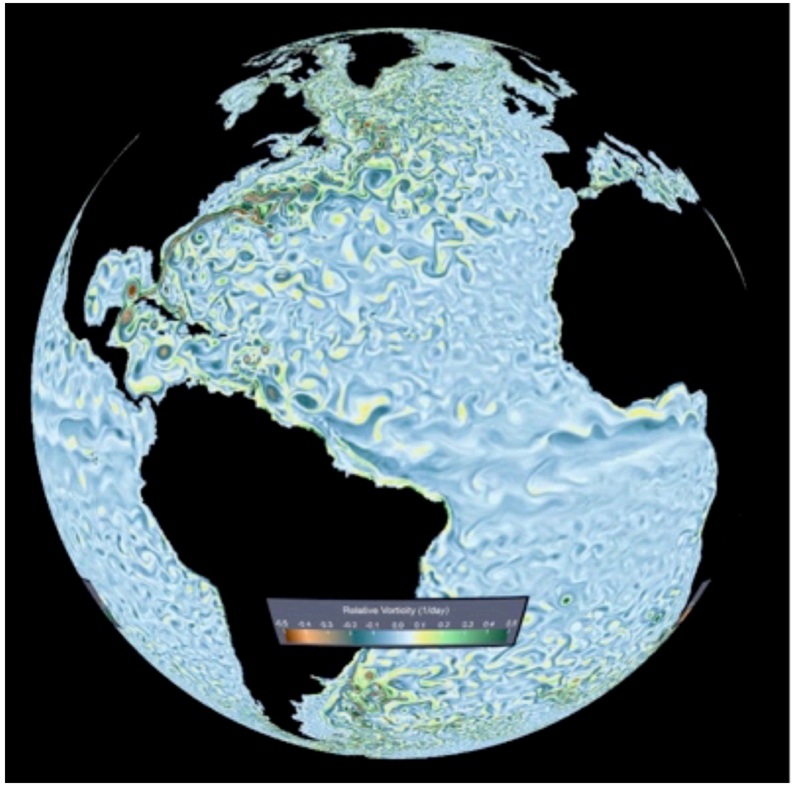
Exploring small scales of turbulence

B. Dubrulle
CEA Saclay/SPEC/SPHYNX
CNRS UMR 3680

Team BANG: A. Barlet, P. Bragança, A. Cheminet, F. Champagnat, P. Cornic, Ch. Cuvier, F. Daviaud, B. Dubrulle, J-M. Foucault, A. Harikrishnan, J-P. Laval, J. LeBris, B. Leclaire, B. Musci, V. Padilla, C. Wiertel

A. Barral, G. Costa, A. Lopez, Q. Pikeroen, C. Campolina, A. Mailybaev

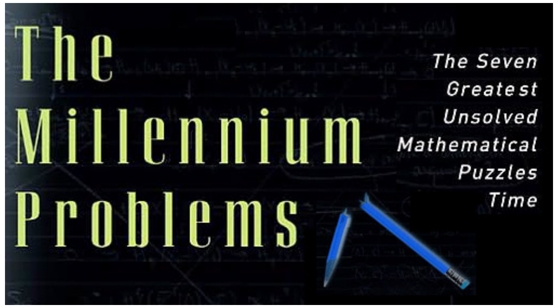
Turbulence



Navier-Stokes Equation

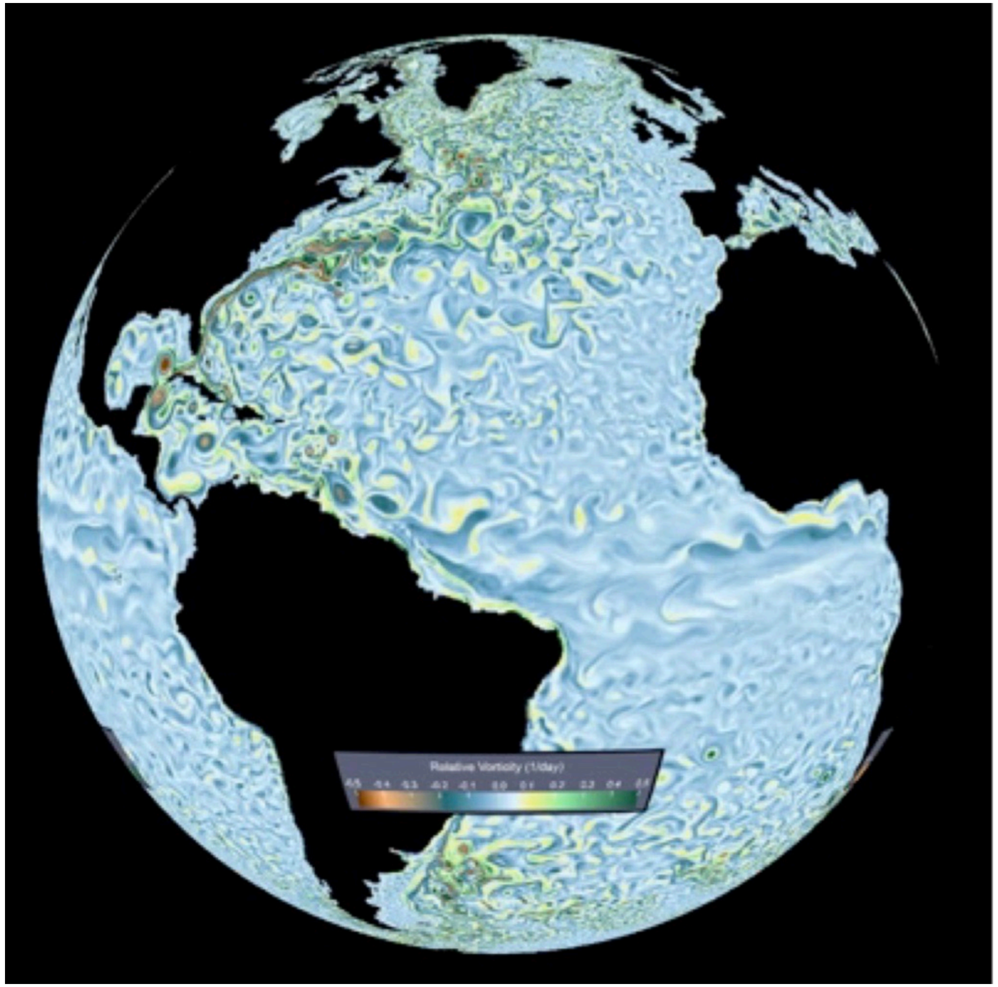
$$\vec{\nabla} \cdot \vec{u} = \vec{0}$$

$$\partial_t \vec{u} + (\vec{u} \cdot \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p + \nu \Delta \vec{u}$$



Are equations well posed? Can they blow-up in finite time?

Turbulence



Kolmogorov Universality

Navier-Stokes Equation

$$\vec{\nabla} \cdot \vec{u} = \vec{0}$$

$$\partial_t \vec{u} + (\vec{u} \cdot \vec{\nabla}) \vec{u} = -\frac{1}{\rho} \vec{\nabla} p + \nu \Delta \vec{u}$$

