Spontaneous Hopf Fibration in the 2HDM

Steven Cotterill

University of Manchester, UK

Based on:
"Spontaneous Hopf Fibration in the Two-Higgs-Doublet Model" – Battye & Cotterill Phys. Rev. Lett. **132** 061601 (2024)

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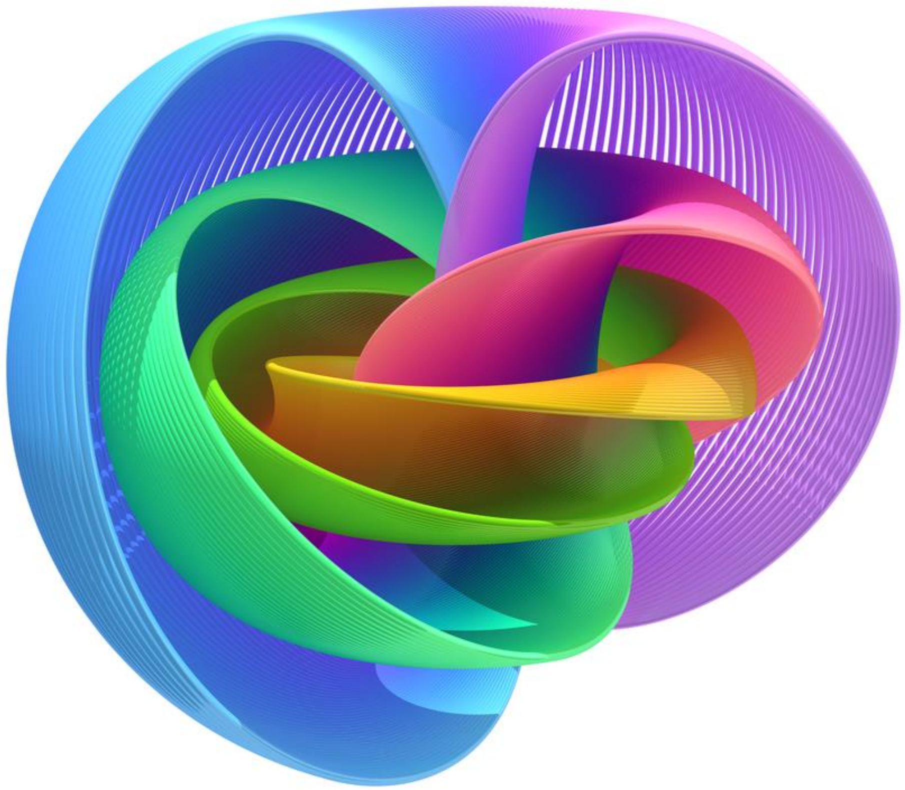


The Two Higgs Doublet Model (2HDM)

- Adds an additional complex scalar doublet to the SM with the general potential.

$$V = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+ \Big[- m_{12}^2(\Phi_1^\dagger\Phi_2) + \lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \text{h.c.}\Big]$$

- 5 scalar particles: h, H, $H_\pm$ and A.
- Neutral vacuum ($f_+ = 0$) for a massless photon.

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \frac{v_{\rm SM}}{\sqrt{2}} e^{\frac{1}{2}i\chi}(\sigma^0 \otimes U_L) \begin{pmatrix} 0 \\ f_1 \\ f_+ \\ f_2 e^{i\xi} \end{pmatrix}$$

- Used in MSSM and DFSZ axion model.
- Experimental constraints from Higgs boson, masses of new particles, FCNCs, etc…

General parameterization without additional symmetries

$$\Phi = \begin{pmatrix} A_{11}e^{i\alpha_{11}} \\ A_{12}e^{i\alpha_{12}} \\ A_{21}e^{i\alpha_{21}} \\ A_{22}e^{i\alpha_{22}} \end{pmatrix}$$

Same as SM →

$$\Phi = \begin{pmatrix} 0 \\ B_{12} \\ B_{21}e^{ib_{21}} \\ B_{22}e^{ib_{22}} \end{pmatrix}$$

Use photon direction →

$$\Phi = \begin{pmatrix} 0 \\ B_{12} \\ B_{21} \\ B_{22}e^{ib_{22}} \end{pmatrix}$$

$$\Phi = \frac{v_{\mathrm{SM}}}{\sqrt{2}} e^{\frac{1}{2}i\chi} (\sigma_0 \otimes U_L) \begin{pmatrix} 0 \\ f_1 \\ f_+ \\ f_2 e^{i\xi} \end{pmatrix}$$

Accidental symmetries of the 2HDM

- Parameter choices → additional "accidental" symmetries.

Symmetry	μ_1^2	μ_2^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
Z_2	–	–	0	–	–	–	–	Real	0	0
$U(1)_{PQ}$	–	–	0	–	–	–	–	0	0	0
$SO(3)_{HF}$	–	μ_1^2	0	–	λ_1	–	$2\lambda_1 - \lambda_3$	0	0	0

"Vacuum Topology of the Two Higgs Doublet Model" – Battye, Brawn & Pilaftsis JHEP **08** 020 (2011)

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} e^{i\eta} & 0 \\ 0 & e^{-i\eta} \end{pmatrix} \text{ or } \begin{pmatrix} \cos\eta_1 e^{i\eta_2} & \sin\eta_1 e^{i\eta_2} \\ -\sin\eta_1 e^{-i\eta_3} & \cos\eta_1 e^{-i\eta_2} \end{pmatrix} \text{ on } \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

Parameterization with accidental symmetries

$$\begin{pmatrix} 0 \\ f_1 \\ f_+ \\ f_2 e^{i\xi} \end{pmatrix} \xrightarrow[\begin{pmatrix} e^{\frac{1}{2}\xi} & 0 \\ 0 & e^{-\frac{1}{2}\xi} \end{pmatrix}]{U_H \otimes U_L} \begin{pmatrix} 0 \\ f_1 \\ f_+ \\ f_2 \end{pmatrix} \xrightarrow[\begin{pmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{pmatrix}]{U_H \otimes U_L} \begin{pmatrix} 0 \\ k \\ k_+ \\ 0 \end{pmatrix}$$

