

Georg Maierhofer - Structure-preserving low-regularity integrators for dispersive nonlinear equations

Friday 24 May 2024 11:00 (45 minutes)

Dispersive nonlinear partial differential equations can be used to describe a range of physical systems, from water waves to spin states in ferromagnetism. The numerical approximation of solutions with limited differentiability (low regularity) is crucial for simulating fascinating phenomena arising in these systems including emerging structures in random wave fields and dynamics of domain wall states, but it poses a significant challenge to classical algorithms. Recent years have seen the development of tailored low-regularity integrators to address this challenge.

Inherited from their description of physical systems many such dispersive nonlinear equations possess a rich geometric structure, such as a Hamiltonian formulation and conservation laws. To ensure that numerical schemes lead to meaningful results, it is vital to preserve this structure in numerical approximations. This, however, results in an interesting dichotomy: the rich theory of existent structure-preserving algorithms is typically limited to classical integrators that cannot reliably treat low-regularity phenomena, while most prior designs of low-regularity integrators break geometric structure in the equation. In this talk, we will outline recent advances incorporating structure-preserving properties into low-regularity integrators. Starting from simple discussions on the nonlinear Schrödinger and the Korteweg–de Vries equation we will discuss the construction of such schemes for a general class of dispersive equations before demonstrating applications of these algorithms including to nonlinear Schrödinger equations with random dispersion, and to the simulation of low-regularity vortex filaments. This is joint work with Yvonne Alama Bronsard, Jianbo Cui, Valeria Banica, Yvain Bruned, and Katharina Schratz.