Homogeneity

$$\delta u = u(x + \ell) - u(x)$$

Karman-Howarth-Monin

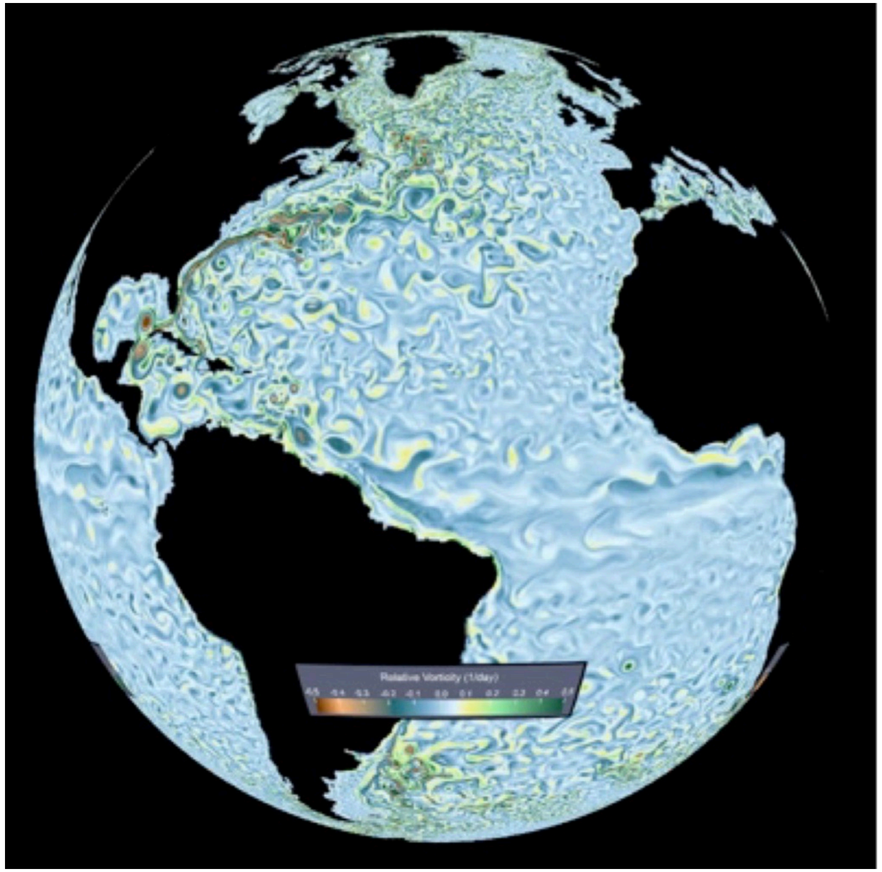
$$\frac{1}{4}\partial_t E + \epsilon = -\frac{1}{4}\nabla_\ell < (\delta u)^3 > + \frac{1}{2}\nu \Delta_\ell E + P_{inj}$$

$$\delta u = (\epsilon \ell)^{1/3} \qquad \eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

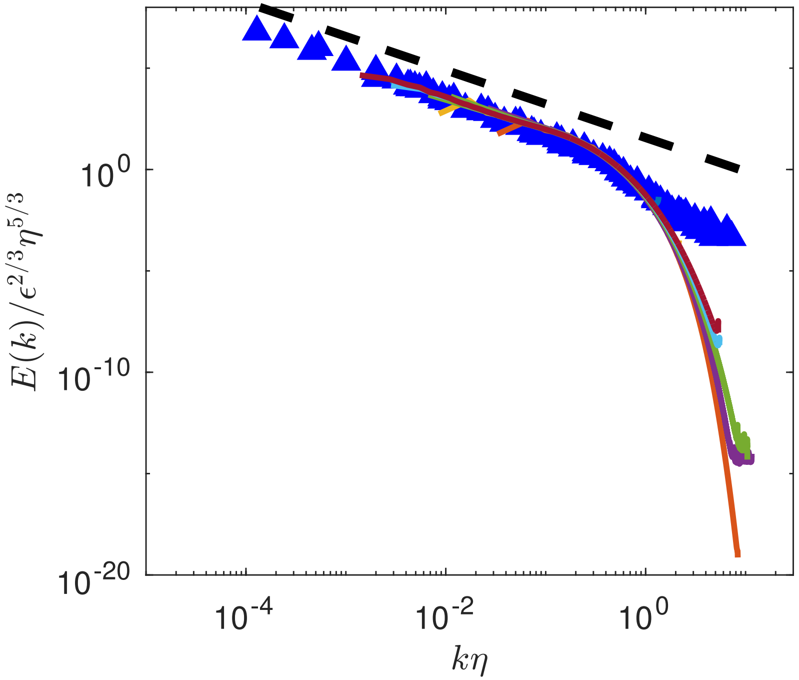
Scale invariance

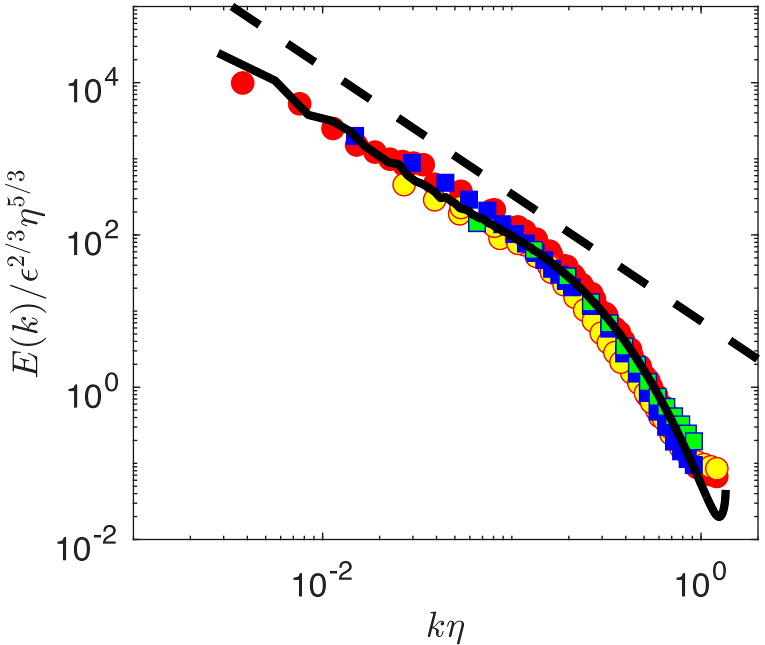
$$\frac{E(k)}{\epsilon^{2/3}\eta^{-5/3}} = C_K \, (k\eta)^{-5/3}$$