Same matrix for both

Same structure but different angles

$$\Phi = \frac{v_{\rm SM}}{\sqrt{2}} (U_H \otimes U_L) \bar{\Phi}$$

$\xrightarrow{U(1)_{PQ}}$ $\bar{\Phi} = \begin{pmatrix} 0 \\ f_1 \\ f_+ \\ f_2 \end{pmatrix}$

$\xrightarrow{SO(3)_{HF}}$ $\bar{\Phi} = \begin{pmatrix} 0 \\ k \\ k_+ \\ 0 \end{pmatrix}$

Bilinear forms

- Can rewrite the potential in terms of $R^\mu = \Phi^\dagger(\sigma^\mu \otimes \sigma^0)\Phi$,

$$V = -\frac{1}{2}M_\mu R^\mu + \frac{1}{4}L_{\mu\nu}R^\mu R^\nu$$

- There is also $n^a = -\Phi^\dagger(\sigma^0 \otimes \sigma^a)\Phi$ associated with isospin rotations.
- Vectors associated with the map SU(2) → SO(3): $U^\dagger \sigma^a U = \mathcal{R}^{ab}\sigma^b$.
- $R_+ = R_\mu R^\mu$ tracks the neutral vacuum condition (needs to be zero for a massless photon).

$$\text{Topological solitons:} \begin{cases} \mathbb{Z}_2, & \pi_0(S^0 \times S^3) = \mathbb{Z}_2 \Longrightarrow \text{Domain walls} & R^1 \to -R^1 \\ \mathrm{U}(1)_{\mathrm{PQ}}, & \pi_1(S^1 \times S^3) = \mathbb{Z} \Longrightarrow \text{Strings} & R^1, R^2 \text{ rotations} \\ \mathrm{SO}(3)_{\mathrm{HF}}, & \pi_2(S^2 \times S^3) = \mathbb{Z} \Longrightarrow \text{Monopoles} & R^a \text{ rotations} \end{cases}$$

"Minkowski space structure of the Higgs potential in the two-Higgs-doublet model" – Ivanov Phys. Rev. D **75**, 035001 (2007)

Previous topological solitons in the 2HDM

$$\Phi = \frac{v_{\mathrm{SM}}}{\sqrt{2}} e^{\frac{1}{2}i\chi}(\sigma^0 \otimes U_L) \begin{pmatrix} 0 \\ f_1 \\ f_+ \\ f_2 e^{i\xi} \end{pmatrix}$$

Domain wall

$$\Phi = \frac{v_{\mathrm{SM}}}{\sqrt{2}} \begin{pmatrix} 0 \\ f_1(x) \\ 0 \\ f_2(x) \end{pmatrix}$$

String

$$\Phi = \frac{v_{\mathrm{SM}}}{\sqrt{2}} \begin{pmatrix} 0 \\ f_1(r) \\ 0 \\ f_2(x) e^{i\theta} \end{pmatrix}$$

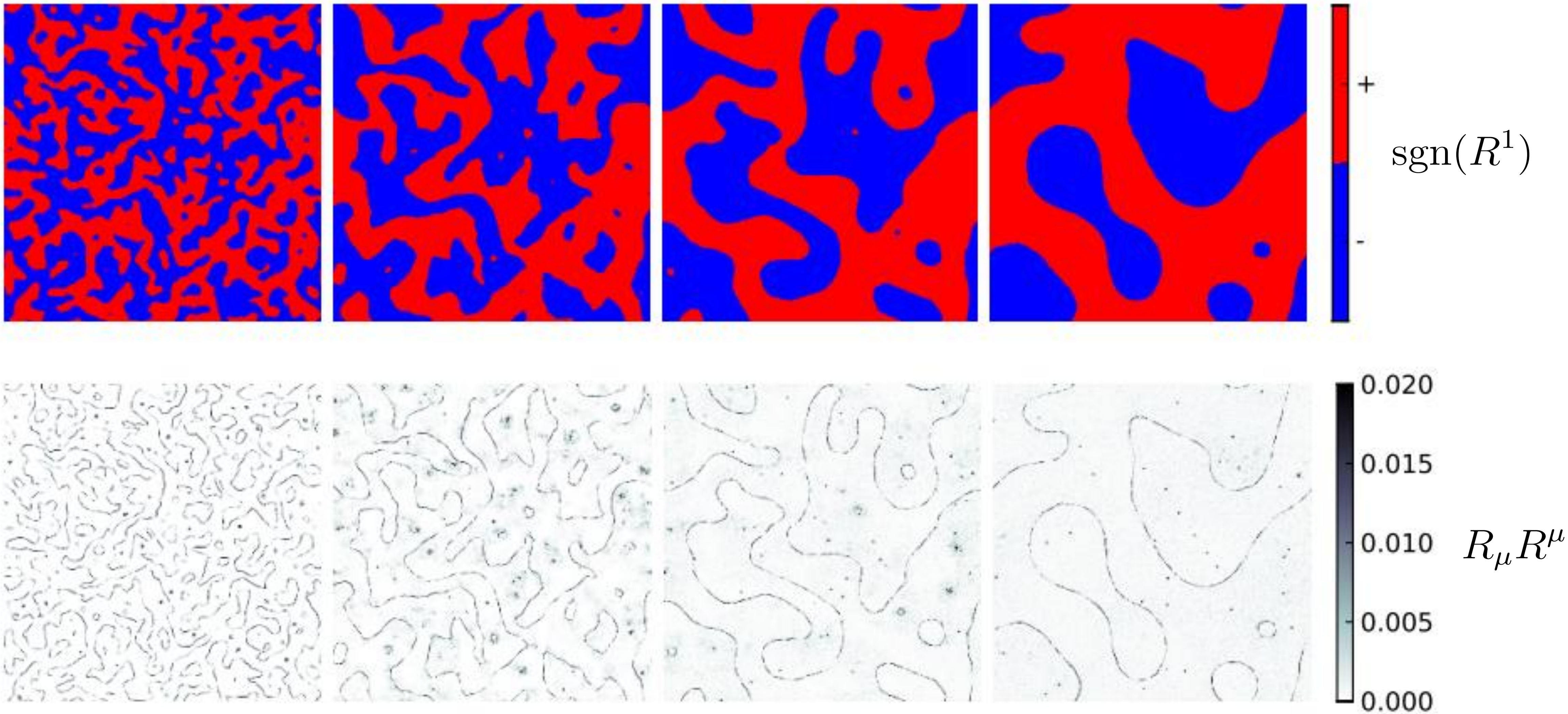
Monopole

$$\Phi = \frac{v_{\mathrm{SM}}}{\sqrt{2}} \begin{pmatrix} 0 \\ k(r)\cos\theta \\ 0 \\ k(r)\sin\theta e^{i\phi} \end{pmatrix}$$

All have $R_\mu R^\mu = 0$ everywhere.

