Asian and European
Schools in Mathematics

A gentle introduction to data assimilation and HPC Applications on an HPC platform

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Project 2: Lotka-Volterra model, derivation of the tangent model

System of ordinary differential equations
and initial conditions:

$$\begin{cases} x' = ax - bxy \\ y' = -cy + dxy \end{cases} \qquad \begin{matrix} x(t=0) = x_0 \\ y(t=0) = y_0 \end{matrix} \qquad a, b, c, d = \text{const}$$

Solution:

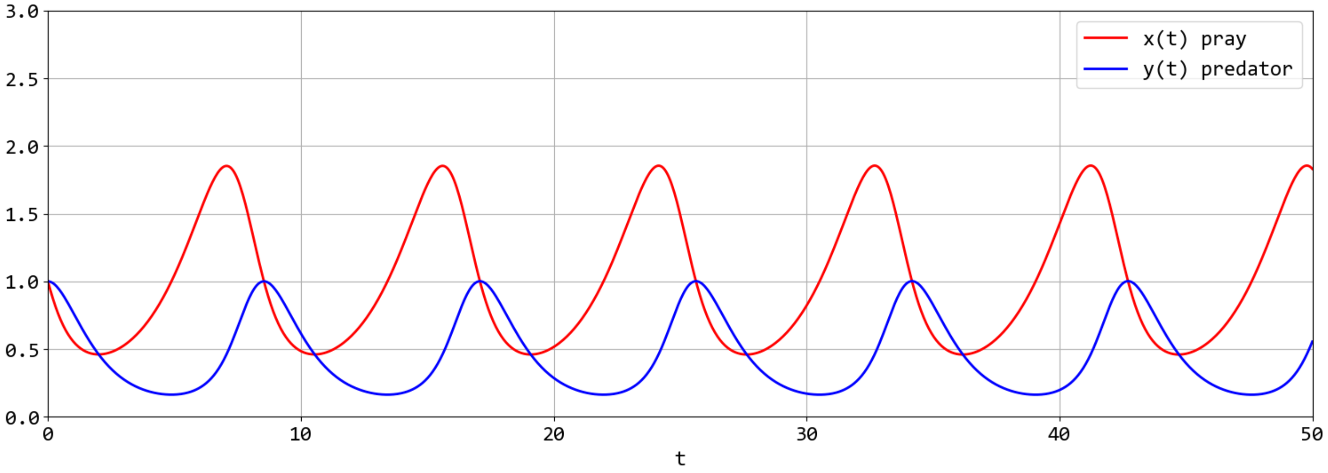


Fig. 1

State space

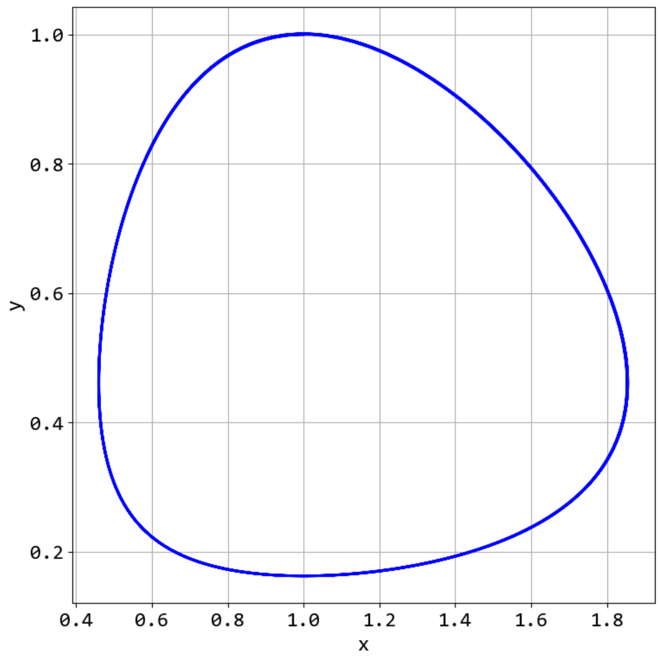


Fig. 2

$$\frac{\partial}{\partial s} \equiv \partial_s$$

Differential equations

$$\begin{cases} \partial_a x' = x + a\,\partial_a x - b(y\,\partial_a x + x\,\partial_a y) \\ \partial_b x' = a\,\partial_b x - xy - b(y\,\partial_b x + b\,x\,\partial_b y) \\ \partial_c x' = a\,\partial_c x - b(y\,\partial_c x + x\,\partial_c y) \\ \partial_d x' = a\,\partial_d x - b(y\,\partial_d x + x\,\partial_d y) \\ \partial_a y' = -c\,\partial_a y + d(x\,\partial_a y + y\,\partial_a x) \\ \partial_b y' = -c\,\partial_b y + d(x\,\partial_b y + y\,\partial_b x) \\ \partial_c y' = -y - c\,\partial_c y + d(x\,\partial_c y + y\,\partial_c x) \\ \partial_d y' = -c\,\partial_d y + xy + d(x\,\partial_d y + y\,\partial_d x) \end{cases}$$

Initial conditions

$$\begin{cases} \partial_\theta x(t=0) = 0 \\ \partial_\theta y(t=0) = 0 \end{cases}$$

