On Strong Asymptotics of Multiple Orthogonal Polynomials for Angelesco Systems

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Let ω be the arcsine distribution of a closed interval Δ and μ be any positive Borel measure there. Denote by $P_n(z)$ the n-th monic orthogonal polynomials w.r.t. μ, i.e.,

$$\int x^k P_n(x) d\mu(x) = 0\,, \qquad k \in \{0, \dots, n-1\}\,,$$

where $P_n(x) = x^n + \dots$.It is a classical Szegö that if $v \in L^1(\omega)$, where $d\mu = v d\omega + d\mu^s$ and μ^s is singular to the Lebesgue measure, then

$$P_n(z) = \exp\left\{ n \int \log(z-x) d\omega(x) \right\} \frac{G_\mu(\infty)}{G_\mu(z)}$$

locally uniformly in $\overline{\mathbb{C}}\backslash\Delta$, where $G_\mu(z)$ is the Szegö function of μ, i.e., it is an outer function in $\overline{\mathbb{C}}\backslash\Delta$ such that $G_\mu(\infty) > 0$ and $|G_{\mu\pm}(x)|^2 = v(x)$ a.e. on Δ. In this talk an extension of this result to multiple orthogonal polynomials for Angelesco systems of measures will be discussed. More precisely, positive Borel measures $\mu_1, \dots, \mu_d$ form an Angelesco system if $\Delta_1 < \Delta_2 < \dots \Delta_d$, where Δ_i is a convex hull for supp μ_i. For these systems, given a multi-index $\vec{n} \in \mathbb{N}^d$, there exists a unique monic polynomial $P_{\vec{n}}(z)$ of degree $|\vec{n}| = n_1 + \dots + n_d$ such that

$$\int x^k P_{\vec{n}}(x) d\mu_i(x) = 0\,, \qquad k \in \{0, \dots, n_i - 1\}\,, \qquad i \in \{1, 2, \dots, d\}\,.$$

Strong asymptotics of these polynomials is derived along ray sequences $\mathcal{N}(\vec{c}) = \{\vec{n} : n_i/|\vec{n}| \to c_i \text{ for each } i\}$, where $\vec{c} = (c_1, \dots, c_d)$, $c_i > 0$ and $c_1 + \cdot + c_d = 1$. It turns out that in general Szegö condition $v_i \in L^1(\omega_i)$, where $d\mu_i = v d\omega_i + d\mu_{ii}^s$ and ω_i is the arcsine distribution of Δ_i, is not sufficient to get the desired asymptotics. The proof utilizes certain generalization of results of Totik on asymptotics of orthogonal polynomials with varying weighs and de la Calle Ysern and Lopez Lagomasino on asymptotics of orthogonal polynomials on the unit circle with reciprocal polynomial weights.

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