How to create a horizon in the lab
and
the route to measure entanglement in experiments

(and possibly some more…)

Quantum Optics group
Laboratoire Kastler Brossel, Paris

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Avenues in QFTCS 24/01/2025

Today’s talk

The propagation of waves in nonlinear media may be controlled to engineer situations where the waves propagate as though they were on an effectively curved geometry, like around an apparent horizon or in an inflating universe. This enables the experimental study of field theories on curved geometries.

Controlled propagation of waves → effective geometry → linearised excitations
(engineered nonlinearity) (curvature) (quantum field)

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Spatial change in geometry: apparent horizon, ergosurface, light ring

Temporal change in geometry: cosmology

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(engineered nonlinearity) (curvature) (quantum field)

Spatial change in geometry: apparent horizon, ergosurface, light ring

Temporal change in geometry: cosmology

Spontaneous emission from the vacuum

Correlated/entangled waves

Dynamical instabilities (can go beyond linearised excitations → backreaction)

Almeida *et al.* EPJH **48** 15 (2023)

In a quantum fluid $\psi = \sqrt{n}e^{-i(\omega t+\phi(r))}$

Fluid velocity $\mathrm{v}_0 = \frac{\hbar}{m}\nabla\phi_0$ Speed of sound $\mathrm{c}_s \propto \sqrt{\frac{gn_0}{m}}$

m – mass
g – interaction constant
n_0 – mean field density

Wave eq for collective excitations of quantum fluid $\psi = (\psi_0 + \psi_1)e^{-i(\omega t+\phi(r))}$

$$-\partial_t\left(\frac{n_0}{c_s^2}(\partial_t n_1 + v_0\nabla n_1)\right) + \nabla\left(n_0\nabla n_1 - \frac{n_0 v_0}{c_s^2}\partial_t n_1 + v_0\nabla n_1\right) = 0$$

Relativistic form of wave eq for collective excitations: $|\eta|^{-1/2}\,\partial_\mu\left(\sqrt{|\eta|}\eta^{\mu\nu}\partial_\nu\psi_1\right) = 0$

with $$\eta_{\mu\nu} = \begin{pmatrix} -(c_s^2 - \boldsymbol{v_0}^2) & -v_o^x & -v_o^y \\ -v_o^x & 1 & 0 \\ -v_o^y & 0 & 1 \end{pmatrix}$$

Surface gravity

$$\kappa = \frac{1}{2c_s(x)}\frac{d}{dx}[v_0^2(x) - c_s^2(x)]|_{x_H}$$

Motion of collective excitations in inhomogeneous fluid flow ↔ scalar field on curved spacetime

Control parameters: $\boldsymbol{v_0}$, $\boldsymbol{c_s}$

Unruh *PRL* **46** 1351 (1981), Visser *Class Quant Grav* **15** 1767 (1998)

Effective spacetime in spatially inhomogeneous fluid flows

In a (quantum) fluid

Fluid velocity $\mathrm{v}_0 = \frac{\hbar}{m}\nabla\phi_0$ Speed of sound $\mathrm{c}_s \propto \sqrt{\frac{gn_0}{m}}$

m – mass
g – interaction constant
n_0 – mean field density

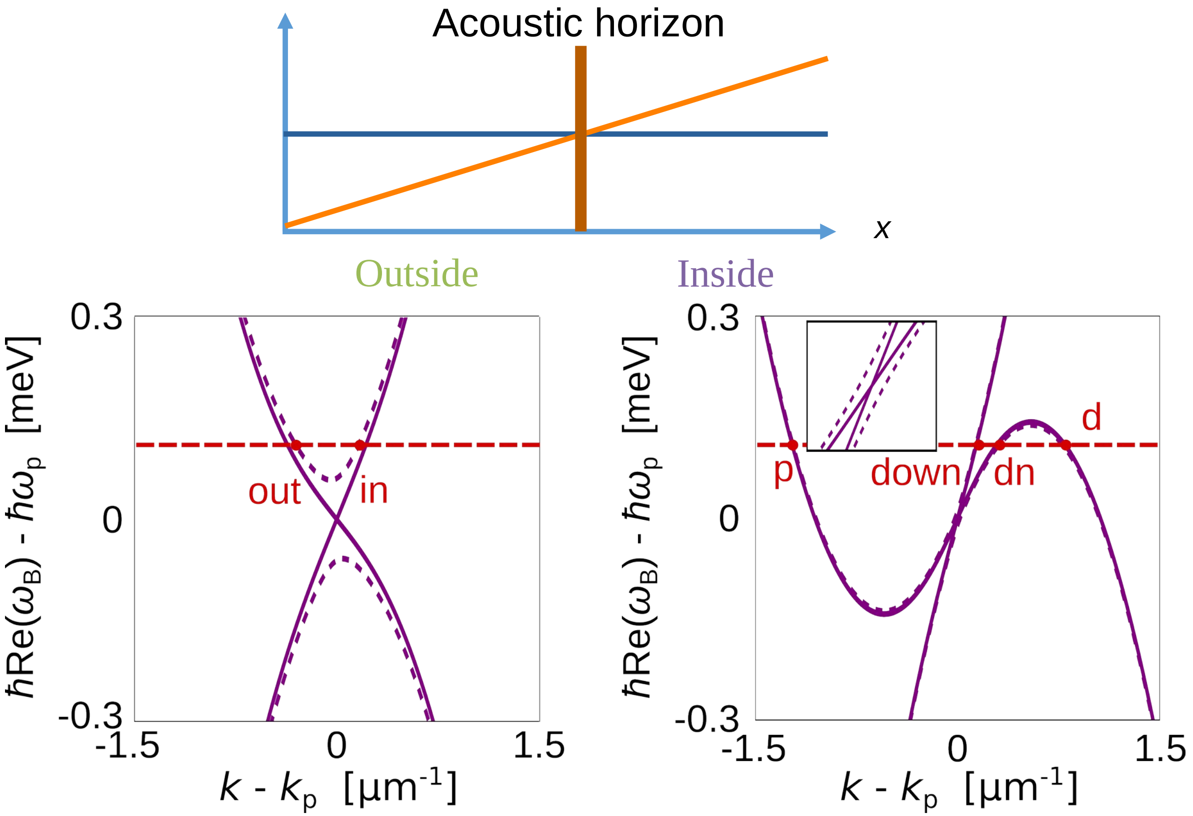
Possible geometries with $\eta_{\mu\nu} = \begin{pmatrix} -(c_s^2 - \boldsymbol{v_0}^2) & -v_o^x & -v_o^y \\ -v_o^x & 1 & 0 \\ -v_o^y & 0 & 1 \end{pmatrix}$

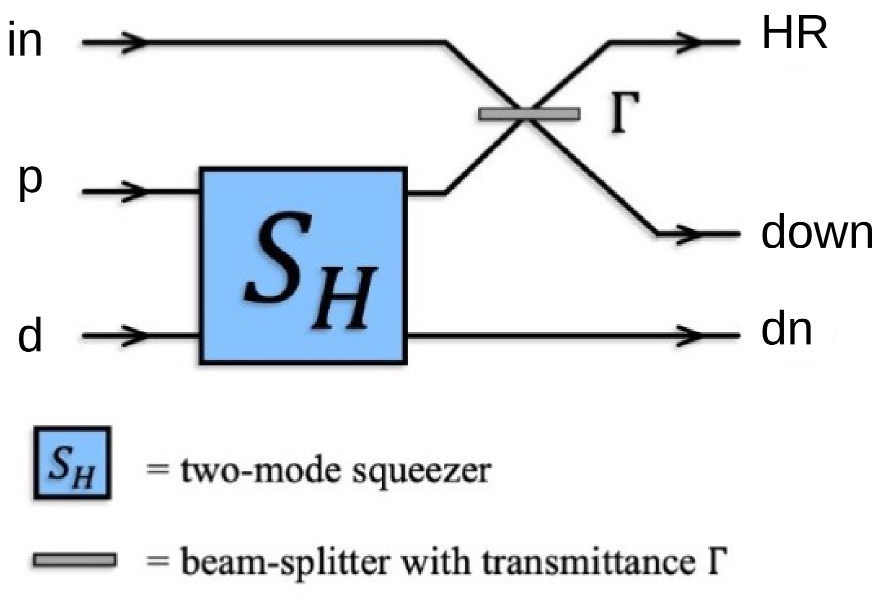
(i) transsonic flow along 1 spatial dimension → stationary 1D spacetime
Horizon where $\mathrm{v}_0 = c_s$

(ii) radially transsonic flow in 2 spatial dimensions → stationary spherically symmetric 2D spacetime
Horizon where $\mathrm{v}_r = c_s$

(iii) radially and azimuthally transsonic flow in 2 spatial dimensions → stationary rotating spacetime
Horizon where $\mathrm{v}_r = c_s$
Ergosurface where $|\boldsymbol{v_0}| = c_s$

Theory: Jacquet *et al* in prep 2025 + EPJD **76** 152 (2022),
Exp: Falque *et al* arXiv:2311.01392





S_H controlled by surface gravity

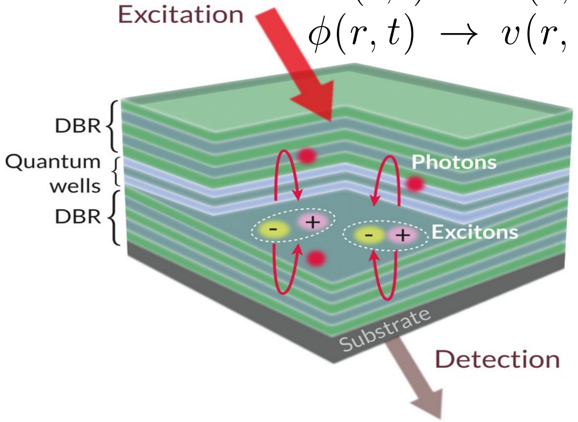
$$\kappa = \frac{1}{2c_s(x)}\frac{d}{dx}\left[v_0^2(x) - c_s^2(x)\right]\Big|_{x_H}$$

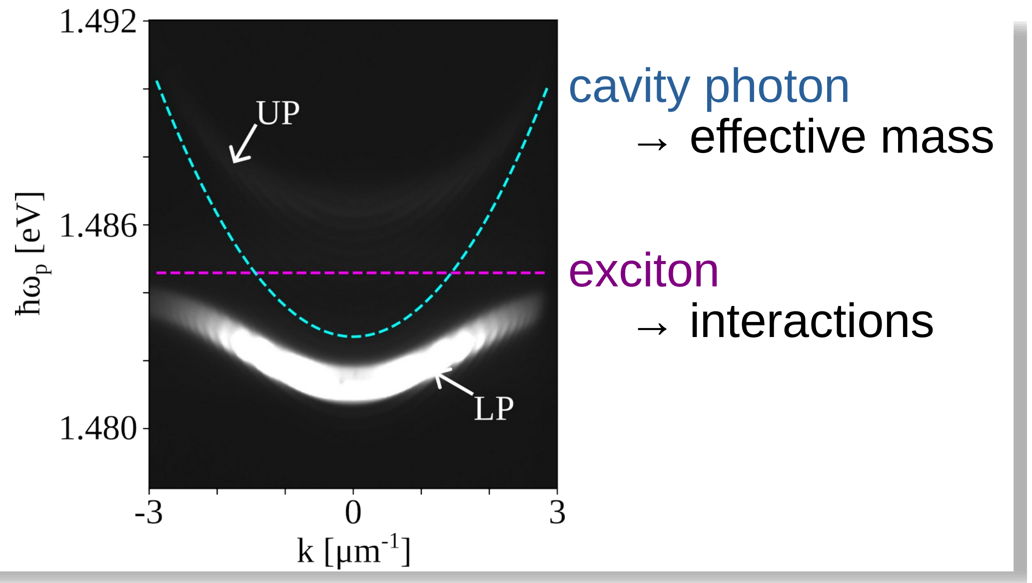
Agullo *et al* "Event horizons are tunable factories of quantum entanglement" Int. Jour. Mod. Phys. D **31** 2242008 (2022)

Microcavity polaritons

Carusotto and Ciutti RMP **85** 299 (2013)

$$\mathrm{I}(\mathrm{r,t}) \rightarrow n(r,t)$$
$$\phi(r,t) \rightarrow v(r,t)$$





Polaritons = photons dressed with material excitations that live in the cavity plane

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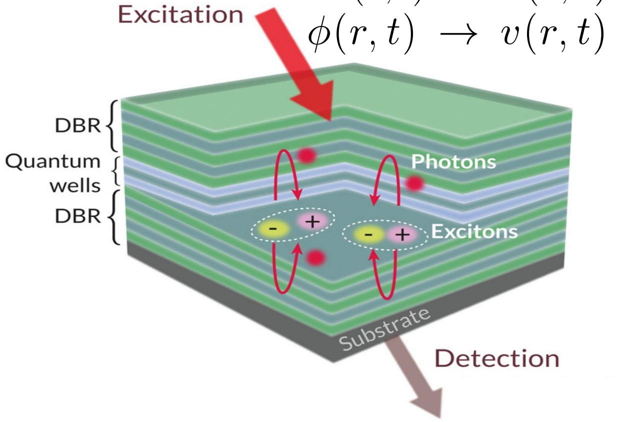
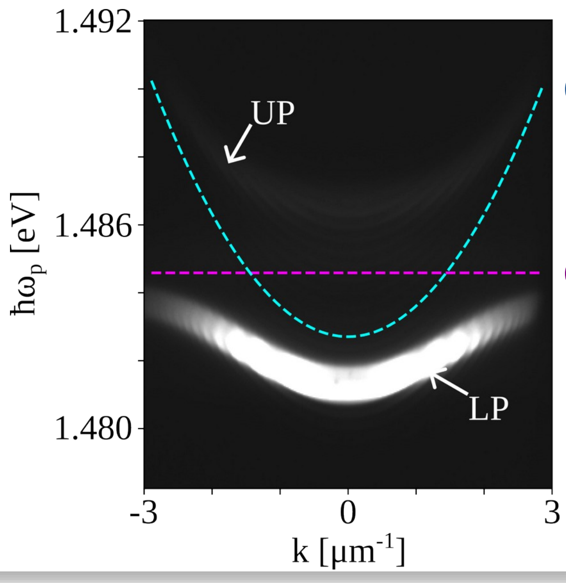




cavity photon
→ effective mass

exciton
→ interactions

Polaritons = photons dressed with material excitations that live in the cavity plane

