Structured Matrix Days

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https://indico.math.cnrs.fr/event/1185/

Computation of centrality and communicability indices using generalized matrix functions

by Francesca Arrigo

Network models are nowadays ubiquitous in the natural, information, social, and engineering sciences.

It is well known that real-world networks are characterized by some "hidden" structural properties that make them very different from both regular graphs and completely random graphs [4]. Real networks frequently exhibit a highly skewed degree distribution, small diameter, and high clustering coefficient. Moreover, highly optimized undirected networks usually have a large *total communicability* [3], meaning that they are good at exchanging information.

In this talk, we describe how to extend the concept of total communicability to the case of directed graphs (see [1]). We further provide guidelines to efficiently select a small number of modifications to the connections in the network aimed to tune the new indices. We show that one of the most effective techniques is based on the concept of *generalized matrix functions* [5, 2], whose definition is based on replacing the Jordan canonical form of A with its compact singular value decomposition, and evaluating the function at the positive singular values of A, if defined. We describe several computational approaches based on variants of Golub–Kahan bidiagonalization algorithm to compute or estimate bilinear forms involving generalized matrix functions.

Extensive numerical studies are presented to assess the effectiveness of the proposed methods.

References

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A Superfast Randomized Algorithm to Decompose Binary Forms

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Symmetric Tensor Decomposition is a major problem that arises in areas such as signal processing, statistics, data analysis and computational neuroscience. It is equivalent to write a homogeneous polynomial in n variables of degree D as a sum of D-th powers of linear forms, using the minimal number of summands. This minimal number is called the *rank* of the polynomial/tensor.

We consider the decomposition of binary forms; this corresponds to the decomposition of symmetric tensors of dimension 2 and order D. This problem is historically linked to Hankel matrices. In the 19th century, Sylvester related the decomposition of binary forms to the kernel of a Hankel matrix. Recently, this relation was generalized to arbitrary symmetric tensors.

We combine Sylvester's approach with results from *structured linear algebra* and techniques from *linear recurrent sequences*. This combination provides simple constructive proofs of known properties and leads to an algorithm with a *softly linear* arithmetic complexity bound. The key ingredient is the efficient computation of the kernel of a set of Hankel matrices.To the best of our knowledge, the previously known algorithms have quadratic complexity bounds. We compute a symbolic minimal decomposition in $O(M(D)\log(D))$ arithmetic operations, where $M(D)$ is the complexity of multiplying two polynomials of degree D. We can approximate the terms of the decomposition, with an error of $2^{-\varepsilon}$, in $O\big(D\log^2(D)\big(\log^2(D)+\log(\epsilon)\big)\big)$ arithmetic operations. To bound the size of the representation of the coefficients in the decomposition, we bound the algebraic degree of the problem by the rank. When the input polynomial has integer coefficients our algorithm performs, up to poly-logarithmic factors, $\tilde{O}(D\ell + D^4 + D^3\tau)$ bit operations, where τ is the maximum bitsize of the coefficients and $2^{-\ell}$ is the relative error of the terms in the decomposition.

Joint work with Jean-Charles Faugère, Ludovic Perret, and Elias Tsigaridas.

Algorithme rapide pour la résolution d'un système de Toeplitz bande avec une application en restauration d'images

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Résumé: *Nous avons introduit un algorithme rapide pour la résolution d'un système linéaire de Toeplitz bande [1]. Cette approche est basée sur l'extension de la matrice donnée par plusieurs lignes en dessus, de plusieurs colonnes à droite et d'attribuer des zéros et des constantes non nulles dans chacune de ces lignes et de ces colonnes de telle façon que la matrice augmentée a la structure d'une matrice triangulaire inférieure de Toeplitz. Dans ce travail, nous nous intéressons à la résolution d'un système de Toeplitz bande par blocs de Toeplitz bandes. Ceci étant primordiale pour établir la connexion de nos algorithmes à des applications en restauration d'images. Un algorithme rapide basé sur [1] a été introduit. La stabilité de l'algorithme a été discutée et son efficacité a été aussi justifiée.*

Mots-clés : *Matrices de Toeplitz, matrices bandes, matrice triangulaire inférieure, étude d'erreur, stabilité numérique, réduction cyclique, restauration d'images.*

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Pathways to the Toeplitz matrix exponential

Robert Luce (École Polytechnique Fédérale Lausanne)

An important structural feature of Toeplitz matrices is that they have low rank representations as solutions of certain matrix Stein or Sylvester equations. Using these low rank properties we present techniques for the computation of the full matrix exponential of a Toeplitz matrix in quadratic complexity, i.e., with run time and storage proportional to the output. An interesting application for our algorithm is the solution of an PIDE arising from option pricing of an asset in Merton's jump diffusion model.

Improved generators for quasiseparable matrices

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The class of quasiseparable matrices is defined by a pair of bounds, called the quasiseparable orders, on the ranks of the sub-matrices entirely located in their strictly lower and upper triangular parts. These arise naturally in applications, as e.g. the inverse of band matrices, and are widely used for they admit structured representations allowing to compute with them in time linear in the dimension. We show, in this paper, the connection between the notion of quasiseparability and the rank profile matrix invariant, presented in [Dumas et al. ISSAC'15]. This allows us to propose an algorithm computing the quasiseparable orders (l, u) in time $O(n^2 s^{w-2})$ where $s = \max(l, u)$ and w is the exponent of matrix multiplication. We then present two new structured representations, a binary tree of PLUQ decompositions, and the Bruhat generator using respectively $O(ns \log(n/s))$ and $O(ns)$ field elements instead of $O(ns^2)$ for the classical generator. We present algorithms computing these representations in time $O(n^2 s^{w-2})$. These representations allow a matrix-vector product in time linear in the size of their representation. Lastly we show how to multiply two such structured matrices in time $O(n^2 s^{w-2})$.

A framework for structured linearizations of matrix polynomials in various bases

Leonardo Robol, Raf Vandebril, Paul Van Dooren

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We present a framework for the construction of linearizations for scalar and matrix polynomials based on dual bases which, in the case of orthogonal polynomials, can be described by the associated recurrence relations. The framework provides an extension of the classical linearization theory for polynomials expressed in non-monomial bases and allows to represent polynomials expressed in product families, that is as a linear combination of elements of the form $\phi_i(\lambda)\psi_j(\lambda)$, where $\{\phi_i(\lambda)\}$ and $\{\psi_j(\lambda)\}$ can either be polynomial bases or polynomial families which satisfy some mild assumptions.

We show that this general construction can be used for many different purposes. Among them, we show how to linearize sums of polynomials and rational functions expressed in different bases. As an example, this allows to look for intersections of functions interpolated on different nodes without converting them to the same basis.

We then provide some constructions for structured linearizations for $\star$-even and $\star$-palindromic matrix polynomials. The extensions of these constructions to $\star$-odd and $\star$-antipalindromic of odd degree is discussed and follows immediately from the previous results.

Newton operator and Shub-Smale alpha-theory in the singular case.

Jean-Claude Yakoubsohn (Institut de Mathématiques de Toulouse)

This work is in collaboration with Marc Giusti from the LIX. The Newton operator is well known in the regular case. We show how to construct an operator which allows to approximate quadratically an isolated root of a polynomial system. We give also the radius of a ball where the quadratic convergence holds (gamma theorem) and a condition which proves the existence of a singular root (alpha-theorem).