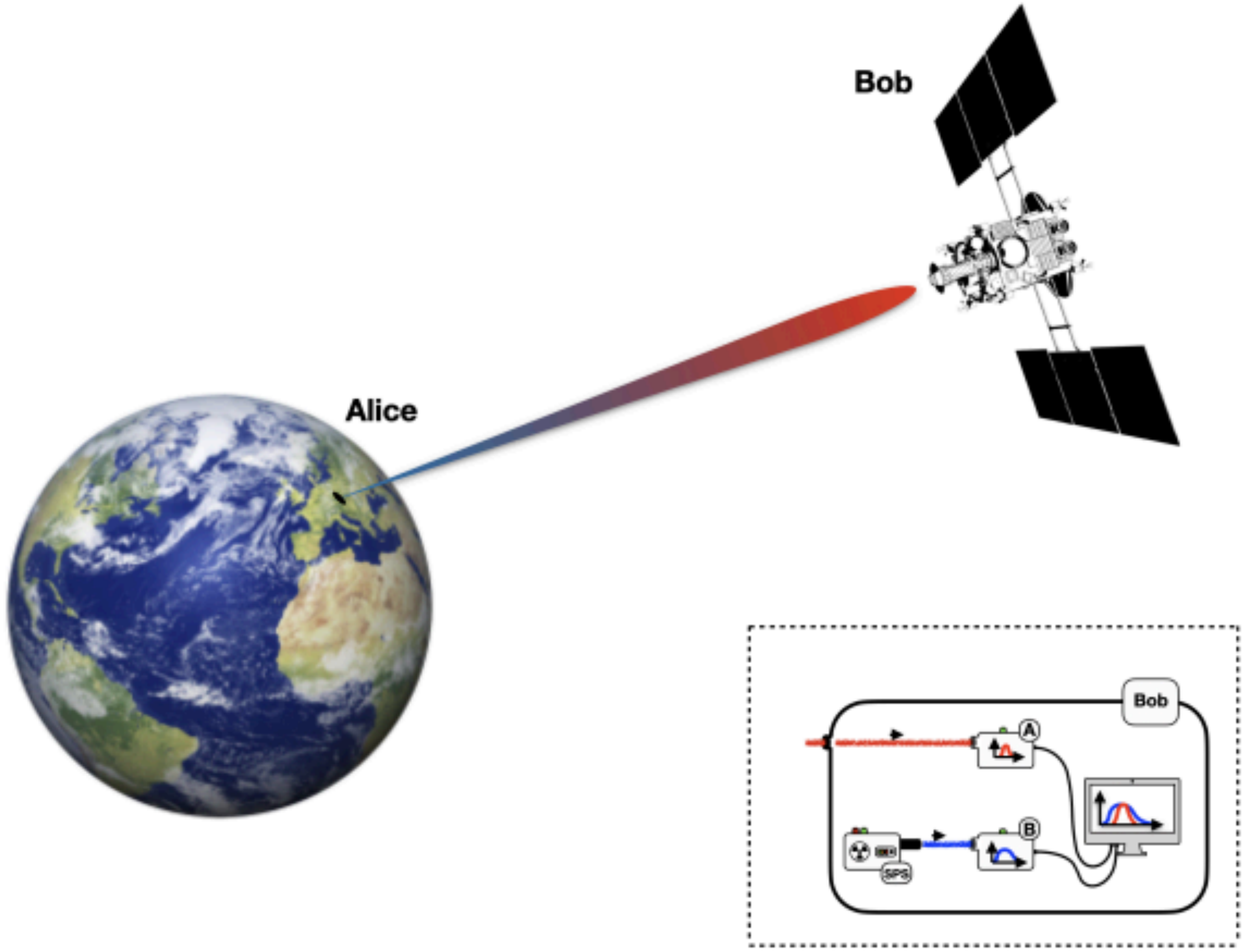
ADVANCES IN QUANTUM DYNAMICS OF PHOTONS IN CURVED SPACETIME

AVENUES 2025 || Civitas Turonum || XXII.I.MMXXV



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Quantum technologies: Earth or Space

Earth based

Earth advantages:
- Cheap(er);
- Reproducible;
- Upgradable.

Earth disadvantages:
- Many sources of noise;
- Small distances;
- Bound to surface.

Quantum technologies: Earth or Space

Relativity?

Satellite based

Space advantages:
- Large distances;
- Microgravity;
- Less noise;

Space disadvantages:
- Very expensive;
- Few-shot experiments;
- Not very flexible once launched.

