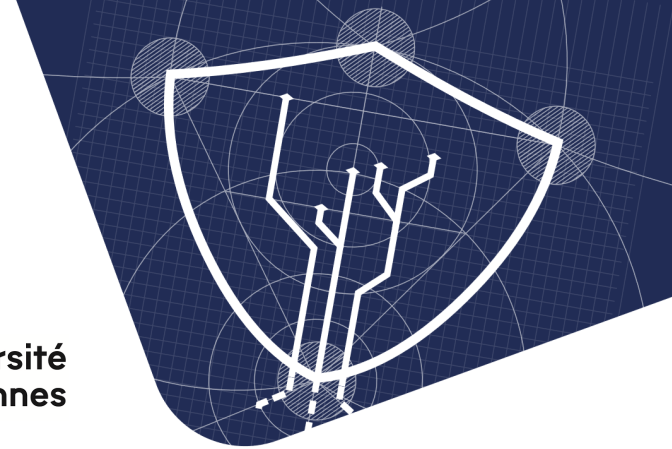

edsi
The Embedded Security Experts

KUDELSKI IOT THINGS

Université de Rennes

April 01, 2025

Post-Quantum Identity-Based Encryption from ML–KEM

Building an IBE scheme from existing blocks

Julien CAM

- **Identity-Based Encryption**
  - Introduction to asymmetric encryption
  - Introduction to Identity-Based Encryption
  - The definition of an IBE scheme
- ID–ML–KEM
- ID–ML–KEM_CKKS19
- Conclusion

Introduction to asymmetric encryption

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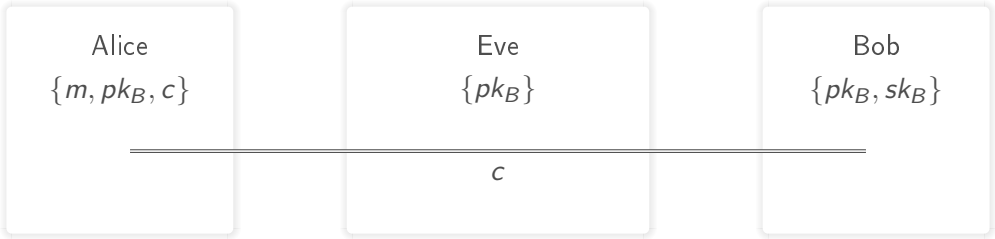

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Introduction to Identity-Based Encryption

- In a traditional asymmetric encryption scheme, a user chooses her secret key, and computes her public key from it with a one-way function.
- In an IBE scheme, a user chooses her public key, and has to find a secret key associated with it.
- Problem: If Alice can compute a secret key associated with identity `Alice` without prior knowledge, then Eve can do the same and obtain the secret key of Alice.
- Therefore, we need an authority to generate the secret keys for the users.
  This authority, called a PKG (*Private Key Generator*) has to be highly trusted.

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The definition of an IBE scheme

Public Recipient's secret PKG's secret

Definition (IBE scheme)

An IBE scheme is made of 4 polynomial-time algorithms:

- $\texttt{Setup}()$: Create the secret key SK and the public key PK of the PKG.
- $\texttt{Extract}(SK, pk)$: Use SK to compute a secret key sk associated with pk.
- $\texttt{Encrypt}(PK, pk, m)$: Encrypt m with the recipient's identity pk.
- $\texttt{Decrypt}(PK, sk, c)$: Use the secret key sk to decrypt the ciphertext c.

such that, for all pk, m and $(PK, SK) = \texttt{Setup}()$:

