IronMaskArithmetic : Comprehensive Verification of Arithmetic Masking Security

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CryptoExperts

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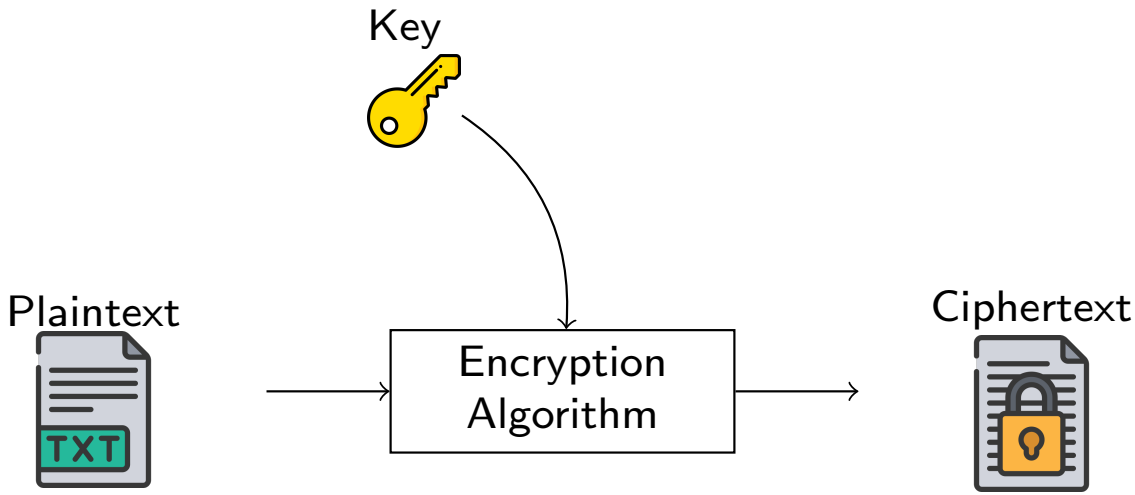
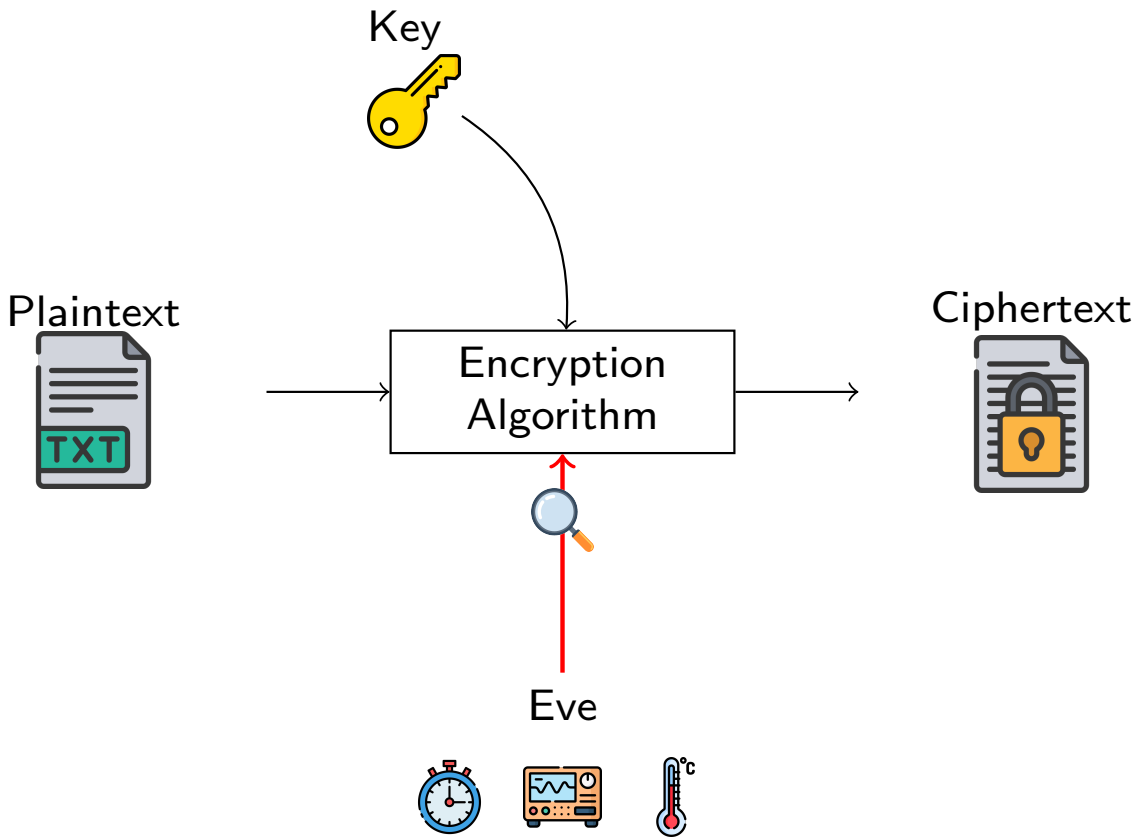
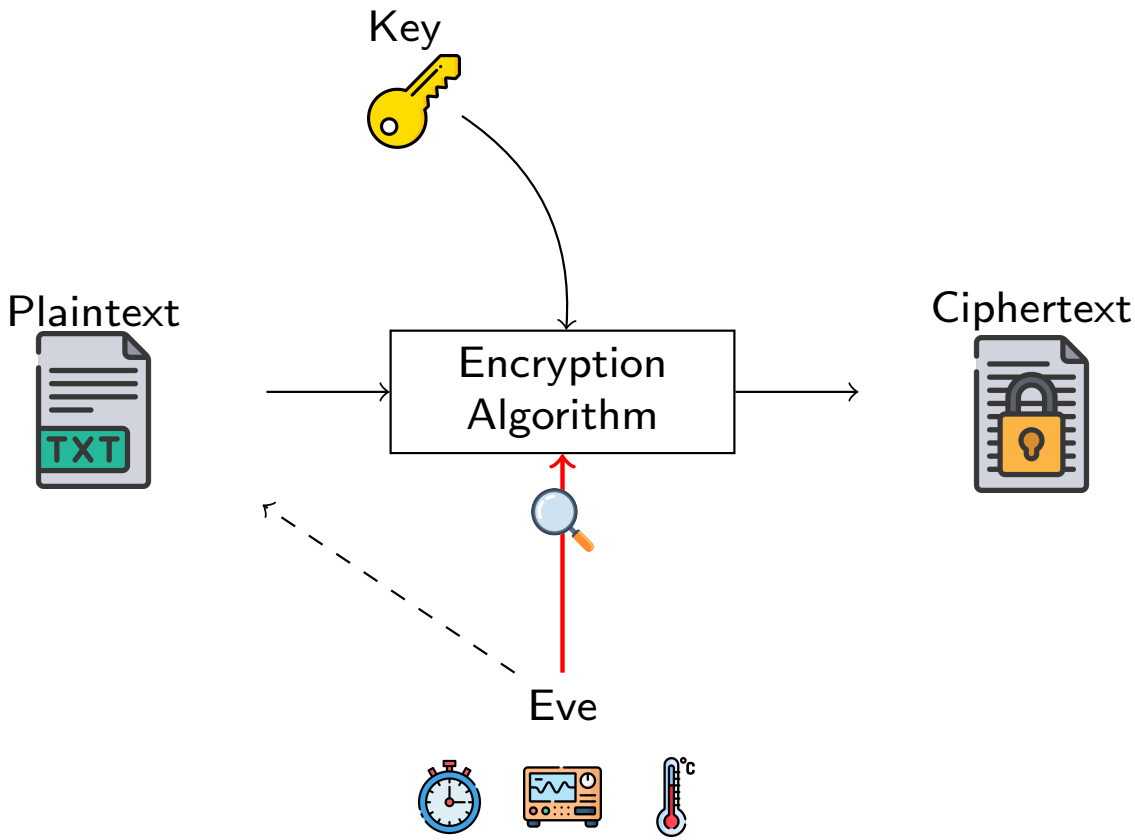
Side Channel Attacks