Dynamics in the cavity plane described by Gross-Pitaevskii (Nonlinear Schrödinger) equation:

$$|\psi|^2$$

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m^*_{LP}} + gn\right)\psi - \frac{i\hbar\gamma}{2}\psi + P(r,t)$$

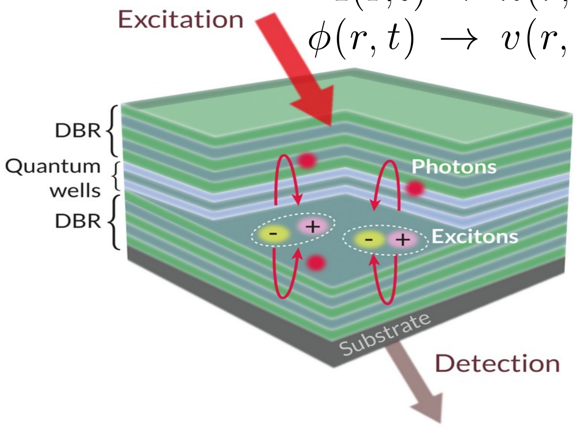
Driven-dissipative dynamics $\rightarrow$ Out-of-equilibrium system

g polariton-polariton interaction constant

γ Losses

P pump

Microcavity polaritons



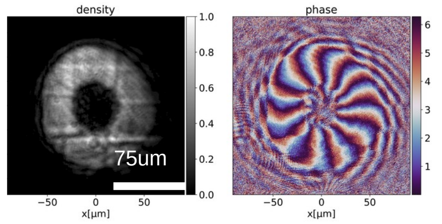
$\mathrm{I(r,t)} \rightarrow n(r,t)$

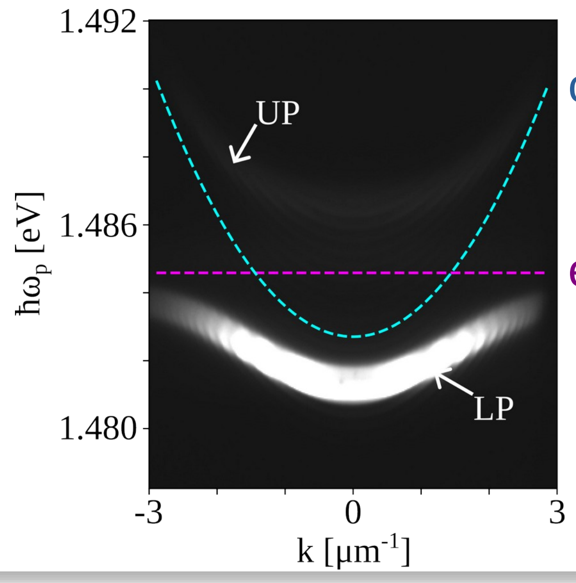
$\phi(r,t) \rightarrow v(r,t)$

SLM to control phase and mode of pump

Imaging photons leaking out of the cavity

$\mathrm{n(r,t)} \rightarrow I(r,t)$ $\mathrm{v(r,t)} \rightarrow \phi(r,t)$





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($n = |\psi|^2$)

Driven-dissipative dynamics → Out-of-equilibrium system

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P pump

Our sample: DBR GaAs, QW InGaAs, Q = 3000, T=4K, $\hbar\gamma/2 = 90\mu eV$

Bogoliubov spectrum out of equilibrium

Falque K *et al.*, arXiv:2311.01392

GPE: $$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m^*_{LP}} + gn\right)\psi - \frac{i\hbar\gamma}{2}\psi + P(r,t)$$

Linearise GPE around steady-state solution $\psi = (\sqrt{n_0} + e^{-\imath\gamma/2}\psi_1)e^{-i(\omega_p t + \phi_p r)}$

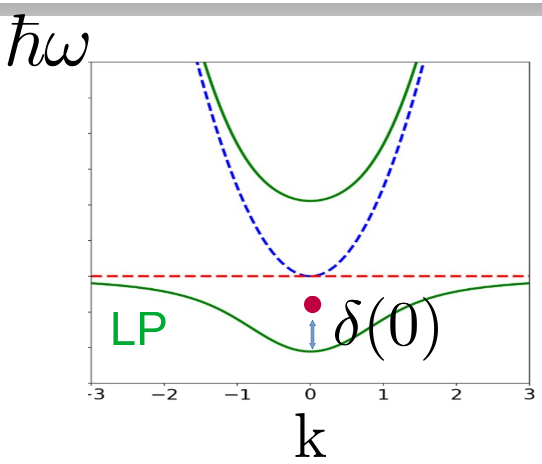
→ Bogoliubov – de Gennes dynamics for ψ_1

WKB dispersion relation $$\omega^{\pm}(\delta k) = \pm\sqrt{(\alpha^2 k^4 + (k^2 + m^2_{det})c_s^2} - \imath\frac{\gamma}{2}$$

higher order derivatives — pump-dependent mass — spectral linewidth

Quasi-resonant photon injection

$$\delta(k_p) = \omega_p - \omega_0 - \frac{\hbar k_p^2}{2m}$$
$$\delta(0) > \frac{\sqrt{3}}{2}\gamma$$



Pump-dependent mass

$$\mathrm{m}_{det} \propto \delta(0) - gn_0$$

$$\left[\frac{1}{\sqrt{|\eta|}}\partial_\mu\sqrt{|\eta|}\eta^{\mu\nu}\partial_\nu - \frac{(m_{\det})^2}{\hbar^2}\right]\psi_1 = 0$$

Falque K *et al.*, arXiv:2311.01392

Bogoliubov spectrum out of equilibrium

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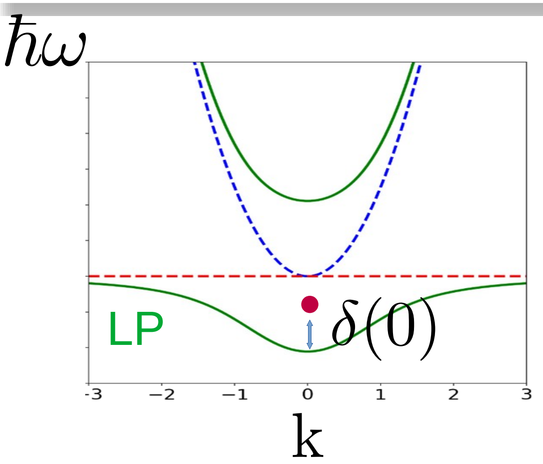
nonlinearities — pump-dependent mass — spectral linewidth



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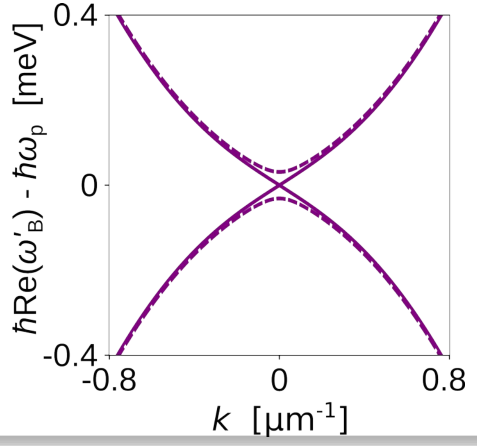


Norm of Bogoliubov excitations

Expansion of acoustic field in terms of excitations

$$\psi_1 = \int d\omega (f_\omega \hat{a}_\omega + f_\omega^* \hat{a}_\omega^\dagger)$$

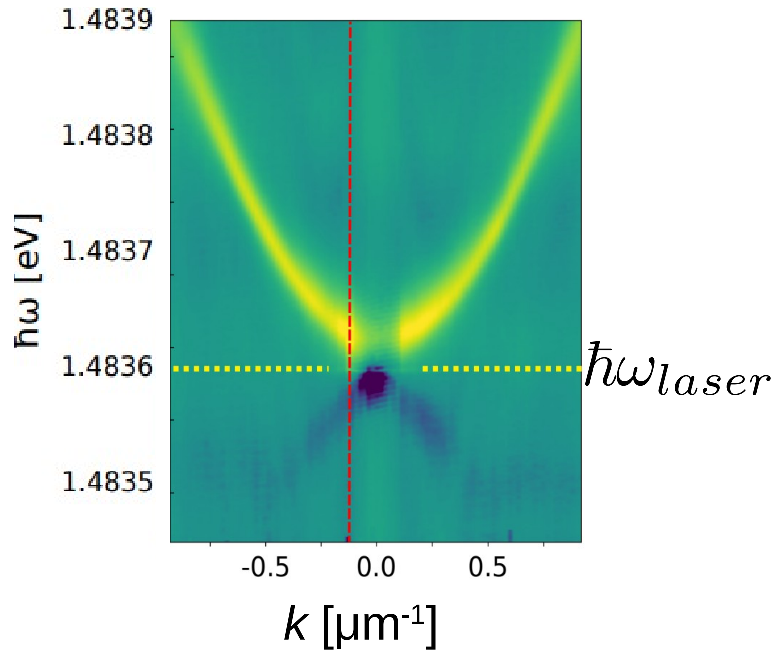
In fluid rest frame, excitations have frequencies

$$\omega^\pm(\delta k) = \pm\sqrt{(\alpha^2 k^4 + k^2 + m_{det}^2 c_s^2} - i\frac{\gamma}{2}$$

Norm of excitations = Noether charge

$$\mathrm{Q}(\mathrm{f}_\omega) = i\int dx (f_\omega^* \partial_t f_\omega - \partial_t f_\omega^* f_\omega)$$

Dispersion relation in fluid rest frame

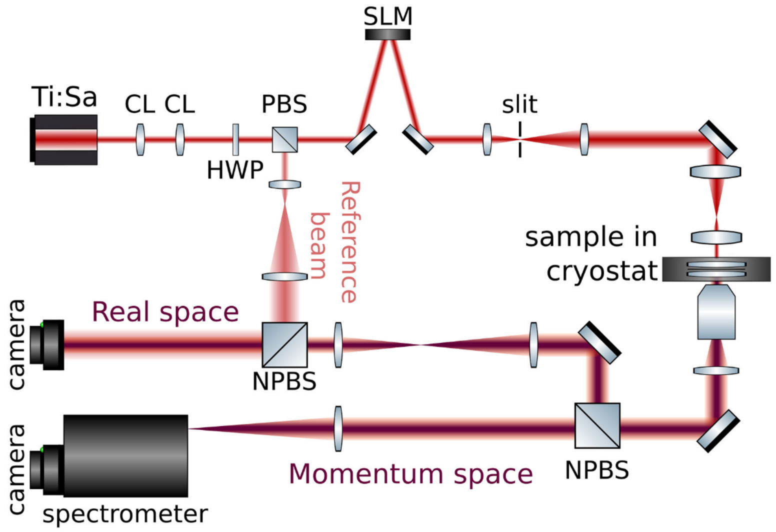


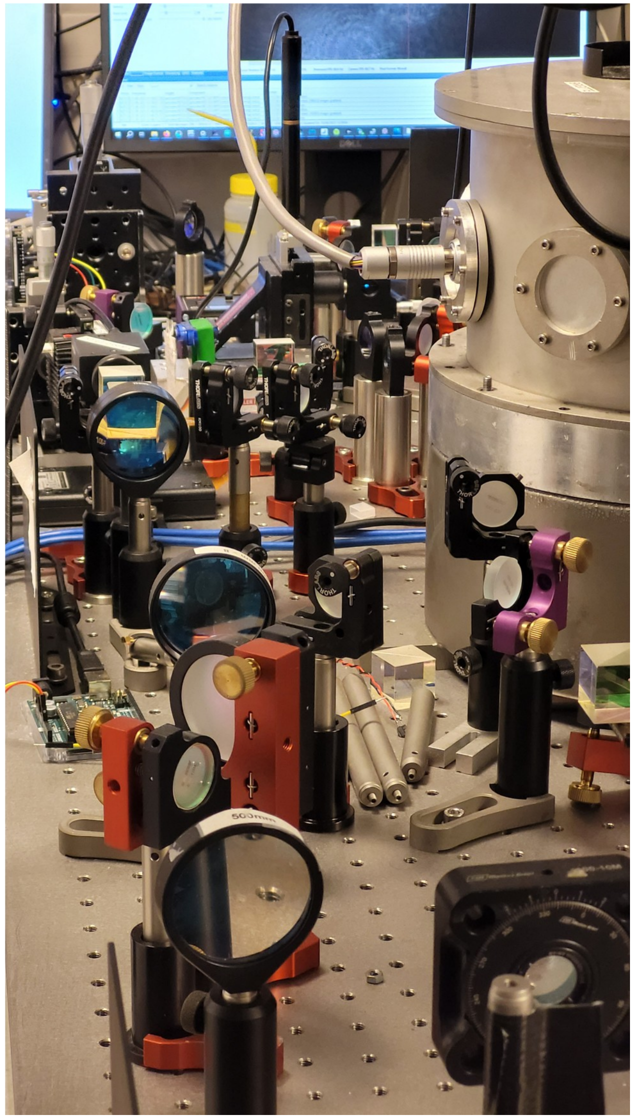
In fluid rest frame:

