

INTRODUCTION TO DATA ASSIMILATION

Mini project 2

Mini-project 2: Lotka-Volterra model, derivation of the tangent model

In this project we study the predator-prey model of Lotka-Volterra.

Consider the following ODE model, that describes the evolution of populations of prey (x) and predators (y) that interact.

$$\begin{cases} x'(t) = ax - bxy, \\ y'(t) = -cy + dxy \end{cases} \quad (1)$$

The initial populations are (x_0, y_0) .

- Implement the solution of this problem. Visualize the solution for various parameters. One can use e.g. $a = 0.6, b = 1.3, c = 1, d = 1$ and $x_0 = 1, y_0 = 1$, or values in a similar ranges.
- Determine analytically the tangent model wrt the parameters a, b, c, d . Implement this tangent model and check the derivatives.
- Fix some true values (a^*, b^*, c^*, d^*) of the parameters, and generate observations (perturbed by some random noise). Recover the parameters starting from arbitrary values (close say up to 20% to the true value), using Gauss-Newton method. What happens if the initial values are far from the true values ?
- (optional)* Same question, where one also aims to recover the initial condition (x_0, y_0) .