Fractional time derivative through the lens of Herglotz-Nevanlinna function theory

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The presence of microstructure in a two-phase composite, such as the porous media, has led to many interesting and challenging mathematical problems. The dynamics occurring near the material interface at the microscopic scale very often leads to a nonlocal-in-time phenomena (aka the memory effect) in the macroscopic description of the dynamics of a composite material. For example, the memory terms in the poroelastic equations and the fractional time derivative (Caputo derivative, Riemann-Liouville derivatives, etc.) appearing in the sub-diffusion equations describing the anomalous diffusion in porous media. On the one hand, these memory terms capture and quantify the microstructure characteristics of the composite materials. On the other hand, they create a challenge in developing a time domain solver for these differential equations with memory terms.

Despite the difference in the appearance of these memory terms, most of them are defined as a time convolution operator with a kernel satisfying athe causality condition, which means their Fourier Laplace transforms have to be a Herglotz-Nevanlinna function. This link can be utilized not only to reveal the physical meaning of these fractional time derivatives but also inform about a time-domain quadrature for handling the convolution term in a numerical solver. In this talk, all the above will be explained and a numerical example will be given.