Chebyshev polynomials and Widom factors

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Let $\mathsf{E} \subset \mathbb{C}$ be an infinite compact set and denote by T_n the minimax (or Chebyshev) polynomials of E, that is, the monic degree n polynomials that minimize the sup-norm on E. A classical result of Szegö states that $\|T_n\|_{\mathsf{E}} \geq \mathrm{Cap}(\mathsf{E})^n$ for all n, a lower bound that doubles when $\mathsf{E} \subset \mathbb{R}$. More recently, Totik proved that for real subsets, $\|T_n\|_{\mathsf{E}}/\mathrm{Cap}(\mathsf{E})^n \to 2$ if and only if E is an interval.

We shall introduce the so-called Widom factors by

$$W_n(\mathsf{E}) := \frac{\|T_n\|_{\mathsf{E}}}{\mathrm{Cap}(\mathsf{E})^n}$$

and pose the question if there are more subsets of the complex plane for which $W_n(\mathsf{E}) \to 2$. It appears the answer is indeed affirmative for certain polynomial preimages. Interestingly, our proof relies on properties of the Jacobi orthogonal polynomials due to Bernstein. We shall also settle a conjecture of Widom concerning Jordan arcs and discuss related open problems.

The talk is based on joint work with B. Eichinger (TU Wien) and O. Rubin (Lund).