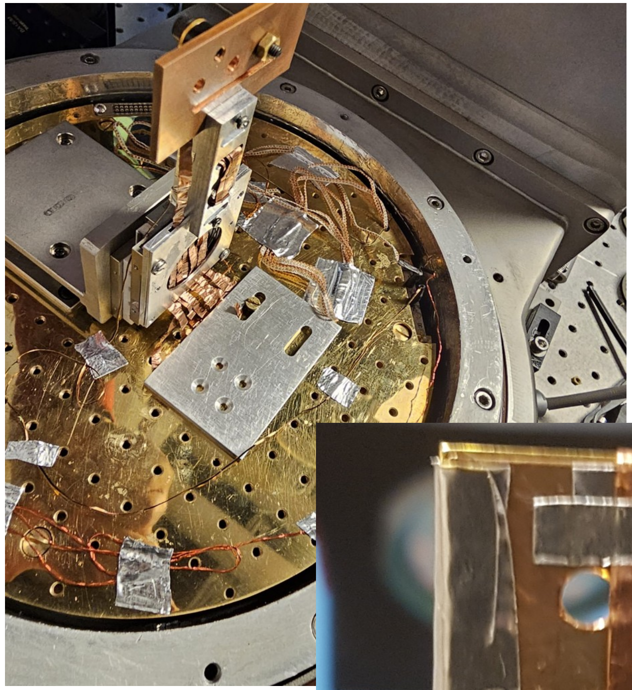
$\omega > \omega_{laser}$ positive-norm mode

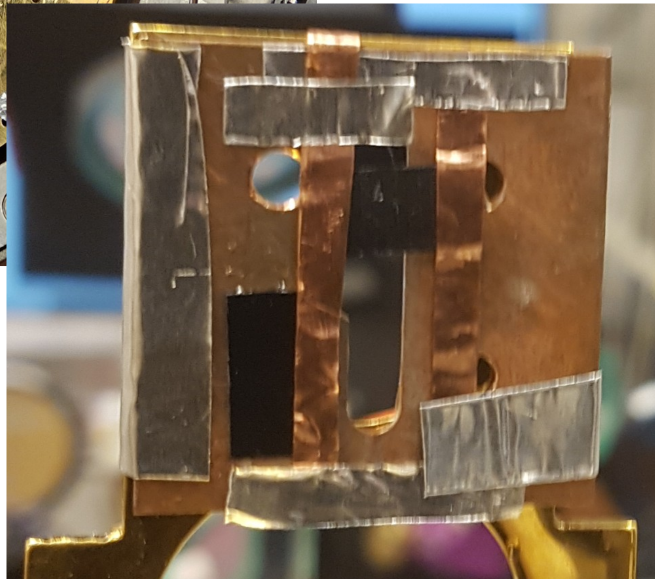
$\omega < \omega_{laser}$ negative-norm mode

Experimental scheme





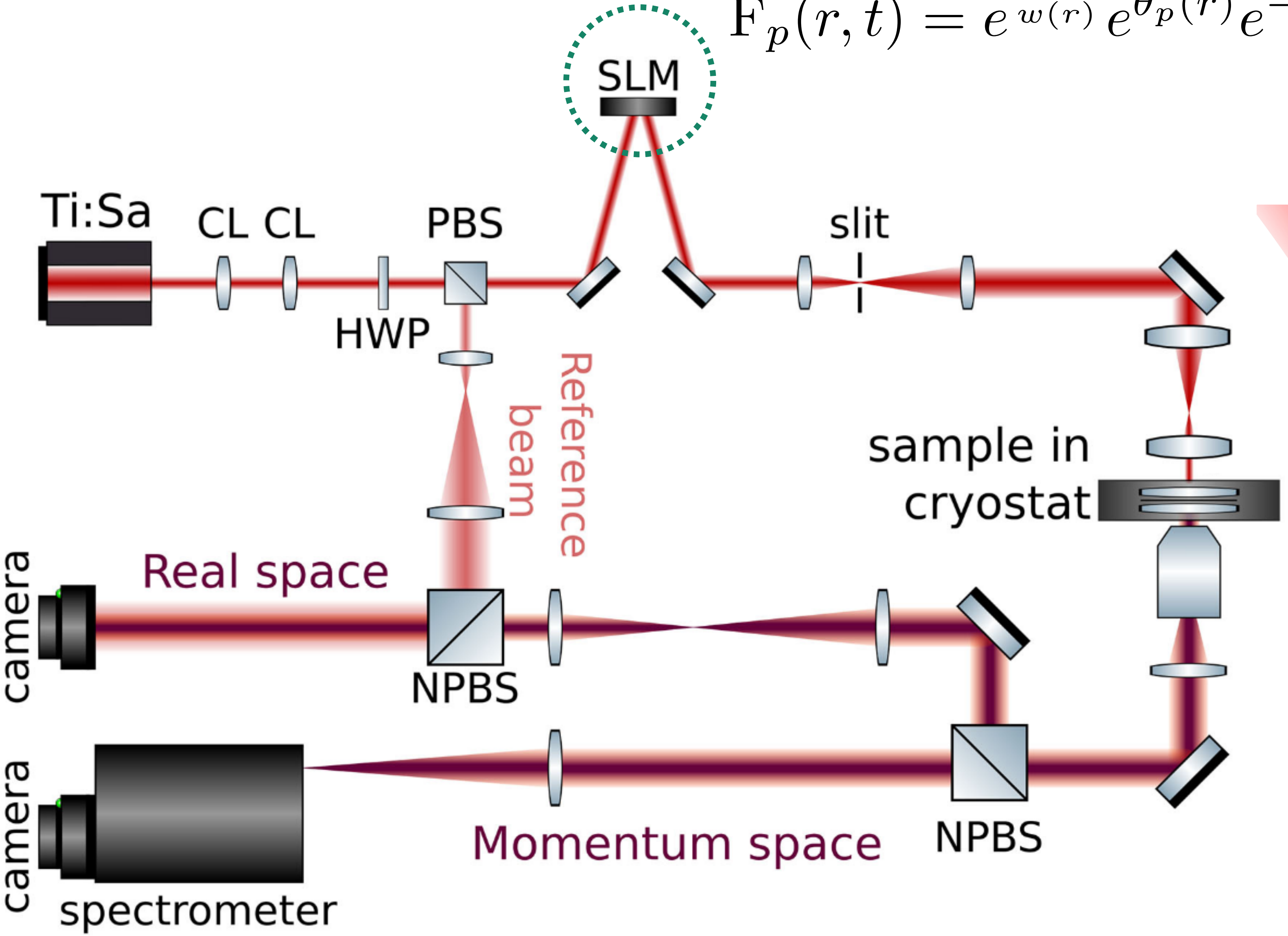


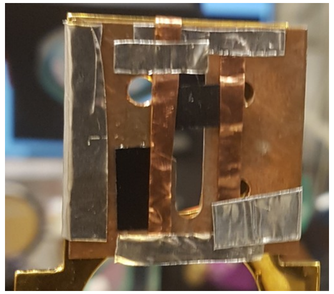


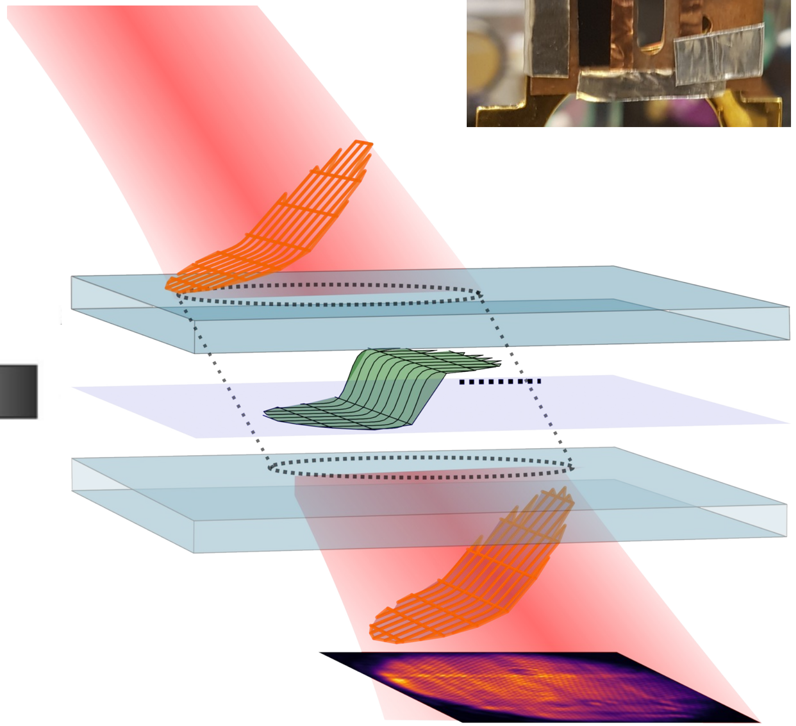
How to create the effective spacetime?

Falque K *et al.*, arXiv:2311.01392

$$F_p(r,t) = e^{\frac{-r^2}{w(r)}} e^{\theta_p(r)} e^{-\imath \omega_p t}$$



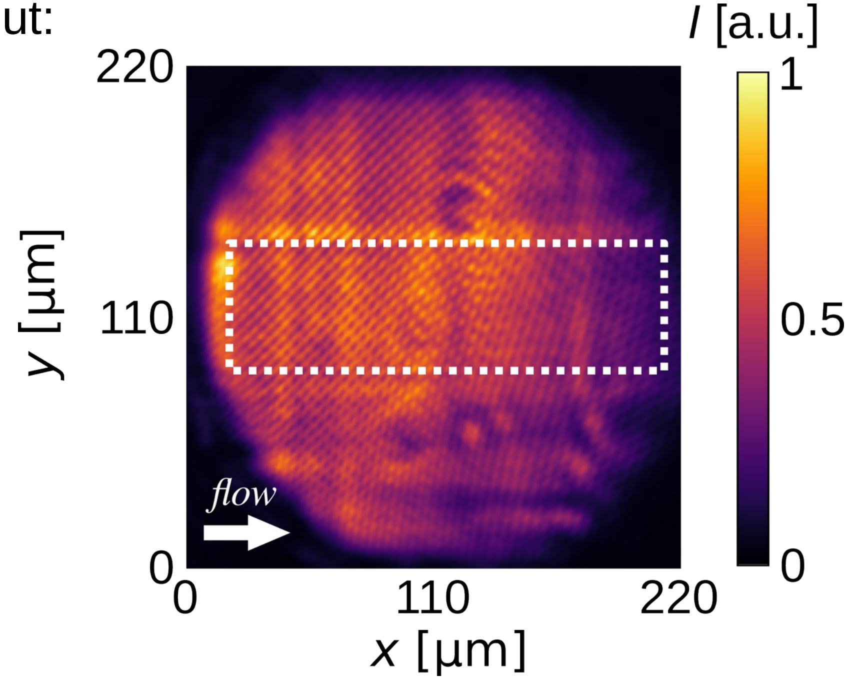




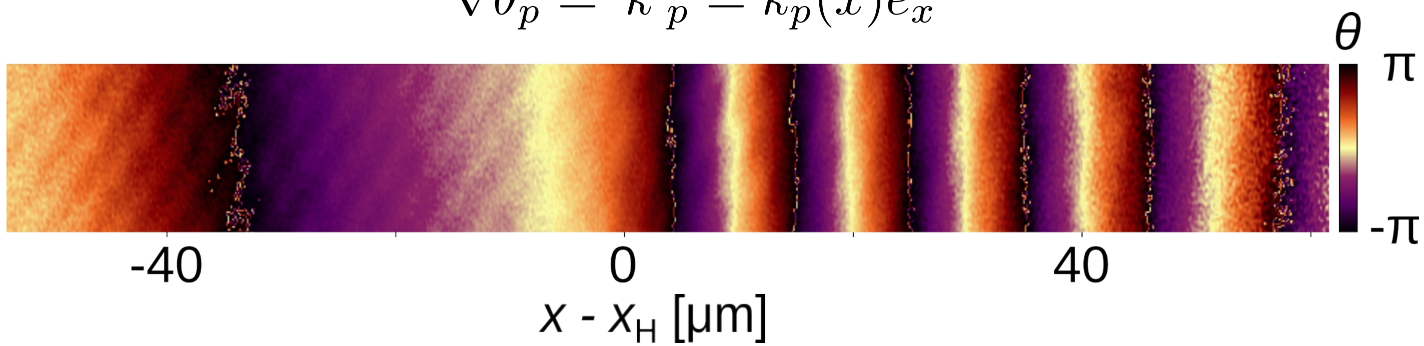
$w(r)=180\mu m$

Falque K *et al.*, arXiv:2311.01392

Output:



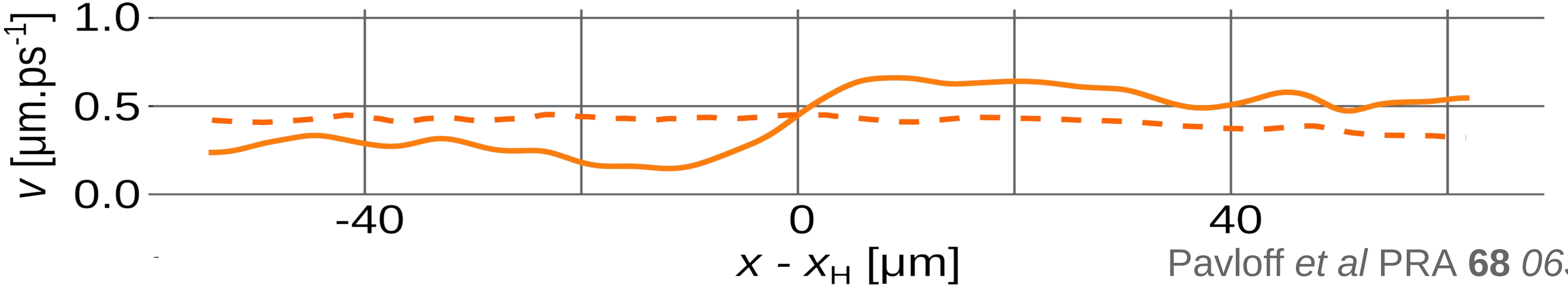
$$\nabla\theta_p = \overrightarrow{k}_p = k_p(x)\vec{e}_x$$



$$\mathrm{v}_0(x) = \frac{\hbar}{m^*}\partial_x\theta_p(x)$$ (solid line)

