmetries

$\chi = \chi^v \partial_v + \chi^\rho \partial_\rho + \chi^A \partial_A$

Newman-Unti gauge

$$\begin{cases} \mathcal{L}_\chi g_{\rho\rho} = 0 \\ \mathcal{L}_\chi g_{\rho v} = 0 \\ \mathcal{L}_\chi g_{\rho A} = 0 \end{cases} \Rightarrow \begin{cases} \chi^v = f(v, x^A) \\ \chi^\rho = -\rho \partial_v f + J \\ \chi^A = Y^A(v, x^A) + I^A \end{cases}$$

+ "near-horizon b^ry conditions"

[LD, Giribet, Gonzalez, Pino '15]

$$\mathcal{L}_\chi g_{vA} = \mathcal{O}(\rho), \quad \mathcal{L}_\chi g_{AB} = \mathcal{O}(\rho^0)$$

$$\mathcal{L}_\chi g_{vv} = \begin{cases} \mathcal{O}(\rho) & \kappa \neq 0 \\ \mathcal{O}(\rho^2) & \kappa = 0 \end{cases}$$

extremal case

Infinite-dimensional symmetries

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$$\mathcal{L}_\chi g_{vv} = \begin{cases} \mathcal{O}(\rho) & \kappa \neq 0 \\ \mathcal{O}(\rho^2) & \kappa = 0 \end{cases} \Rightarrow \begin{cases} f = f(v, x^A) \\ f_{ext} = T(x^A) + v X(x^A) \end{cases}$$

extremal case

« time-dependent supertranslations »

$\& \ \partial_v Y^A = 0$

Carroll symmetries

CONFORMAL CARROLL ALGEBRA ($CCARR_N$)

$$\mathcal{L}_\xi g = \lambda g \qquad \mathcal{L}_\xi n = -\frac{\lambda}{N} n$$

$\nearrow$ BMS = $CCARR_2$

$\mathscr{I}^+$: $n = \partial_u$

$g = g_{AB}\, dx^A dx^B$

$$\Rightarrow \xi = Y^A(x)\partial_A + \left(T(x) + \frac{2}{N}\frac{u}{(d-2)} D_A Y^A\right)\partial_u$$

density of conformal weight $-\frac{2}{N}$

NEWMAN - UNTI ALGEBRA (NU)

$$\mathcal{L}_\xi g = \lambda g$$

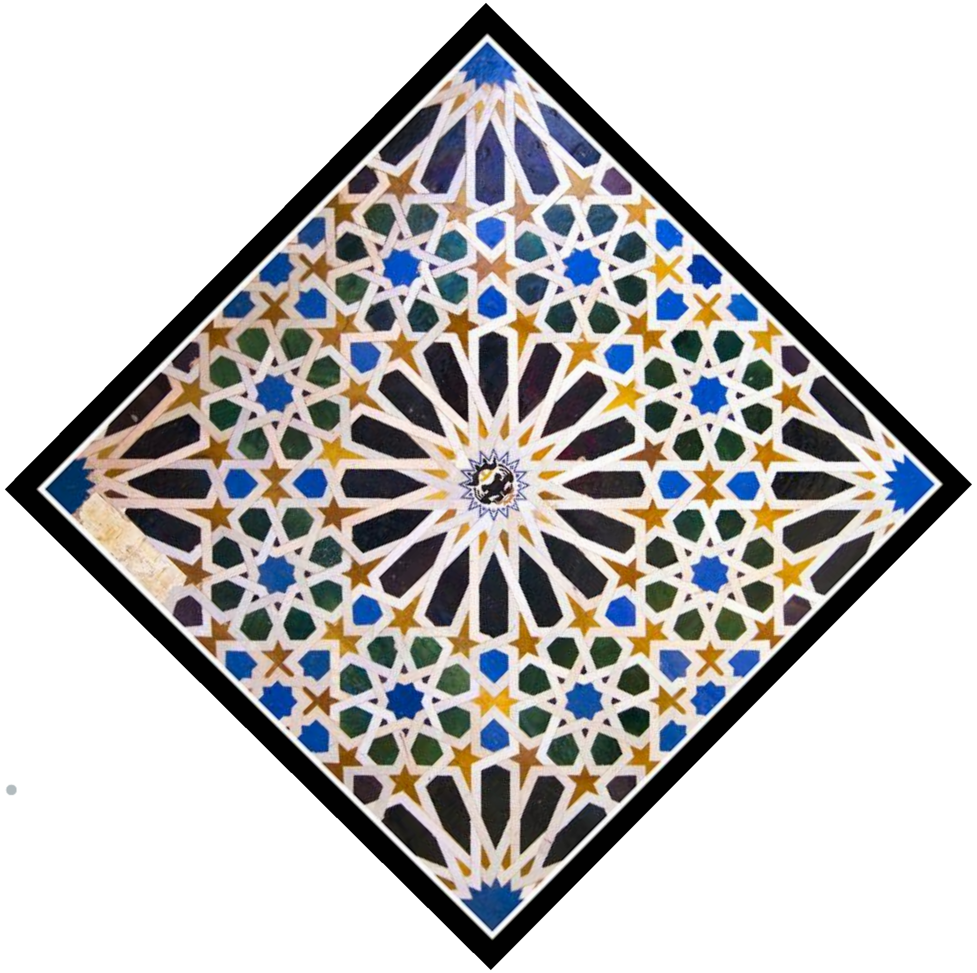
$$\xi = Y^A(x)\partial_A + f(u,x)\partial_u$$