“Vacuum Topology of the Two Higgs Doublet Model” – Battye, Brawn & Pilaftsis JHEP **08** 020 (2011)

Random simulations in the $\mathbb{Z}_2$ case



“Simulations of Domain Walls In Two Higgs Doublet Models” – Battye, Pilaftsis & Viatic JHEP **01** 105 (2021)

Initial attempts for monopoles

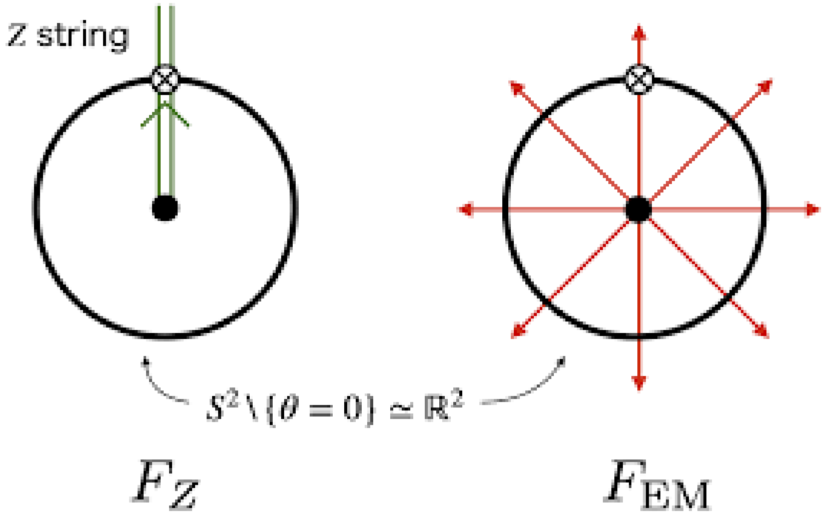
$$\Phi = \frac{v_{\rm SM}}{\sqrt{2}} \begin{pmatrix} 0 \\ k(r)\cos\theta \\ 0 \\ k(r)\sin\theta e^{i\phi} \end{pmatrix} \quad \text{but} \quad \hat{R}^a = \begin{pmatrix} \sin 2\theta \cos\phi \\ \sin 2\theta \sin\phi \\ \cos 2\theta \end{pmatrix}$$

$$\xrightarrow{\hat{R}^a = \hat{r}^a} \quad \Phi = \frac{v_{\rm SM}}{\sqrt{2}} \begin{pmatrix} 0 \\ k(r)\cos\frac{1}{2}\theta \\ 0 \\ k(r)\sin\frac{1}{2}\theta e^{i\phi} \end{pmatrix}$$

Nambu monopole

Both assume that k_+ is zero everywhere, not just in the vacuum.

We will see that it can be improved by using the SM degrees of freedom.



Eto, Hamada & Nitta, Phys. Rev. D **102** 105018 (2020)

Gradient energy

$$D_\mu \Phi = \left[(\sigma^0 \otimes \sigma^0)\partial_\mu + \tfrac{1}{2} i g (\sigma^0 \otimes \sigma^a) W_\mu^a + \tfrac{1}{2} i g'' (\sigma^a \otimes \sigma^0) V_\mu^a \right] \Phi \qquad (g' = 0)$$

Absorb U_H and U_L with a gauge transformation like

$$\tfrac{1}{2} i g \mathcal{W}_\mu^a \sigma^a = U_L^\dagger \partial_i U_L + \tfrac{1}{2} i g W_\mu^a U_L^\dagger \sigma^a U_L$$

$$= e^{\frac{1}{2} i \chi} (U_H \otimes U_L) \left[(\sigma^0 \otimes \sigma^0)\partial_\mu + \tfrac{1}{2} i g \mathcal{W}_\mu^a (\sigma^0 \otimes \sigma^a) + \tfrac{1}{2} i g'' \mathcal{V}_\mu^a (\sigma^a \otimes \sigma^0) \right] \bar{\Phi} \qquad \bar{\Phi} = \begin{pmatrix} 0 \\ k \\ k_+ \\ 0 \end{pmatrix}$$

Nambu monopole:

$$U_H = \begin{pmatrix} \cos \frac{1}{2}\theta & -\sin \frac{1}{2}\theta e^{-i\phi} \\ \sin \frac{1}{2}\theta e^{i\phi} & \cos \frac{1}{2}\theta \end{pmatrix} \qquad (g'' \mathcal{V}_i^a)^2 = \frac{2}{r^2} + \left(\frac{1 - \cos\theta}{r \sin\theta} \right)^2$$

Comes from a=3

Gradient energy

The full expression contains:

$$(g\mathcal{W}_i^a - g''\mathcal{V}_i^a)^2(k-k_+)^2$$

$$[(g\mathcal{W}_i^a + g''\mathcal{V}_i^a)(g\mathcal{W}_i^b + g''\mathcal{V}_i^b)\mathcal{P}^{ab} + (g\mathcal{W}_i^3 - g''\mathcal{V}_i^3)^2](k+k_+)^2 \qquad \mathcal{P}^{ab} = \delta^{ab} - \hat{z}^a\hat{z}^b$$

$$U_H = U_L \Rightarrow g\mathcal{W}_i^a = g''\mathcal{V}_i^a$$

- Cancels the divergence at $\theta = \pi$
- $(k-k_+)^2$ can be non-zero at the centre