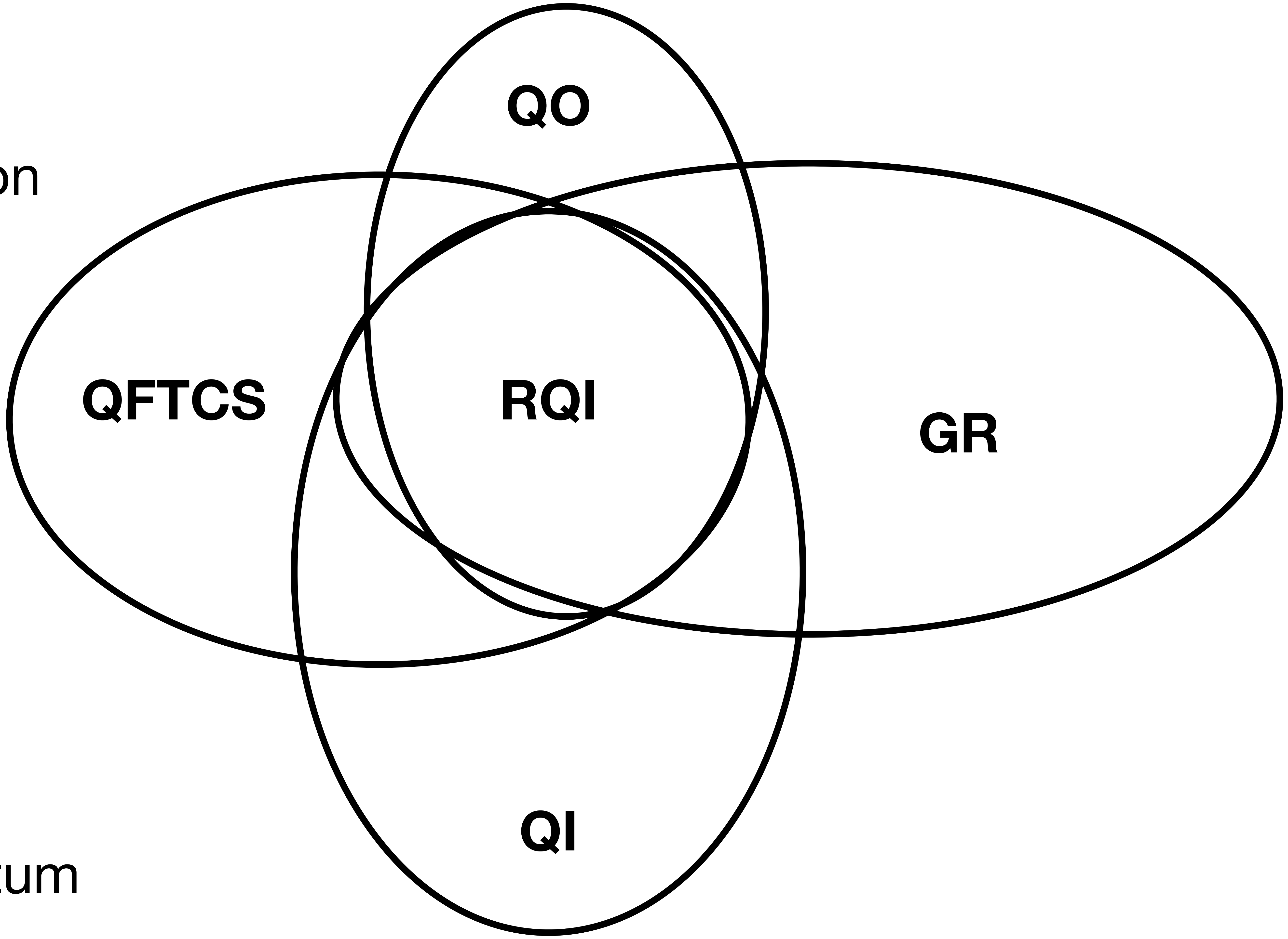
Tools

QO = Quantum Optics

QI = Quantum Information

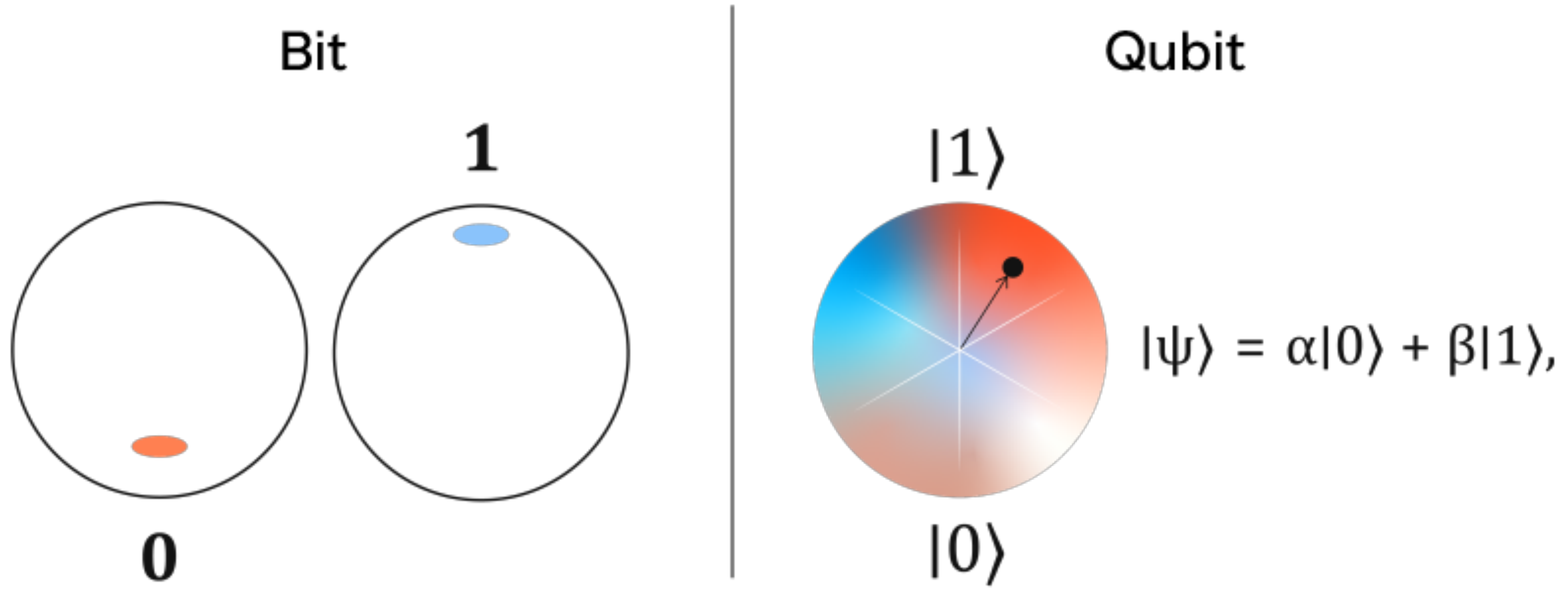
QFTCS = well…

GR = you know…



RQI = Relativistic Quantum Information

Previous state of the art: Photons effectively work as two-level quantum systems (for the purposes of QI)



https://www.qnulabs.com/blog/quantum-101-qubit

Goal: Study the effects of gravity on photons that propagate in curved spacetime

Propagation in flat spacetime

Photon: Excitation of a **quantum field**

We use QFT in flat spacetime

$$\hat{\phi} = \int d\omega [u_\omega \hat{a}_\omega + u^*_\omega \hat{a}^\dagger_\omega]$$

$$\hat{a}^\dagger_{\omega_0} := \int_0^{+\infty} d\omega F_{\omega_0}(\omega) e^{-i\omega(r_A - t_0)} \hat{a}^\dagger_\omega \qquad \text{Alice}$$

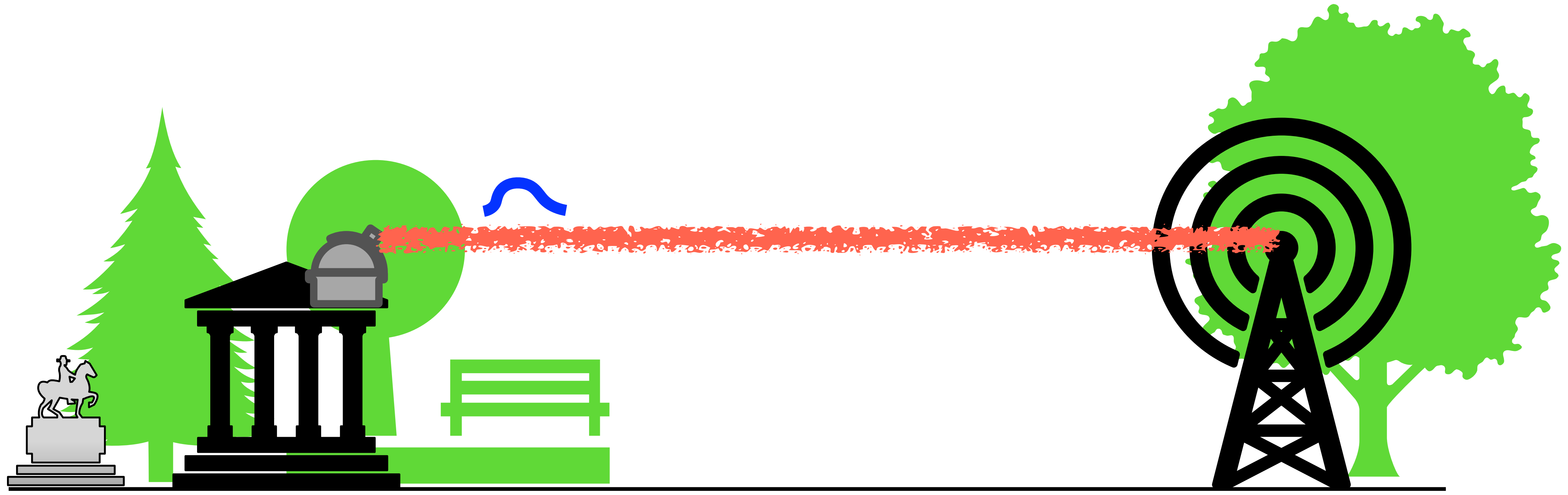
$$\hat{a}^\dagger_{\omega_0} := \int_0^{+\infty} d\omega F'_{\omega'_0}(\omega) e^{-i\omega(r_B - t_B)} \hat{a}^\dagger_\omega \qquad \text{Bob}$$