$\exists$ SUBALGEBRA : $(\mathcal{L}_m)^N \xi = 0$ (NU_N)

$\hookrightarrow \partial_u^N f = 0$

NU_2 ... NU

extremal $\mathcal{H}$ $\quad$ $\mathcal{H}$



Alhambra الحَمْراء tile (13th century)

OUTLINE

① BLACK HOLE HORIZON V.S NULL INFINITY
   A) GEOMETRY
   B) INFINITE SYMMETRIES

② $\mathcal{H}/\mathcal{I}$ «DUALITY»

③ MAP OF ∞ CONSERVED QUANTITES $\mathcal{H}^{\pm} \leftrightarrow \mathcal{I}^{\pm}$

Null infinity as an extremal horizon

$\mathscr{I}$ is an **extremal non-expanding horizon** for the (unphysical) conformally completed spacetime

$$d\tilde{s}^2_{\mathscr{I}^+} = \Omega^2 ds^2_{\mathscr{I}^+}\,, \qquad \Omega = \frac{\alpha}{r}$$

[Ashtekar, Khera, Kolanowski, Lewandowski '22] [Ashtekar, Speziale '24]

- The expansion of all normal vanishes at $\mathscr{I}$ $\longrightarrow$ non-expanding
- $g_{uA} \sim \mathcal{O}(1) \longrightarrow \mathscr{I}$ is non-twisting
- Subcase: no **radiation** $\longleftrightarrow$ **isolated** horizon

$$N_{AB} = -\partial_v \lambda_{AB}$$

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$$d\tilde{s}^2_{\mathscr{I}^+} = \Omega^2 ds^2_{\mathscr{I}^+} = \Omega^2 \left[-F du^2 - 2\,du dr + r^2 \mathcal{H}_{AB} \left(dx^A - \frac{U^A}{r^2} du \right) \left(dx^B - \frac{U^B}{r^2} du \right) \right]$$

spatial inversion

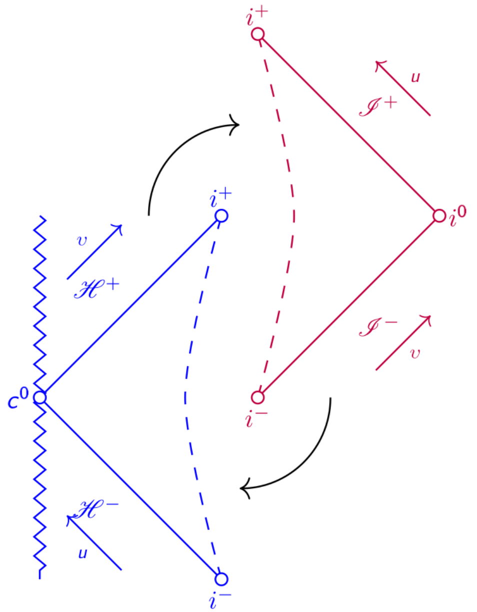
$$r = \frac{\alpha^2}{\rho}\,, \qquad u = v$$

