Polynomial and rational convergence rates for Laplace problems on planar domains

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Laplace problems on planar domains can be solved by means of least-squares expansions associated with polynomial or rational approximations. Here it is shown that, even in the context of an analytic domain with analytic boundary data, the difference in convergence rates may be huge when the domain is nonconvex. Our proofs combine the theory of the Schwarz function for analytic continuation, potential theory for polynomial and rational approximation rates, and the theory of crowding of conformal maps. Polynomials are impossibly slow, but rational functions are amazingly fast, and the AAA-least squares algorithm makes the method practical.