A framework for structured linearizations of matrix polynomials in various bases

Leonardo Robol, Raf Vandebril, Paul Van Dooren

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We present a framework for the construction of linearizations for scalar and matrix polynomials based on dual bases which, in the case of orthogonal polynomials, can be described by the associated recurrence relations. The framework provides an extension of the classical linearization theory for polynomials expressed in non-monomial bases and allows to represent polynomials expressed in product families, that is as a linear combination of elements of the form $\phi_i(\lambda)\psi_j(\lambda)$, where $\{\phi_i(\lambda)\}$ and $\{\psi_j(\lambda)\}$ can either be polynomial bases or polynomial families which satisfy some mild assumptions.

We show that this general construction can be used for many different purposes. Among them, we show how to linearize sums of polynomials and rational functions expressed in different bases. As an example, this allows to look for intersections of functions interpolated on different nodes without converting them to the same basis.

We then provide some constructions for structured linearizations for $\star$-even and $\star$-palindromic matrix polynomials. The extensions of these constructions to $\star$-odd and $\star$-antipalindromic of odd degree is discussed and follows immediately from the previous results.