$$\boxed{d\tilde{s}^2_{\mathscr{I}^+} = ds^2_{\mathscr{H}^+}} \quad \text{with} \quad \mathcal{F} = \alpha^{-2} F\,, \quad \theta^A = -\rho\alpha^{-4} U^A$$

$\kappa = 0$ **extremal**

$\Theta = 0$ **non-expanding**

$\omega_A = 0$ **non-rotating**



Null infinity as an extremal horizon

$\mathscr{I}$ is an **extremal non-expanding horizon** for the (unphysical) conformally completed spacetime

$$d\tilde{s}^2_{\mathscr{I}^+} = \Omega^2 ds^2_{\mathscr{I}^+}\,, \qquad \Omega = \frac{\alpha}{r}$$

This black hole horizon `dual' to null infinity is in general **not part of the physical spacetime**

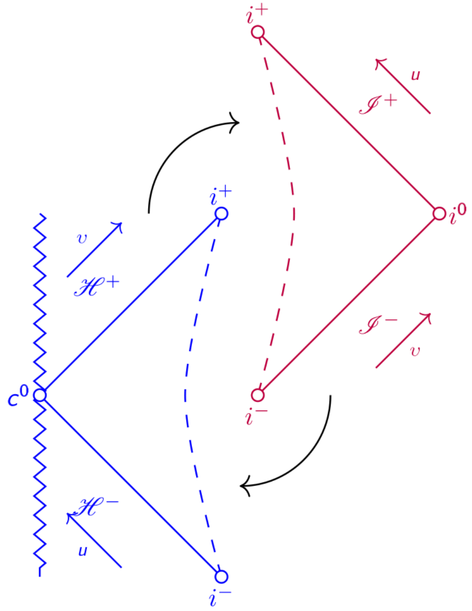
BUT

A consequence of this duality is

If the physical spacetime contains an **extremal non-rotating horizon**, then

the map $\mathscr{H}^{\pm} \longleftrightarrow \mathscr{I}^{\pm}$ should be an **exact isometry** of that spacetime

This 'explains' the **Couch-Torrence symmetry** of **extreme Reissner-Nordstrom (ERN)** black holes



[Agrawal, Charalambous, LD'24]

ERN as a self-dual example: the Couch-Torrence inversion symmetry

Extremal Reissner-Nordström (ERN) black hole

$$ds^2_{\mathrm{ERN}} = -\frac{\Delta(r)}{r^2}dt^2 + \frac{r^2}{\Delta(r)}dr^2 + r^2 d\Omega_2^2 \qquad \Delta(r) = (r-M)^2$$

Couch-Torrence (CT) inversion **symmetry** [Couch, Torrence '84]

$$r \xrightarrow{\mathrm{CT}} \tilde{r} = \frac{Mr}{r-M} \quad : \quad \text{isometry of } \; r^{-2} ds^2_{\mathrm{ERN}}$$

Note: isometry of the conformal metric [Borthwick, Gourgoulhon, Nicolas '23]

CT inversion : maps null infinity to the horizon!

$$r_* \xrightarrow{\mathrm{CT}} -r_* \qquad r_* = r - M - \frac{M^2}{r-M} + 2M \ln\left|\frac{r-M}{M}\right|$$

$$\Rightarrow (v, r, x^A) \xleftrightarrow{\mathrm{CT}} \left(u, \frac{Mr}{r-M}, x^A\right) \quad \Leftrightarrow \quad \boxed{\mathscr{H}^\pm \xleftrightarrow{\mathrm{CT}} \mathscr{I}^\pm}$$