Gauged monopole ansatz

- Full ansatz:

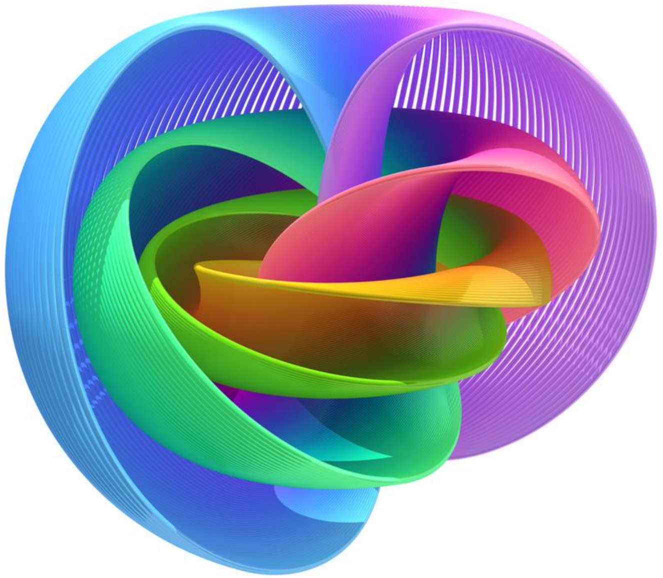
$$\Phi = \frac{v_{\text{SM}}}{2\sqrt{2}} \begin{pmatrix} -(k+k_+)\sin\theta e^{-i\phi} \\ (k-k_+)+(k+k_+)\cos\theta \\ -(k-k_+)+(k+k_+)\cos\theta \\ (k+k_+)\sin\theta e^{i\phi} \end{pmatrix}$$

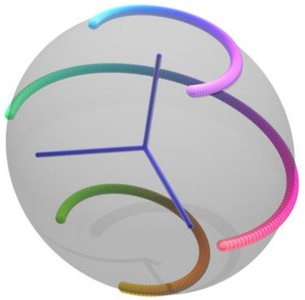
$$gW_i^a = -\frac{1}{r}h(r)\epsilon^a_{ij}\hat{r}^j \qquad g''V_i^a = -\frac{1}{r}H(r)\epsilon^a_{ij}\hat{r}^j$$

$$R^a = n^a = \frac{1}{2}v_{\text{SM}}^2(k^2-k_+^2)\hat{r}^a$$

- Gradient energy causes a “Spontaneous Hopf Fibration” with winding in S^2 but not S^1.

$S^3 \rightarrow S^2$





n^a

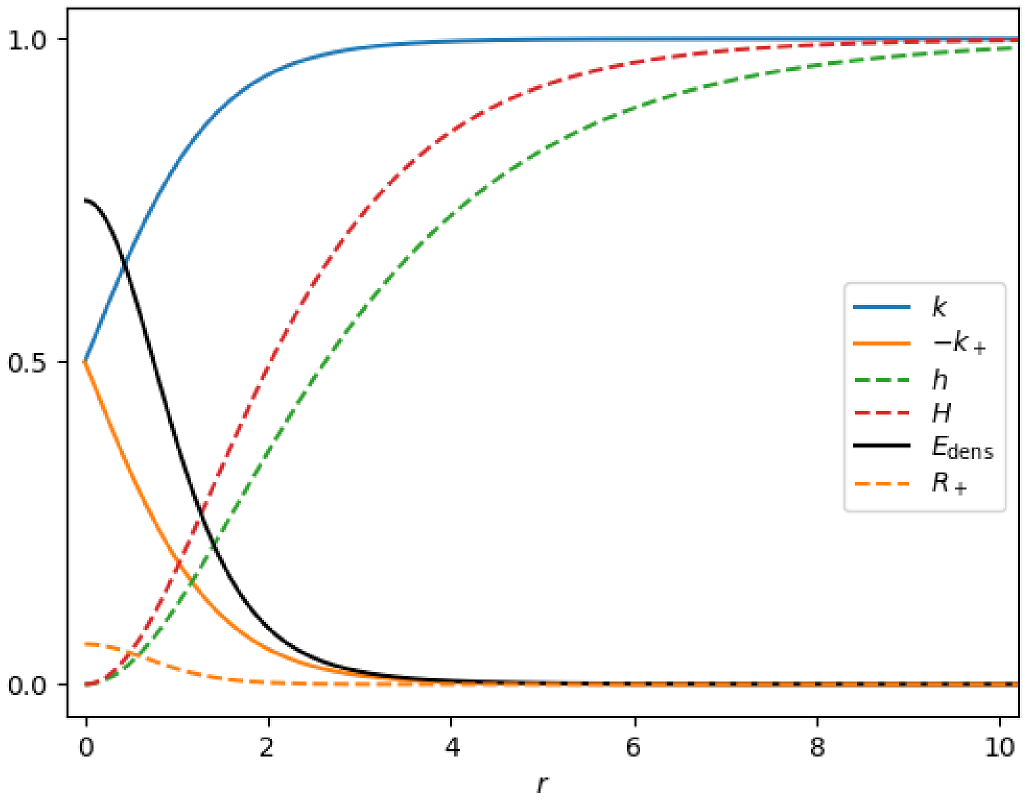
Parameters

- For $SO(3)_{HF}$ there are 3 remaining parameters in the potential:

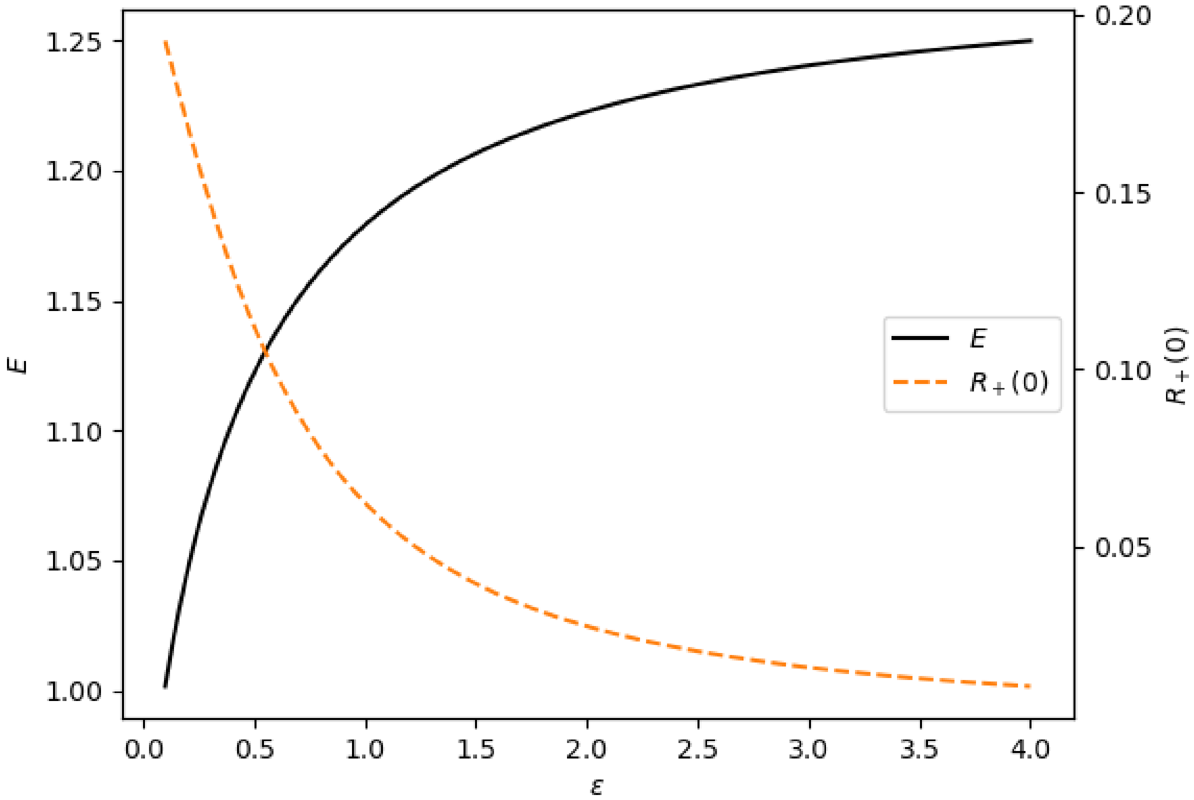
$$\mu_1^2, \lambda_1, \lambda_4$$

Global	Gauged
$\mu_1^2, \lambda_1, \lambda_4$ ↓ Rescale — "Physical" parameterization $\epsilon = M_{H\pm}/M_h$ ($M_H = M_A = 0$)	$\tilde{\lambda}_1 = \lambda_1/g^2$ $\tilde{g}^2 = (g''/g)^2$ $\epsilon = M_{H\pm}/M_h$ (g')

Gauged monopole solution



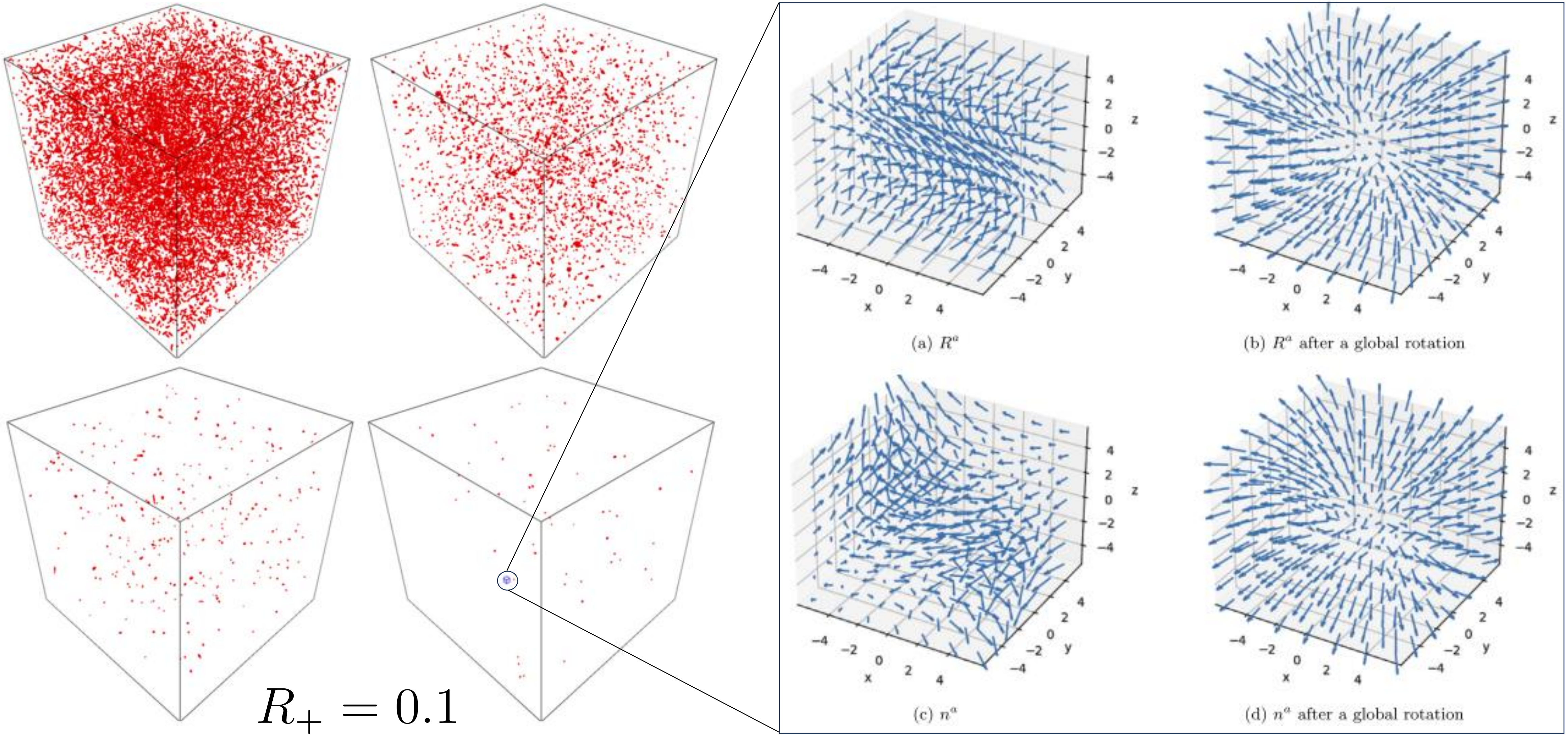
$\tilde{\lambda}_1 = 1 \quad \tilde{g}^2 = 2 \qquad \epsilon = 1$



