Improved generators for quasiseparable matrices

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The class of quasiseparable matrices is defined by a pair of bounds, called the quasiseparable orders, on the ranks of the sub-matrices entirely located in their strictly lower and upper triangular parts. These arise naturally in applications, as e.g. the inverse of band matrices, and are widely used for they admit structured representations allowing to compute with them in time linear in the dimension. We show, in this paper, the connection between the notion of quasiseparability and the rank profile matrix invariant, presented in [Dumas et al. ISSAC'15]. This allows us to propose an algorithm computing the quasiseparable orders (l, u) in time $O(n^2 s^{w-2})$ where $s = \max(l, u)$ and w is the exponent of matrix multiplication. We then present two new structured representations, a binary tree of PLUQ decompositions, and the Bruhat generator using respectively $O(ns \log(n/s))$ and $O(ns)$ field elements instead of $O(ns^2)$ for the classical generator. We present algorithms computing these representations in time $O(n^2 s^{w-2})$. These representations allow a matrix-vector product in time linear in the size of their representation. Lastly we show how to multiply two such structured matrices in time $O(n^2 s^{w-2})$.