ERN as a self-dual example: the Couch-Torrence inversion symmetry

Extremal Reissner-Nordström (ERN) black hole

$$ds^2_{\rm ERN} = -\frac{\Delta(r)}{r^2}dt^2 + \frac{r^2}{\Delta(r)}dr^2 + r^2 d\Omega_2^2 \qquad \Delta(r) = (r-M)^2$$

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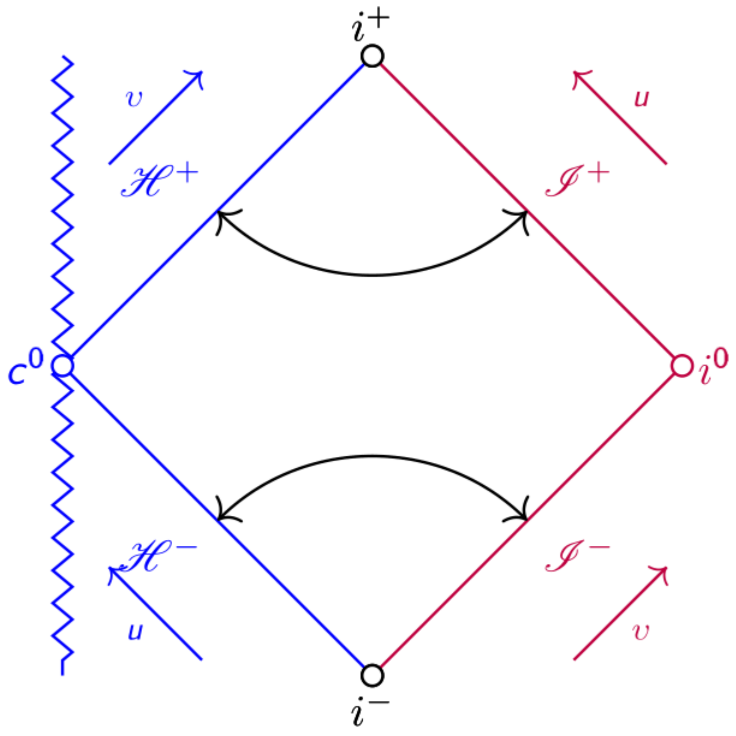
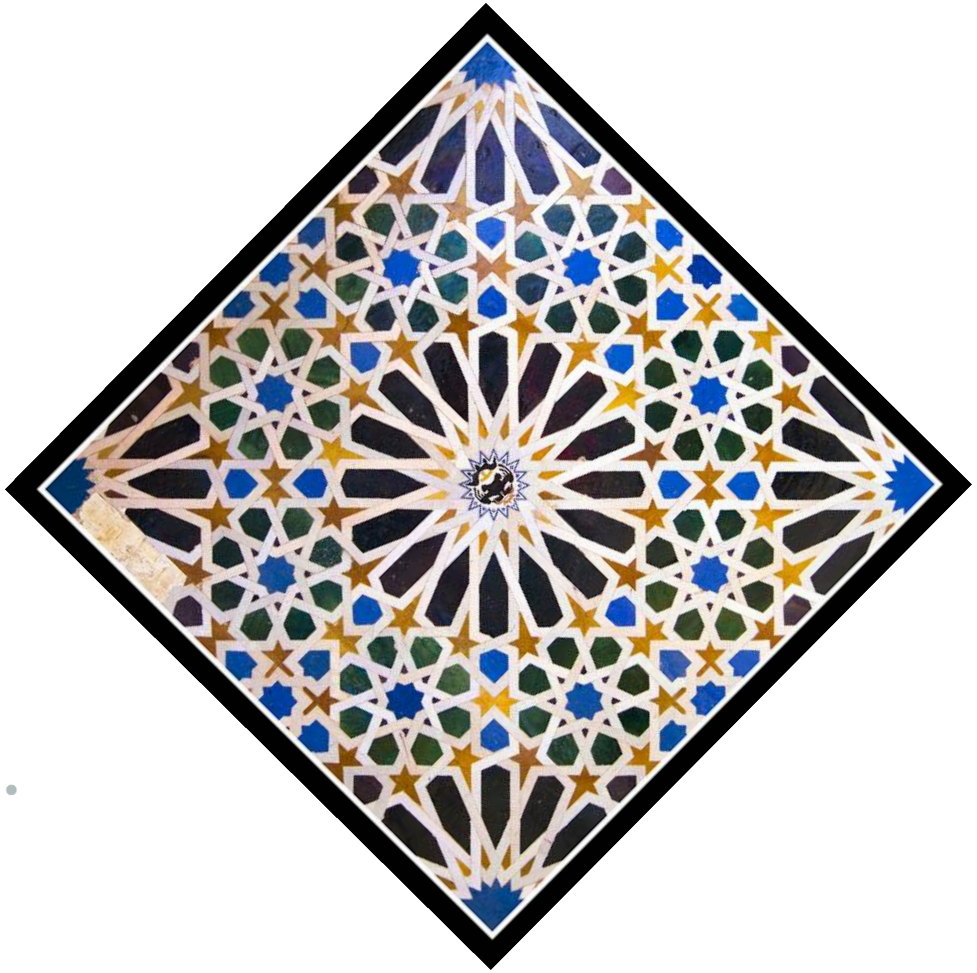




