

# Exponential Volumes in Geometry and Representation Theory

*lundi 16 septembre 2024 14:00 (1 heure)*

Let  $S$  be a topological surface with holes. The moduli space parametrising hyperbolic structures on  $S$  with geodesic boundary, and a given set  $L$  of lengths of boundary circles carries the Weil-Peterson volume form. Its volume is finite. Maryam Mirzakhani proved remarkable recursion formulas for these volumes, related to several areas of Mathematics. In particular the volumes are polynomials in  $L$ . Their leading coefficients are the volumes studied by Maxim Kontsevich in his proof of Witten's conjecture.

However for a surface  $P$  with polygonal boundary, e.g. just a polygon, similar volumes are infinite. We consider a variant of these moduli spaces, and show that they carry a canonical exponential volume form. We prove that exponential volumes are finite, and satisfies unfolding formulas generalizing Mirzakhani's recursions.

There is a generalization of these moduli spaces for any split simple real Lie group  $G$ , with canonical exponential volume forms. When the modular group of the surface  $P$  is finite, the exponential volumes are finite for any  $G$ . We show that when  $P$  are polygons, they can be used to define a commutative algebra of positive Whittaker functions for the group  $G$ .

We define the tropical limits of the exponential volumes.

The tropical limits for surfaces  $S$  with holes and  $SL(2)$  lead to the volumes studied by Kontsevich in his proof of Witten's conjecture.

The tropical limits of the algebra of positive Whittaker functions for any group  $G$  give the algebra of spherical functions for the group  $G(\mathbb{C})$ .

A part of the talk is based on the joint work with Zhe Sun.

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