$$\theta = \{a, b, c, d\}$$

Cost function

$$J(\theta) = \frac{1}{2}\|F(\theta)\|^2$$

$$F_i(\theta) = \mathrm{model}(\theta, t_i) - y_i^{\mathrm{obs}}$$

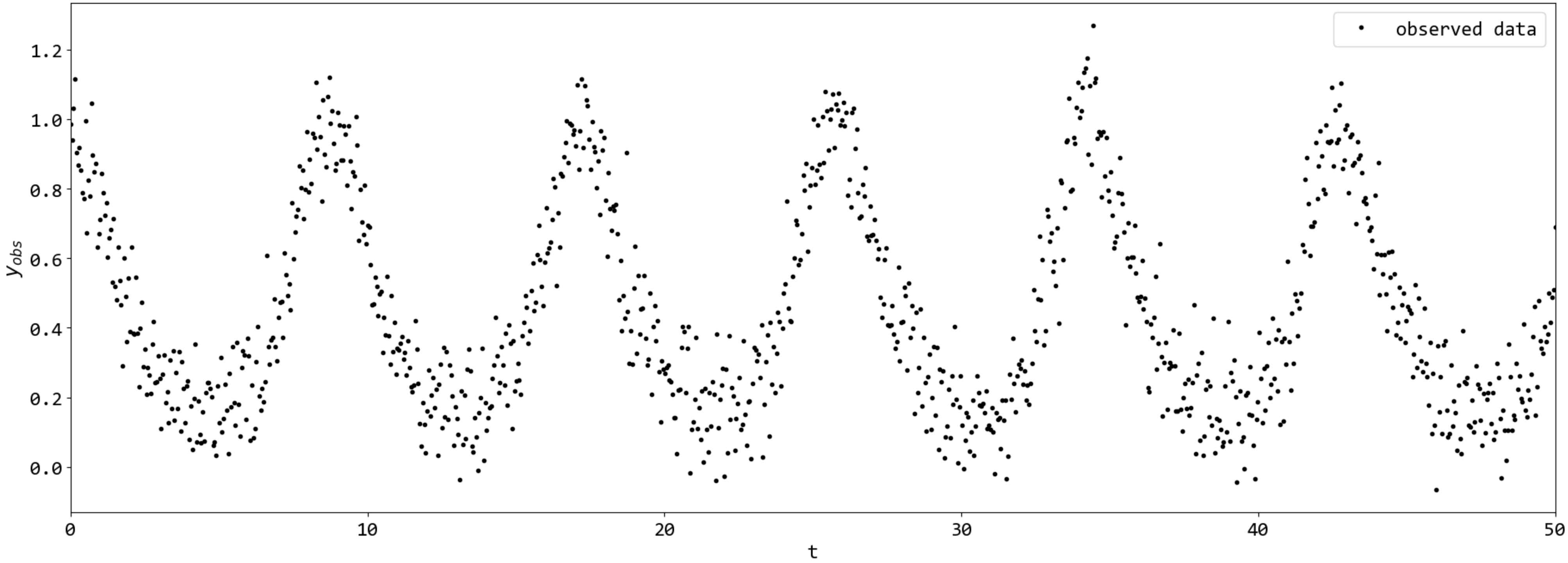
Observed data

noise: Gaussian

$\sigma = 0.1$

mean=0.0

Fig. 3



Results

True values of parameters:
a = 0.6, b = 1.3, c = 1.0, b = 1.0