Side Channel Attacks



Side Channel Attacks



Masking countermeasure

- Replace a sensitive value $\mathbf{x}$ of the Algorithm by a tuple $(\mathbf{x}_1, \ldots, \mathbf{x}_n)$.
- Any set of $(n-1)$ shares is independent from $\mathbf{x}$.
- Arithmetic masking :

$$\mathbf{x} = \sum_{i=1}^{n} \mathbf{x}_i$$

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Gadget

- Operations on sensitives values $\Longrightarrow$ Operations on their respective shares.
- Masking with $n = 2$,

$$\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2$$
$$\mathbf{y} = \mathbf{y}_1 + \mathbf{y}_2$$

- Addition without masking

$$\mathbf{z} \leftarrow \mathbf{x} + \mathbf{y}$$

Gadget Addition

$$\mathbf{z}_1 \leftarrow \mathbf{x}_1 + \mathbf{y}_1$$
$$\mathbf{z}_2 \leftarrow \mathbf{x}_2 + \mathbf{y}_2$$
$$\mathbf{z} \leftarrow \mathbf{z}_1 + \mathbf{z}_2$$

Gadget

- Operations on sensitives values $\Longrightarrow$ Operations on their respective shares.
- Masking with $n = 2$,

$$\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2$$
$$\mathbf{y} = \mathbf{y}_1 + \mathbf{y}_2$$

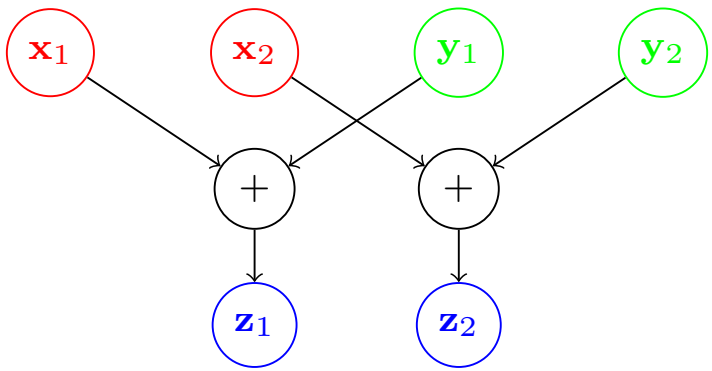
- Multiplication without masking

$$\mathbf{z} \leftarrow \mathbf{x}.\mathbf{y}$$

Gadget Multiplication

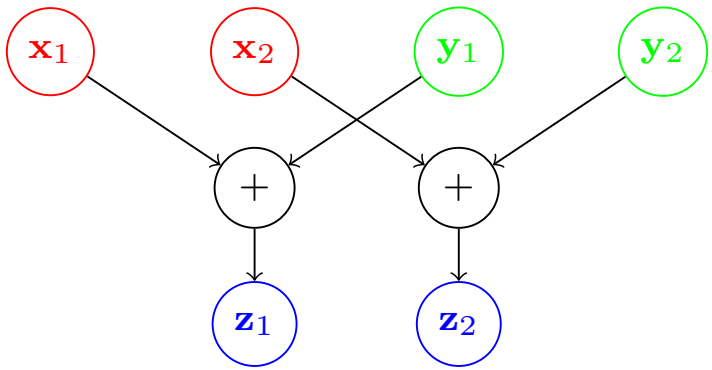
$$r \xleftarrow{\$} \mathbb{K}$$
$$\mathbf{z}_1 \leftarrow \mathbf{x}_1.\mathbf{y}_1 + r + \mathbf{x}_1.\mathbf{y}_2$$
$$\mathbf{z}_2 \leftarrow \mathbf{x}_2.\mathbf{y}_1 - r + \mathbf{x}_2.\mathbf{y}_2$$
$$\mathbf{z} \leftarrow \mathbf{z}_1 + \mathbf{z}_2$$