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- ID–ML–KEM
  - The cost of integrating a new scheme
  - An overview of ML–KEM
  - Inverting the LWE function
  - An overview of an ideal ID–ML–KEM
- ID–ML–KEM_CKKS19
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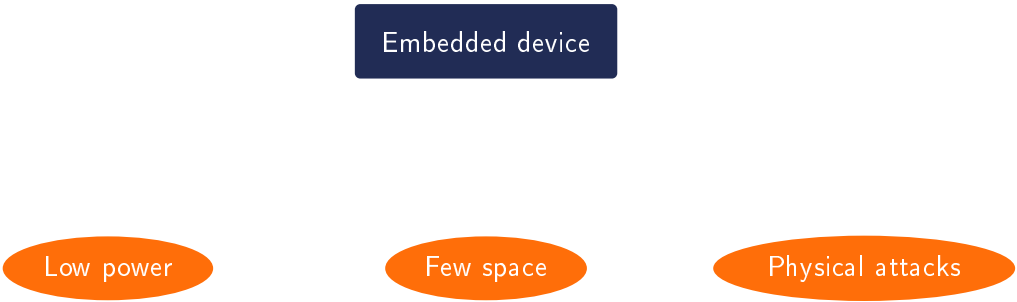

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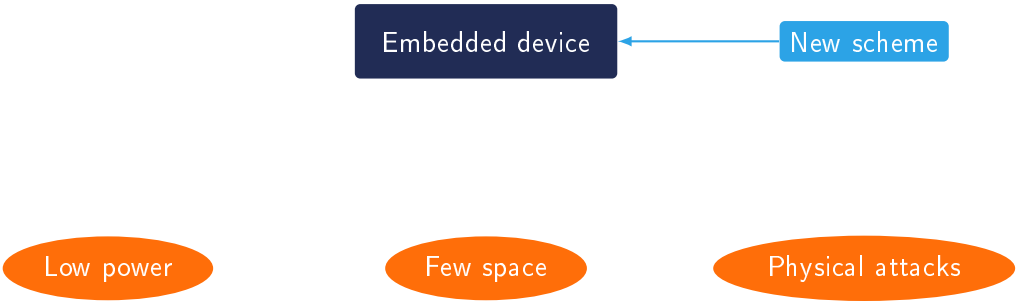

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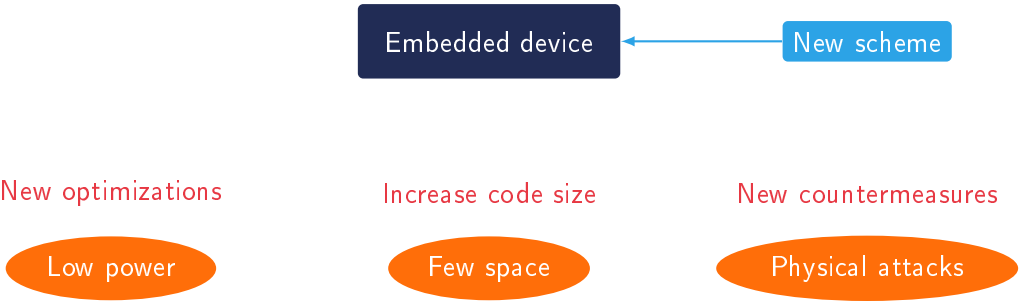

An overview of ML–KEM

KeyGen

Output	PK	$\in R_q^{k \times k}$
Output	pk	$\in R_q^k$
Output	sk	$\in R_q^k$
Such that	$pk - PK \cdot sk$ is small	

Encrypt

Input	PK	$\in R_q^{k \times k}$
Input	pk	$\in R_q^k$
Input	m	$\in R_q$
Output	$u = PK^T r + e_1$	$\in R_q^k$
Output	$v = pk^T r + e_2 + \lfloor q/2 \rceil m$	$\in R_q$
With	$r, e_1 \in R_q^k,\ e_2 \in R_q$ from CBD	

Decrypt

Input	sk	$\in R_q^k$
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Input	v	$\in R_q$
Output	$m' = \left\lfloor \frac{v - sk^T u}{q/2} \right\rceil$	$\in R_q$