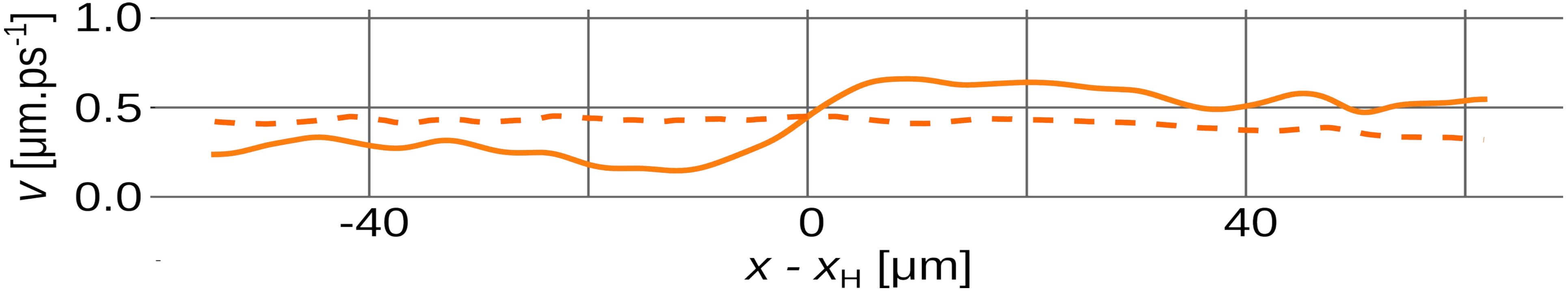
$$\mathrm{v}_0(x) = \frac{v_\mathrm{d}-v_\mathrm{u}}{2}\,tanh\left(\frac{x-x_\mathrm{H}}{w_\mathrm{H}}\right) + \frac{v_\mathrm{d}+v_\mathrm{u}}{2}$$

$$\mathrm{c}_s(x) = \sqrt{\frac{gn_0(x)}{m}}$$ (dashed line)



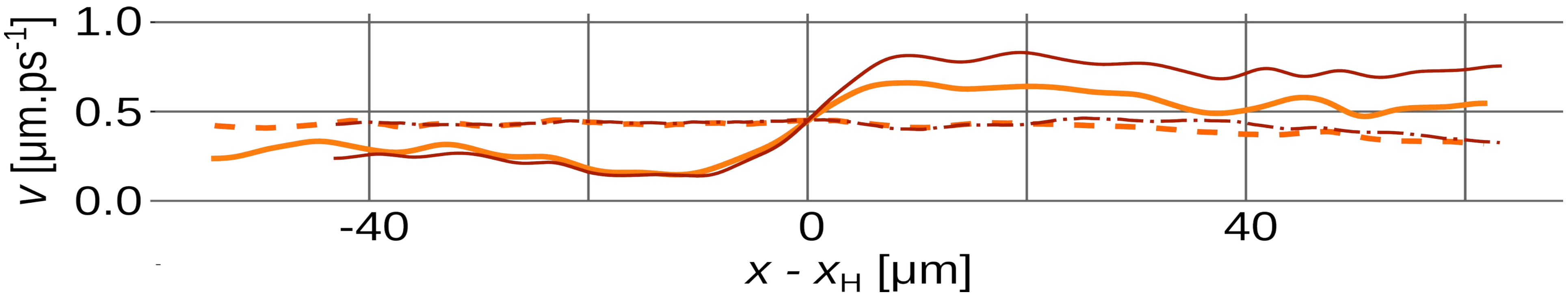
Pavloff *et al* PRA **68** *063608 (2003)*

Falque K *et al.*, arXiv:2311.01392



(solid line)	$v_u = 0.27\mu m.ps^{-1}$	$v_d = 0.53\mu m.ps^{-1}$
(dashed line)	$c_s = 0.4\mu m.ps^{-1}$	$c_s = 0.4\mu m.ps^{-1}$
$M = v/c_s$	0.6	1.4

Falque K *et al.*, arXiv:2311.01392

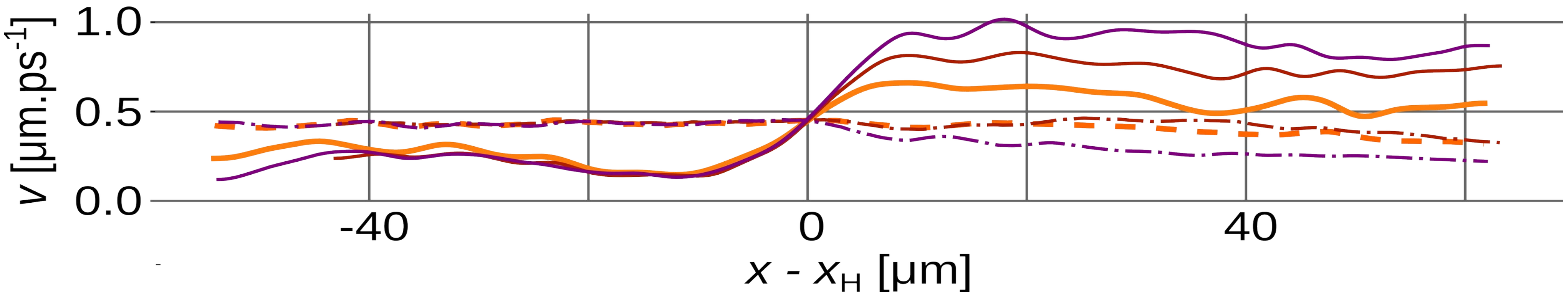


$M = v/c_s$	0.6	1.4
$M = v/c_s$	0.6	1.9

$$v_0(x) = \frac{v_d - v_u}{2} \, tanh\left(\frac{x - x_H}{w_H}\right) + \frac{v_d + v_u}{2}$$

Tuning the effective spacetime

Falque K *et al.*, arXiv:2311.01392



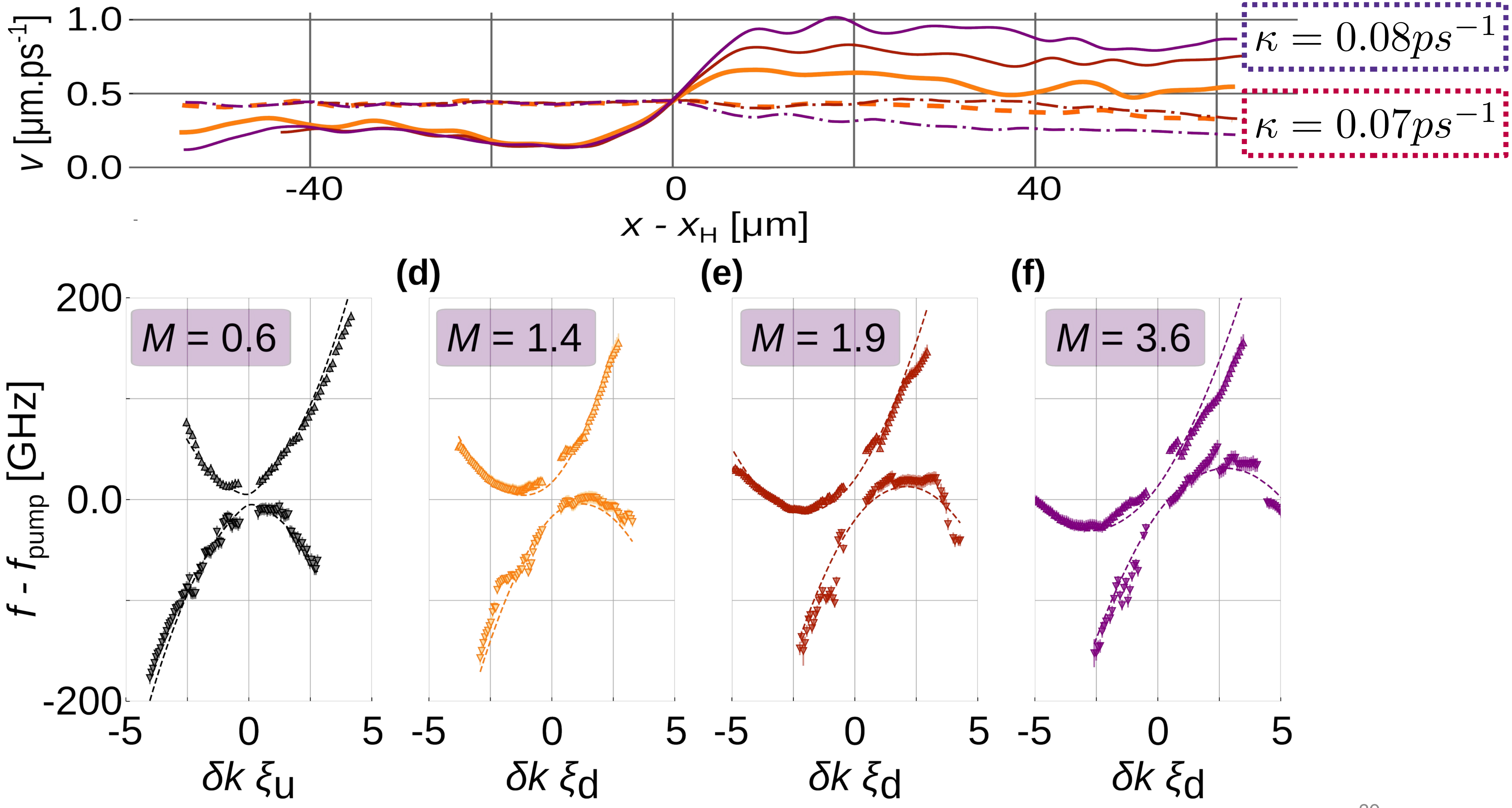
$M = v/c_s$	0.6		1.4
$M = v/c_s$	0.6	$0.07 \mathrm{ps}^{-1}$	1.9
$M = v/c_s$	0.6	$0.08 \mathrm{ps}^{-1}$	3.6

Strength of emission controlled by

$$\kappa = \frac{1}{2c_s(x)} \frac{d}{dx}\left[v_0^2(x) - c_s^2(x)\right]\Big|_{x_H}$$

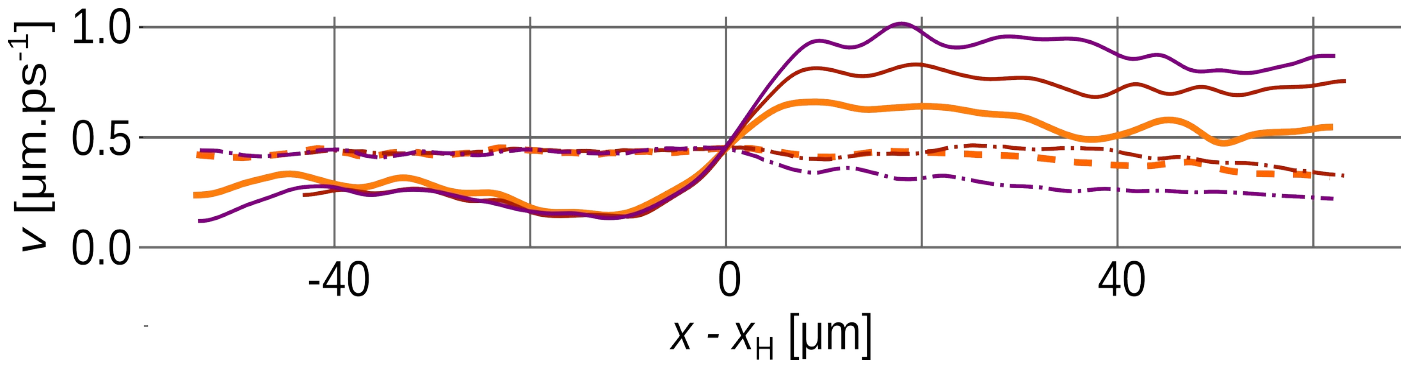
Bogoliubov spectrum in transsonic fluid

Falque K *et al.*, arXiv:2311.01392

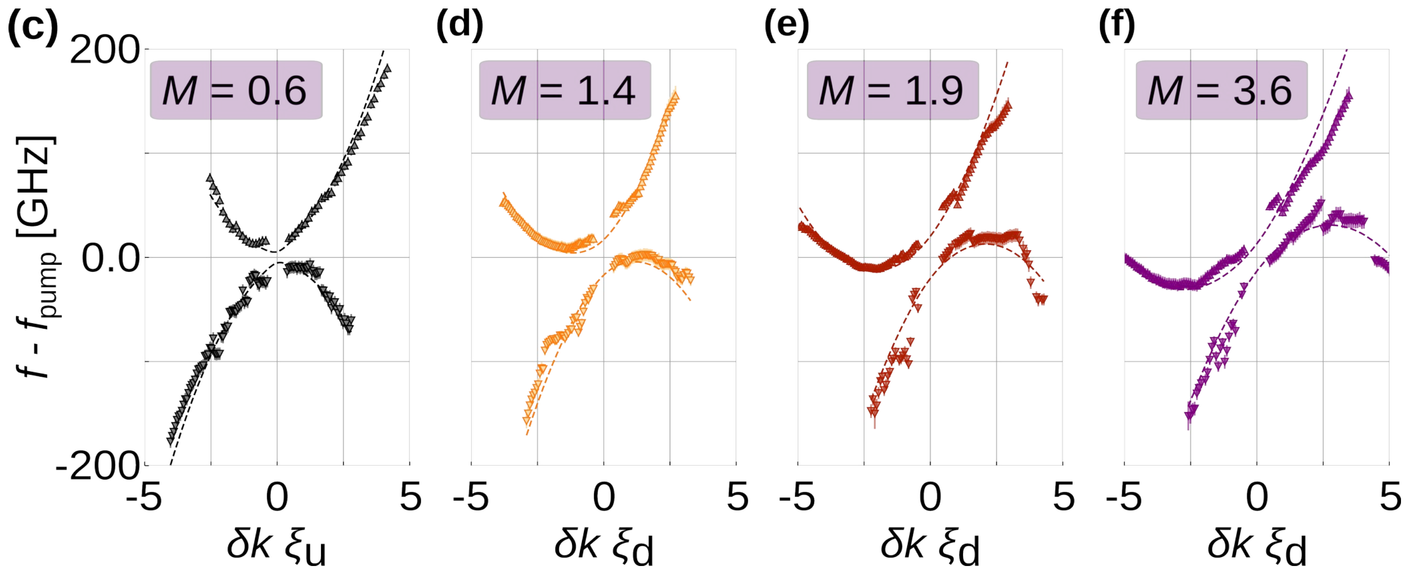


in another system -- PRL 124 141101 (2020)