OUTLINE

① BLACK HOLE HORIZON
V.S NULL INFINITY
A) GEOMETRY
B) INFINITE SYMMETRIES

② $\mathcal{H}$ / $\mathcal{I}$ « DUALITY »

③ MAP OF ∞ CONSERVED QUANTITES $\mathcal{H}^{\pm} \leftrightarrow \mathcal{I}^{\pm}$

Matching of near-$\mathscr{I}$ charges to near-$\mathscr{H}$ charges

Massless **scalar perturbations** on ERN black hole

$$\Box_{\mathrm{ERN}}\Phi = 0$$

- Near $\mathscr{I}^+$ expansion:

$$\Phi \sim \frac{1}{r}\sum_{n=0}^{\infty}\frac{\Phi^{(n)}\left(u,x^A\right)}{r^n} \qquad \Phi^{(n)}\left(u,x^A\right) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\Phi^{(n)}_{\ell m}\left(u\right){}_0Y_{\ell m}\left(x^A\right)$$

$$r^2\Box_{\mathrm{ERN}} = \partial_r\left(r-M\right)^2\partial_r - 2r\partial_u\partial_r r + 2\eth\eth'$$

$$\boxed{N_{\ell m} = \sum_{n=1}^{\ell+1}(-1)^{\ell+1-n}\frac{n}{\ell+1}\binom{\ell}{n-1}M^{\ell+1-n}\Phi^{(n)}_{\ell m}\left(u\right) \quad \Rightarrow \quad \partial_u N_{\ell m} = 0\,, \quad \ell \geq 0}$$

infinite tower of conserved quantities

NP conserved quantities [Newman, Penrose '65 '68]

Matching of near-$\mathscr{I}$ charges to near-$\mathscr{H}$ charges

Massless **scalar perturbations** on ERN black hole

$$\Box_{\mathrm{ERN}}\Phi = 0$$

- Near $\mathscr{H}^+$ expansion:

$$\Phi \sim \sum_{n=0}^{\infty} \hat{\Phi}^{(n)}\left(v, x^A\right)\left(\frac{\rho}{M}\right)^n \qquad \hat{\Phi}^{(n)}\left(v, x^A\right) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} \hat{\Phi}^{(n)}_{\ell m}(v)\, {}_0Y_{\ell m}\left(x^A\right)$$

$$(M+\rho)^2\Box_{\mathrm{ERN}} = \partial_\rho\, \rho^2 \partial_\rho + 2\,(M+\rho)\,\partial_v \partial_\rho\,(M+\rho) + 2\eth\eth'$$

$$\boxed{A_{\ell m} := \hat{\Phi}^{(\ell+1)}_{\ell m}(v) + \frac{2\ell+1}{\ell+1}\hat{\Phi}^{(\ell)}_{\ell m}(v) + \frac{\ell}{\ell+1}\hat{\Phi}^{(\ell-1)}_{\ell m}(v) \quad \Rightarrow \quad \partial_v A_{\ell m} = 0\,, \quad \ell \geq 0}$$

infinite tower of conserved quantities

Aretakis charges [Aretakis ‘11]