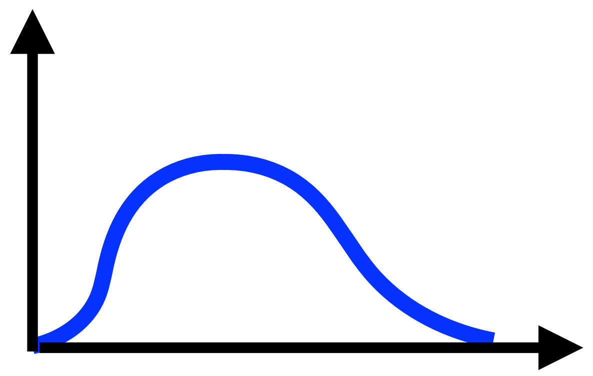
Approximation: pulse 1-dimenstional

$$\int d^3k F_{k_0}(k_x, k_y, k_z) \approx \int d\omega F_{\lambda_0}(\omega)$$

Propagation in flat spacetime

Photon: Excitation of a **quantum field**





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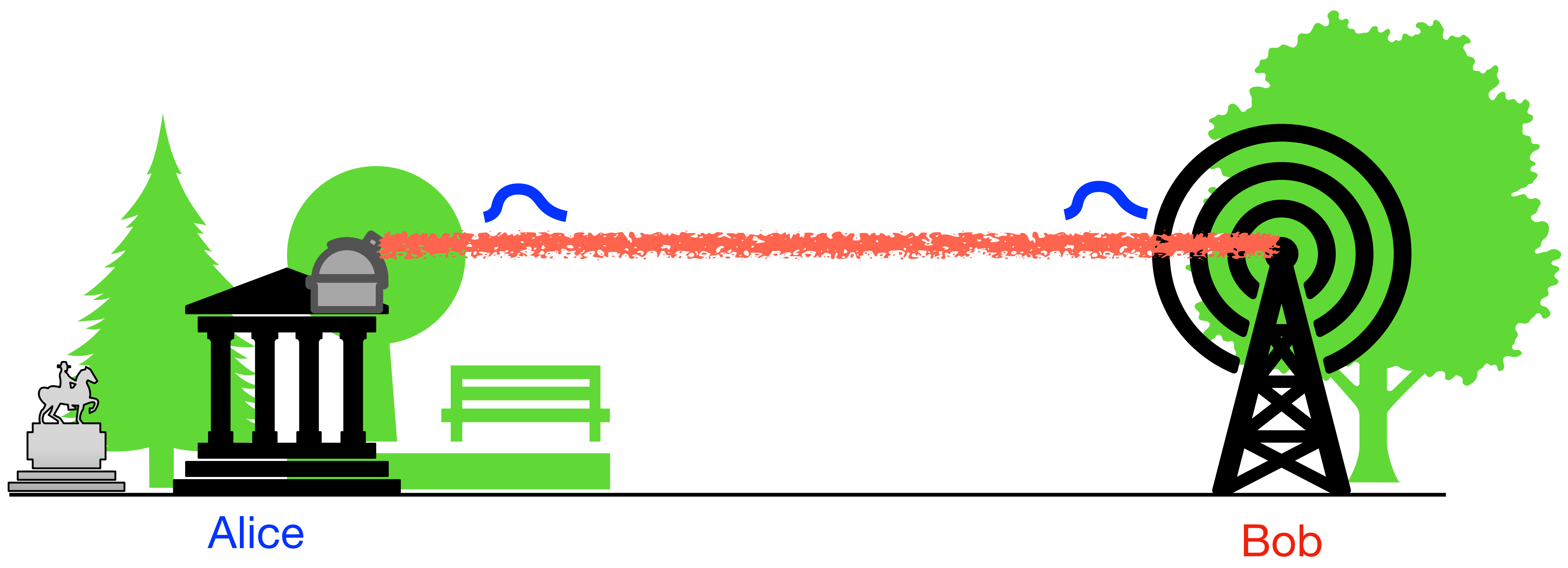
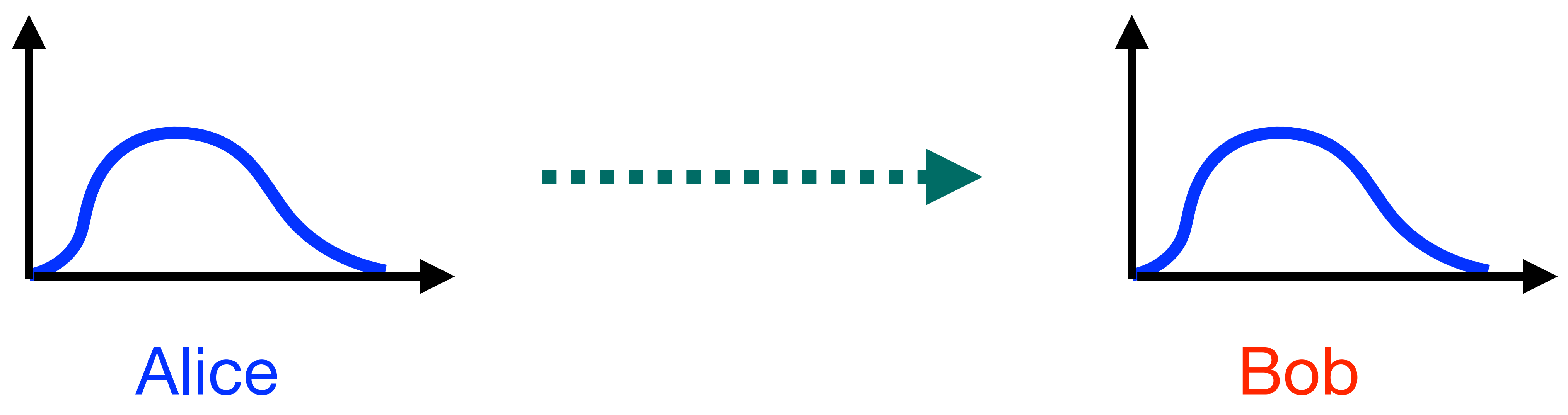
Photon: Excitation of a **quantum field**





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$$\hat{a}^\dagger_{\omega_0} := \int_0^{+\infty} d\omega F'_{\omega'_0}(\omega) e^{-i\omega(r_B - t_B)} \hat{a}^\dagger_\omega \qquad \text{Bob}$$

Alice wave packet as measured **locally** by Bob

$$F'_{\omega'_0}(\omega) = F_{\omega_0}(\omega)$$

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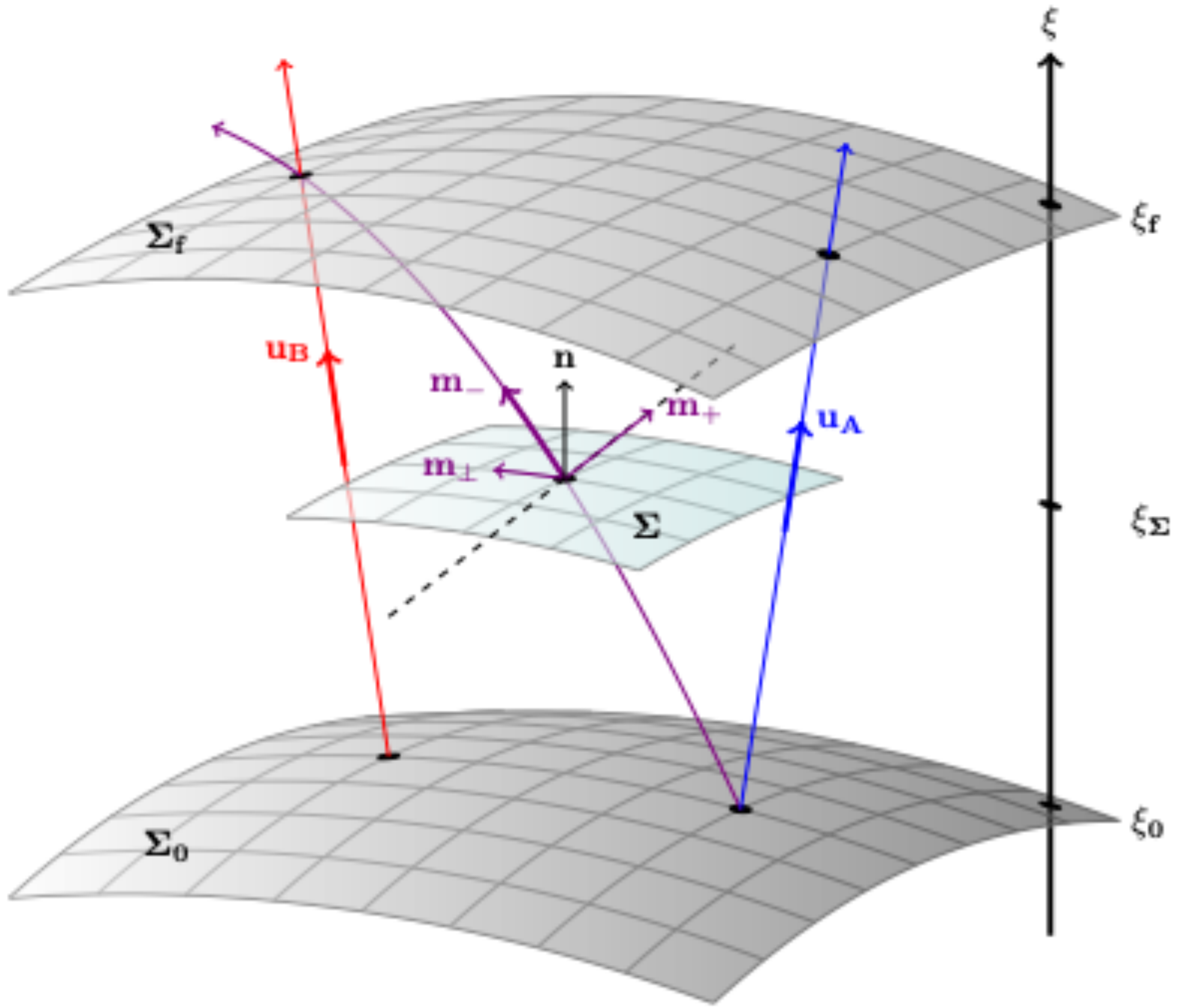
Propagation in (weakly) curved spacetime