Energy and $R_+ = R_\mu R^\mu$ as a function of ϵ.

Global monopole simulations

$\epsilon = 0.35$



$R_+ = 0.1$

“Global monopoles in the two-Higgs-doublet-model” – Battye, Cotterill & Viatic Phys. Lett B **844** 138091 (2023)

Particle masses

- Fix vacuum values, $k = 1 \quad k_+ = 0$, and look at energy again.
- Get the usual massless photon and charged weak bosons.
- Additional two massive particles with mass $\frac{1}{2} g''^2 v_{\rm SM}^2$
- Plus bosons from the mass matrix:

$\frac{1}{8} v_{\rm SM}^2 \begin{pmatrix} Z_i & \mathcal{V}_i^3 \end{pmatrix} \begin{pmatrix} \tilde{g}^2 & \tilde{g} g'' \\ \tilde{g} g'' & g''^2 \end{pmatrix} \begin{pmatrix} Z^i \\ \mathcal{V}^{i3} \end{pmatrix}$ where $Z_i = \frac{1}{\tilde{g}}(g' \mathcal{Y}_i - g \mathcal{W}_i^3)$ and $\tilde{g}^2 = g^2 + g'^2$

Modified Z boson

$$M_Z^2 = \frac{1}{4}(\tilde{g}^2 + g''^2) v_{\rm SM}^2$$

New massless particle

Difficult to explain

Strings

- The potential is less restricted so there are more model parameters.
- Reminders:

$$\begin{pmatrix} e^{i\eta} & 0 \\ 0 & e^{-i\eta} \end{pmatrix} \qquad R^1, R^2 \text{ rotations} \qquad \Phi = \frac{v_{\text{SM}}}{\sqrt{2}} e^{\frac{1}{2}i\chi}(U_H \otimes U_L) \begin{pmatrix} 0 \\ f_1 \\ f_+ \\ f_2 \end{pmatrix}$$

- Simple approach of $\hat{R}^a = \hat{r}^a$ for the 2-vector gives

$$\Phi = \frac{v_{\text{SM}}}{\sqrt{2}} \begin{pmatrix} 0 \\ f_1 e^{-\frac{1}{2}i\theta} \\ f_+ e^{\frac{1}{2}i\theta} \\ f_2 e^{\frac{1}{2}i\theta} \end{pmatrix}$$

Similar issue to the Nambu monopole!

Gradient energy

- Gradient energy ($V_i^a \to V_i^3$) is much more complicated.
- Still has terms like

$$(g\mathcal{W}_i^3 - g''\mathcal{V}_i^3)^2(f_1^2 + f_+^2) \qquad (g\mathcal{W}_i^3 + g''\mathcal{V}_i^3)^2 f_2^2$$

- But also, complications like current terms

$$g\mathcal{W}_i^2(f_2\partial_i f_+ - f_+\partial_i f_2)$$

- Result: divergence can still cancel, but the two gauge fields no longer have the same structure.

Parameters

- For $U(1)_{PQ}$ there are 6 parameters in the potential:

$$\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4$$

Global	Gauged
$\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ Rescale ↓ "Physical" parameterization α, β Angles between perturbations in Higgs basis and mass eigenstates $\delta = M_H/M_h, \quad \epsilon = M_{H\pm}/M_h$ $(M_A = 0)$	The same, plus: $\tilde{\lambda}_1 = \lambda_1/g^2$ $\tilde{g}^2 = (g''/g)^2$ (g')

Gauged string solution

$$\Phi = \frac{v_{\rm SM}}{\sqrt{2}} \begin{pmatrix} 0 \\ f_1 \\ f_+ \\ f_2 e^{i\theta} \end{pmatrix}$$

$$\hat{R}^a = \hat{n}^a = \hat{r}^a \,, \text{ where } a \in [1,2]$$

But it's easier to absorb the phase into the gauge fields.

$$gW_i^a = -\frac{1}{r}\left[h_1(r)\hat{x}^a + (1 - h_3(r))\hat{z}^a\right]\hat{\theta}_i + h_2(r)\hat{y}^a\hat{r}_i,$$