Circuits



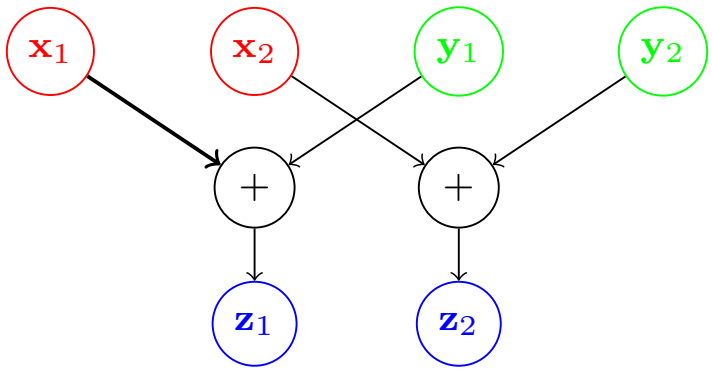
- **Circuits**
  - Directed acyclic graph.
  - Each vertex is an input/output share or arithmetic gate.
  - Each edge is a wire of the circuit.

t-probing model



- **t-Probing Model**
  - Up to t wires of the circuits leak its value.
  - Secure if any leakages sets is independant from the secrets.

t-probing model

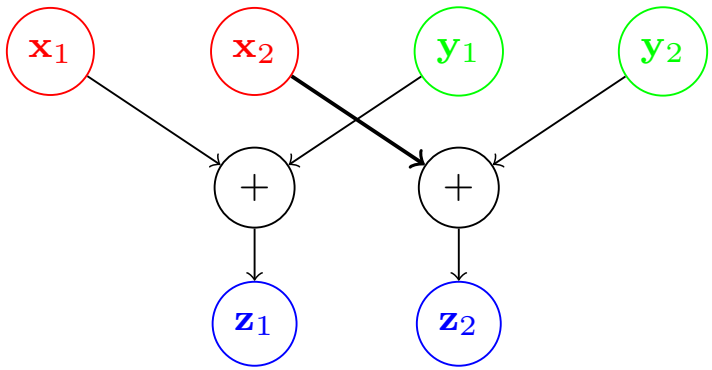


- **t-Probing Model**
  - Up to t wires of the circuits leak its value.
  - Secure if any leakages sets is independant from the secrets.

Example : 1-probing Secure ?

- Leaks the value of $\mathbf{x}_1$.
- Don't leak the value of $\mathbf{x}_2 \implies$ independent from $\mathbf{x}$.
- Don't leak shares values of $\mathbf{y} \implies$ independent from $\mathbf{y}$.

t-probing model

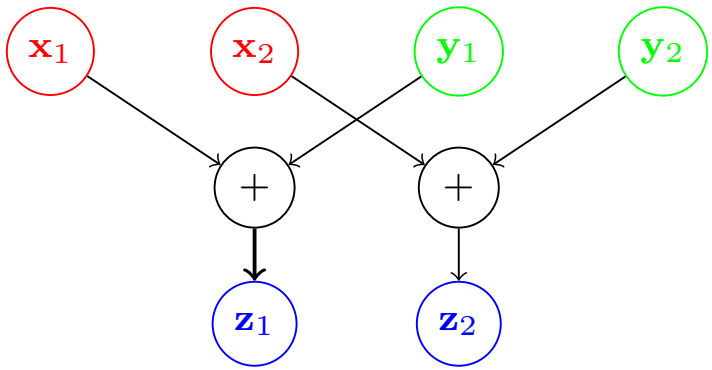


- **t-Probing Model**
  - Up to t wires of the circuits leak its value.
  - Secure if any leakages sets is independant from the secrets.