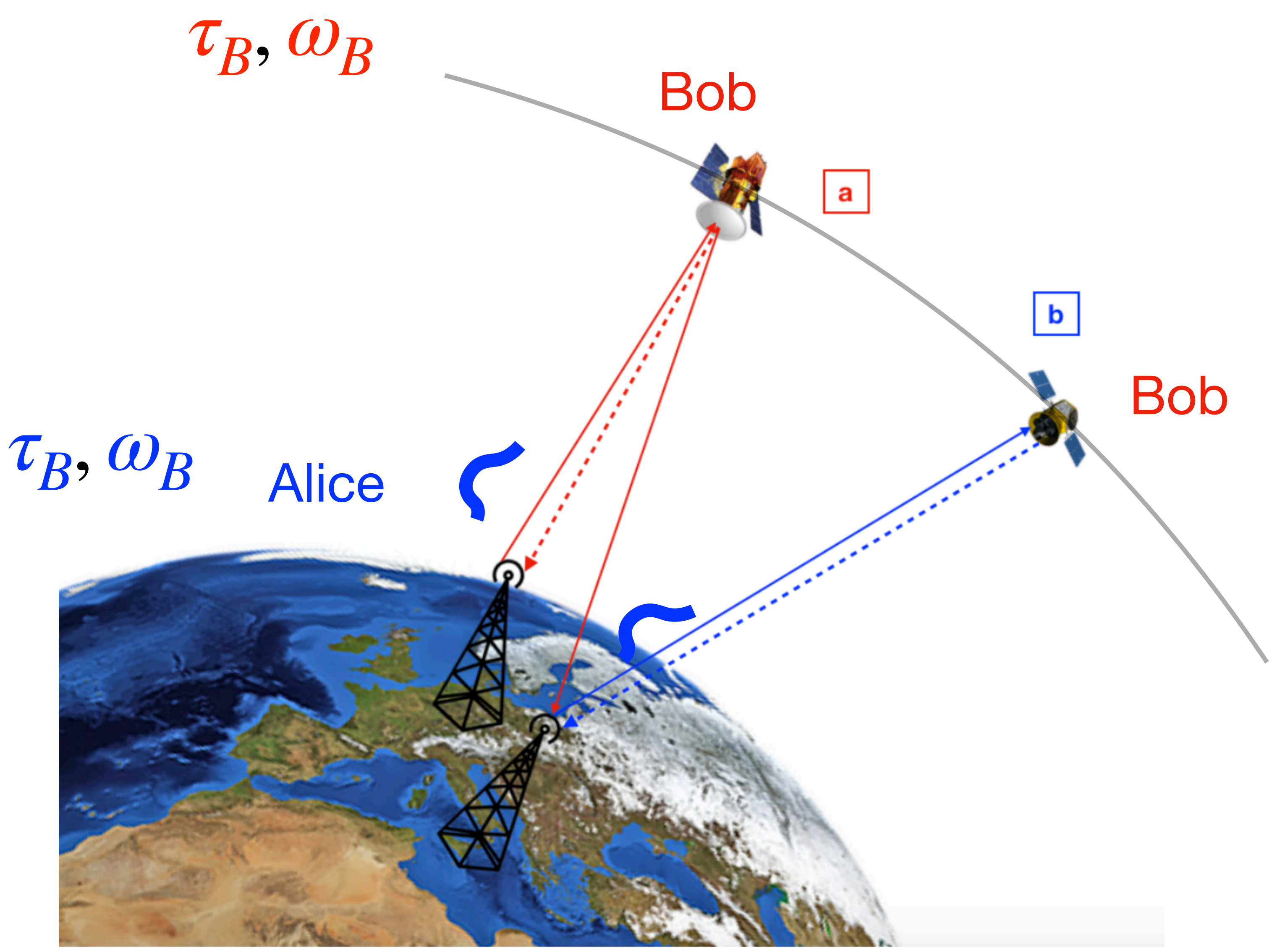
Decomposition of 4-vectors

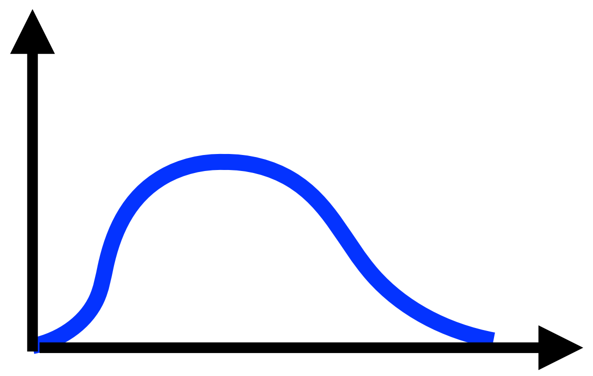
- Alice's **trajectory**
- Bob's **trajectory**
- Photon's **trajectory**

Parameters

- "Time" parameter ξ
- Foliation hypersurfaces Σ

Propagation in (weakly) curved spacetime




Alice

$$f(r_A) = 1 - \frac{r_S}{r_A}$$

We use QFT in flat spacetime

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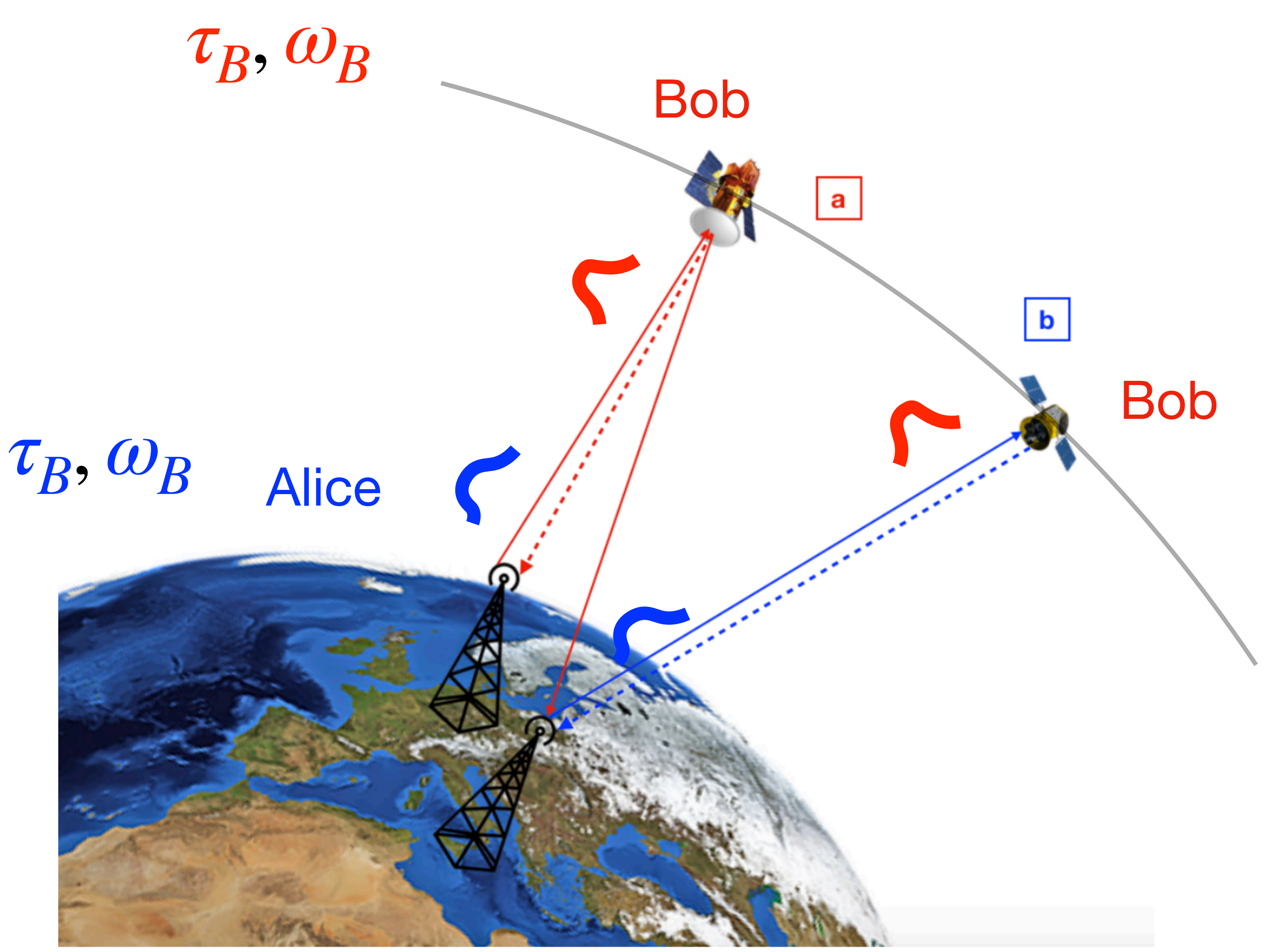
$$\hat{a}^\dagger_{\omega'_0} := \int_0^{+\infty} d\omega F'_{\omega'_0}(\omega) e^{-i\omega(r_B - t_B)} \hat{a}^\dagger_\omega \qquad \text{Bob}$$

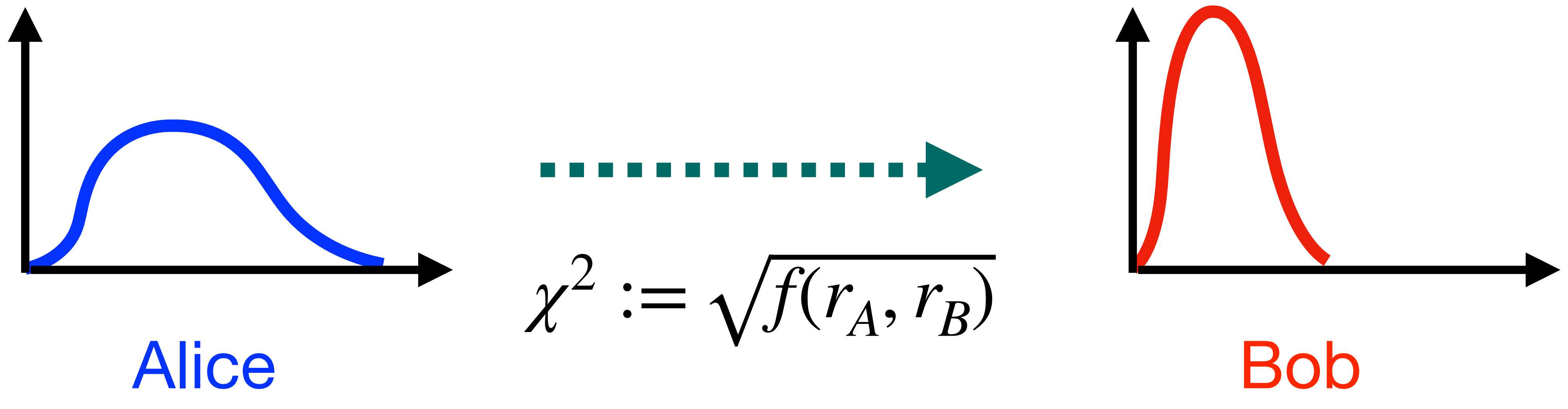
Proper time/frequency relation between Alice & Bob

$$\tau_B = \frac{f(r_B)}{f(r_A)} \tau_A \qquad \omega_B = \frac{f(r_A)}{f(r_B)} \omega_A$$

