On Systems of Interacting Particles

Rapport sur les contributions

https://indico.math.cnrs.fr/e/12097

ID de Contribution: **1** Type: **Non spécifié**

Riesz Gases and their Phase Diagrams in Dimension One

jeudi 6 juin 2024 10:00 (1 heure)

$Riesz\ gases$ form an important class of point processes in statistical physics, consisting in an infinite number of particles interacting through an inverse power-law repulsive pair potential of homogeneity s. These objects appear in many unexpected mathematical situations and seem to be sort of universal. An interesting although very challenging question both theoretically and numerically is that of the existence of phase transitions with respect to the temperature and the homogeneity parameter s. In this talk, I will present what is (un)known at the current time on these models focusing mainly on the case of the dimension one. I will then present numerical evidences which support the existence of a phase transition in dimension one when the parameter s ranges between two remarkable models, namely the one-dimensional $Coulomb\ gas$ and the $Dyson\ log\text{-}gas$, and discuss the nature of this transition. Many questions will be left open, only hopefully to spur new contributions in the future.

Orateur: LELOTTE, Rodrigue (CERMICS, ENPC)

ID de Contribution: **2** Type: **Non spécifié**

Constrained Dynamics on Measures

jeudi 6 juin 2024 11:00 (1 heure)

Stochastic diffusions are widely used to model physical phenomena, the noise being useful to account for average effects which need not being specified. However, the proposed model is always an approximation that cannot exactly reproduce all the features of the real system (mean, variance, higher order moment…).
Starting from the Gibbs conditioning principle, this talk presents a systematic way to constrain the law of a diffusion at each time. A careful regularity analysis is done on the corrected process. Quantitative stability is then investigated when perturbing the constraint, showing the robustness of the correction procedure. This work is a collaboration with Giovanni Conforti and Julien Reygner.

Orateur: CHAINTRON, Louis-Pierre (DMA, ENS)

ID de Contribution: **3** Type: **Non spécifié**

Mean-Field Limits for Singular Interactions Using Modulated Energy, and its Limitations

jeudi 6 juin 2024 13:30 (1 heure)

We consider particles whose interaction potential is singular, which is the case of the Coulomb potential. In such a situation, it is more difficult to account for the relative independence of the particles in the limit of their large number, a property known as “molecular chaos”. We will see that the modulated energy method provides a fairly robust answer to this question, and we will also see its limitations.

Orateur: CHODRON DE COURCEL, Antonin (IHES)

ID de Contribution: **4** Type: **Non spécifié**

Quasi-Potential for the one Dimensional SSEP in Weak Contact with Reservoirs

jeudi 6 juin 2024 14:30 (1 heure)

We derive a formula for the quasi-potential of a one-dimensional symmetric exclusion process in weak contact with reservoirs. The interaction with the boundary is so weak that, in the diffusive scale, the density profile evolves as the one of the exclusion process with reflecting boundary conditions. In order to observe an evolution of the total mass, the process has to be observed in a longer time-scale, in which the density profile becomes immediately constant. This work is a collaboration with Claudio Landim (IMPA, Rio de Janeiro).

Orateur: VELASCO, Sonia (MAP5, Paris-Cité)

ID de Contribution: **5** Type: **Non spécifié**

A Keller-Segel System with Spatial Competition

jeudi 6 juin 2024 16:00 (1 heure)

In this presentation, we will introduce and study a Keller-Segel type model describing the time evolution of the spatial distribution of a population of cells subject to three mechanisms.
1) Each cell produces a chemical substance which diffuses in the space and attracts all the other cells (chemotaxis). This corresponds to an attractive and singular mean-field interaction.
2) The cells diffuse in space.
3) The cells are subject to cell division, i.e. there are births in the population, and more importantly, to a local spatial competition. This means that a cell present in an area where there are a lot of other cells will have a high probability of dying.
The standard Keller-Segel model, which corresponds to mechanisms 1) and 2), has already been extensively studied. At the microscopic level, it can be described by a stochastic system of N particles in mean-field interaction.
At the macroscopic level, the system is described by a non-linear PDE that appears as the limit in distribution of the particle system when N tends to infinity: this is the propagation of chaos phenomenon. We will focus on the difficulties coming from the demographic mechanism 3), both at the level of the particle system and of the PDE. Of course, there are not only difficulties: the spatial competition can counterbalance the singular attraction, which can typically make the PDE explode in finite time if the attraction is strong enough in front of the diffusion. In the presence of a sufficiently strong spatial competition, this explosion phenomenon no longer occurs. The propagation of chaos will be tackled in the framework of moderately interacting particle systems.

Orateur: CAVALLAZZI, Thomas (CentraleSupélec, Paris-Saclay)