# Computation of centrality and communicability indices using generalized matrix functions

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Network models are nowadays ubiquitous in the natural, information, social, and engineering sciences.

It is well known that real-world networks are characterized by some "hidden" structural properties that make them very different from both regular graphs and completely random graphs [4]. Real networks frequently exhibit a highly skewed degree distribution, small diameter, and high clustering coefficient. Moreover, highly optimized undirected networks usually have a large *total communicability* [3], meaning that they are good at exchanging information.

In this talk, we describe how to extend the concept of total communicability to the case of directed graphs (see [1]). We further provide guidelines to efficiently select a small number of modifications to the connections in the network aimed to tune the new indices. We show that one of the most effective techniques is based on the concept of *generalized matrix functions* [5, 2], whose definition is based on replacing the Jordan canonical form of $A$ with its compact singular value decomposition, and evaluating the function at the positive singular values of $A$, if defined. We describe several computational approaches based on variants of Golub–Kahan bidiagonalization algorithm to compute or estimate bilinear forms involving generalized matrix functions.

Extensive numerical studies are presented to assess the effectiveness of the proposed methods.

## References

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