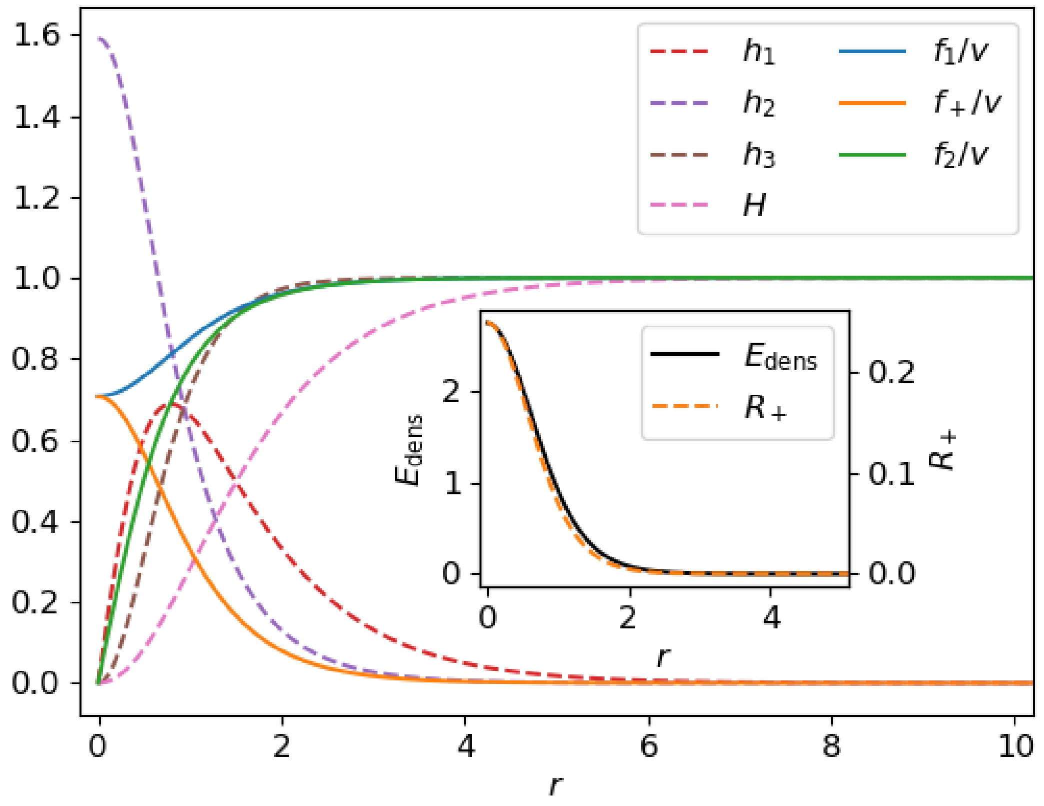
$$g''V_i^3 = -\frac{1}{r}(1 - H(r))\hat{\theta}_i$$

Parameters

$\tilde{\lambda}_1 = 1$ $\tilde{g}^2 = 2$

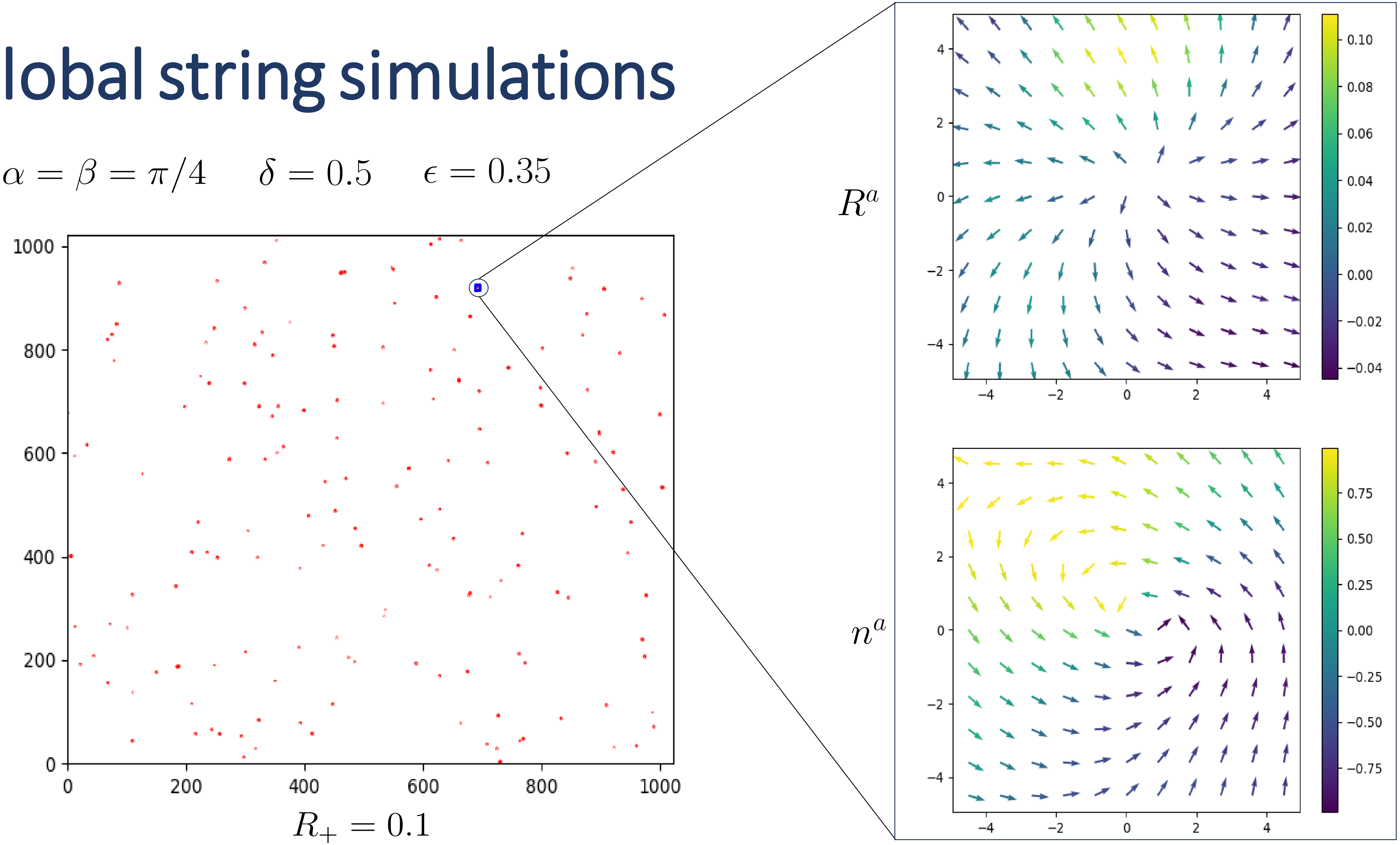
$\alpha = \beta = \pi/4$ "Alignment limit"

