Potential and flux reconstructions for optimal a priori and a posteriori error estimates

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Given a scalar-valued discontinuous piecewise polynomial, a “potential reconstruction” is a piecewise polynomial that is trace-continuous, i.e., H^1-conforming. It is best obtained via a conforming finite element solution of local homogeneous Dirichlet problems on patches of elements sharing a vertex. Similarly, given a vector-valued discontinuous piecewise polynomial not satisfying the target divergence, a “flux reconstruction” is a piecewise polynomial that is normal-trace-continuous, i.e., $\boldsymbol{H}$(div)-conforming, and has the target divergence. It is best obtained via local homogeneous Neumann problems on patches of elements, using the mixed finite element method. These concepts are known to lead to guaranteed, locally efficient, and polynomial-degree-robust a posteriori error estimates. We show that they also allow to devise stable local commuting projectors that lead to *p*-robust equivalence of global-best approximation over the whole computational domain using a conforming finite element space with local- (elementwise-)best approximations without any continuity requirement along the interfaces and without any constraint on the divergence. Therefrom, optimal *hp* approximation / a priori error estimates under minimal elementwise Sobolev regularity follow.