But why? An example



Controlled curvature of the horizon independently from asymptotic spacetime properties



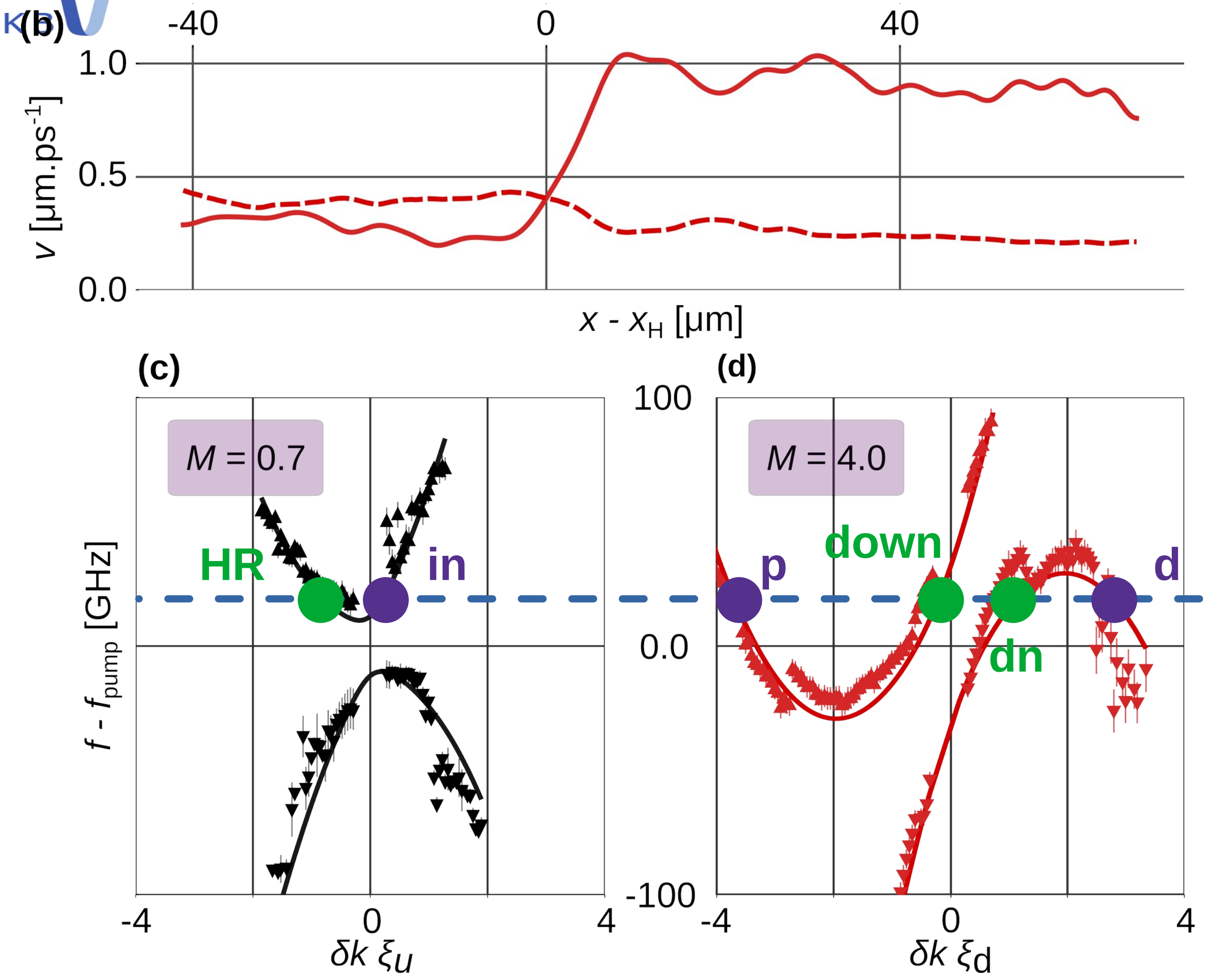
Controllable, space-dependent field mass:

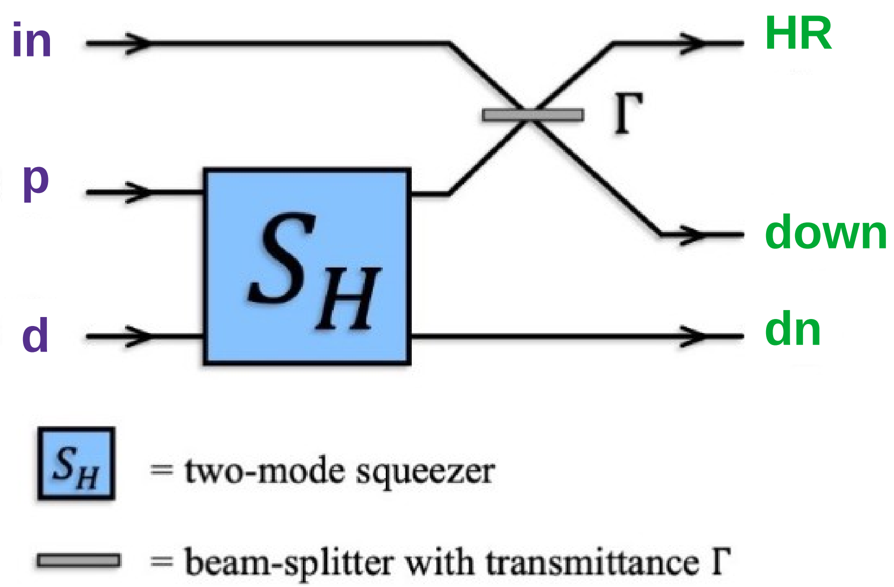
Open/close mass gap → control tunnelling across horizon

Del Porro F *et al.*, arXiv:2406.14603

→ measure amplitudes
→ behaviour of entanglement?

Entanglement by the Hawking effect





S_H controlled by surface gravity $\kappa = 0.11ps^{-1}$

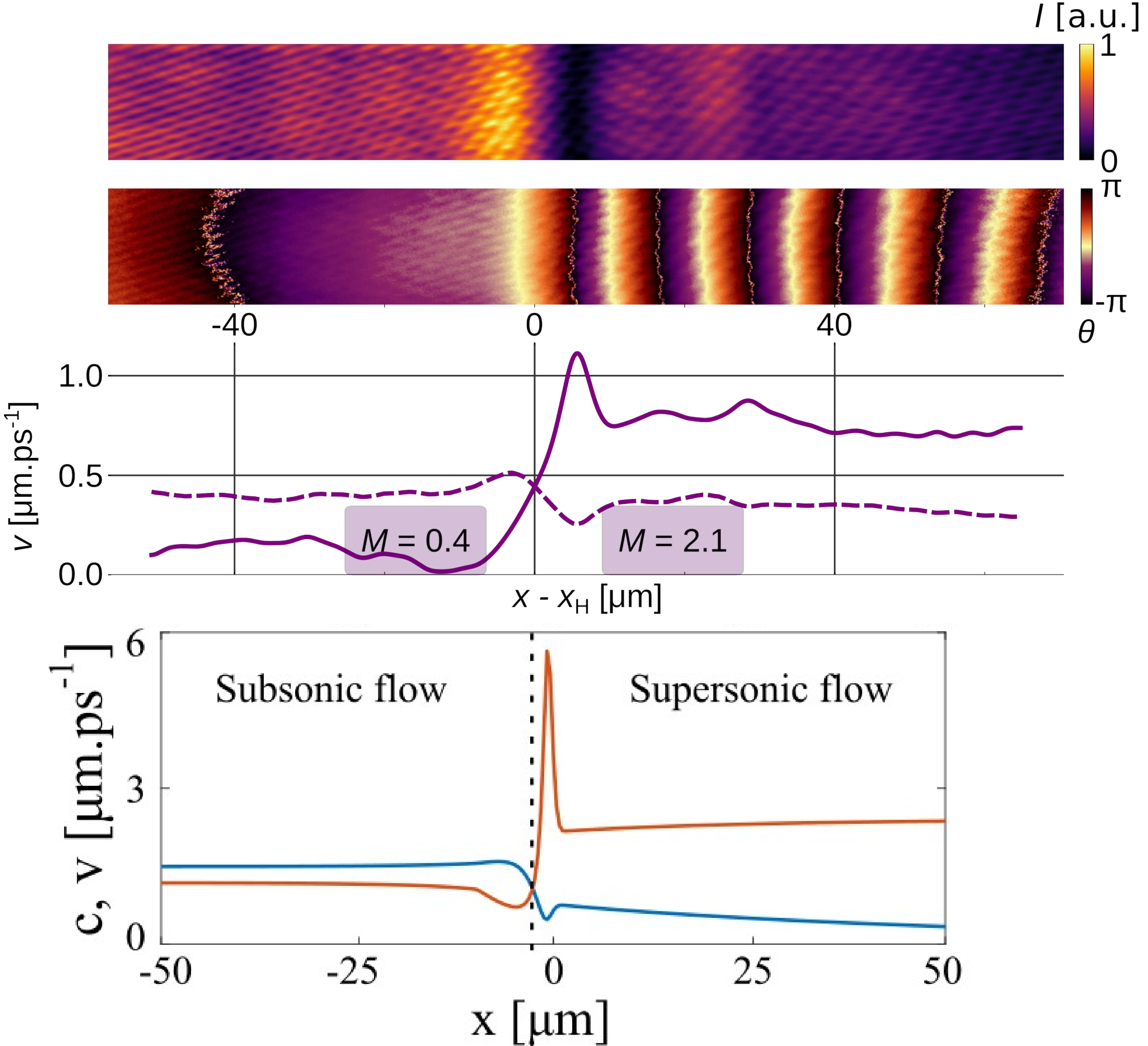
But what about Γ?

Open question: how does the degree of entanglement vary with κ and M_u, M_d?

Direction of propagation: group velocity $\partial\omega/\partial k$

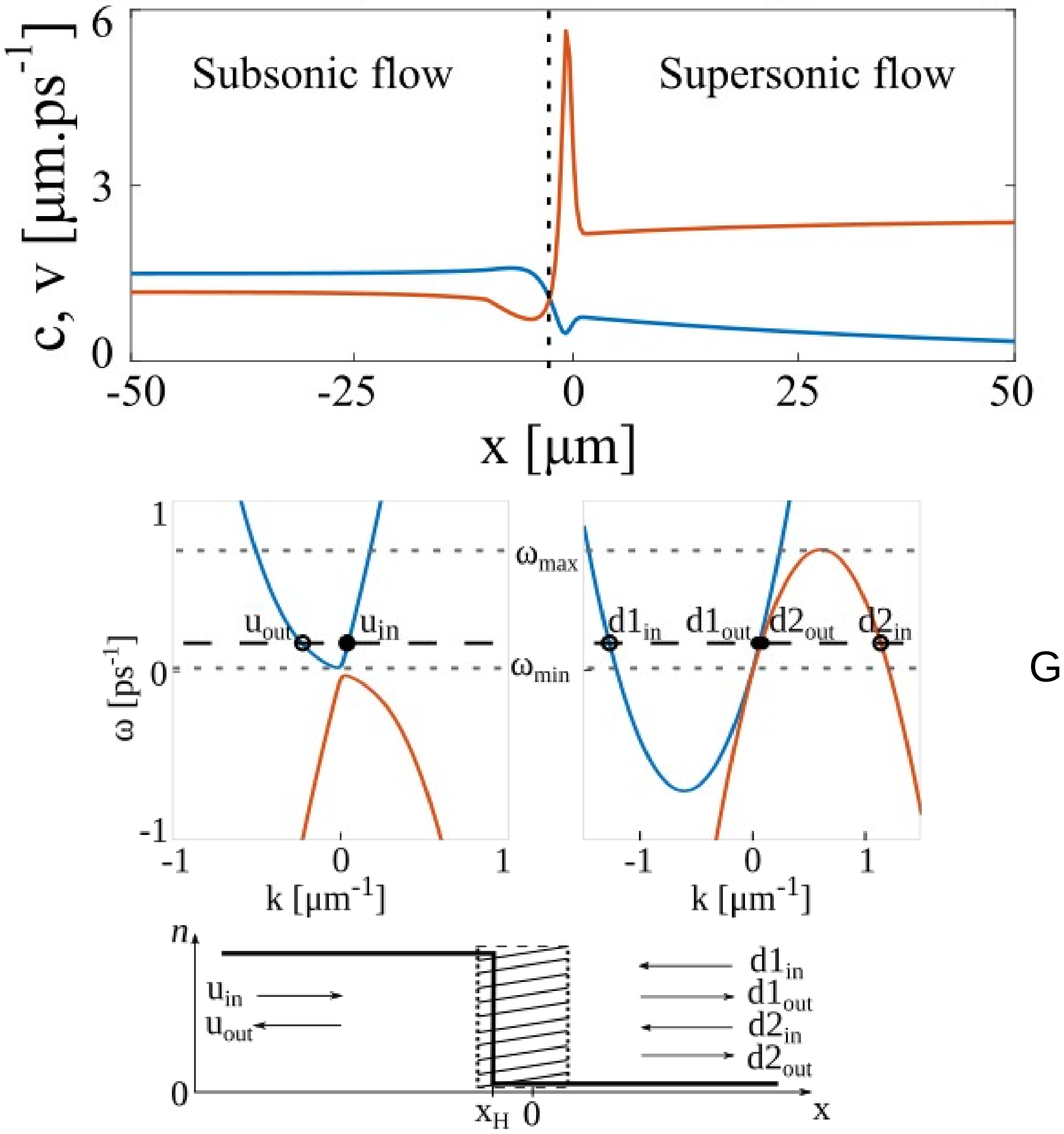
Hawking effect due to scattering on stationary potential