Test of K41 universality

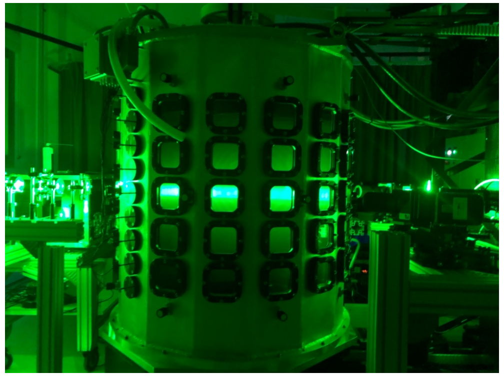


Ocean



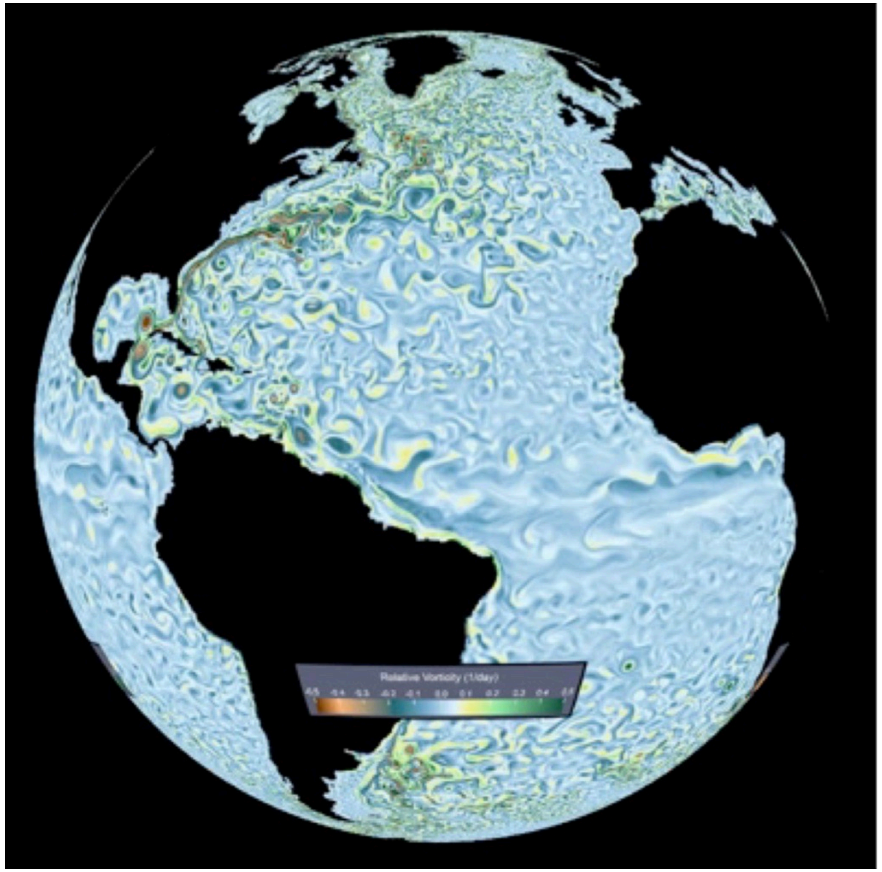


DNS

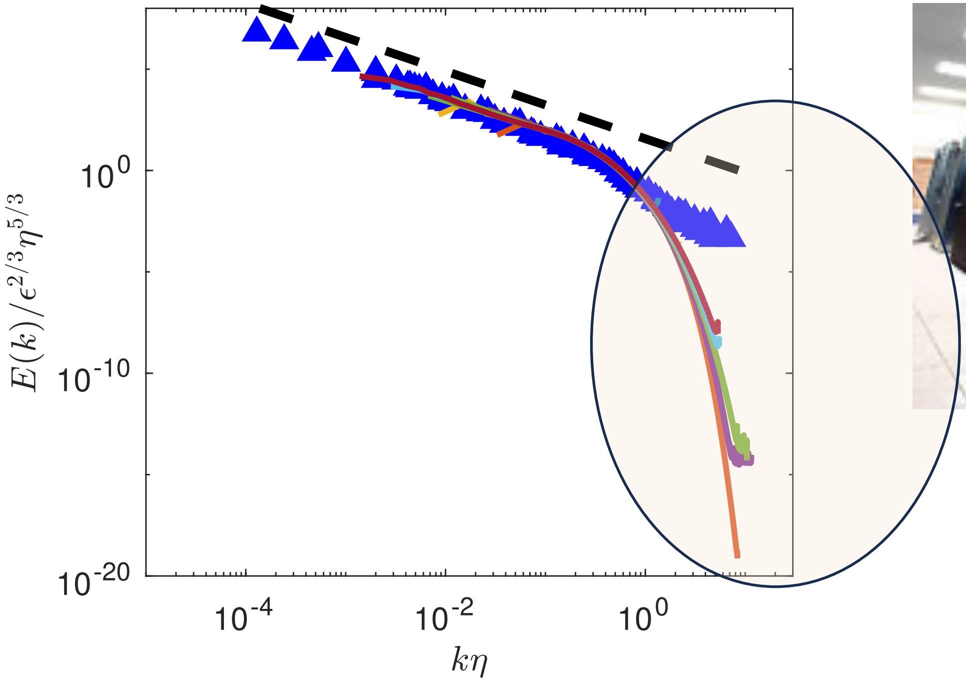


Experiments

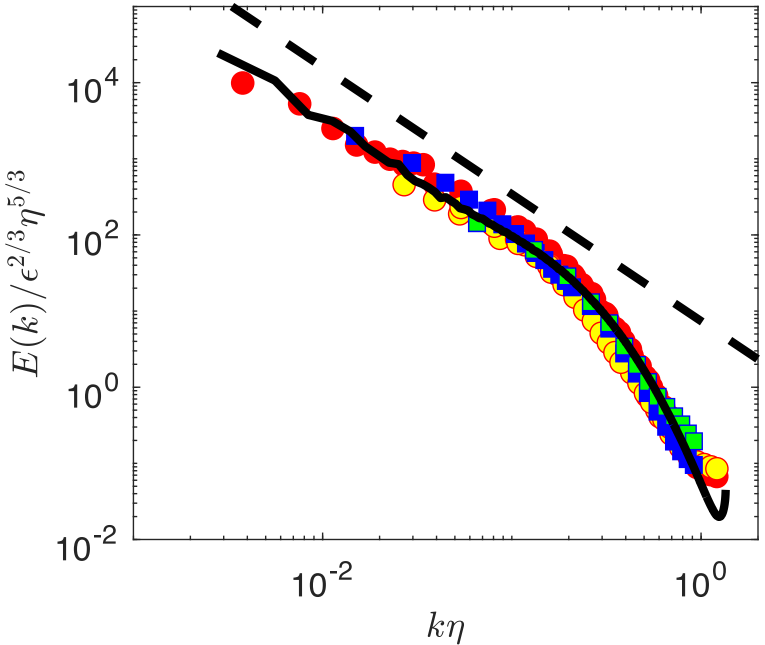
Test of K41 universality

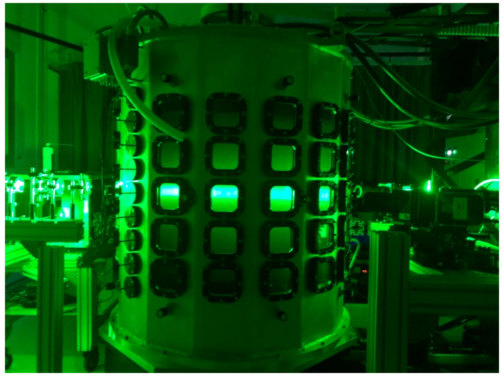


Ocean



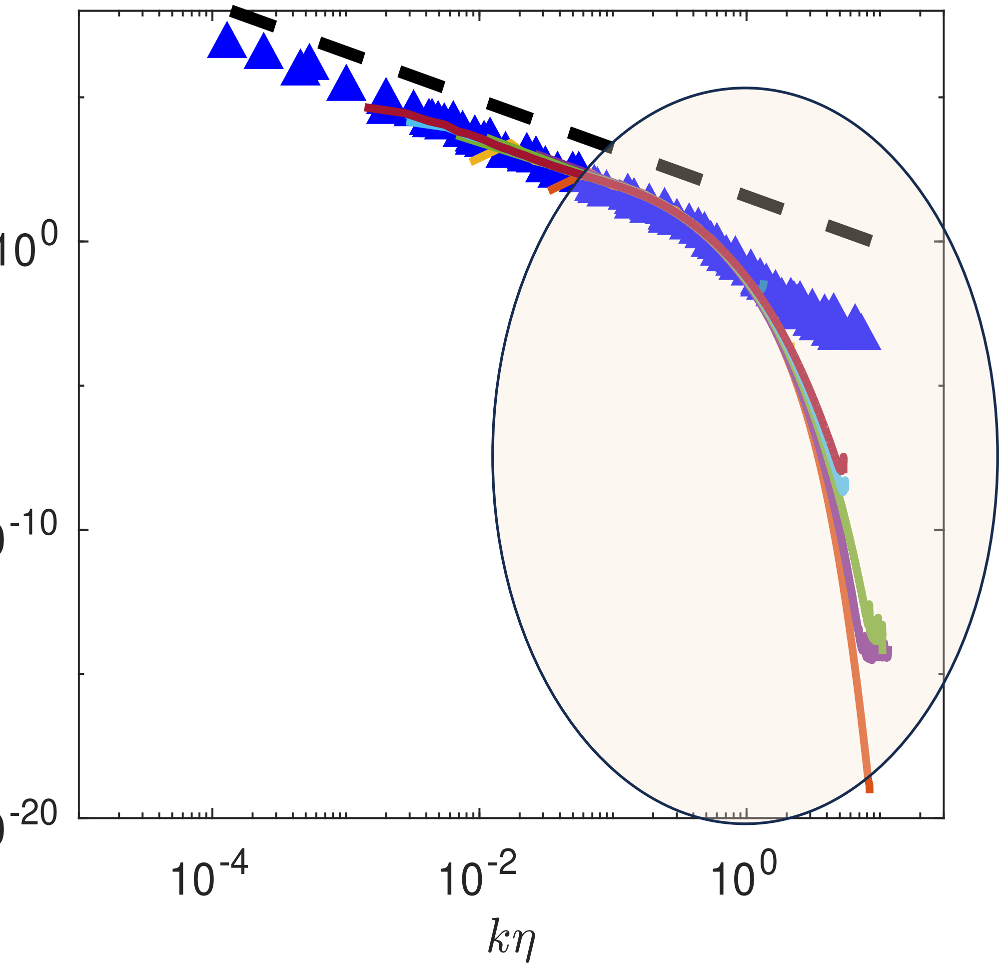
DNS





Experiments

Breaking of K41 Universality



K41 appears broken in the dissipating range!