Propagation in (weakly) curved spacetime



$$f(r_A, r_B) = \frac{1 - \frac{3M}{r_B}}{1 - \frac{2M}{r_A}}$$



$$\chi^2 := \sqrt{f(r_A, r_B)}$$

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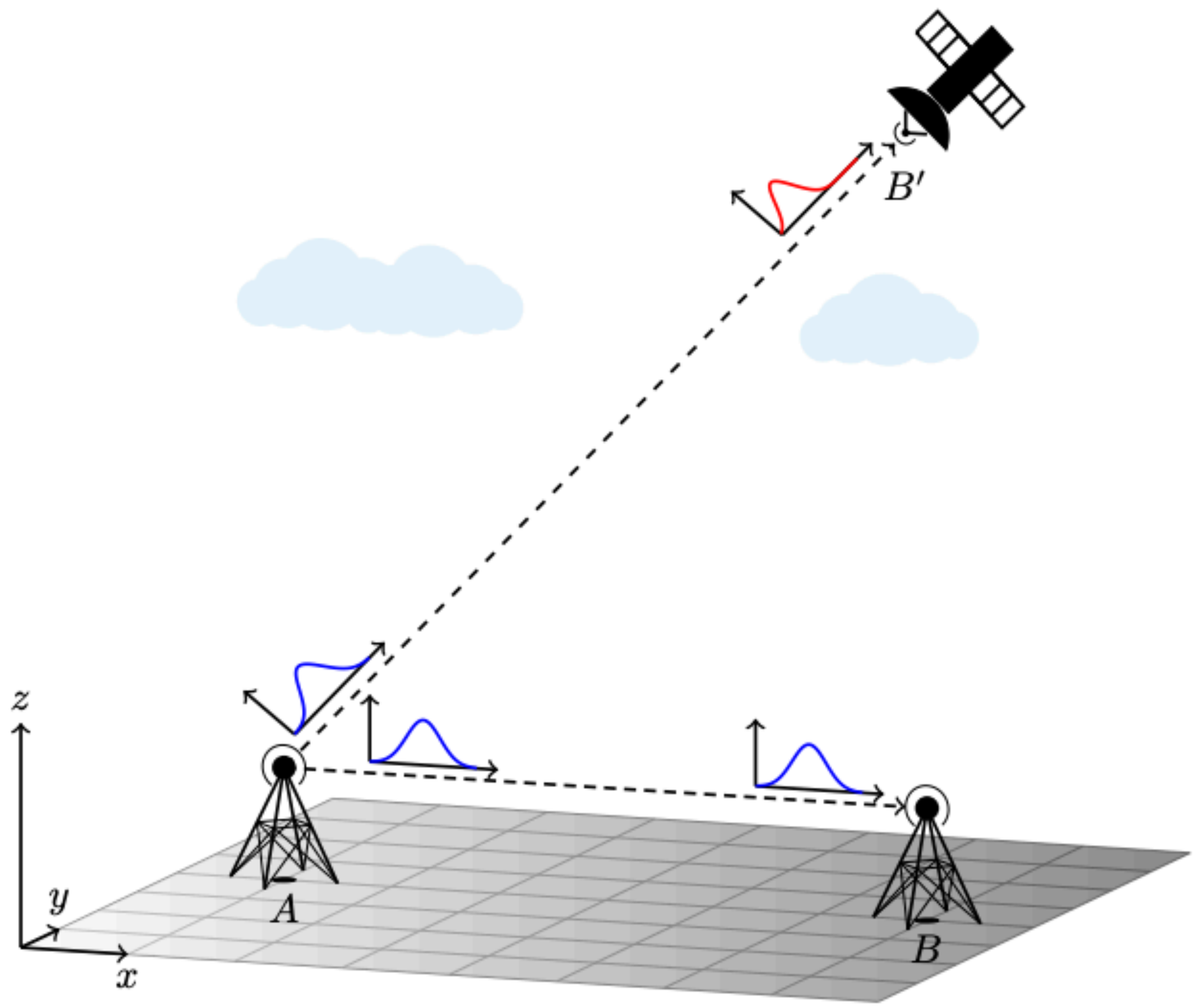
Alice wave packet as measured **locally** by Bob

$$F'_{\omega'_0}(\omega) = \sqrt[4]{f(r_A, r_B)} F_{\omega_0}(\sqrt{f(r_A, r_B)}\,\omega)$$

Propagation in (weakly) curved spacetime



Potential wave packet: Gaussian profile

$$F_{\omega_0}(\omega) = C\, e^{-\frac{(\omega-\omega_0)^2}{4\sigma^2}} \longrightarrow F'_{\omega'_0}(\omega) = \chi C\, e^{-\frac{(\omega-\omega_0/\chi^2)^2}{4\sigma^2/\chi^2}}$$

$$\chi^2 := \sqrt{f(r_A, r_B)}$$

It is important to note that "any" protocol operated between Alice and Bob will result in the receiver (Bob) to witness effects that depend on the overlap of the two wave packets

$$\Delta := \left| \int_{-\infty}^{+\infty} d\omega F'_{\omega'_0}(\omega) F^*_{\omega_0}(\omega) \right|$$

This overlap, for example, measures **how well** can we distinguish two single photons.

$$\Delta := \langle 1_{F'_{\omega'_0}} | 1_{F_{\omega_0}} \rangle$$

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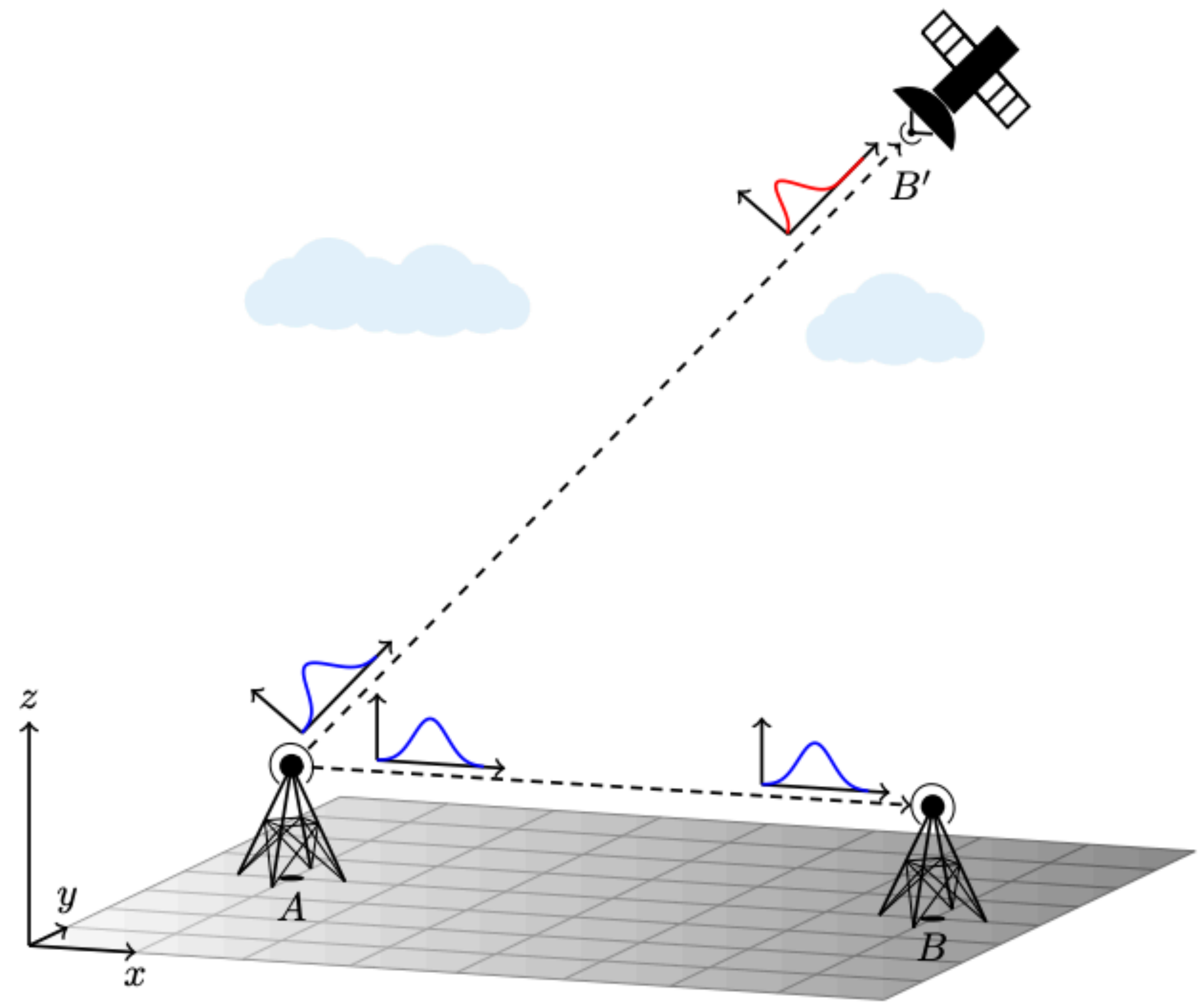
Propagation in (weakly) curved spacetime