Example : 1-probing Secure ?

- Leaks the value of $\mathbf{x}_2$.
- Don't leak the value of $\mathbf{x}_1 \implies$ independent from $\mathbf{x}$.
- Don't leak shares values of $\mathbf{y} \implies$ independent from $\mathbf{y}$.

t-probing model

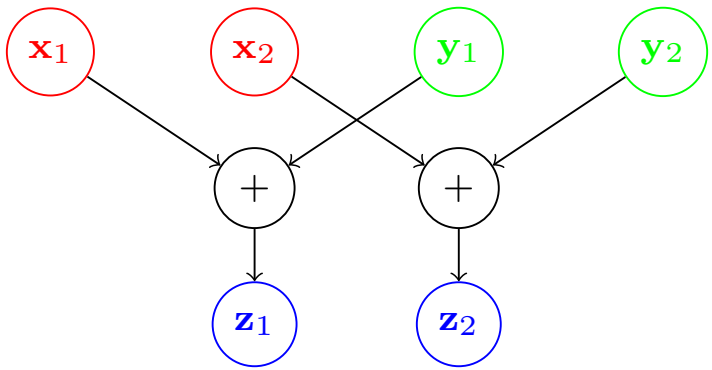


- **t-Probing Model**
  - Up to t wires of the circuits leak its value.
  - Secure if any leakages sets is independant from the secrets.

Example : 1-probing Secure ?

- Leaks the value of $\mathbf{x}_1$ and $\mathbf{y}_1$.
- Don't leak the value of $\mathbf{x}_2 \implies$ independent from $\mathbf{x}$.
- Don't leak the value of $\mathbf{y}_2 \implies$ independent from $\mathbf{y}$.

t-probing model

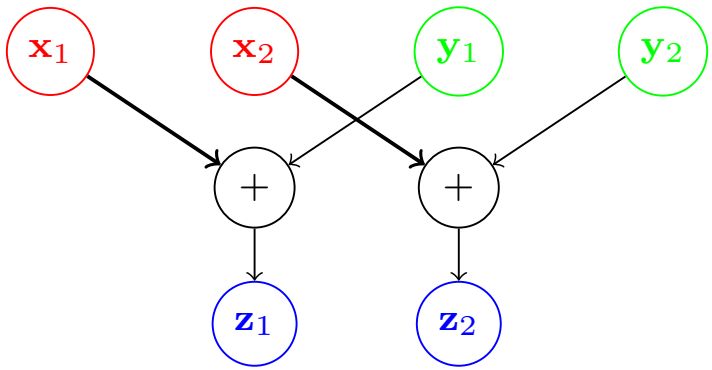


- **t-Probing Model**
  - Up to t wires of the circuits leak its value.
  - Secure if any leakages sets is independant from the secrets.

Example : 1-probing Secure ?

- All combination of 1 wire $\Longrightarrow$ independent from the secrets.
- The circuit is 1-probing secure.

t-probing model

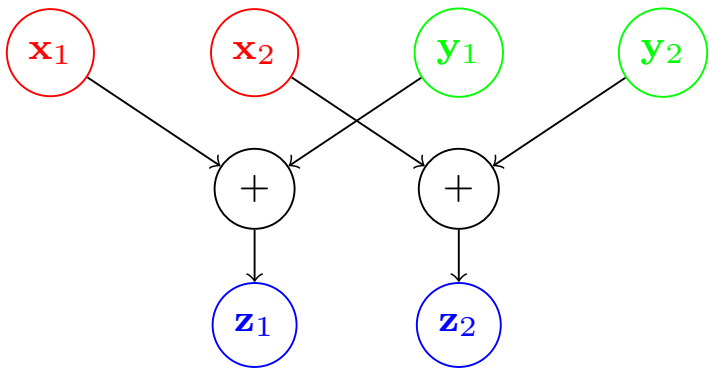


- **t-Probing Model**
  - Up to t wires of the circuits leak its value.
  - Secure if any leakages sets is independant from the secrets.

