Introduction to data assimilation
Mini project 1

Mini-project 1: SIR epidemiologic model with lock-down effect

We propose here an extension of the SIR epidemiologic model (see practical day 2), and take into account the effect of a lock-down. The lock-down affects the transmission coefficient τ. We generate synthetic data, and aim to reconstruct quantitatively the effect on τ.

We consider the following ODE system (SIR model) that models the spread of an epidemics in a population:

$$\begin{cases} S'(t) = -\tau S(t)I(t), \\ I'(t) = \tau S(t)I(t) - \nu I(t) \\ R'(t) = \nu I(t), \qquad\qquad t \in [0,T], \end{cases} \tag{1}$$

together with the initial condition: $S(t=0) = N - I_0, I(t=0) = I_0, R(t=0) = 0$.

In this exercise we fix the time interval [0,T] = [0,250], and the following values: $N = 6e7, I_0 = 1$. The values of y are computed on every integer point in this interval (observations available every day).
The parameters values are $\theta^* = (\tau^*, \nu^*)$ close to $(3.10^{-9}, 10^{-1})$ up to a factor of e.g. 10.
A public intervention (e.g. lock down) affects the spread force τ and multiplies it by a factor $f \in (0,1)$ between the known dates $t_1 = 80$ and $t_2 = 130$. In other words in the system (1) the parameter τ is replaced by $\tau(t)$ where:

$$\begin{cases} \tau(t) = \tau^*, & t \in [0, t_1] \\ \tau(t) = f\tau^*, & t \in [t_1, t_2] \\ \tau(t) = \tau^*, & t \in [t_2, T] \end{cases}$$

a) Modify the function that implements the SIR model (from practical day 2) to incorporate the effect of multiplying τ by the factor f between the dates t_1 and t_2. Run the simulations with different values of the reduction factor f, and observe the effect on $I(t)$

b) Modify the function Dy to compute also the derivative wrt f. Check the derivative.

c) Now the solution depends on 4 parameters: (τ, ν, I_0, f).
Simulate the data with a factor $f^* = 0.1$.
Implement the Gauss-Newton algorithm to estimate the parameters. (Note: due to convergence issues, some adaptation of Gauss-Newton may be required. The simplest to implement is to use a step 0.1 instead of "optimal" step 1)
Add noise and test the robustness of the method.

d) Same question with another value for the factor f^*.

e) *(optional)* Explore numerically the influence of the time interval $[t_1, t_2]$ on the identifiability of the parameter f.