Tuning the effective spacetime



Dip in density == resonator

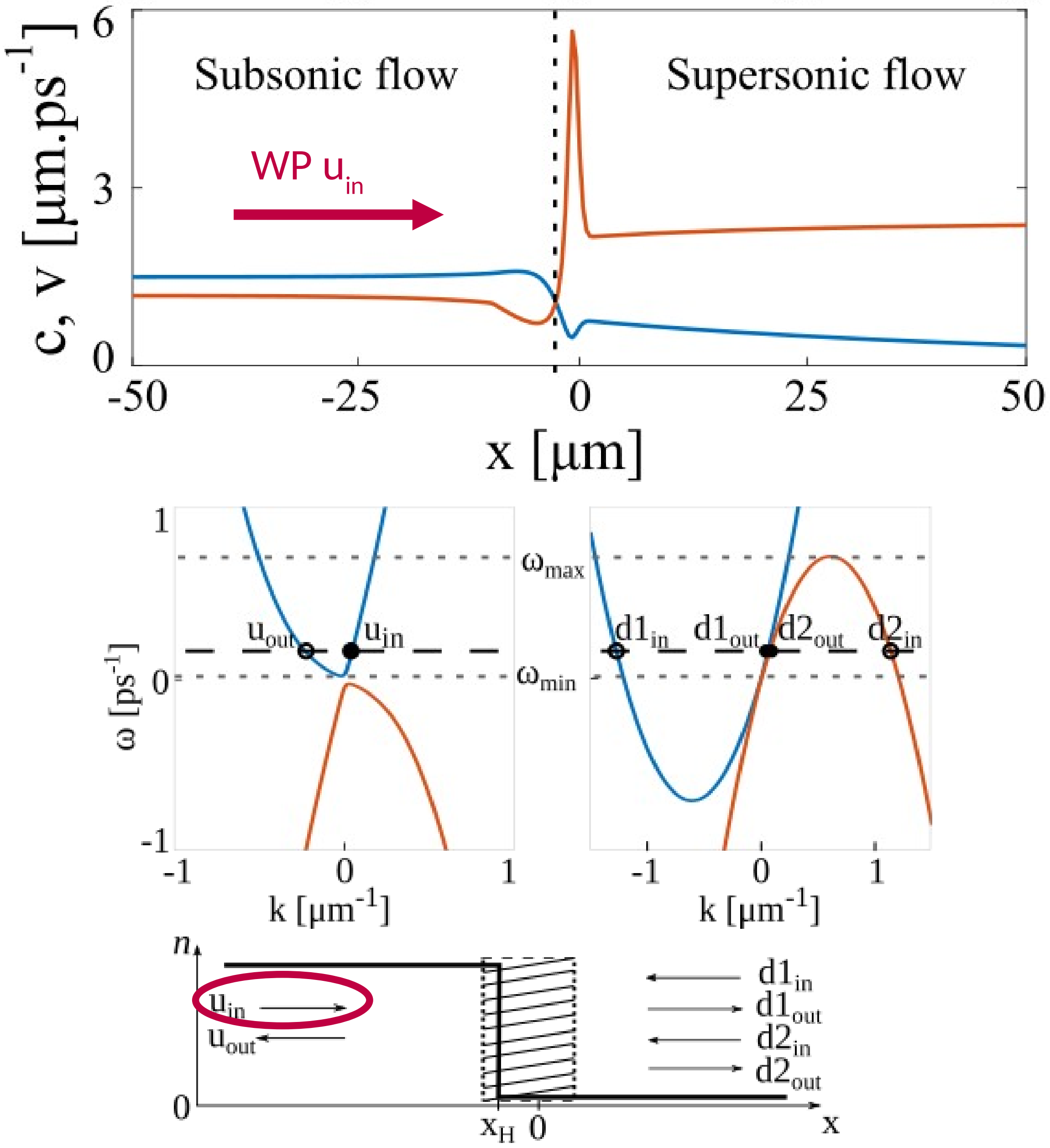
Modes of the system



Dip in density == resonator

Group velocity of modes → propagation w.r.t horizon

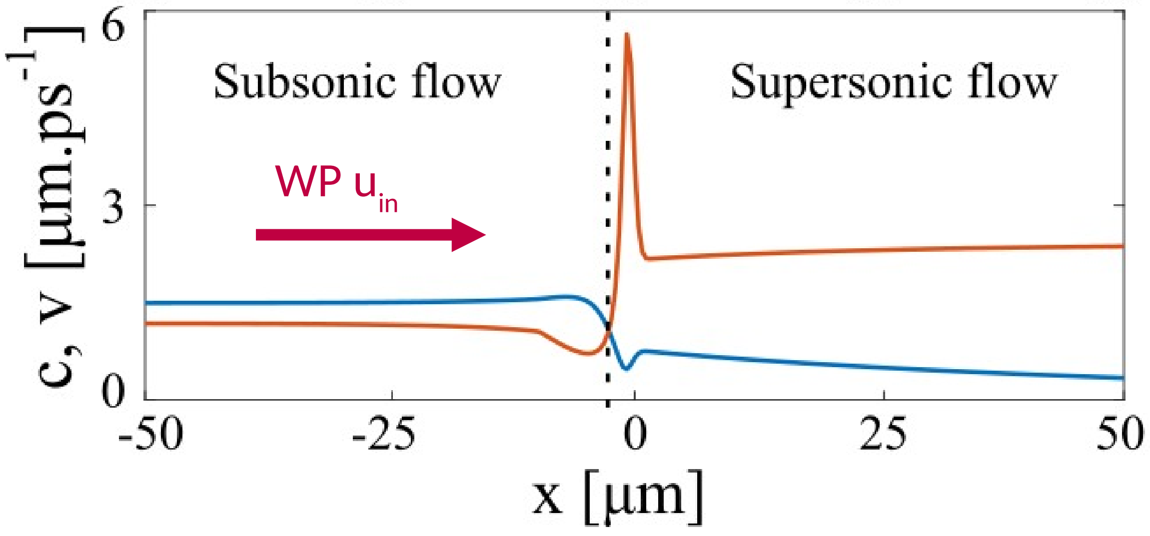
Jacquet *et al* PRL **130** 111501 (2023)



Send wavepacket u_{in} toward horizon:

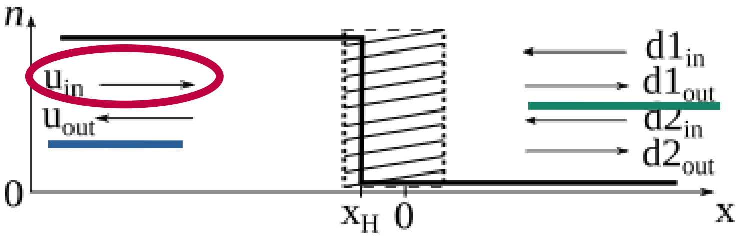
Dip in density == resonator

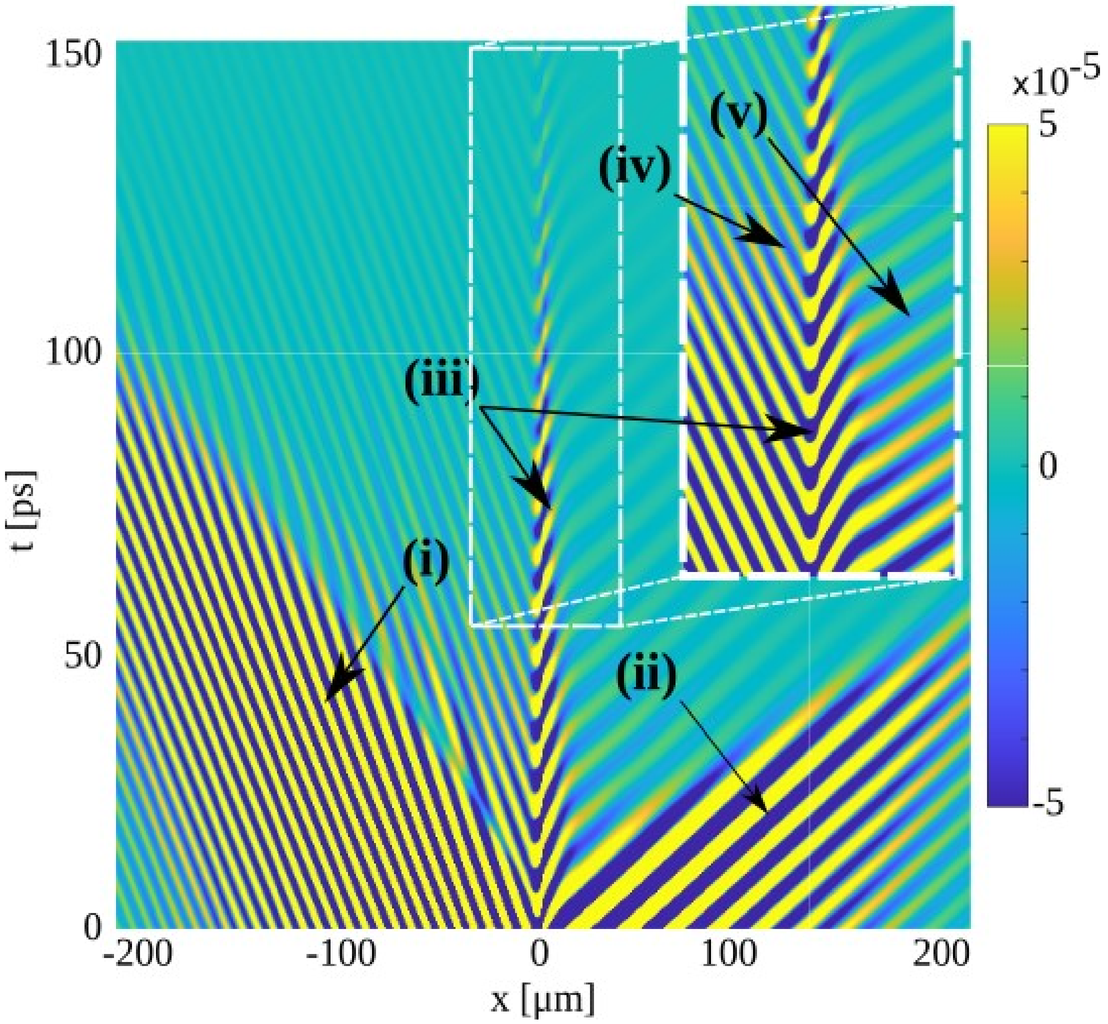
Perturbing the horizon



Send wavepacket u_{in} toward horizon:

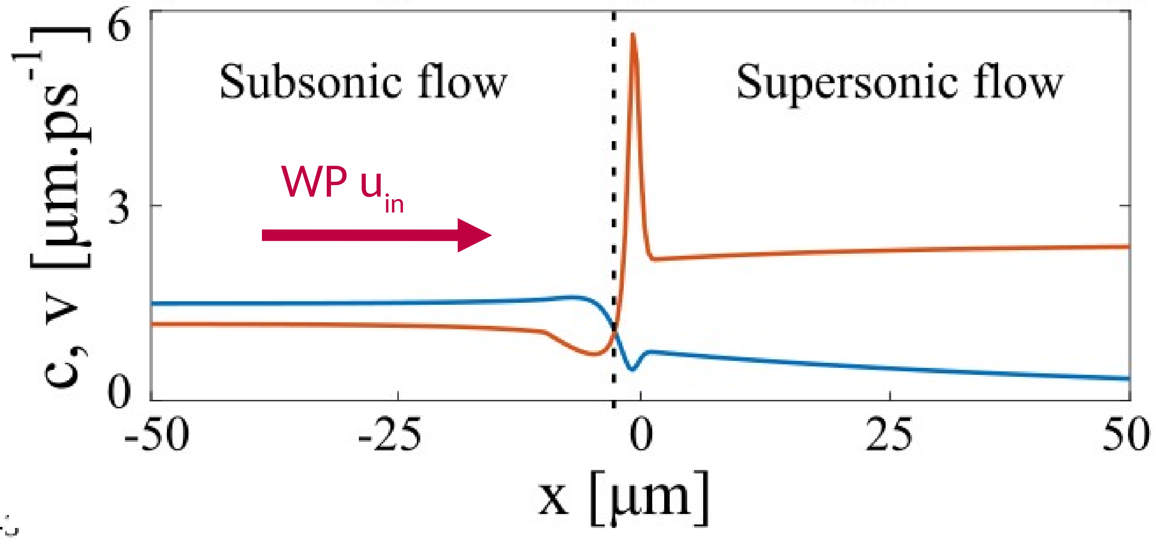
(i) reflection
(ii) transmission
(iii) density @horizon oscillates and dampens
(iv) density @horizon couples with mode propagating outward
(v) density @horizon couples with mode propagating inward

