

**On Giuseppe Veronese and
non-Archimedean
mathematics**

**Rapport sur les
contributions**

ID de Contribution: 1

Type: **Non spécifié**

Veronese's non-Archimedean continuity

vendredi 6 décembre 2024 09:30 (1 heure)

The paper focuses on the history of the “Archimedes axiom”, a name that was first given by Stolz in 1882 to a mathematical sentence occurring in Euclid’s Book V to express an intuitive content concerning quantity measurement in proportion theory. The name was later associated to the analysis of continuity in Hilbert’s *Grundlagen*, where it first expressed continuity tout court, and subsequently just a component of continuity, as in the second edition of the text, where Hilbert included it between the axioms of the fifth group together with a maximal axiom. It will be shown that beyond Euclid and Hilbert, a plethora of lesser-known authors essentially contributed to the axiomatic formulation of continuity (Stolz, but also Veronese and Baldus, for example). It will be claimed that alternative axiomatic formulations of the Archimedean axiom not only offered more rigorous formulations of some intuitive notion of continuity but allowed the latter to be specified and decomposed into simpler components. Different axiomatic presentations, which have followed one another historically and often emerged in a distant dialogue between various authors, have gradually clarified and sometimes broken down into distinct elements the intuitive content initially associated with the idea of measuring magnitudes, and then progressively refined with reference to certain minimal properties of continuity.

Orateur: CANTÙ, Paola (CNRS, Centre d’Épistémologie Gilles Gaston Granger, Université Aix-Marseille)

ID de Contribution: 2

Type: **Non spécifié**

Giuseppe Veronese: non-Archimedean numbers and non-Archimedean geometries

vendredi 6 décembre 2024 10:45 (1 heure)

My talk will be divided into two parts: in the first and briefer one, I will explain the non-Archimedean numerical system of infinite and infinitesimal numbers devised by Giuseppe Veronese (1854-1917) in the long Introduction (more than 200 pages) to his monumental *Fondamenti di geometria a più dimensioni e a più specie di unità rettilinee* (1891). Relying on his non-Archimedean numbers Veronese founds the non-Archimedean geometry. He offers a new definition of segment, congruence and rigid movement. He also highlights the concept of multiple of a segment which requires a profound modification of this notion. Granted these basic elements, Veronese edifies a whole system of geometry with peculiar features. Most of my talk will be dedicated to explaining the basic characteristic of Veronese's geometrical system.

Orateur: BUSSOTTI, Paolo (Università degli Studi di Udine)

ID de Contribution: 3

Type: **Non spécifié**

The impact of the discovery of non-Archimedean fields on fundamental aspects of the Parallel Postulate

vendredi 6 décembre 2024 12:00 (1 heure)

If the world were Archimedean, then statements such as the Euclidean parallel postulate, the existence of rectangles, the fact that perpendiculars to the sides of a right triangle intersect, would all be equivalent. Aristotle's axiom, that the distances between two legs of an angle grow without bounds, would be simply true in all models of absolute geometry if the Archimedean axiom were to hold. We will take a look at a whole new world which opens only in the absence of the Archimedean axiom, the world of the creatures that lie beneath the surface of the ocean that is Euclidean geometry and the ocean floor that is absolute geometry.

Orateur: PAMBUCCIAN, Victor (Arizona State University)

ID de Contribution: 4

Type: **Non spécifié**

Revisiting the reception history of Veronese's non-Archimedean geometry: Fondamenti di geometria between modernism and counter-modernism

vendredi 6 décembre 2024 14:45 (1 heure)

Saša Popović, Revisiting the reception history of Veronese's non-Archimedean geometry: Fondamenti di geometria between modernism and counter-modernism

Abstract. Following his important contributions to the projective geometry of hyperspaces from the 1870s and 80s, Giuseppe Veronese (1854–1917) introduced non-Archimedean geometry between 1889 and 1891, and elaborated it in a series of seminal publications over the next two decades, up to 1908/9. Initially, Veronese's results were widely discussed by leading mathematicians (e.g. T. Levi-Civita, G. Peano, G. Cantor, D. Hilbert, M. Dehn, L. E. J. Brouwer, C. S. Peirce, H. Poincaré, H. Hahn, etc.), as well as in major reference journals (e.g. the *Jahrbuch*), mathematical encyclopaedias and lexicons of the time. His work also quickly garnered the attention of philosophers working on the foundations of mathematics (e.g. the Marburg neo-Kantians). However, shortly after this initial burst of enthusiasm for Veronese's new geometry, already at the beginning of the 1920s we can see that discussions in both mathematical and philosophical circles started shifting to other mathematical subjects and, consequently, to other authors, resulting in a somewhat peculiar situation that Veronese's results seem to be almost entirely unknown by contemporary mathematicians, philosophers and historians of mathematics alike. This striking asymmetry in the initial and final stages of the reception of Veronese's theory will be at the heart of my talk. I will indicate what were the key factors which negatively impacted further dissemination and development of Veronese's ideas, as well as who were the "main culprits" for what may be considered a *damnatio memoriae* of Giuseppe Veronese in the actual practice of contemporary (non-Archimedean) mathematics, as well as in contemporary historiography. In 1990, Mehrtens proposed an analysis of the modernization process of fin-de-siècle mathematics in terms of a conflict between "the moderns" and the "counter-moderns". We shall see how Veronese's work fits into the broader modernist geometrical foundational programmes, especially Hilbert's, and, what is more, I will try to show that Veronese's *Fondamenti di geometria* (1891) not only represents a clear example of the "perfect pre-Hilbert style" of doing mathematics, but that there is a continuous developmental path from Veronese's geometrical investigations from the 1890's all the way up to late 20th century non-Archimedean theories such as, e.g., A. Robinson's Non-Standard Analysis from the 1960s and J. H. Conway's theory of surreal numbers from the 1970s.

Orateur: POPOVIĆ, Saša (Mathematical Institute of the Serbian Academy of Sciences and Arts, University of Rijeka Centre for Logic and Decision Theory)

ID de Contribution: 5

Type: **Non spécifié**

TBA (exposé en ligne)

vendredi 6 décembre 2024 16:00 (1 heure)

TBA

Orateur: EHRlich, Philip (University of Ohio)