Values calculated by tangent model:
a = 0.5, b = 1.091, c = 1.0189, b = 1.208

Initial guesses:
a = 0.608
b = 1.440
c = 1.056
b = 0.949

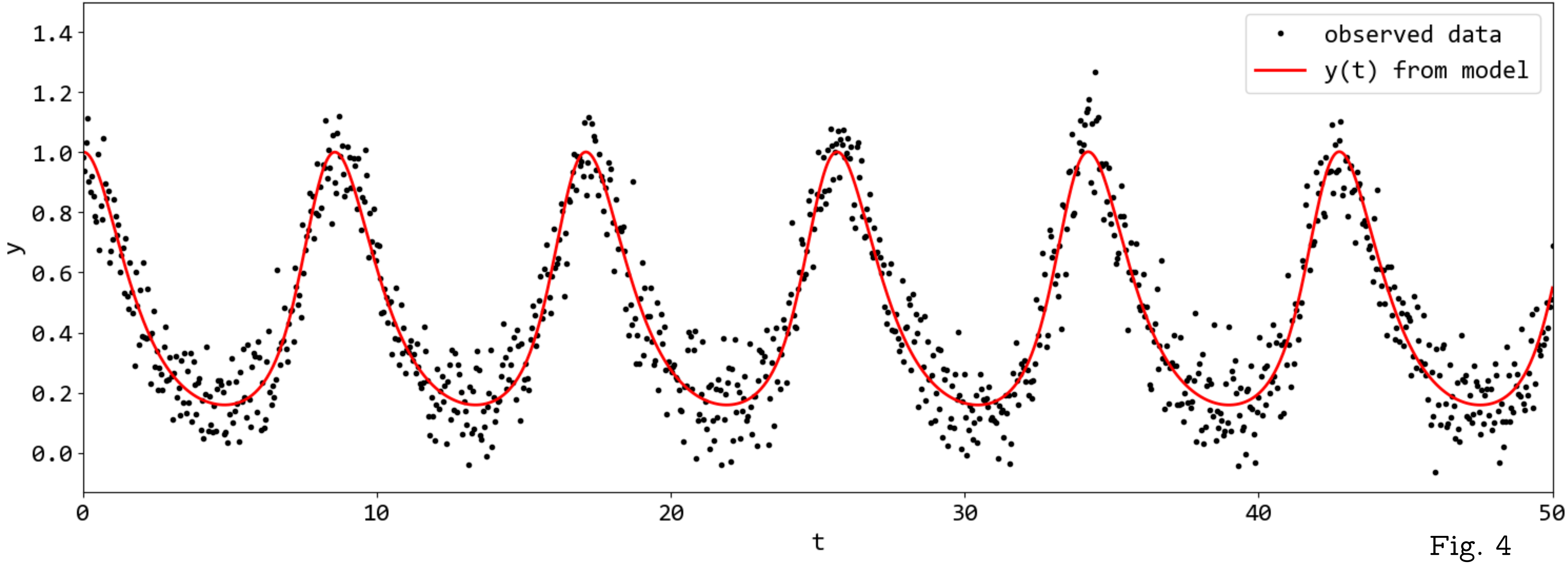


Fig. 4