Inverting the LWE function

- To generate keys with ML–KEM:
  1. Sample a uniform matrix PK
  2. Sample small random sk and e
  3. Compute $pk = PK \times sk + e$

- Finding e and sk from PK and pk is hard (LWE assumption)

- It is however possible knowing some *trapdoor* SK such that:
  - SK has small entries
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- So, an ID–ML–KEM requires a trapdoor for inverting the LWE function

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An overview of an ideal ID–ML–KEM

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Output $SK \in ?$

Output $PK \in R_q^{k \times k}$

Such that SK is a LWE-trapdoor for PK

Extract

Input $SK \in ?$

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The CKKS19 scheme

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Output	SK	$\in \mathbb{Z}^{(k+1)n \times (k+1)n}$
Output	PK	$\in R_q^k$
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Extract

Input	SK	$\in \mathbb{Z}^{(k+1)n \times (k+1)n}$
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Input	PK	$\in R_q^k$
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Output	$u = r \cdot PK + e_1$	$\in R_q^k$
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From CKKS19 to ID–ML–KEM_CKKS19

- The encryption and decryption follow a structure similar to ML–KEM
- The public keys are not of the form we wanted for ID–ML–KEM
- The parameters are not the ones used by ML–KEM

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Master public key PK	Matrix in $R_q^{2\times 2}$	Vector (h_1, h_2) in R_q^2
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Solution: define $PK = \begin{pmatrix} h_1 & h_2 \\ 0 & 0 \end{pmatrix}$ and $pk = \begin{pmatrix} H(\texttt{ID}) \\ 0 \end{pmatrix}$

From CKKS19 to ID–ML–KEM_CKKS19

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n	q	$\mathbb{Z}_q$ arithmetic	NTT	`BaseCaseMultiply`	Security
512	524 288	New but easy	???	???	82 bits
256	3 329	ML–KEM	$R_q \simeq (\mathbb{Z}_q^2)^{128}$	ML–KEM	70 bits
256	8 380 417	ML–DSA	$R_q \simeq (\mathbb{Z}_q)^{256}$	ML–DSA	41 bits
512	3 329	ML–KEM	$R_q \simeq (\mathbb{Z}_q^4)^{128}$	New	132 bits
1 024	8 380 417	ML–DSA	$R_q \simeq (\mathbb{Z}_q)^{1024}$	ML–DSA	141 bits

From CKKS19 to ID–ML–KEM_CKKS19

- The encryption and decryption follow a structure similar to ML–KEM
- The public keys can be adapted to match our expectations for ID–ML–KEM
- Some standard parameters can be found: $n = 1\,024$ and $q = 8\,380\,417$

The probability of a decryption failure

Definition

A decryption failure is a case where, for a legitimate master key pair (SK, PK), a message m and a legitimate user public key pk:

$$\texttt{Decrypt}(PK,\ \texttt{Extract}(SK, pk),\ \texttt{Encrypt}(PK, pk, m)) \neq m$$

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- e_k and sk are sampled from $\mathcal{N}(0, \sigma^2)$
- e_1, e_2 and r are sampled from Centered Binomial Distributions
- e_u and e_v are errors caused by compression and decompression