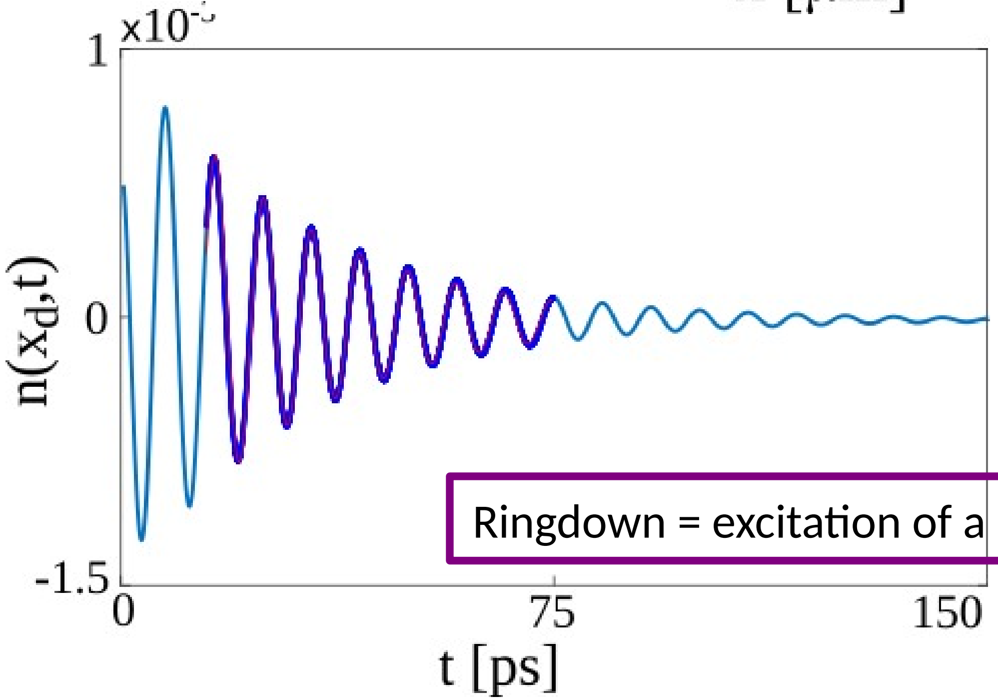


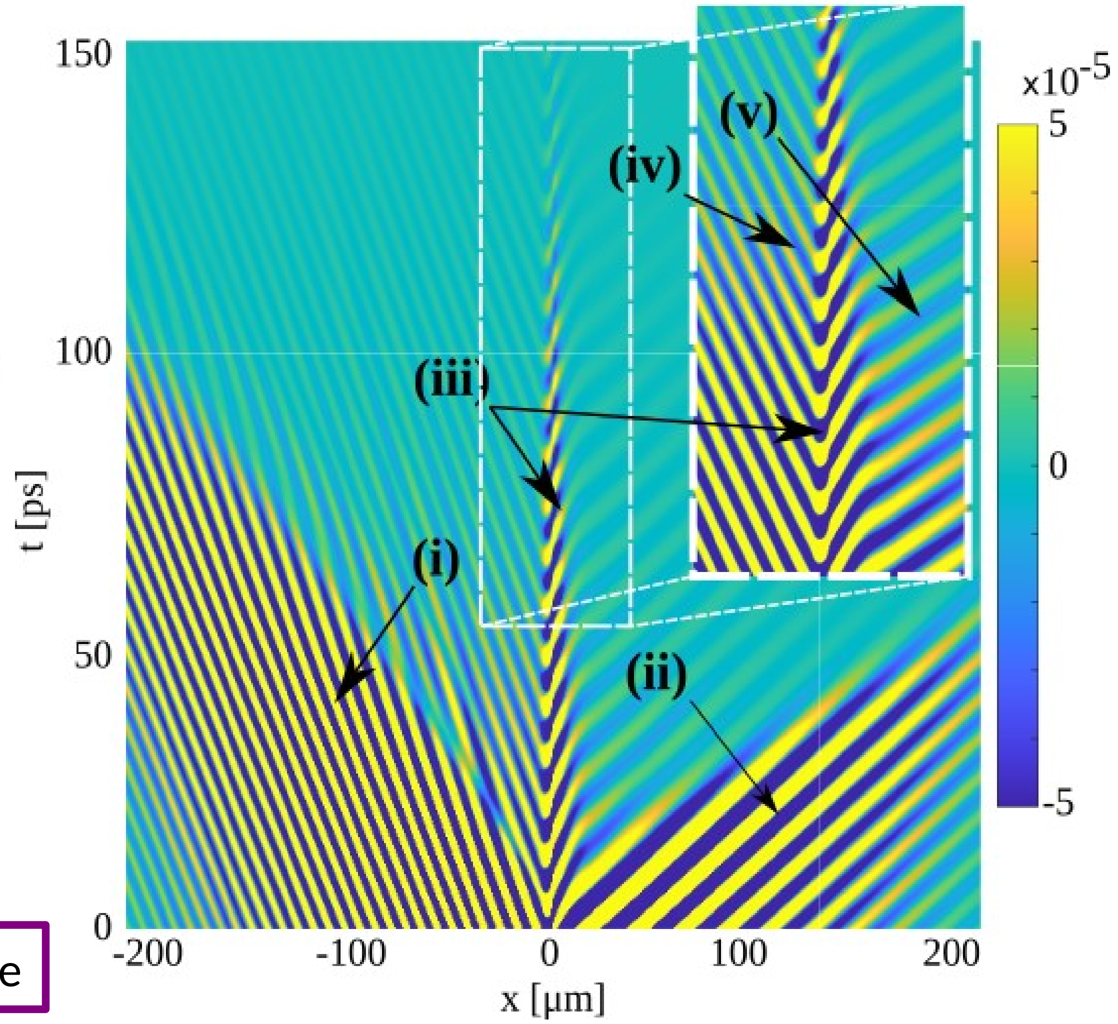


Perturbing the horizon

Jacquet *et al* PRL **130** 111501 (2023)







Ringdown = excitation of a quasi-normal mode

$\mathrm{Re}(\omega)$ Frequency of oscillation

$\mathrm{Im}(\omega)$ Decay rate < (bare polariton lifetime)

Scattering of vacuum fluctuations: long and strong Hawking correlations

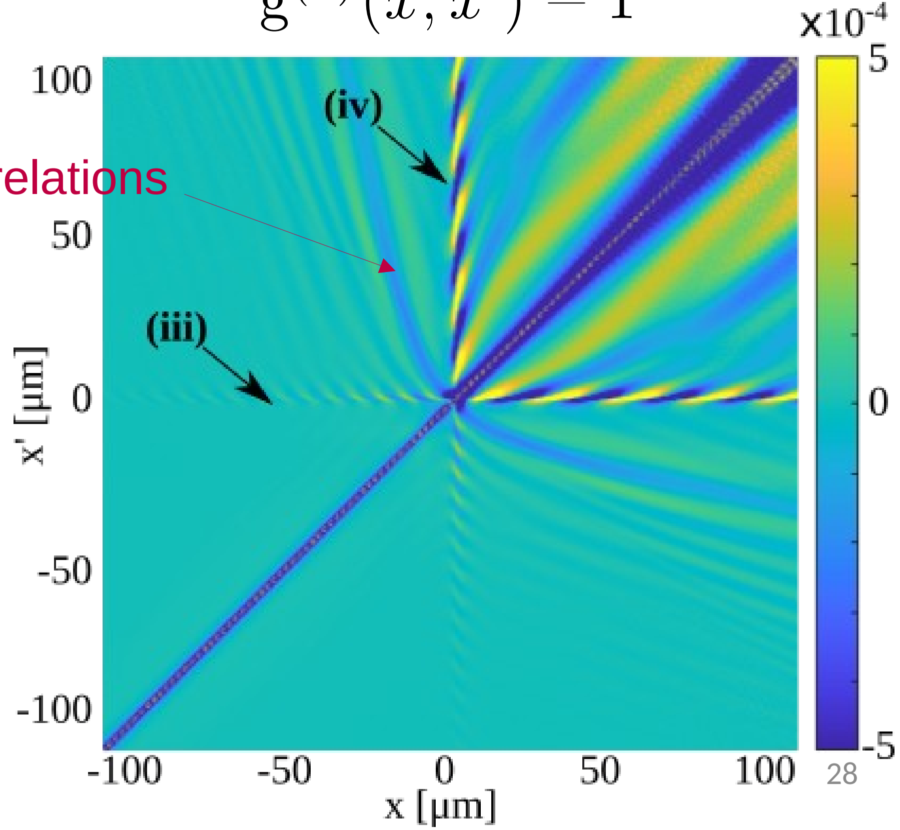
Numerical simulation: Truncated Wigner Approximation
(1 billion realisations)

Measure equal time correlations

$$g^{(2)}(x,x') = \frac{\langle \Psi(x)^\dagger \Psi(x')^\dagger \Psi(x) \Psi(x') \rangle}{\langle \Psi(x)^\dagger \Psi(x) \rangle \langle \Psi(x')^\dagger \Psi(x') \rangle}$$

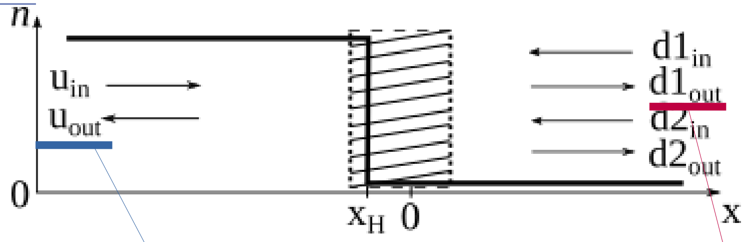
$$g^{(2)}(x,x') - 1$$

Hawking correlations



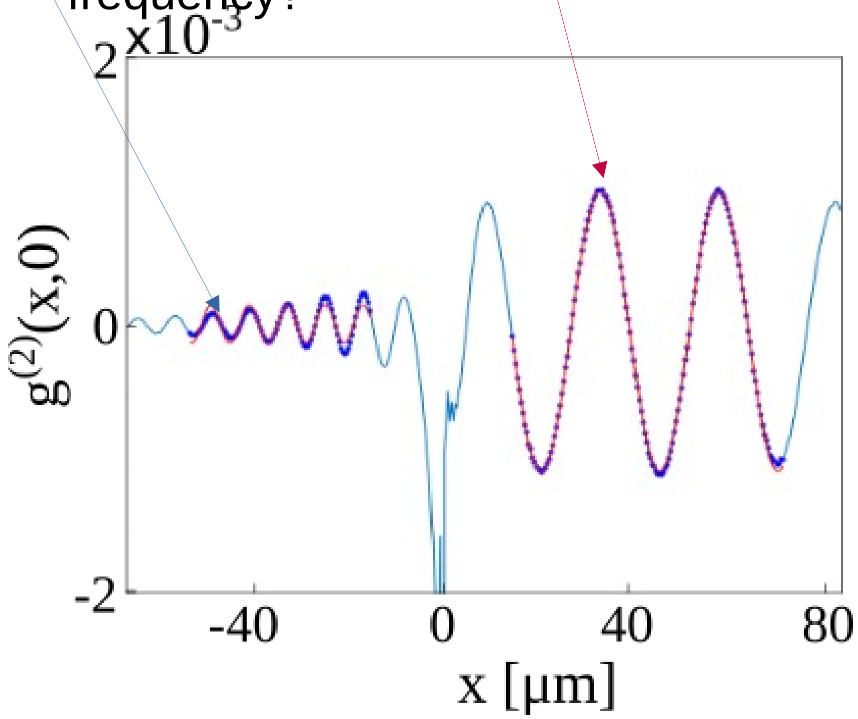
(iii) horizon - outside
(iv) horizon - inside

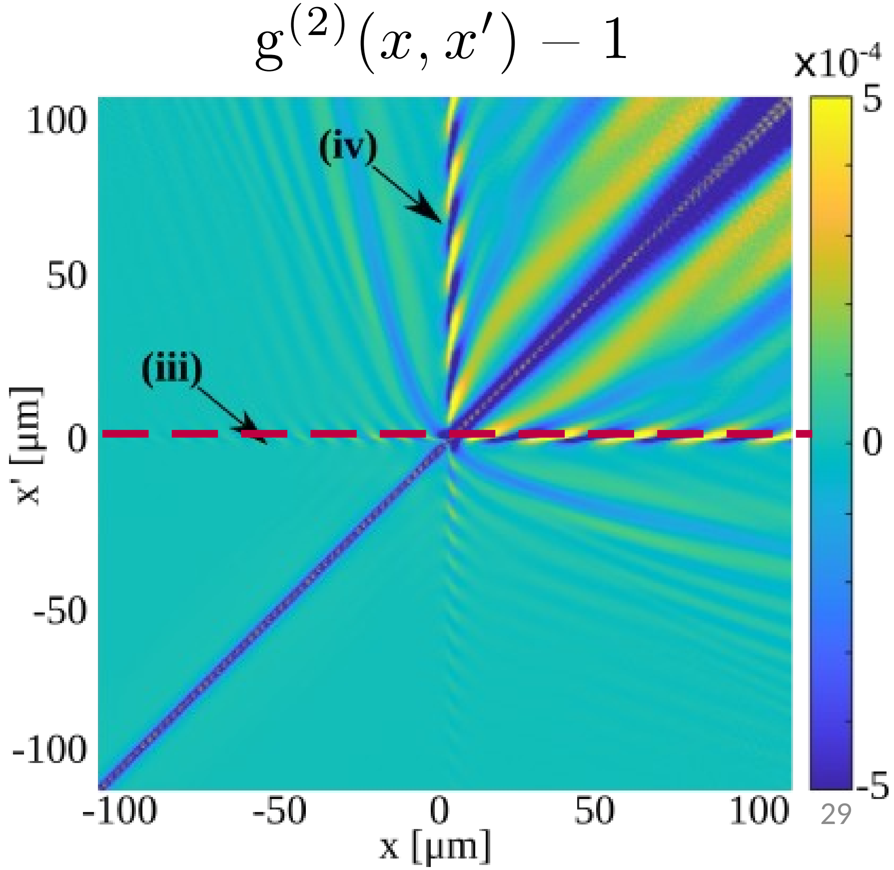
Scattering of vacuum fluctuations: horizon correlations