$\delta = 2$ $\epsilon = 1$



Global string simulations

$\alpha = \beta = \pi/4 \qquad \delta = 0.5 \qquad \epsilon = 0.35$



Neutrality violation mass analysis

- Neglect gradient energy to estimate effective mass

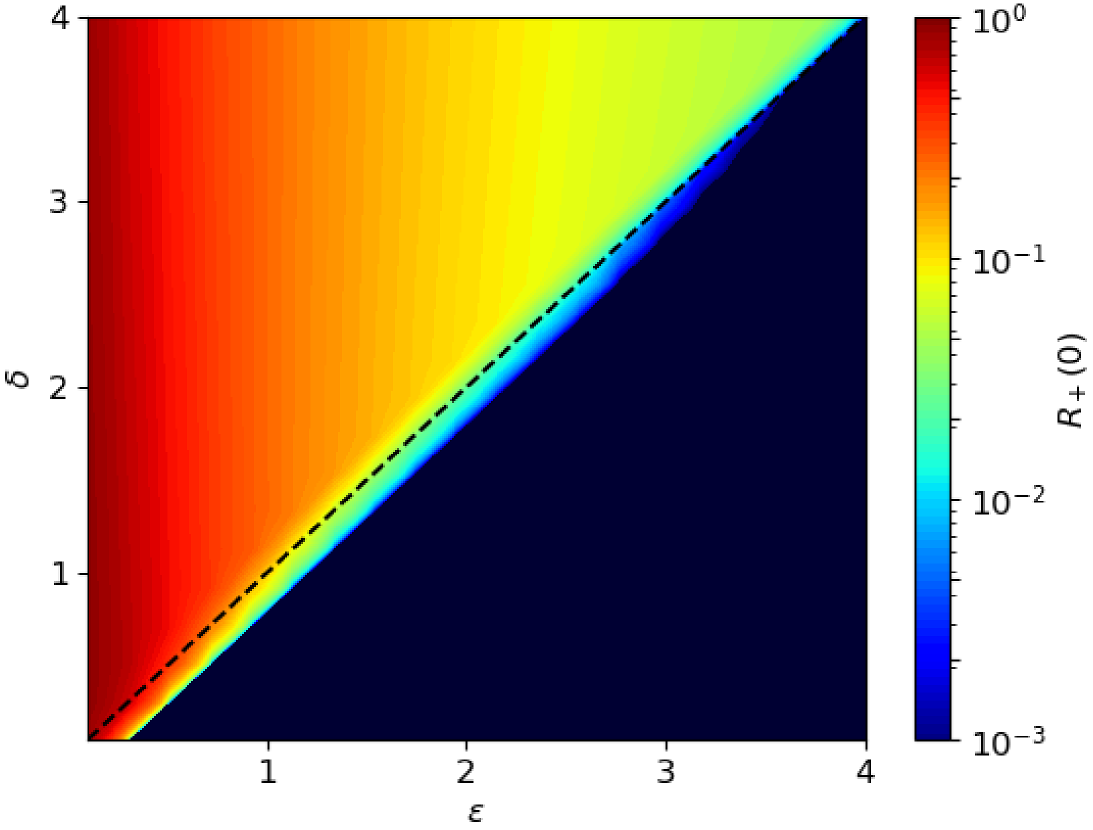
- Monopoles, assuming $k = k_+ = 0$ at the core:

$$m^2_{k_+}(0) = -\frac{1}{2}\tilde{\lambda}_1$$

- Strings, assuming $f_1 = 1$ and $f_+ = f_2 = 0$ at the core:

$$m^2_{f_+}(0) = \tilde{\lambda}_1\left(\frac{\epsilon^2 - \delta^2}{1 + \delta^2}\right)$$

(when $\alpha = \beta = \pi/4$)



Particle masses

- Vacuum values: $f_1 = \cos\beta \quad f_+ = 0 \quad f_2 = \sin\beta$
- Massless photon, charged weak bosons and two neutral gauge bosons from the mass matrix:

$$\frac{1}{8} v_{\mathrm{SM}}^2 \begin{pmatrix} Z_i & \mathcal{V}_i^3 \end{pmatrix} \begin{pmatrix} \tilde{g}^2 & \tilde{g} g'' \cos 2\beta \\ \tilde{g} g'' \cos 2\beta & g''^2 \end{pmatrix} \begin{pmatrix} Z^i \\ \mathcal{V}^{i3} \end{pmatrix}$$

$$M_\pm^2 = \frac{M_W^2}{2\cos^2\theta_W \cos^2\theta_C} \left[1 \pm \sqrt{1 - \sin^2 2\theta_C \sin^2 2\beta)} \right] \qquad \tan\theta_C = \frac{g''}{\sqrt{g^2 + g'^2}} \qquad \tan\theta_W = \frac{g'}{g}$$

- Potentially viable near $\beta = \frac{\pi}{4}$:

$$M_Z^2 = \frac{M_W^2}{\cos^2\theta_W} \qquad M_{Z'}^2 = \frac{M_W^2 \tan^2\theta_C}{\cos^2\theta_W}$$

Summary

- Spontaneous Hopf fibration of the SM 3-sphere is enforced by energetics for 2HDM solitons.
- This mechanism is not necessarily specific to the 2HDM.
- Possible to have localised neutrality violation in the core of defects.
- Local non-zero photon mass → new interactions?
- Are there phenomenologically viable models with solitons in the 2HDM?

"Minkowski space structure of the Higgs potential in the two-Higgs-doublet model" – Ivanov Phys. Rev. D **75**, 035001 (2007)

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"Simulations of Domain Walls In Two Higgs Doublet Models" – Battye, Pilaftsis & Viatic JHEP **01** 105 (2021)

"Topological structure of a Nambu monopole in two-Higgs-doublet models: Fiber bundle, Dirac's quantization, and a dyon" – Eto, Hamada & Nitta, Phys. Rev. D **102** 105018 (2020)

"Global monopoles in the two-Higgs-doublet-model" – Battye, Cotterill & Viatic Phys. Lett B **844** 138091 (2023)

"Spontaneous Hopf Fibration in the Two-Higgs-Doublet Model" – Battye & Cotterill Phys. Rev. Lett. **132** 061601 (2024)