Matching of near-$\mathscr{I}$ charges to near-$\mathscr{H}$ charges

Conserved quantities near $\mathscr{H}^+$

$$A_{\ell m} = \hat{\Phi}^{(\ell+1)}_{\ell m}(v) + \frac{2\ell+1}{\ell+1}\hat{\Phi}^{(\ell)}_{\ell m}(v) + \frac{\ell}{\ell+1}\hat{\Phi}^{(\ell-1)}_{\ell m}(v)$$

Conserved quantities near $\mathscr{I}^+$

$$N_{\ell m} = \sum_{n=1}^{\ell+1} (-1)^{\ell+1-n} \frac{n}{\ell+1} \binom{\ell}{n-1} M^{\ell+1-n} \Phi^{(n)}_{\ell m}(u)$$

Couch-Torrence (CT) inversion: $\hat{\Phi}\left(v,\rho,x^A\right) \xrightarrow{\text{CT}} \tilde{\Phi}\left(u,r,x^A\right) = \left(\frac{M}{r-M}\right)\hat{\Phi}\left(v\mapsto u, \rho\mapsto \frac{M^2}{r-M}, x^A\right)$

if $\Box_{\mathscr{H}^+}\hat{\Phi}\left(v,\rho,x^A\right)=0$, then $\Box_{\mathscr{I}^+}\tilde{\Phi}\left(u,r,x^A\right)=0$

$$\rightarrow \quad \boxed{M^{\ell+2}A_{\ell m} = N_{\ell m}}$$

Map between Aretakis and Newman-Penrose **conserved quantities**

[Bizon, Friedrich ’12][Lucietti, Murata, Reall, Tanahashi ‘12] [Fernandes, Ghosh, Virmani ‘20]

Linearized gravitational perturbations on ERN

$$\psi_0^{(j)} = \begin{cases} \Phi & \text{for } j = 0 \\ \phi_0 & \text{for } j = 1 \\ \Psi_0 & \text{for } j = 2 \end{cases}$$

- Near $\mathscr{I}^+$ expansion:

$$r^2 \Box_{\text{ERN}} = \frac{1}{(r-M)^{2j}} \partial_r (r-M)^{2(j+1)} \partial_r - \frac{2}{r^{2j-1}} \partial_u \partial_r r^{2j+1} + 2\left(\eth\bar{\eth}' + j\right)$$

$$\psi_0^{(j)} \sim \frac{1}{r^{2j+1}} \sum_{n=0}^{\infty} \frac{\psi_0^{(j,n)}\left(u, x^A\right)}{r^n} \qquad \psi_0^{(j,n)}\left(u, x^A\right) = \sum_{\ell=j}^{\infty} \sum_{m=-\ell}^{\ell} \psi_{0;\ell m}^{(j,n)}(u) \; {}_{+j}Y_{\ell m}\left(x^A\right)$$

$$\boxed{N_{\ell m}^{(j)} = \sum_{n=1}^{\ell-j+1} (-1)^{\ell-j+1-n} \frac{n}{\ell-j+1} \binom{\ell+j}{n+2j-1} M^{\ell-j+1-n} \psi_{0;\ell m}^{(j,n)}(u) \;\Rightarrow\; \partial_u N_{\ell m}^{(j)} = 0\,, \quad \ell \geq j}$$

infinite tower of conserved quantities

NP conserved quantities [Newman, Penrose '65 '68]

Linearized gravitational perturbations on ERN

$$\psi_0^{(j)} = \begin{cases} \Phi & \text{for } j = 0 \\ \phi_0 & \text{for } j = 1 \\ \Psi_0 & \text{for } j = 2 \end{cases}$$

