A Superfast Randomized Algorithm to Decompose Binary Forms

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Symmetric Tensor Decomposition is a major problem that arises in areas such as signal processing, statistics, data analysis and computational neuroscience. It is equivalent to write a homogeneous polynomial in n variables of degree D as a sum of D-th powers of linear forms, using the minimal number of summands. This minimal number is called the *rank* of the polynomial/tensor.

We consider the decomposition of binary forms; this corresponds to the decomposition of symmetric tensors of dimension 2 and order D. This problem is historically linked to Hankel matrices. In the 19th century, Sylvester related the decomposition of binary forms to the kernel of a Hankel matrix. Recently, this relation was generalized to arbitrary symmetric tensors.

We combine Sylvester's approach with results from *structured linear algebra* and techniques from *linear recurrent sequences*. This combination provides simple constructive proofs of known properties and leads to an algorithm with a *softly linear* arithmetic complexity bound. The key ingredient is the efficient computation of the kernel of a set of Hankel matrices.To the best of our knowledge, the previously known algorithms have quadratic complexity bounds. We compute a symbolic minimal decomposition in $O(M(D)\log(D))$ arithmetic operations, where $M(D)$ is the complexity of multiplying two polynomials of degree D. We can approximate the terms of the decomposition, with an error of $2^{-\varepsilon}$, in $O\big(D\log^2(D)\big(\log^2(D)+\log(\epsilon)\big)\big)$ arithmetic operations. To bound the size of the representation of the coefficients in the decomposition, we bound the algebraic degree of the problem by the rank. When the input polynomial has integer coefficients our algorithm performs, up to poly-logarithmic factors, $\tilde{O}(D\ell + D^4 + D^3\tau)$ bit operations, where τ is the maximum bitsize of the coefficients and $2^{-\ell}$ is the relative error of the terms in the decomposition.

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