Flatness approach for the boundary controllability of a system of heat equations

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In this talk, we are interested in the boundary controllability of the following system coupling two heat equations:

$$\begin{cases} \partial_t y_1 - \partial_x^2 y_1 = 0 & \text{in } (0,T)\times(0,1), \\ \partial_t y_2 - \dfrac{1}{d}\partial_x^2 y_2 = y_1 & \text{in } (0,T)\times(0,1), \\ y_1(t,0)=0, \quad y_1(t,1)=u(t) & \text{in } (0,T), \\ y_2(t,0)=y_2(t,1)=0 & \text{in } (0,T), \\ y_1(0,\cdot)=y_1^0, \quad y_2(0,\cdot)=y_1^0 & \text{in } (0,1). \end{cases} \tag{0.1}$$

In [1], using an observation inequality and the method of moments, the authors prove (among other thing) that:

- If $d=1$, the system is null controllable at any time $T>0$.
- If $\sqrt{d}\notin\mathbb{Q}$ there exist an explicit $T_d\in[0,+\infty]$ (given by the condensation index of the eigenvalues of our operator) such that the system is null controlable at any time $T>T_d$.

We will show, in the case $d=1$ and in the case $T_d<+\infty$, how it is possible to retrieve the controllability of the system using the flatness method. It allows an explicit resolution of the problem which lead to the construction of smooth controls (Gevrey of order σ for any $\sigma\in(1,2)$).

References

[1] Farid Ammar Khodja, Assia Benabdallah, Manuel González-Burgos, and Luz de Teresa, *Minimal time for the null controllability of parabolic systems: the effect of the condensation index of complex sequences*, J. Funct. Anal. **267** (2014), no. 7, 2077–2151.