Potential wave packet: Gaussian profile

$$F_{\omega_0}(\omega) = C\, e^{-\frac{(\omega-\omega_0)^2}{4\sigma^2}} \quad \longrightarrow \quad F'_{\omega_0'}(\omega) = \chi C\, e^{-\frac{(\omega-\omega_0/\chi^2)^2}{4\sigma^2/\chi^2}}$$

$$\chi^2 := \sqrt{f(r_A, r_B)}$$

hysically viable results in the **weak gravity regime**:

$$\delta := \frac{1-\frac{3M}{r_B}}{1-\frac{2M}{r_A}} - 1 \ll 1 \qquad \delta^2 \ll \frac{\omega_0^2}{\sigma^2}\delta^2 \ll 1$$

$$\Delta \approx 1 - \frac{\omega_0^2}{8\sigma^2}\delta^2$$

In our case this occurs

$$\delta \sim 10^{-10}; \quad \omega_0 \sim 4 \times 10^{14}\text{Hz} \qquad \sigma \sim 10^{6}\text{Hz}$$

Transformation induced by gravitational redshift

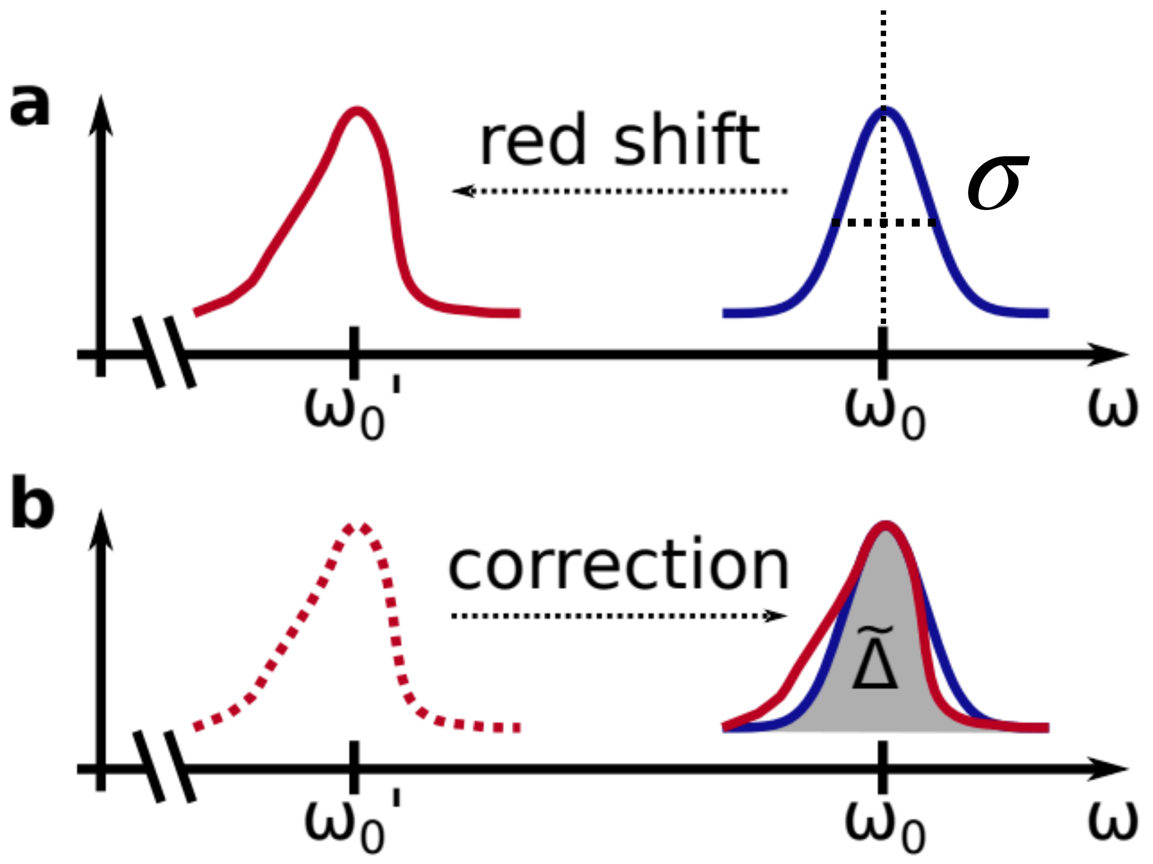
Q: what is the nature of the transformation?

Transformation induced by gravitational redshift

FIRST ASPECT

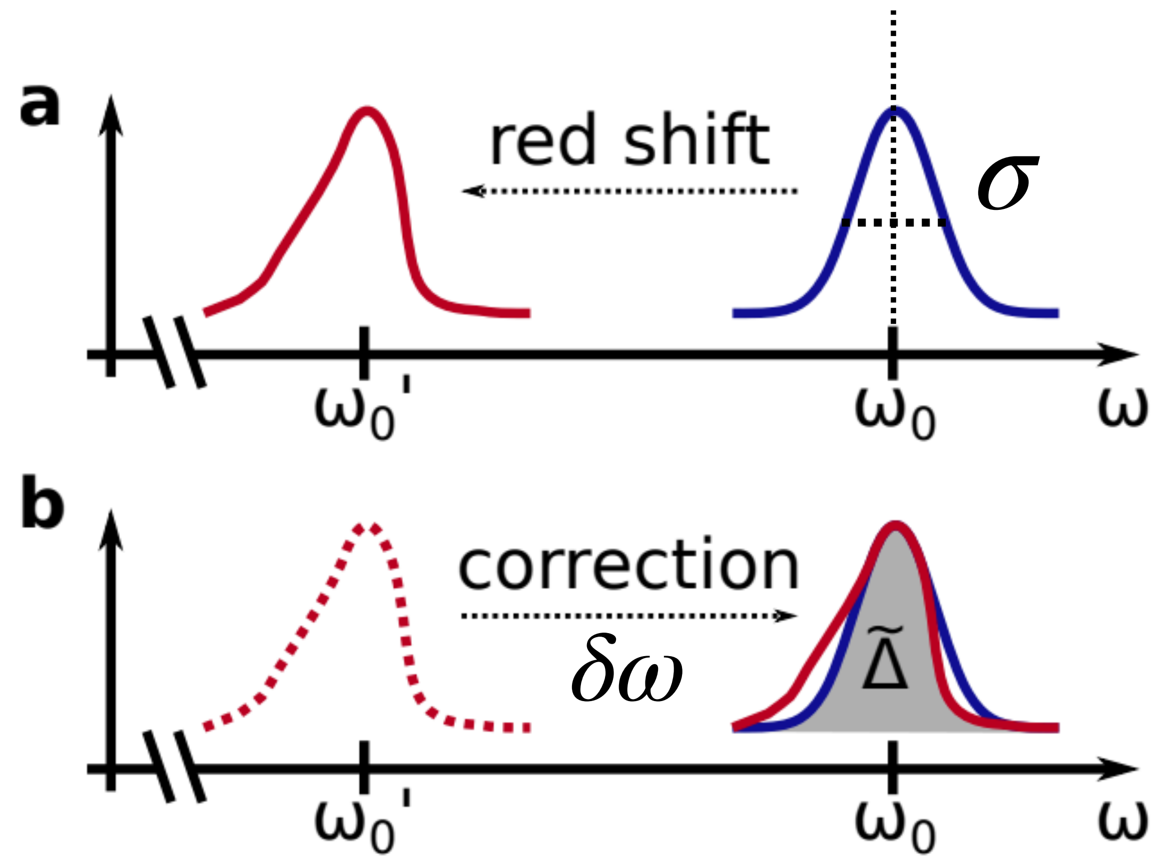
Can we distinguish between genuine distortion and rigid translation?