Example : 2-probing Secure ?

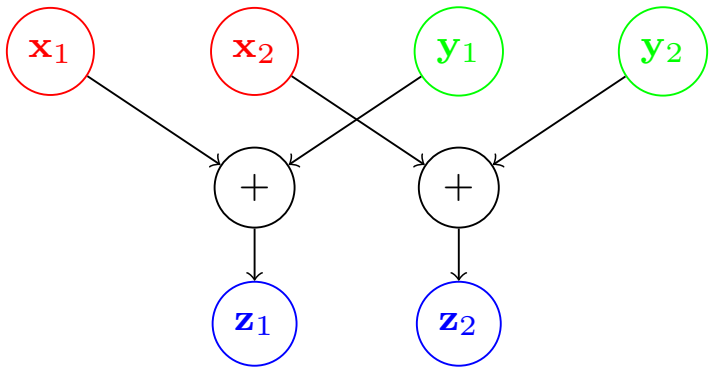
- Leaks the values of $\mathbf{x}_1$ and $\mathbf{x}_2$.
- $\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2 \implies$ we can find $\mathbf{x}$ from the values of $\mathbf{x}_1$ and $\mathbf{x}_2$.
- The circuit is not 2-probing secure.

(p,ε)-random-probing-model



- **(p, ε)-Random-Probing Model**
  - Every wires leak its value with a probability $p \in [0, 1]$.
  - Secure if there is a negligeable probability ($\leq \varepsilon$) to get information on the secrets.

(p,ε)-random-probing-model



- **(p, ε)-Random-Probing Model**
  - Every wires leak its value with a probability $p \in [0, 1]$.
  - Secure if there is a negligeable probability ($\leq \varepsilon$) to get information on the secrets.

$$f(p) := \sum_{i=1}^{|C|} c_i p^i (1-p)^{|C|-i} \leq \varepsilon$$

$c_i \implies$ Failure Tuples of i wires.

Verification tool

Random Probing Security :

$$f(p) = \sum_{i=1}^{|C|} c_i p^i (1-p)^{|C|-i}$$

$c_i \implies$ Failure Tuples of i wires.

- Verification exponential in the size of the circuit.
- Automatic Tools $\implies$ faster security verification.

IronMask

- IronMask : Formal tool to verify the security in the t-probing model and (p, ε)-random probing model.
- Circuits on the boolean field $\mathbb{F}_2$.
- Comprehensive : Compute each coefficient c_i.

IronMaskArithmetic

- IronMask : Formal tool to verify the security in the t-probing model and (p, ε)-random probing model.
- Circuits on the boolean field $\mathbb{F}_2$.
- Comprehensive : Compute each coefficient c_i.

- IronMaskArithmetic: An extension of the formal tool IronMask .
- Circuits on any arithmetic field $\mathbb{F}_q$ with q prime.

Parsing gadgets

```
#SHARES 2
#IN a b
#RANDOMS r01 rr01
#OUT e

c0 = a0 + r01
c1 = a1 + r01

d0 = b0 + rr01
d1 = b1 + rr01

e0 = c0 + d0
e1 = c1 + d1
```

(a) Parsing with IronMask

```
#SHARES 2
#IN a b
#RANDOMS r01 rr01
#OUT e
#CAR 3329

c0 = a0 + 2 r01
c1 = a1 + -2 r01

d0 = b0 + 3 rr01
d1 = b1 + -3 rr01

e0 = c0 + d0
e1 = c1 + d1
```

(b) Parsing with IronMaskArithmetic

IronMask - Exhaustive Approach

- To find failure tuples of size i :
  - Consider a tuple.
  - Determine if it is a failure tuple.
  - Reitere for all the tuples of size i.
- $\binom{|C|}{i}$ tuples to evaluate.