Navier-Stokes Equation

$$\vec{\nabla}\bullet\vec{u} = 0$$

$$\partial_t \vec{u} + (\vec{u}\bullet\vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p + \nu\Delta\vec{u}$$

Homogeneity

$$\delta u = u(x+\ell) - u(x)$$

Karman-Howarth-Monin

$$\frac{1}{4}\partial_t E + \epsilon = -\frac{1}{4}\nabla_\ell < (\delta u)^3 > + \frac{1}{2}\nu\Delta_\ell E + P_{inj}$$

$$\delta u = (\epsilon\ell)^{1/3}$$

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

Scale invariance

$$\frac{E(k)}{\epsilon^{2/3}\eta^{-5/3}} = C_K\,(k\eta)^{-5/3}$$

Breaking of K41 Universality

$$\vec{\nabla} \bullet \vec{u} = 0$$

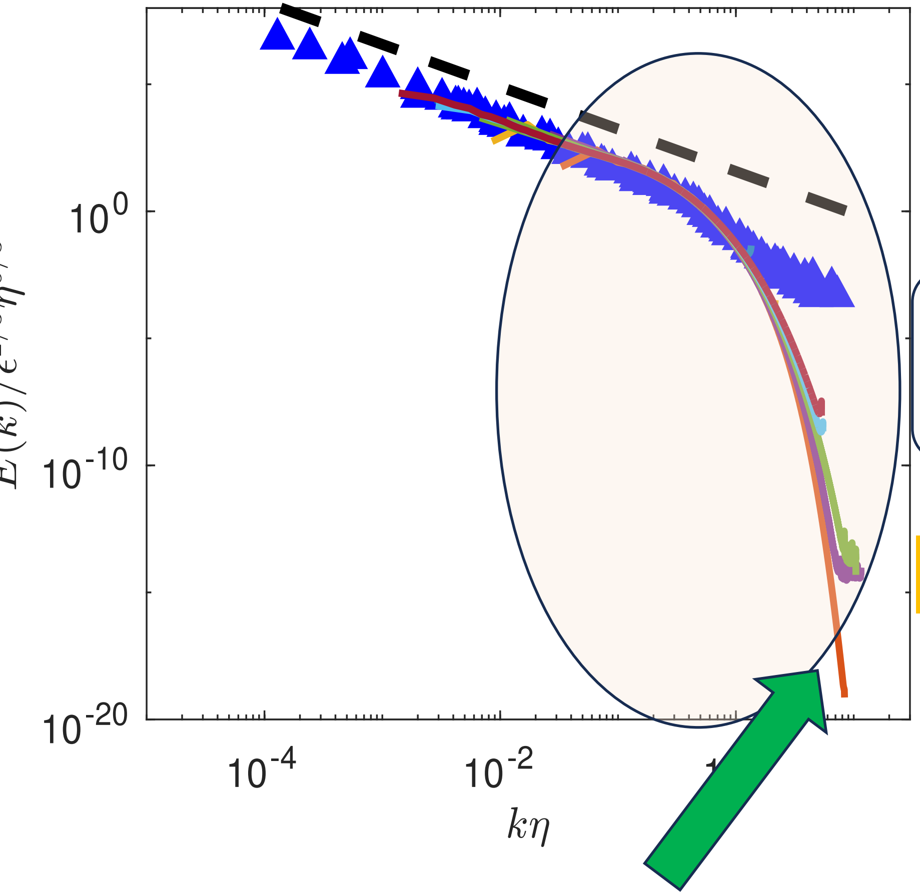
$$\partial_t \vec{u} + (\vec{u} \bullet \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p + \nu \Delta \vec{u}$$

Navier-Stokes Equation

~~Homogeneity~~

Local Karman-Howarth-Monin

$$\delta u = u(x+\ell) - u(x)$$



$$\frac{1}{2}\partial_t E_\ell + \partial_j J_j \quad = -\frac{1}{4}\int \nabla\phi^\ell(\xi)\cdot\delta\mathbf{u}(\delta u)^2 - \frac{\nu}{2}\int \nabla^2\phi^\ell(\xi)(\delta u)^2 d\xi \quad (5)$$

Local Scale invariance

$$\delta u \sim \ell^h \qquad \eta_h \sim \nu^{-1/(1+h)}$$

Paladin and Vulpiani

Signature of local self similar solution with h<1/3

Kolmogorov
h=1/3

$$h < \frac{1}{3} \Longrightarrow \eta_h < \eta_K$$

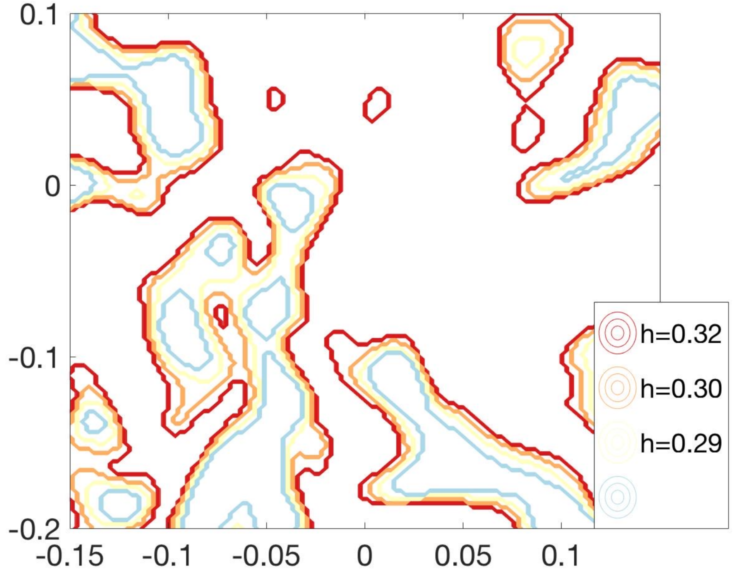
Local invariance and Multifractal

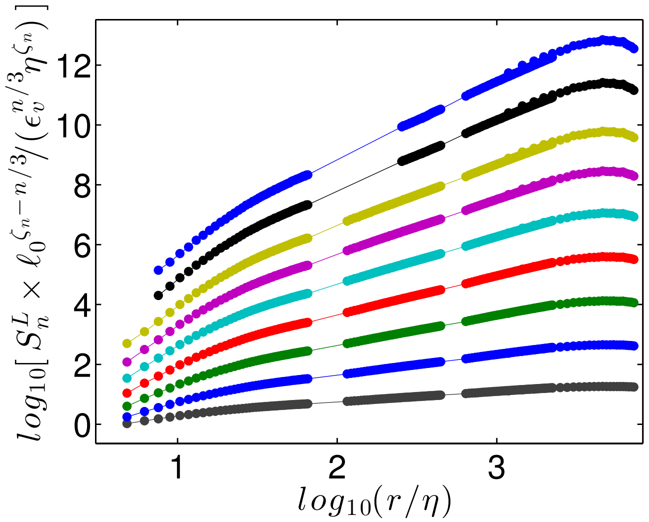
Heuristic interpretation of Parisi&Frisch (1985) $\delta u \sim \ell^{h(x)}$

over a fractal set of Codimension

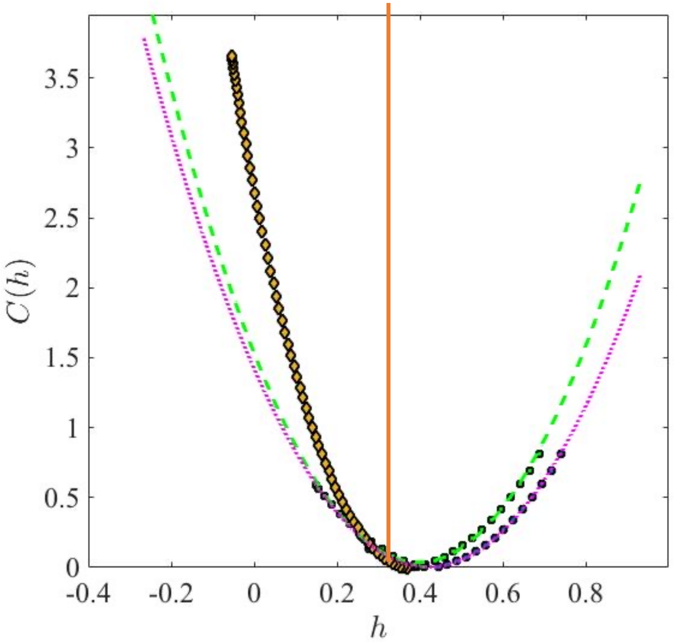
Probabilistic interpretation, using large deviations

$$\text{Prob}\left[\ln(\delta u) = h\ln(\ell/L)\right] \sim_{\ell\to 0} e^{\ln(\ell/L)C(h)} = \left(\frac{\ell}{L}\right)^{C(h)},$$