Transformation induced by gravitational redshift



$$F'(\omega)_{\omega_0'} = \chi F\left(\frac{\chi^2\omega - \omega_0}{\sigma}\right) \qquad \text{with} \qquad \chi^2\omega - \omega_0 = \chi^2\left[(\omega - \omega_0) + \frac{\chi^2 - 1}{\chi^2}\omega_0\right]$$

Transformation induced by gravitational redshift



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Locally perform translations

$$\omega \rightarrow \omega + \delta\omega$$

$$\bar{z} := (\chi^2 - 1)z_0 + \chi^2\delta z$$

$$(z := \omega/\sigma)$$

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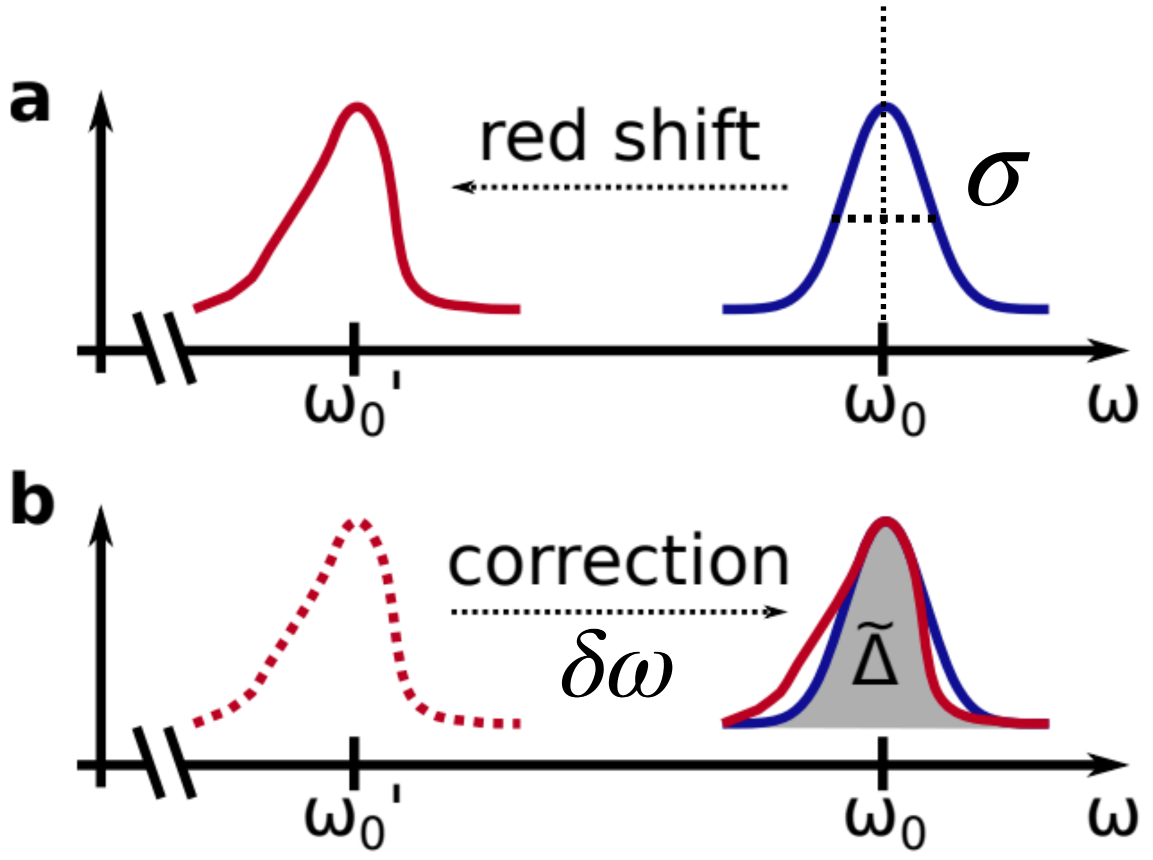
Transformation induced by gravitational redshift



$$\Delta \rightarrow \tilde{\Delta} := \left| \int dz f(\chi z + \bar{z}) f(z/\chi) e^{-i(\psi(\chi z + z_0 + \bar{z}) - \psi(z/\chi + z_0))} \right|$$

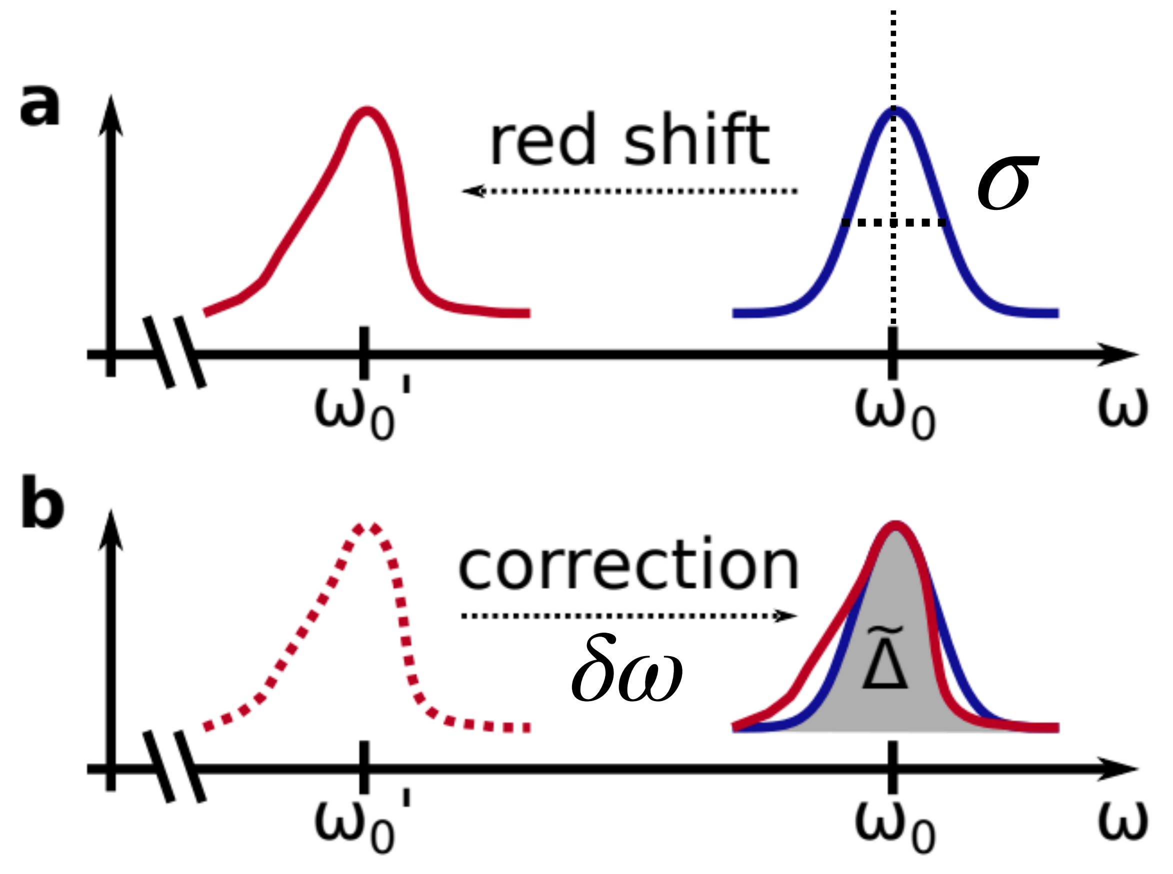
$$\Delta := \left| \int_{-\infty}^{+\infty} d\omega F'_{\omega_0'}(\omega) F^*_{\omega_0}(\omega) \right|$$

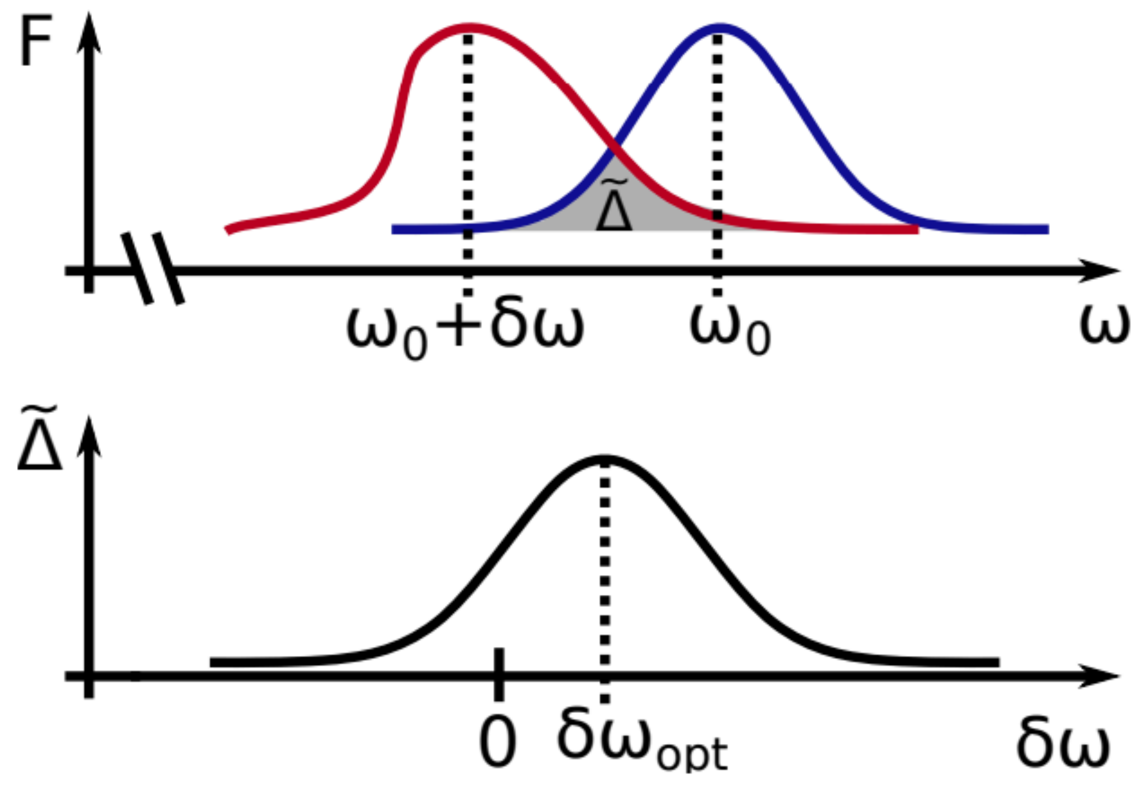
$$F(z) := (\sigma)^{-1/2} f(z) e^{-i\psi(z)}$$

$$\bar{z} := (\chi^2 - 1) z_0 + \chi^2 \delta z$$

$$(z := \omega/\sigma)$$

Genuine distortion/rigid shift





(**)