IronMaskArithmetic- Constructive Approach - Incompressible tuple

- q_1, q_2, q_3 leaked values of the circuit.
- (q_1, q_2, q_3) is a failure tuple.
- All sub tuples of (q_1, q_2, q_3) are not failure tuples.

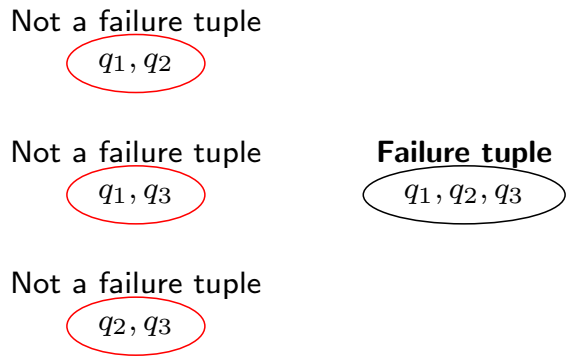
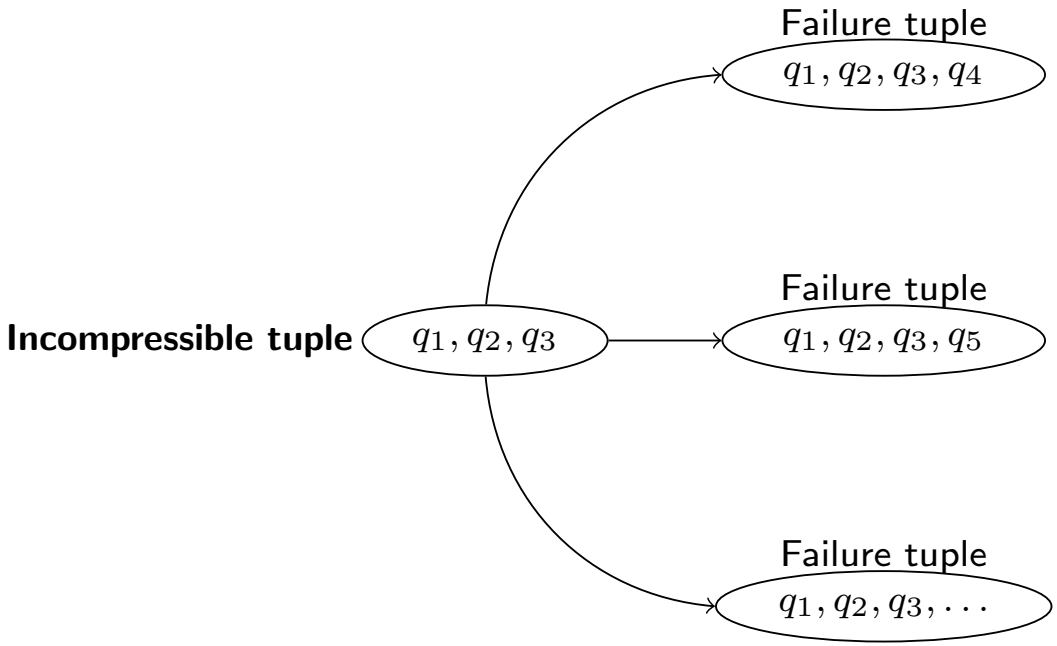