Conclusion

- With Identity-Based Encryption, fewer messages are required
- IBE avoids the need to verify certificates (no signature scheme required)
- A scheme based on ML–KEM allows:
  - Relying on some of the most trusted lattice-based assumptions
  - Using code that is already optimized, implemented and protected against physical attacks
- The CKKS19 scheme can be modified to reuse ML–KEM's `Encrypt` and `Decrypt`
- We can found parameters that allows to achieve:
  - Standard level of security (141 bits)
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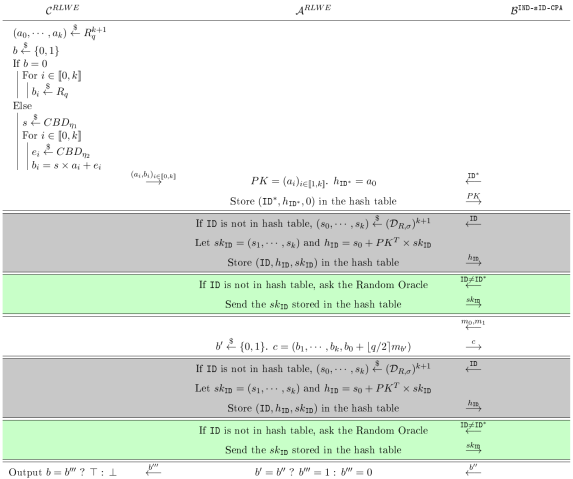
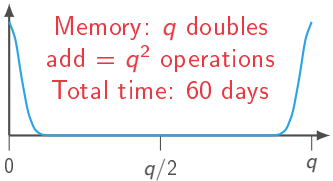
ID–ML–KEM_CKKS19 - Proof of security



Optimizing the computation of the failure probability



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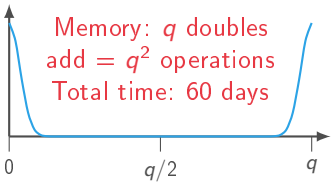
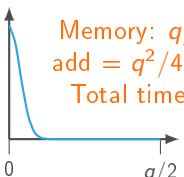
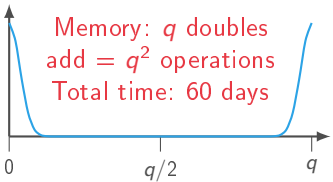
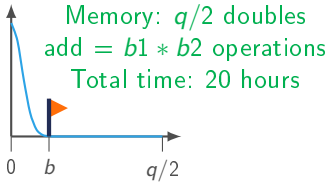
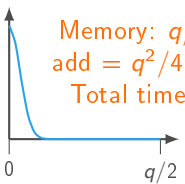




Optimizing the computation of the failure probability







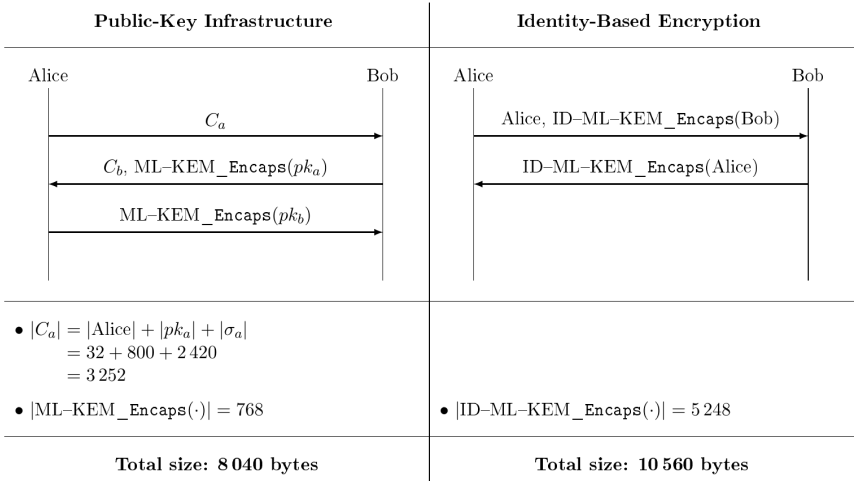
Compression parameters

d_u	23	20	19	18	17	19					18
d_v	4	23				10	5	3	2	1	3
$-\log_2(\mathbb{P}(\text{fail}))$	∞	∞	388	103	20	388	348	228	104	0	61
Size (in Bytes)	6 400	8 064	7 808	7 552	7 296	6 144	5 504	5 248	5 120	4 992	4 992