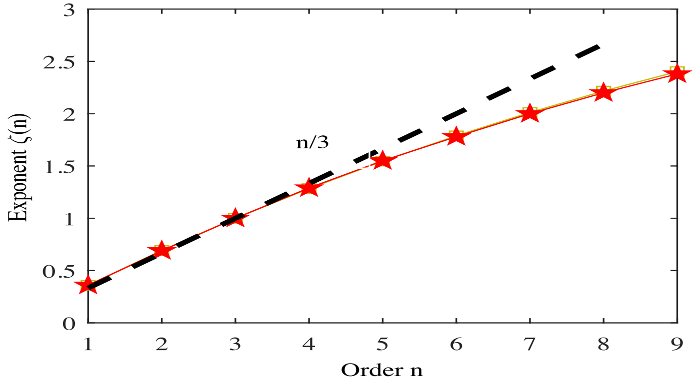
C(h): large deviation function of h



Shift from *Global* Scale invariance to *Local* scale invariance

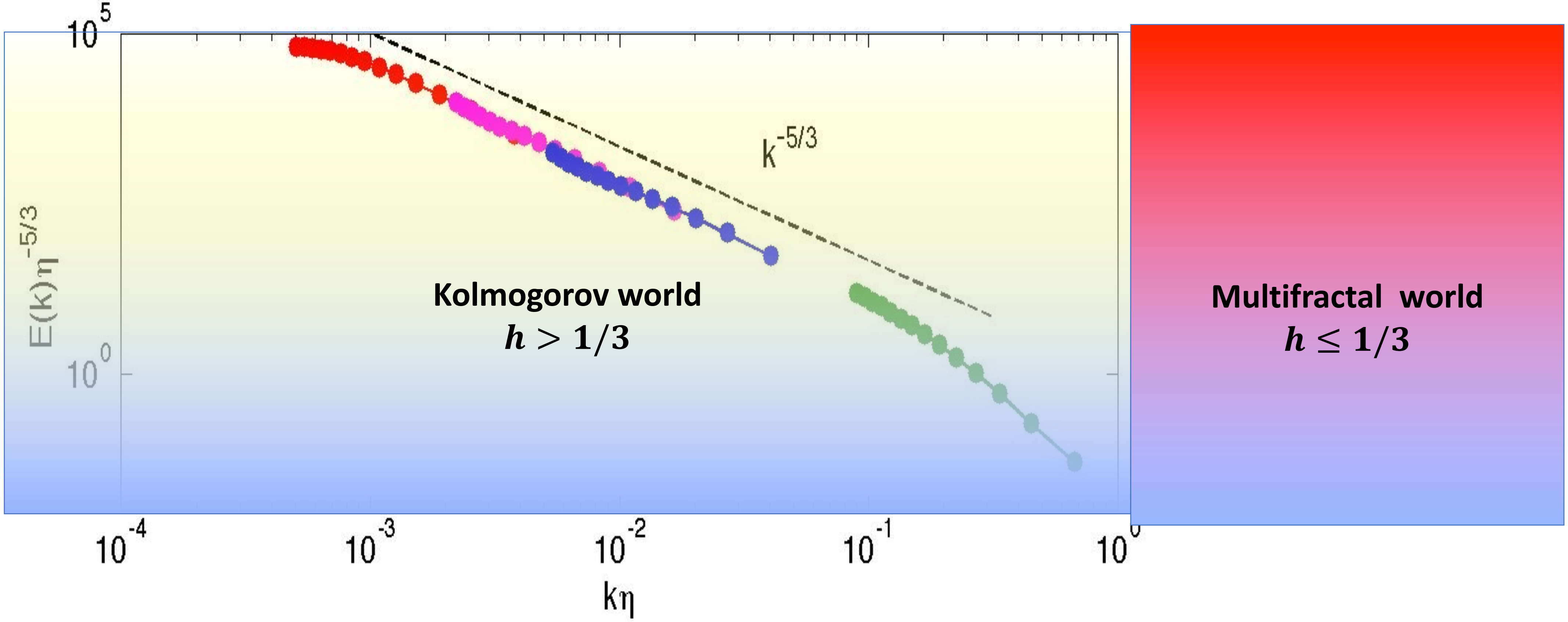
$$\delta u \sim \ell^{1/3} \Longrightarrow S_p(\ell) \sim \ell^{p/3}$$

$$S_p(\ell) = <(\delta u)^p> \sim \ell^{p/3}$$



'.-F. Muzy, E. Bacry & A. Arneodo (1991) Phys. Rev. Lett. ***67***, 3515-3518.
Kestener & Arneodo, (2004) Phys. Rev. Lett. ***93***, 044501

The new world beyond Kolmogorov scale



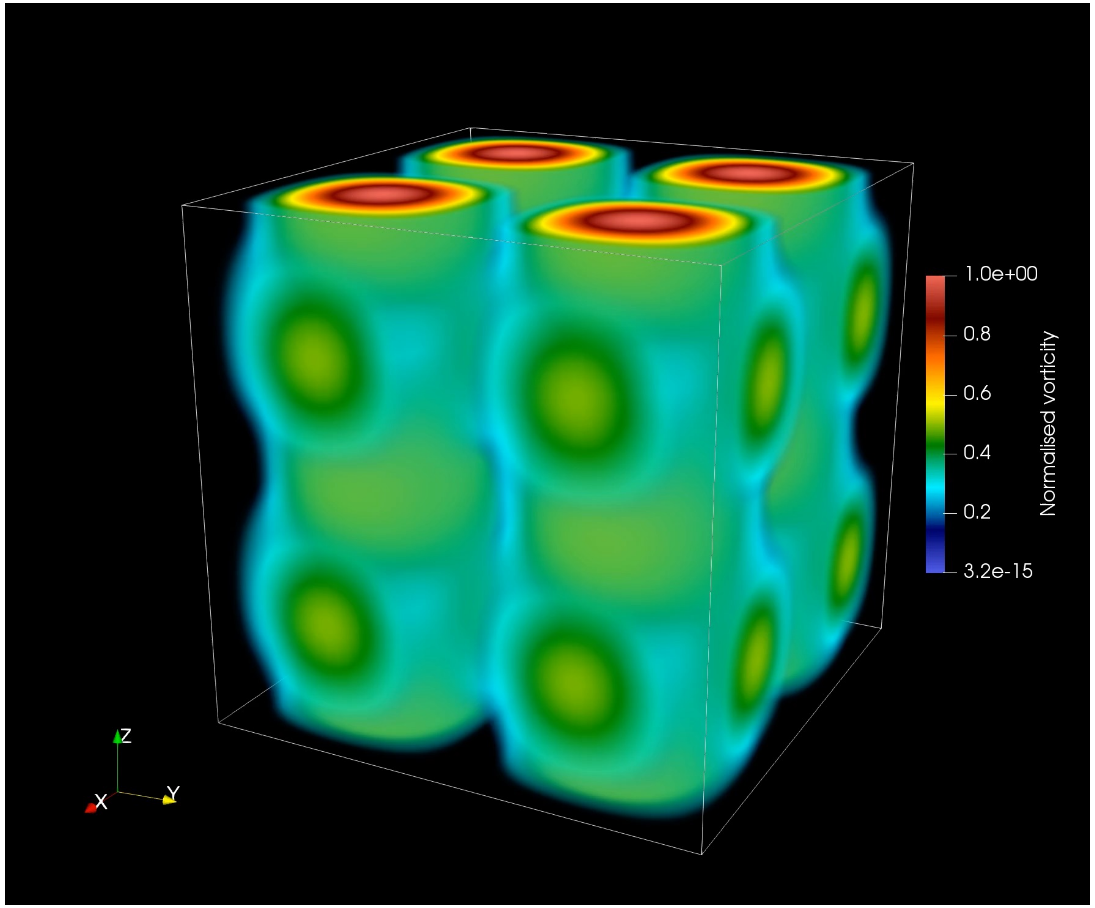
To observe these structures necessity to go at very large resolution!!!
-> **Challenge for DNS and experiments!**