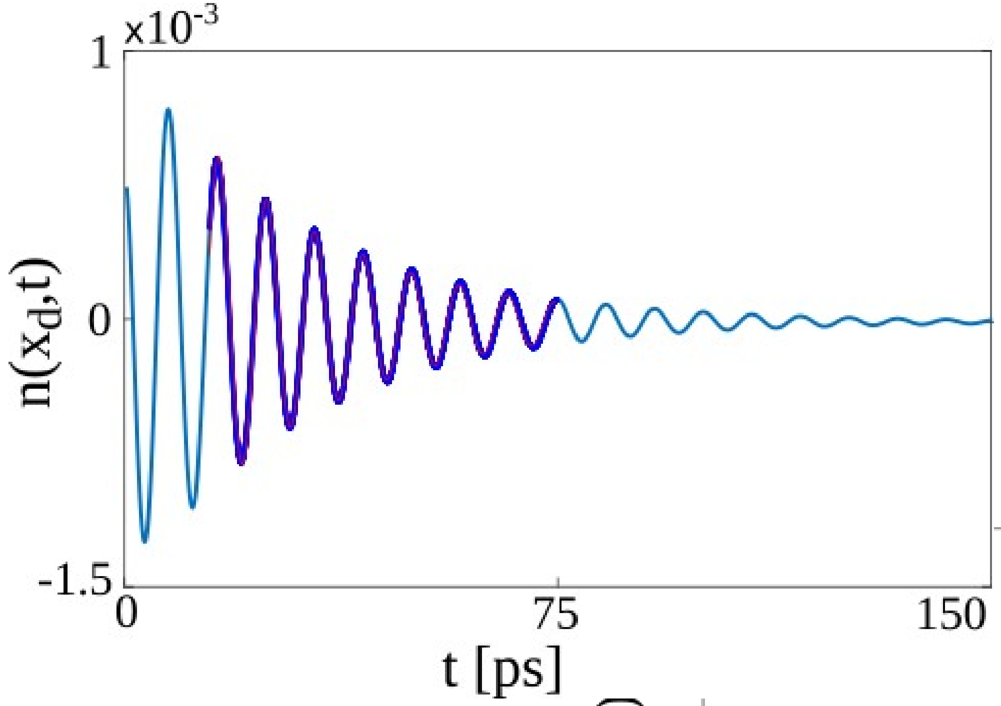
(iii) horizon - outside
(iv) horizon - inside

Spatial frequency?





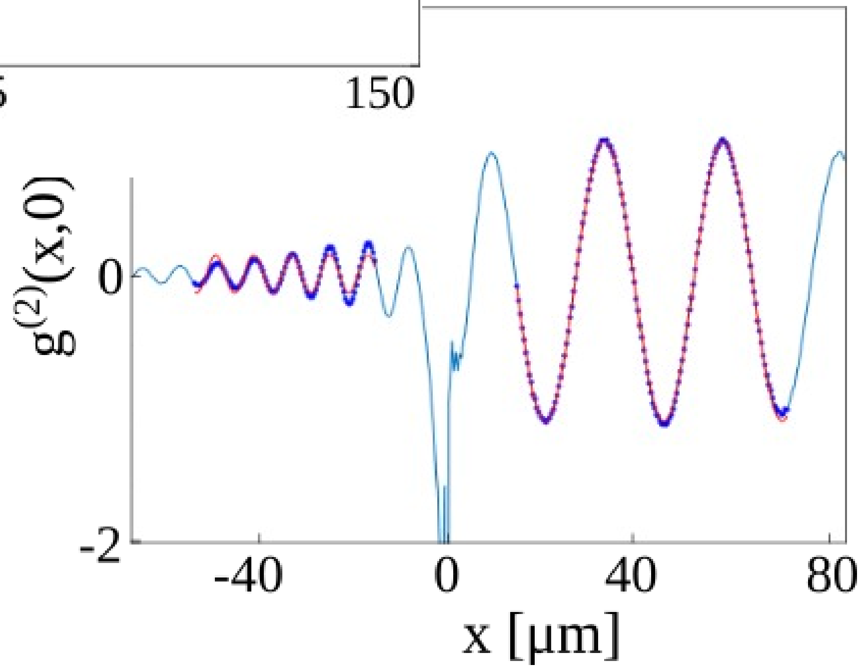
Scattering of vacuum fluctuations: excitation of a quasi normal mode



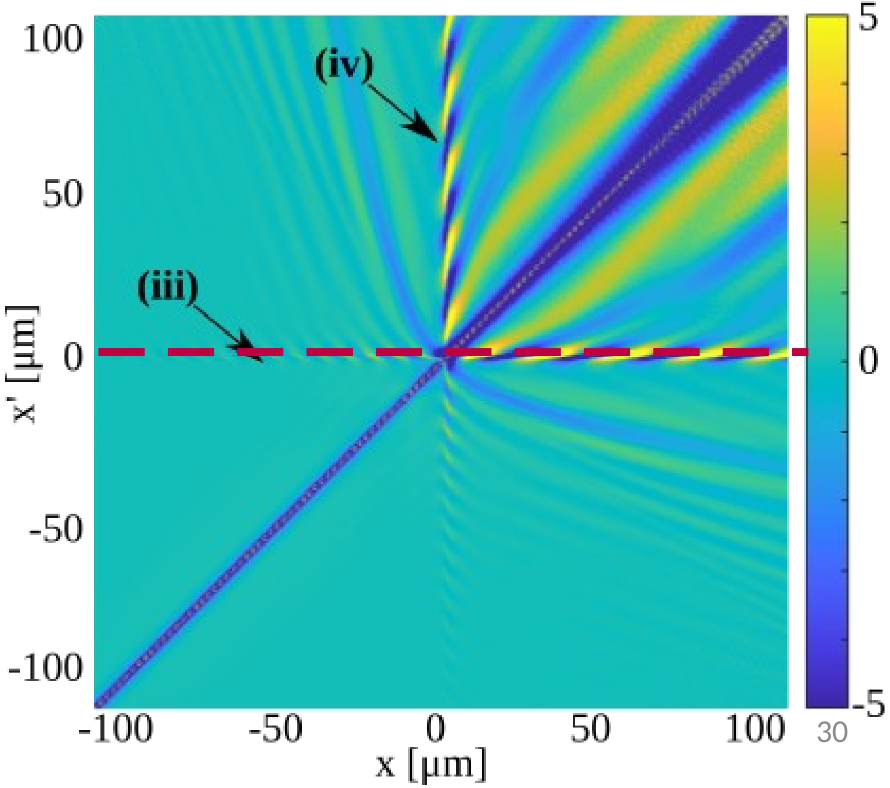
$\mathrm{Re}(\omega)$ Frequency of oscillation
$\mathrm{Im}(\omega)$ Decay rate < γ

(iii) horizon - outside
(iv) horizon - inside

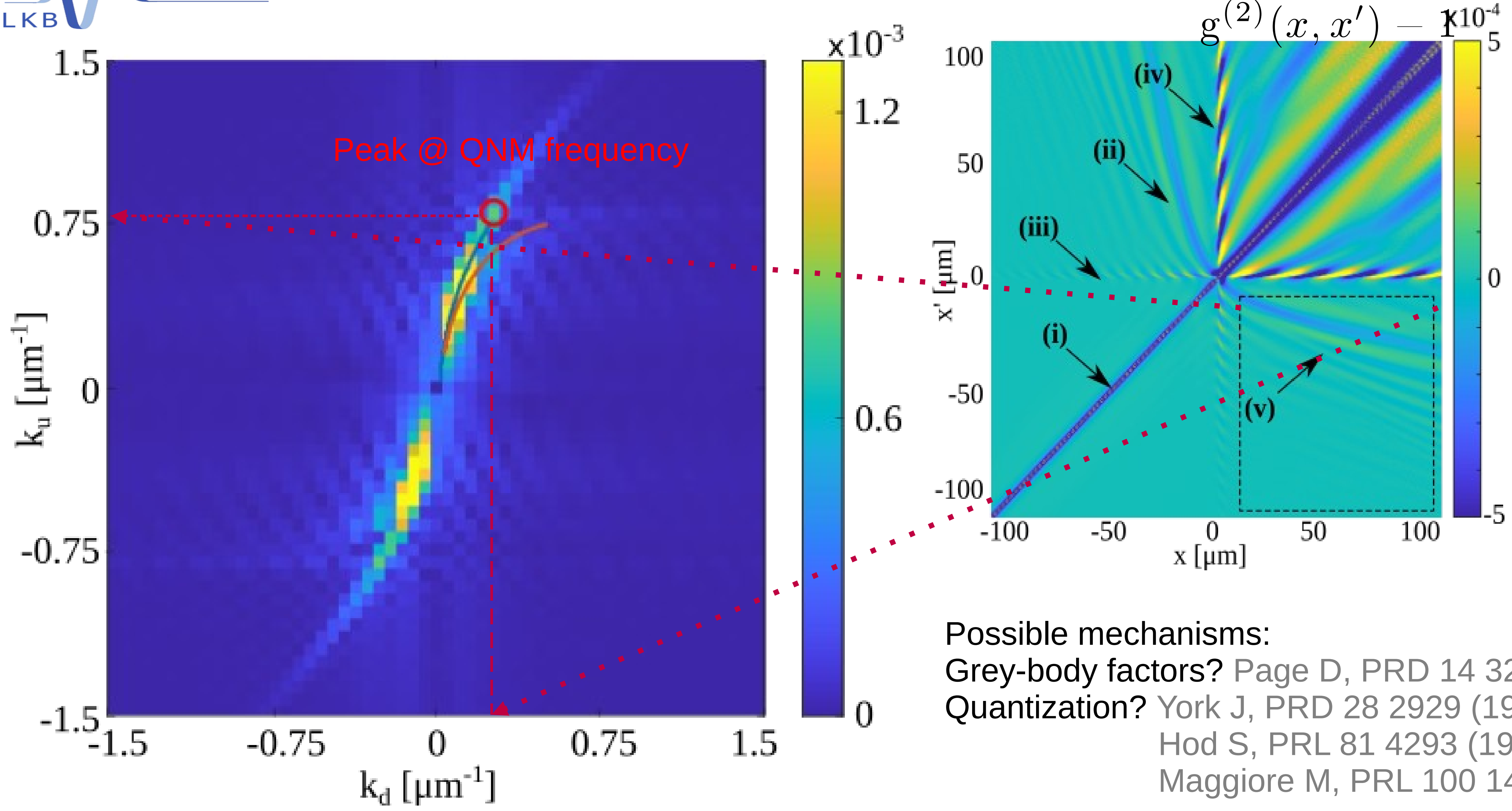
→ excitation of a quasi-normal mode



$g^{(2)}(x, x') - 1$



Spectral modulation

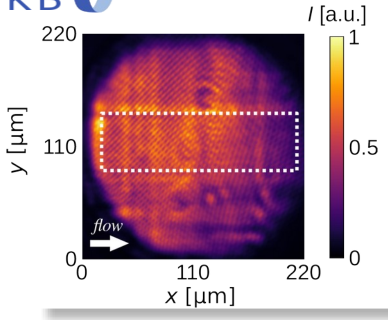


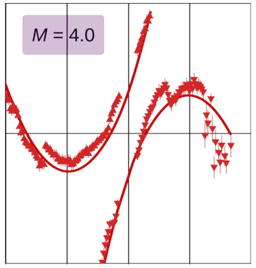
Possible mechanisms:

Grey-body factors? Page D, PRD 14 3260 (1976)

Quantization? York J, PRD 28 2929 (1983)
Hod S, PRL 81 4293 (1998)
Maggiore M, PRL 100 141301 (2008)

Today: only linearised excitations on an effective 1D geometry

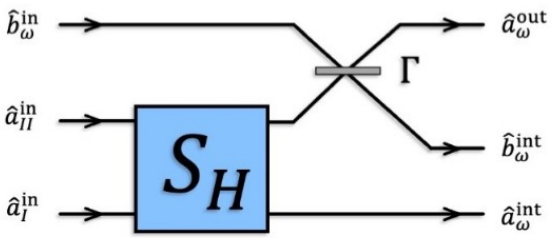




Experiments with polaritons

- High-resolution method to measure spectrum — PRL **129** 103601 2022, PRB **107** 174507 2023
- All-optical control of curvature
  - tunable surface gravity κ → observe two-mode squeezing
  - Measurement of spectrum → QFT

Experiment arXiv:2311.01392
Theory: EPJD **76** 152 2022
PRL **130** 111501 2023

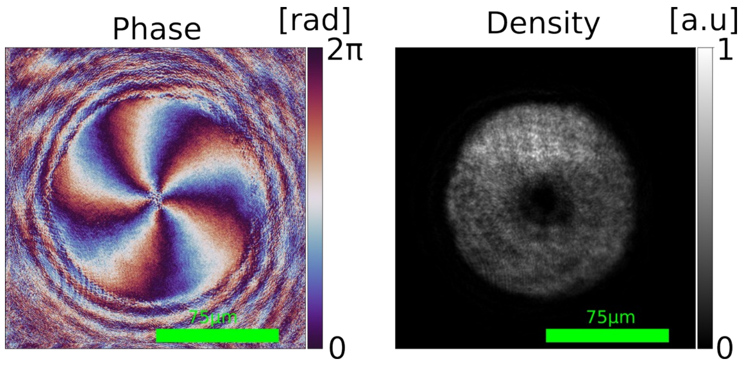


Quantum optics experiments

- Measure phase and density → access full field statistics and dynamics
- Homodyne detection to enhance signal strength and measure quantum correlations
- Enhance strength of emission and degree of entanglement by probing with squeezed state

I Agullo *et al* PRL **128** 091301 2022
PRD **110** 025021 2024

Where do we go from here?



Entanglement in rotating geometries?

Theory: PRD **109** 105024 2024

Winter school analogue gravity/cosmology in Benasque 7th - 17th January 2026