ID–ML–KEM_CKKS19 - Sizes

Element	Nature	Size (bytes)		
Master secret key	$\mathbb{Z}^{(k+1)n\times(k+1)n}$	$\texttt{sizeof}(\mathbb{Z})\times(k+1)^2\times n^2$	$=$	18 874 368
Master SK seed	$\mathbb{B}^{\lambda/8}$	$\lambda\,/\,8$	$=$	32
Master SK GSO	$\mathbb{R}^{(k+1)n\times(k+1)n}$	$\texttt{sizeof}(\mathbb{R})\times(k+1)^2\times n^2$	$=$	75 497 472
Master public key	T_q^k	$\lceil\log_2(q)\rceil\times n\times k\,/\,8$	$=$	5 888
User secret key	T_q^k	$\lceil\log_2(q)\rceil\times n\times k\,/\,8$	$=$	5 888
User public key	R_q	$\lceil\log_2(q)\rceil\times n\,/\,8$	$=$	2 944
User identity	$\mathbb{B}^{\lambda/8}$	$\lambda\,/\,8$	$=$	32
Ciphertext	$\mathbb{B}^{n(d_uk+d_v)/8}$	$n\times(d_u\times k+d_v)\,/\,8$	$=$	5 248

The communications



APPENDIX

All algorithms

Setup()

Output: Master secret key $SK \in R_q^{3\times 3}$
Output: Master public key $PK \in R_q^2$
1: Sample from $[\mathcal{D}_{\mathbb{Z},\sigma_j}]^n$ the coefficients $f_{i,j}$ of $S = \begin{pmatrix} f_{1,1} & f_{1,2} \\ f_{2,1} & f_{2,2} \\ f_{3,1} & f_{3,2} \end{pmatrix}$ ▷ S_i is S without its i^{th} row
2: $d = (\det(S_1), -\det(S_2), \det(S_3))$
3: Find small F_1, F_2 and F_3 in R such that: $\sum_{i=1}^{3} d_i \cdot F_i = q$ ▷ The MNTRU equation
4: **return** $SK = \begin{pmatrix} f_{1,1} & f_{1,2} & F_1 \\ f_{2,1} & f_{2,2} & F_2 \\ f_{3,1} & f_{3,2} & F_3 \end{pmatrix}$ and $PK = (d_2 d_1^{-1}, d_3 d_1^{-1})$

Extract(SK, pk)

Input: Master secret key $SK \in R_q^{3\times 3}$
Input: User's public key $pk \in R_q$
Output: User's secret key $sk \in R_q^2$
1: **if** sk has already been computed **then**
2: **return** sk ▷ Outputting two sk for the same pk breaks security
3: **else**
4: $\begin{pmatrix} s_0 \\ sk_1 \\ sk_2 \end{pmatrix} = \begin{pmatrix} pk \\ 0 \\ 0 \end{pmatrix} - \texttt{GaussianSampler}\left(SK, \sigma, \begin{pmatrix} pk \\ 0 \\ 0 \end{pmatrix}\right)$
5: **return** $sk = (sk_1, sk_2)$
6: **end if**

Encrypt(PK, pk, m)

Input: Master public key $PK \in R_q^2$
Input: User's public key $pk \in R_q$
Input: Message $m \in R_q$
Output: Ciphertext $c \in R_q^2 \times R_q$
1: Define $A = \begin{pmatrix} PK_1 & PK_2 \\ 0 & 0 \end{pmatrix}$ and $t = \begin{pmatrix} pk \\ 0 \end{pmatrix}$
2: Sample $r \in R_q^2$ from CBD_{η_1}
3: Sample $e_1 \in R_q^2$ from CBD_{η_2}
4: Sample $e_2 \in R_q$ from CBD_{η_2}
5: **return** $(A^T r + e_1, t^T r + e_2 + \lfloor q/2 \rceil m)$

Decrypt(sk, c)

Input: User's secret key $sk \in R_q^2$
Input: Ciphertext $c \in R_q^2 \times R_q$
Output: Message $m' \in R_q$
1: $w = c_2 - sk^T c_1$
2: **return** $m' = \lfloor 2w/q \rceil$

APPENDIX

The BKZ basis reduction algorithm and the security of CKKS19

- The most efficient way to solve lattice problems is to "reduce" a basis with BKZ.
- To do that, we reduce the basis by blocks of size β.
  To summarize, BKZ solves SVP in each sublattice generated by β vectors.
- Increasing β means:
  - More information at each iteration $\Rightarrow$ better output
  - Solving SVP in higher dimension $\Rightarrow$ higher running time.