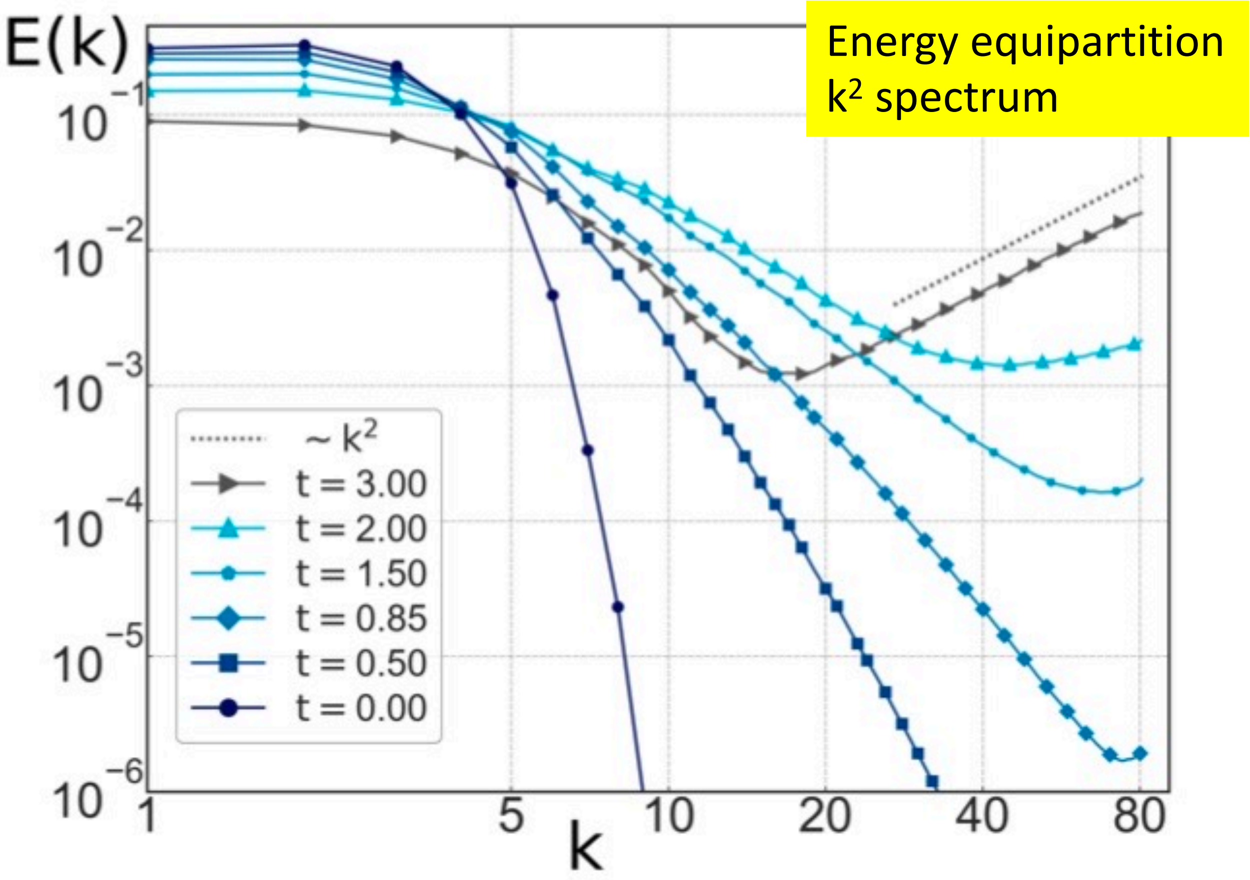
Part II: What can we say about those singularities?

Numerical approach

Can we check convergecy to singularity using DNS?

$$\partial_t \hat{u}_i + I k_j \hat{u}_j * \hat{u}_i = -I k_i \hat{P}$$





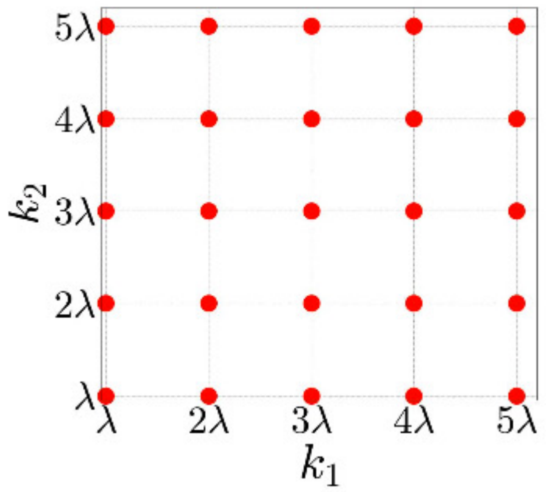
Any truncated Euler equation ends in equilibrium state with k^2 spectrum

Cichowlas et al, PRL, 2005

from DNS to log-lattices

$$\partial_t \hat{u}_i + I k_j \hat{u}_j * \hat{u}_i = -I k_i \hat{P}$$

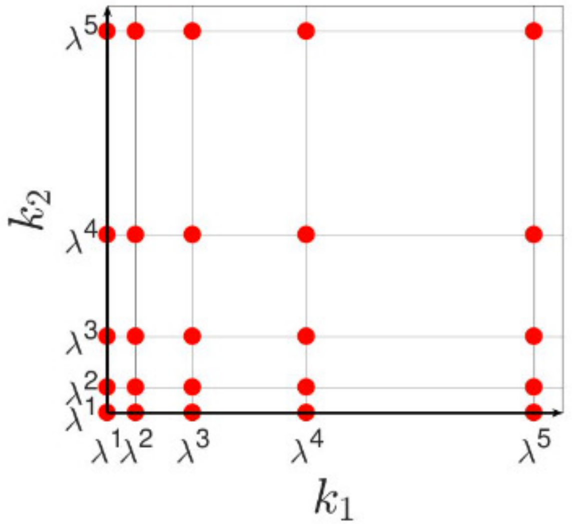
Fourier grid



N^3 modes

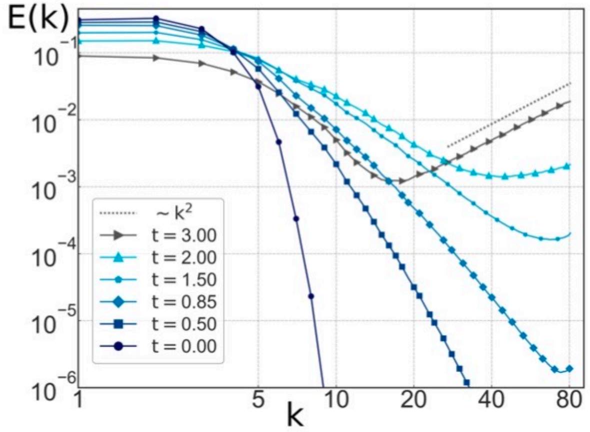
10^{24}

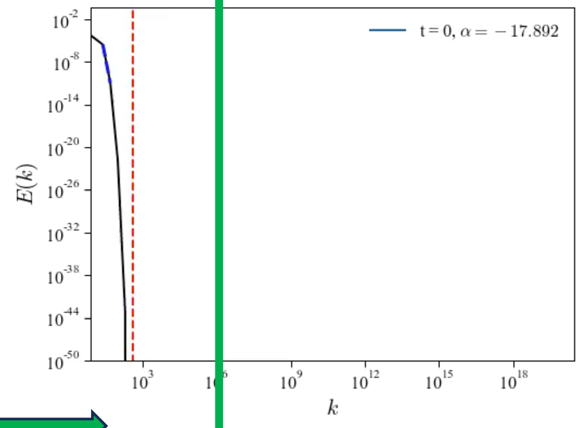
Log grid



$(\mathrm{Log}(N))^3$ modes

$6250 = 17^3$





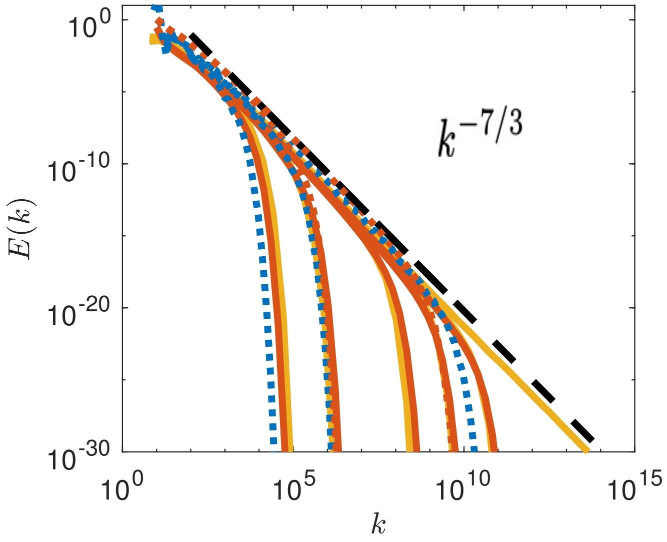
Max of existing DNS

Campolina&Mailybaev, 2018

Blow-up in Euler 3D on log-lattice (adaptative grid)