Results

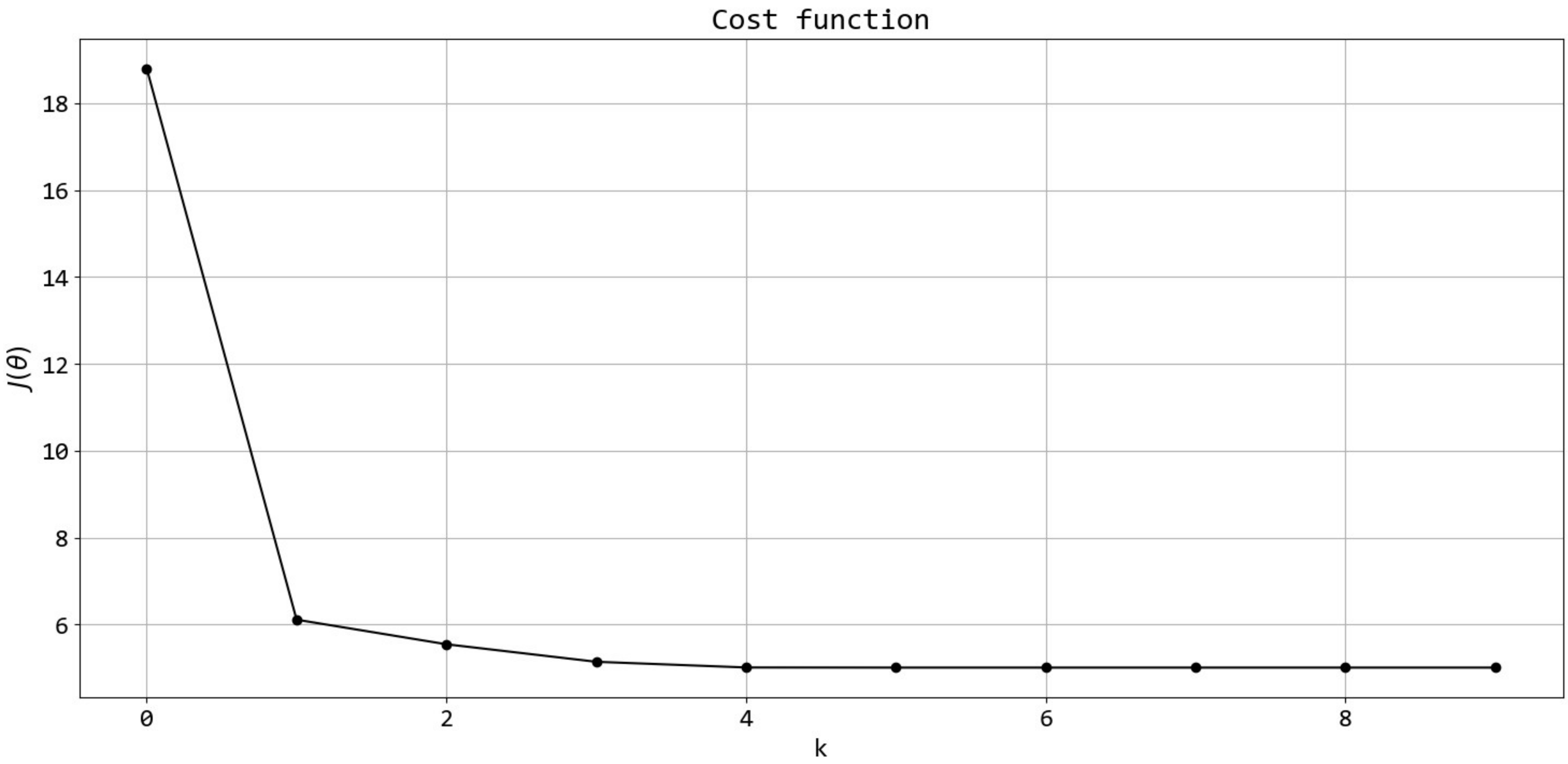


Fig. 5

Results

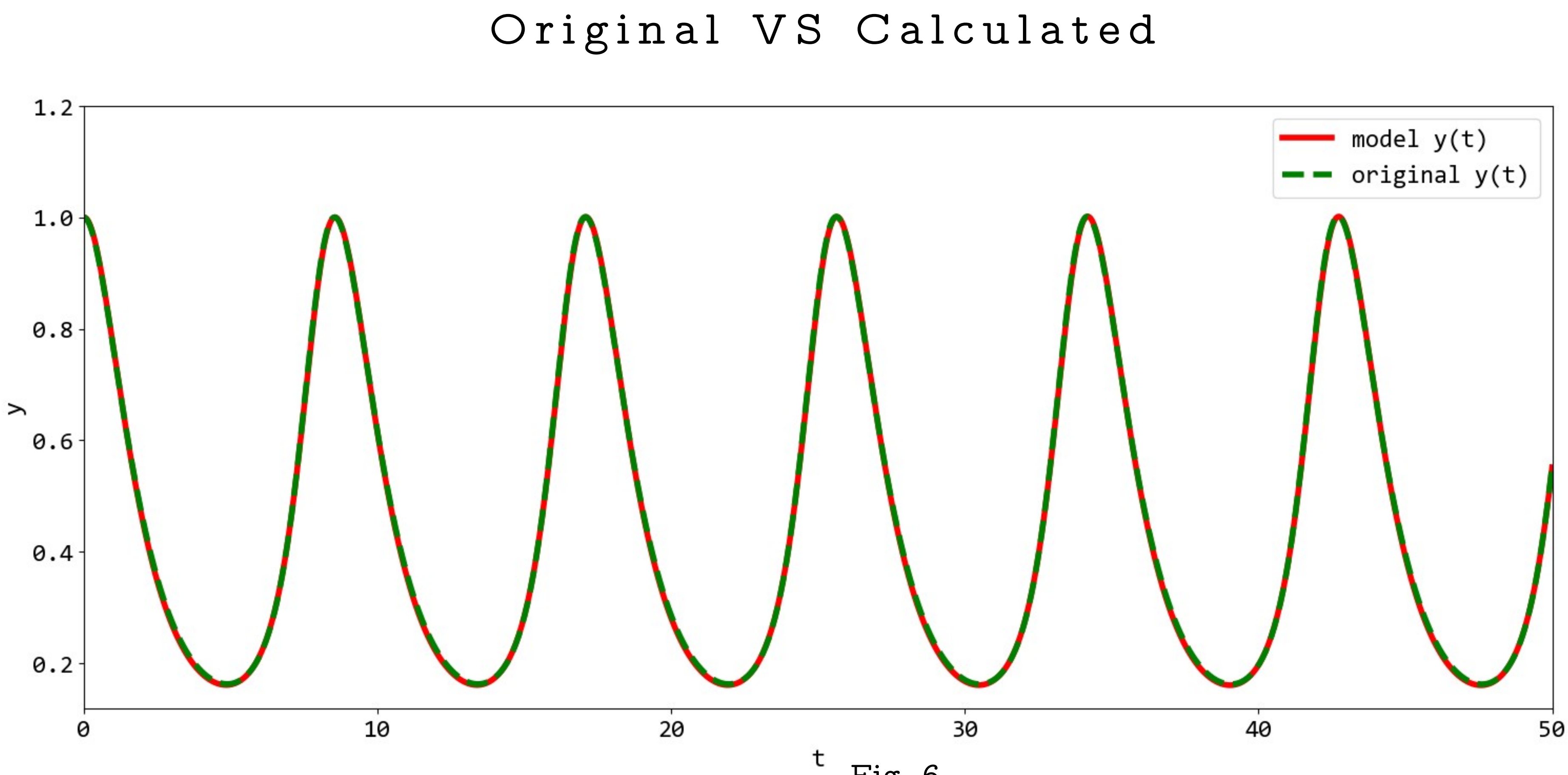


Fig. 6

Thank you for your attention!

მადლობა ყურადღებისთვის!