Figure: Illustration of an incompressible tuple

Constructive Approach - Incompressible tuple



Constructive approach - Example

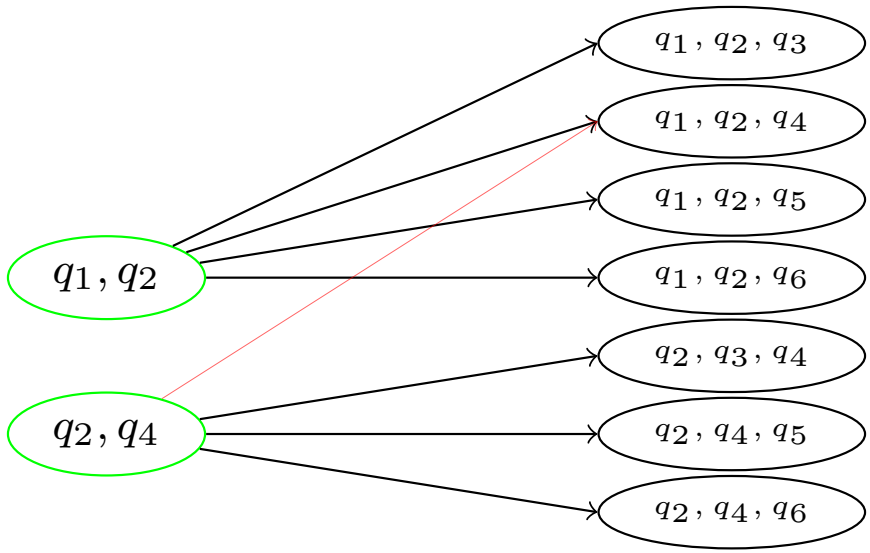
Example:

- 6 wires.
- 3 incompressible failure tuples : (q_1, q_2), (q_2, q_4) and (q_4, q_5, q_6)
- $c_3 =?$

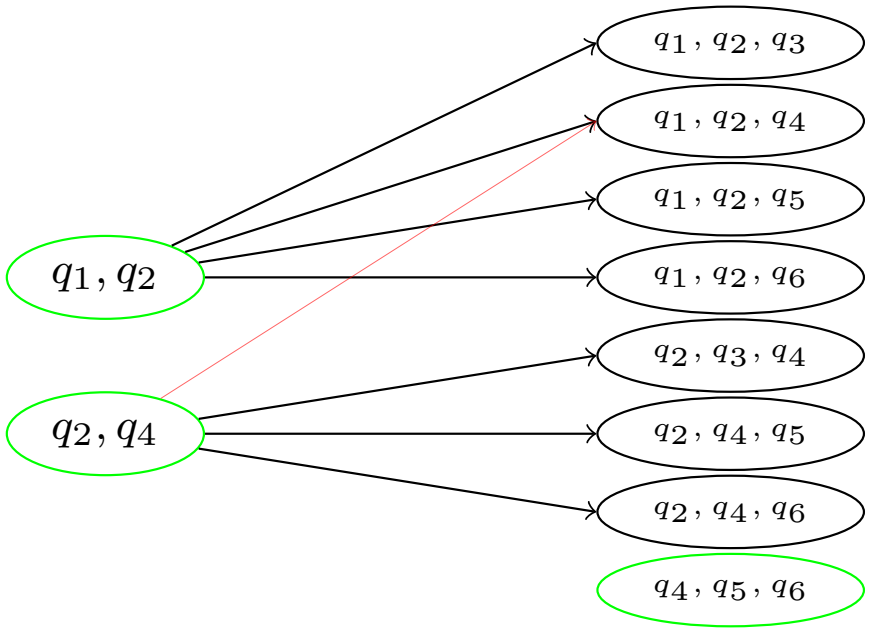
q_1, q_2

q_2, q_4

Constructive approach - Example



Constructive approach - Example



$$c_3 = 8$$

Timing performance

	IronMask (Car 2)	IronMaskArithmetic(Car 3329)
Add_3_shares RP -c 4	0s	0s
Add_7_shares SNI -t 6	8s	4s
Copy_6_shares RPC -c 5 -t 2	1min 20s	1 min 9s
Mult_6_shares NI -t 5	1s	4min 7s
Mult-ref_5_shares RP -c 4	1s	10s

Thanks for your attention

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