$$\partial_t \hat{u}_i + I k_j \hat{u}_j * \hat{u}_i = -I k_i \hat{P}$$

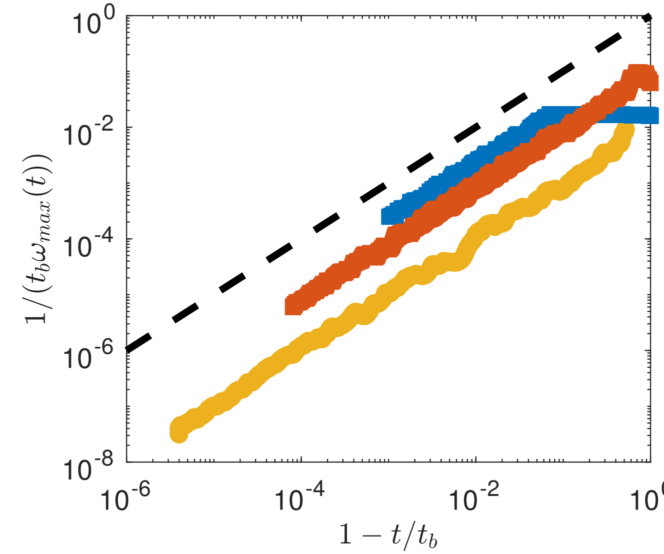
No viscosity: Energy is conserved



λ=2

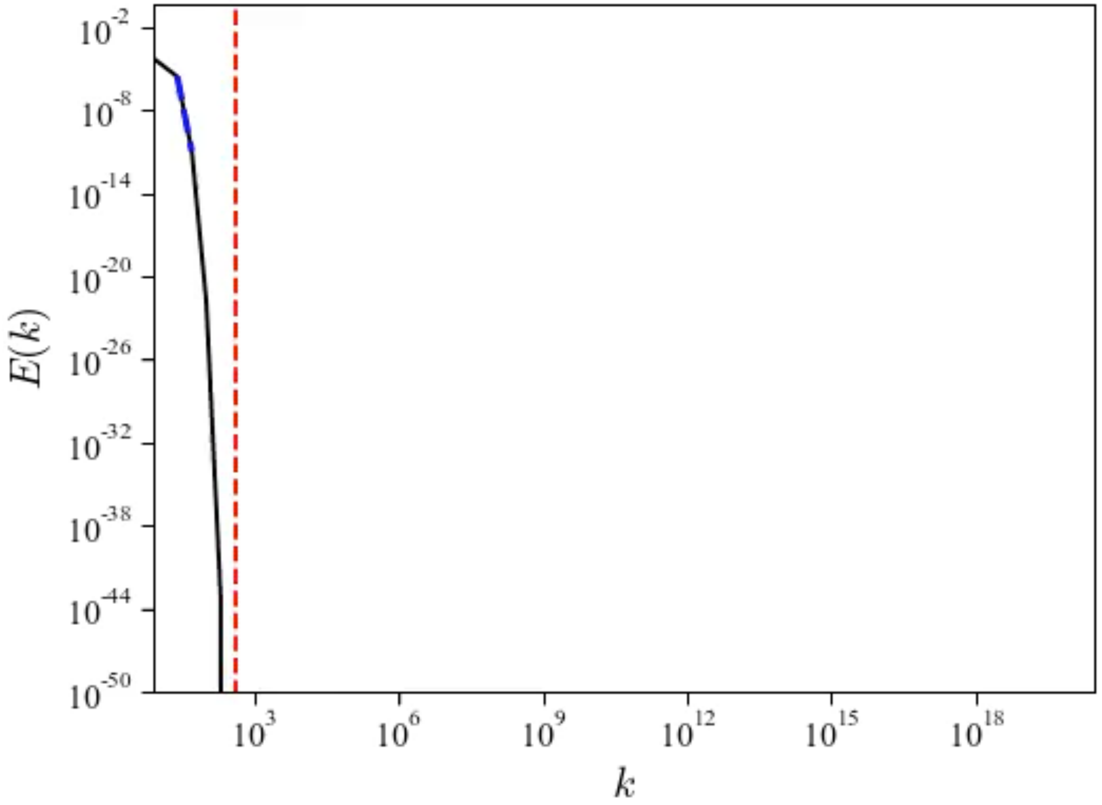
λ=1.6

λ=1.3



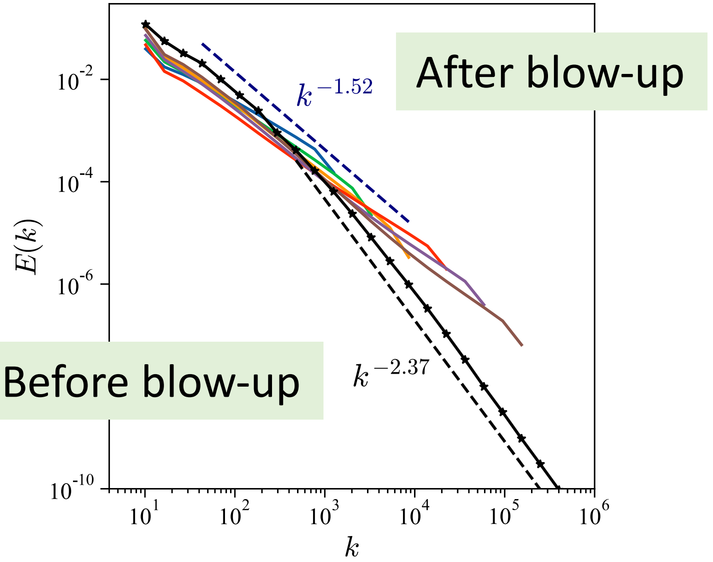
Formation of a
Finite-time singularity

$$\omega \sim \frac{1}{t_b - t}$$

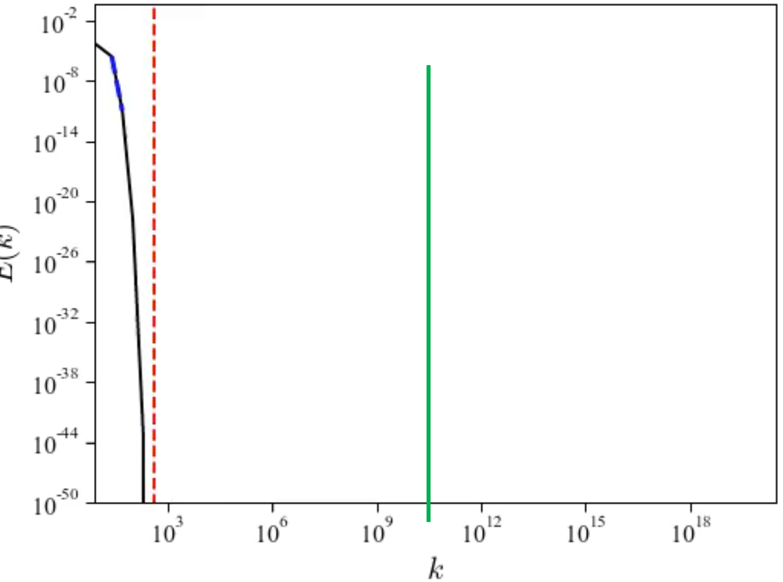


Campolina&Mailybaev, 2018

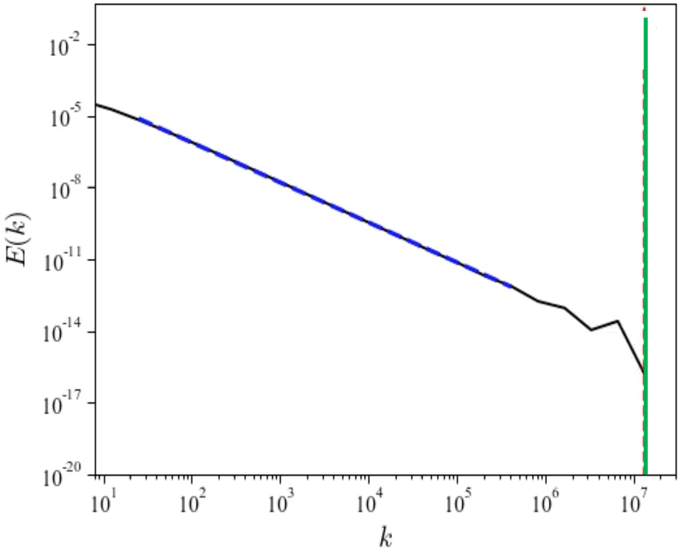
After the blow-up: convergency to another solution (dissipative)

