A Keller-Segel System with Spatial Competition

jeudi 6 juin 2024 16:00 (1 heure)

In this presentation, we will introduce and study a Keller-Segel type model describing the time evolution of the spatial distribution of a population of cells subject to three mechanisms.

1) Each cell produces a chemical substance which diffuses in the space and attracts all the other cells (chemotaxis). This corresponds to an attractive and singular mean-field interaction.

2) The cells diffuse in space.

3) The cells are subject to cell division, i.e. there are births in the population, and more importantly, to a local spatial competition. This means that a cell present in an area where there are a lot of other cells will have a high probability of dying.

The standard Keller-Segel model, which corresponds to mechanisms 1) and 2), has already been extensively studied. At the microscopic level, it can be described by a stochastic system of N particles in mean-field interaction.

At the macroscopic level, the system is described by a non-linear PDE that appears as the limit in distribution of the particle system when N tends to infinity: this is the propagation of chaos phenomenon. We will focus on the difficulties coming from the demographic mechanism 3), both at the level of the particle system and of the PDE. Of course, there are not only difficulties: the spatial competition can counterbalance the singular attraction, which can typically make the PDE explode in finite time if the attraction is strong enough in front of the diffusion. In the presence of a sufficiently strong spatial competition, this explosion phenomenon no longer occurs. The propagation of chaos will be tackled in the framework of moderately interacting particle systems.

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