- Near $\mathscr{H}^+$ expansion:

$$(M+\rho)^2 \Box_{\text{ERN}} = \frac{1}{\rho^{2j}} \partial_\rho \, \rho^{2(j+1)} \partial_\rho + \frac{2}{(M+\rho)^{2j-1}} \partial_v \partial_\rho \, (M+\rho)^{2j+1} + 2\,(\eth\eth' + j)$$

$$\psi_0^{(j)} \sim \frac{1}{r^{2j+1}} \sum_{n=0}^{\infty} \frac{\hat{\psi}_0^{(j,n)}\left(v, x^A\right)}{r^n} \qquad \hat{\psi}_0^{(j,n)}\left(u, x^A\right) = \sum_{\ell=j}^{\infty} \sum_{m=-\ell}^{\ell} \hat{\psi}_{0;\ell m}^{(j,n)}(v) \, {}_{+j}Y_{\ell m}\left(x^A\right)$$

$$A_{\ell m}^{(j)} := \hat{\psi}_{0;\ell m}^{(j,\ell-j+1)} + \frac{2\ell+1}{\ell-j+1} \hat{\psi}_{0;\ell m}^{(j,\ell-j)} + \frac{\ell+j}{\ell-j+1} \hat{\psi}_{0;\ell m}^{(j,\ell-j-1)} \quad \Rightarrow \quad \partial_v A_{\ell m}^{(j)} = 0\,, \quad \ell \geq j$$

New infinite tower of near-horizon **conserved quantities** [Agrawal, Charalambous, LD ‘24]

Summary and outlook

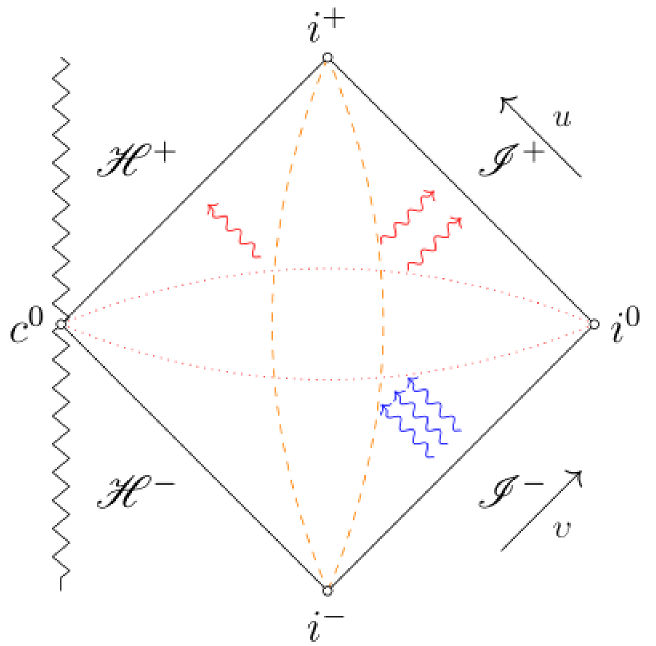
- We explored a **geometric duality** between null infinity and black hole horizons

  'null infinity is an extremal isolated horizon for the conformally completed asymptotically flat spacetime'

- $\mathscr{I}$ and its 'dual' black hole horizon are generically not part of the same spacetime

  EXCEPTION : Extreme RN, hence the 'surprising' Couch-Torrence exact isometry

$$\boxed{\mathscr{H}^{\pm} \overset{\text{CT}}{\longleftrightarrow} \mathscr{I}^{\pm}}$$



- Under the CT inversion**, Aretakis** conserved quantities and **Newman-Penrose** charges are in 1:1 correspondence

→ **We found a novel infinite tower of conserved quantities for spin-two perturbations**

↓

Proof of instability of ERN under gravitational perturbations?

Beyond the ERN case? Derivation of other tower of charges?

Overarching goal: obtain the *global* form of all conservation laws in asymptotically flat spacetimes with black holes