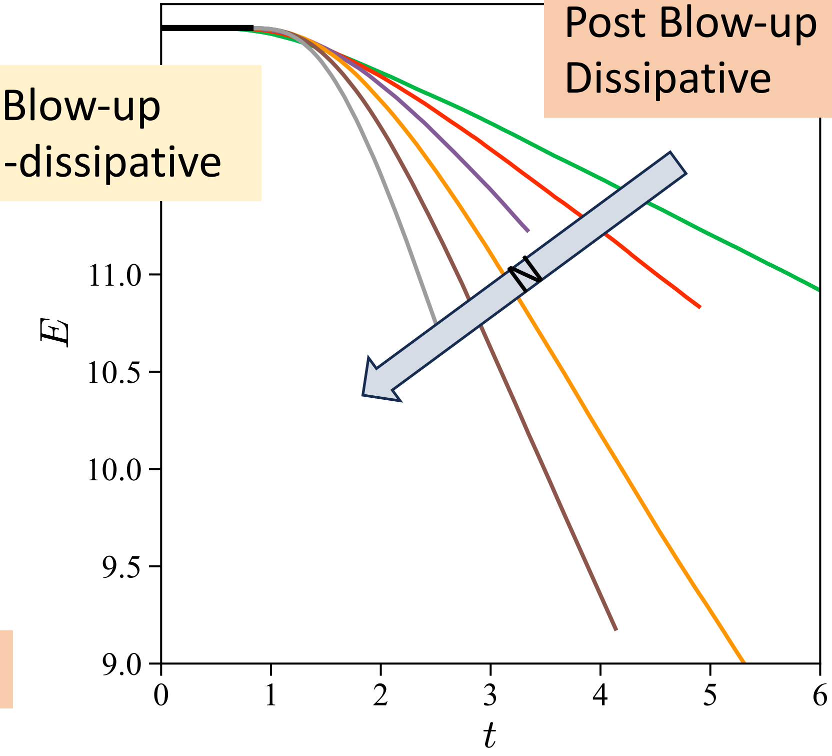


Before blow-ip



After blow-up





Regularization: : impose a boundary in scale space and add a noise of decreasing intensity

We fix the maximal wavenumber $k = \lambda^N$

$$dW = U(\lambda^N)d\eta; \quad < d\eta^2 >= 1; \quad < d\eta = 0 >$$

We look at limit $N \to \infty$

Singularity induces dissipation

Costa et al, 2024

Where does the dissipation come from??

Theory

Local energy budget

Duchon&Robert. Nonlinearity (2000),

$$\vec{\nabla} \bullet \vec{u} = \vec{0}$$

$$\partial_t \vec{u} + (\vec{u} \bullet \vec{\nabla}) \vec{u} = -\frac{1}{\rho} \vec{\nabla} p + \nu \Delta \vec{u}$$

Weak solutions

Viscous dissipation

$$\frac{1}{2}\partial_t \mathbf{u}^2 + \mathrm{div}\left(\mathbf{u}\left(\frac{1}{2}\mathbf{u}^2 + p\right) - \nu \nabla \mathbf{u}\right) = D(u) - \nu (\nabla \mathbf{u})^2$$

(

$$D(u) = \lim_{\ell \to 0} \frac{1}{4} \int_{r \le \ell} d^3 r \; \nabla \phi_\ell(r) \cdot \delta u_r |\delta u_r|^2$$

$$\delta u \sim \ell^h$$

$$D(u)[x] \propto \lim_{\ell \to 0} \ell^{3h-1}$$

Inertial dissipation:

Possible energy dissipation by singularities!:

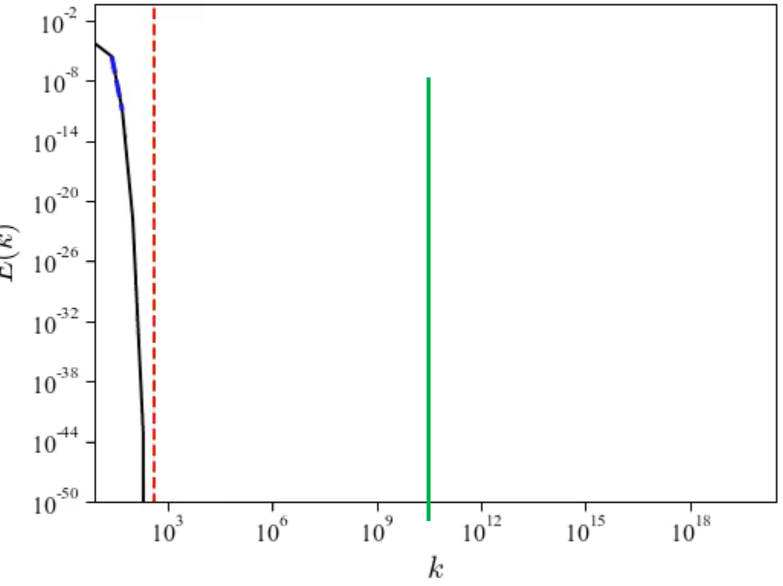
If $h > 1/3$ → Euler equation conserves energy,
Dissipation in Navier-Stokes by viscosity.
(Eyink 1994, Constantin et al, 1994)

If $h \le 1/3$ → Dissipation through irregularities (singularities)

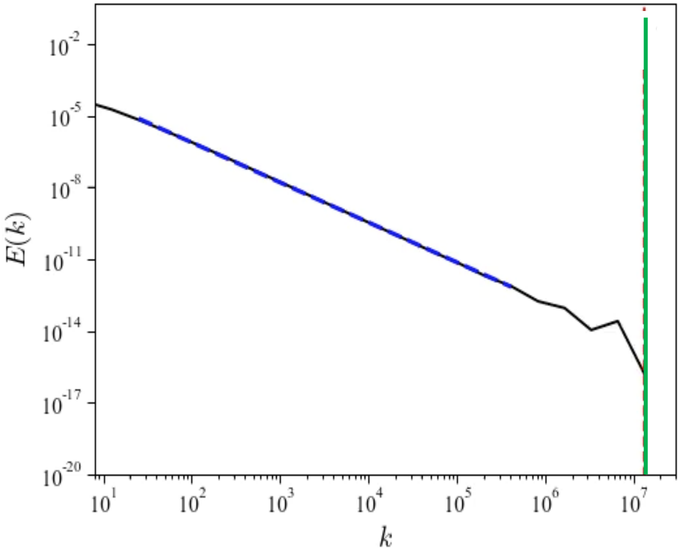
Without viscosity ! ***(Isett, 2018)***

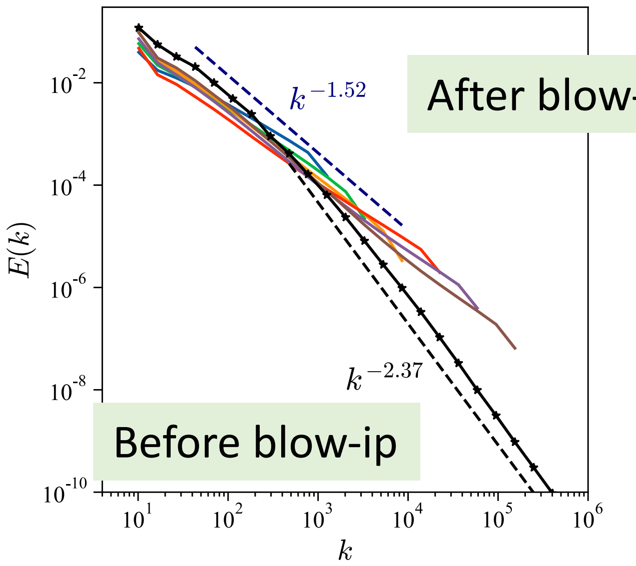
After the blow-up: convergency to another solution (dissipative)

Before blow-ip



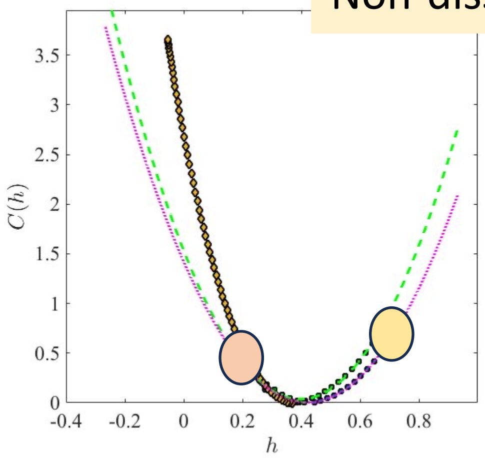
After blow-up





Post Blow-up
Dissipative

Pre-Blow-up
Non-dissipative



Regularization: **: impose a boundary in scale space and add a noise of decreasing intensity**

We fix the maximal wavenumber $k = \lambda^N$

$$dW = U(\lambda^N)d\eta; \quad < d\eta^2 >= 1; \quad < d\eta = 0 >$$

We look at limit $N \to \infty$

Singularity induces dissipation

Costa et al, 2024