- Breaking CKKS19 requires to solve a lattice problem with a certain quality.
- Reaching this quality requires at least a block size β.
- Using such a block size requires a running-time of at least 2^{λ}.
- So CKKS19 provides a λ-bit security.

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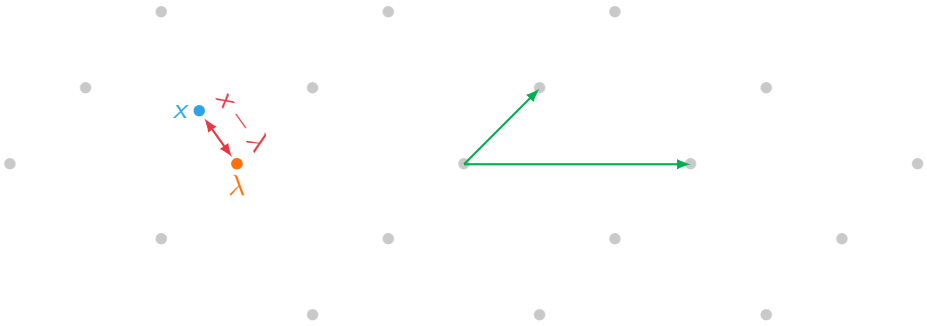
Structured variants of LWE

Plain	Module	Ring
$\begin{pmatrix} 8 & 6 & -1 & 3 \\ -1 & 5 & 7 & -4 \\ 4 & -1 & 9 & 1 \\ -1 & -2 & 6 & 0 \end{pmatrix}$	$\begin{pmatrix} 8 & 6 & -1 & 3 \\ -6 & 8 & -3 & -1 \\ 4 & -1 & 9 & 1 \\ 1 & 4 & -1 & 9 \end{pmatrix}$	$\begin{pmatrix} 8 & 6 & -1 & 3 \\ -3 & 8 & 6 & -1 \\ 1 & -3 & 8 & 6 \\ -6 & 1 & -3 & 8 \end{pmatrix}$
—	$\begin{pmatrix} \times_{6X+8} & \times_{3X-1} \\ \times_{-X+4} & \times_{X+9} \end{pmatrix}$	$\times_{3X^3-X^2+6X+8}$

Deterministic - Output the closest lattice element



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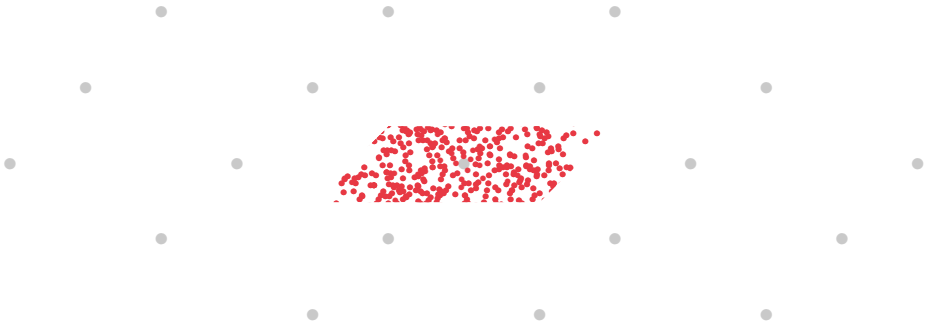

Randomized - Output a relatively close lattice element