(*)

$$\tilde{\Delta} := \left| \int dz f(\chi z + \bar{z}) f(z/\chi) e^{-i(\psi(\chi z + z_0 + \bar{z}) - \psi(z/\chi + z_0))} \right|$$

OPTIMIZE	OBTAIN	
$\delta z_{\text{opt}}(\bar{z})$	Effective redshift	(*)
$\tilde{\Delta}_{\text{opt}}(\bar{z})$	Effective genuine deformation	(**)

$$\bar{z} := (\chi^2 - 1) z_0 + \chi^2 \delta z$$

$$(z := \omega/\sigma)$$

Genuine distortion/rigid shift: effect of quantum coherence

We send two type of states:

Pure state: $\hat{\rho}_\text{p} := |1_A\rangle\langle 1_A|$ → $\hat{\rho}_\text{p}$

Mixed state: $\hat{\rho}_\text{m}$ **"benchmark state"** → $\hat{\rho}_\text{m}$

$$F(z) = 1/\sqrt[4]{2\pi}\exp[-z^2/4 - i\tilde{\phi} z].$$

$$\tilde{\Delta}^{\text{Ga}}_{\text{p,opt}} = \frac{\sqrt{2}\chi}{\sqrt{1+\chi^4}} e^{-\frac{(\chi^2-1)^2}{\chi^4+1}\tilde{\phi}^2}$$

$$\tilde{\Delta}^{\text{Ga}}_{\text{m,opt}} = \frac{\sqrt{2}\chi}{\sqrt{1+\chi^4}}.$$

consider the near-Earth regime, we

$$\tilde{\Delta}^{\text{Ga}}_{\text{p,opt}} \approx 1 - (1 + 2\tilde{\phi}^2)\,\delta_1^2$$

$$\tilde{\Delta}^{\text{Ga}}_{\text{m,opt}} \approx 1 - \delta_1^2,$$

$\chi = 1 + \delta_1$

$\delta_1 \ll 1$

$$\tilde{\phi} z \to \tilde{\phi}^2 z^2$$

$$\tilde{\Delta}^{\text{Ga}}_{\text{p,opt}} = \frac{\sqrt{2}\chi}{\sqrt{1+\chi^4}} \frac{e^{-4\frac{(\chi^2-1)^2}{\chi^4+1}\frac{\tilde{\phi}^4}{\xi(\tilde{\phi})}z_0^2}}{\sqrt{\xi(\tilde{\phi})}} e^{256\frac{a_1^2(\tilde{\phi})}{a_2(\tilde{\phi})}}$$

$$\tilde{\Delta}^{\text{Ga}}_{\text{m,opt}} = \frac{\sqrt{2}\chi}{\sqrt{1+\chi^4}}.$$

ally, considering again the near-Earth regime, we l

$$\tilde{\Delta}^{\text{Ga}}_{\text{p,opt}} \approx 1 - (1 + 32\tilde{\phi}^4 + 8\tilde{\phi}^4 z_0^2)\delta_1^2 + 2^9\frac{\tilde{\phi}^4 z_0^2 \delta_1^4}{1+16\tilde{\phi}^4}$$

$$\tilde{\Delta}^{\text{Ga}}_{\text{m,opt}} \approx 1 - \delta_1^2.$$

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Genuine distortion/rigid shift: effect of quantum coherence

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Mixed state: $\hat{\rho}_{\mathrm{m}}$ **"benchmark state"** → $\hat{\rho}_{\mathrm{m}}$

Window states $$|1_n(\theta)\rangle := \int_{(n-1/2)\sigma}^{(n+1/2)\sigma} d\omega e^{i\theta\omega/\sigma} |1_\omega\rangle$$

$$\hat{\rho}_{\mathrm{p}} = \int d\omega d\omega' F_A^*(\omega) F_A(\omega') |1_\omega\rangle\langle 1_{\omega'}|$$ → $$\hat{\rho}_{\mathrm{m}} \sum_n \rho_{nn} |1_n(0)\rangle\langle 1_n(0)|$$

$$\hat{\rho}_{\mathrm{p}} = \begin{pmatrix} \rho_{\omega\omega} & \rho_{\omega\omega'} & \dots \\ \rho^*_{\omega\omega'} & \rho_{\omega'\omega'} & \dots \\ \dots & \dots & \dots \end{pmatrix}$$

Project on windows states and integrate phase →

$$\hat{\rho}_{\mathrm{m}} = \begin{pmatrix} \rho_{11} & 0 & \dots \\ 0 & \rho_{22} & \dots \\ \dots & \dots & \dots \end{pmatrix}$$

Such that: $$\langle 1_n(0)|\hat{\rho}_{\mathrm{p}}|1_n(0)\rangle \approx \langle 1_n(0)|\hat{\rho}_{\mathrm{m}}|1_n(0)\rangle$$

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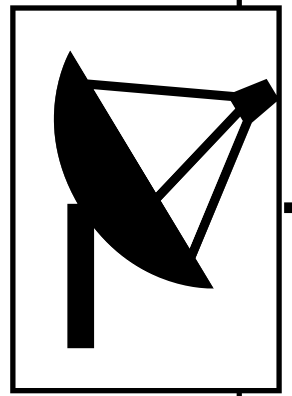

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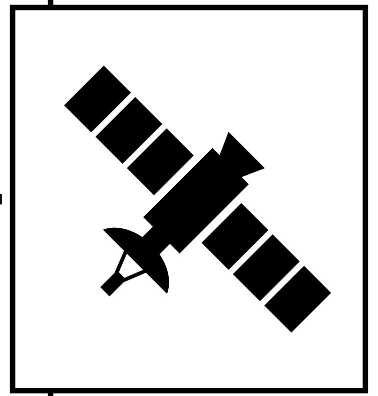

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$$\tilde{\Delta}^{\rm Ga}_{\rm m,opt} \approx 1 - \delta_1^2.$$

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Curved spacetime as a mode mixer

Extended photon

$$\hat{A}_{\omega_0} := \int d\omega F_{\omega_0}(\omega) e^{-i\psi(\omega)} \hat{a}_\omega$$

SECOND ASPECT

The photonic operators are mixed linearly

Curved spacetime as a mode mixer

Extended photon

$$\hat{A}_{\omega_0} := \int d\omega F_{\omega_0}(\omega) e^{-i\psi(\omega)} \hat{a}_\omega$$

Gravitational redshift is:

* **NOT** a unitary transformation on sharp frequencies $\hat{a}_{\chi^2\omega} \neq \hat{U}^\dagger(\chi)\hat{a}_\omega\hat{U}(\chi)$

* a unitary transformation on **finite-bandwidth** photons $\hat{A}_{\omega_0'} = \hat{U}^\dagger(\chi)\hat{A}_{\omega_0}\hat{U}(\chi)$

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Curved spacetime as a mode mixer

Extended photon

$$\hat{A}_1 \equiv \hat{A}_{\omega_0} := \int d\omega F_{\omega_0}(\omega) e^{-i\psi(\omega)} \hat{a}_\omega$$

$$\hat{A}_2 \equiv \hat{A}_{\tilde{\omega}_0} := \int d\omega \tilde{F}_{\tilde{\omega}_0}(\omega) e^{-i\tilde{\psi}(\omega)} \hat{a}_\omega$$

$$\hat{\mathbb{X}}' := \hat{U}^\dagger(\chi)\,\hat{\mathbb{X}}\,\hat{U}(\chi) \equiv \boldsymbol{U}(\chi)\,\hat{\mathbb{X}}.$$

$$\hat{\mathbb{X}} = (\hat{A}_1, \ldots, \hat{A}_N, \ldots)^{Tp}$$

$$\boldsymbol{U}(\chi)\boldsymbol{U}^\dagger(\chi) = 1$$

Curved spacetime as a mode mixer

Extended photon

$$\hat{A}_{\omega_0} := \int d\omega F_{\omega_0}(\omega) e^{-i\psi(\omega)} \hat{a}_\omega$$

$$\hat{A}_{\tilde{\omega}_0} := \int d\omega \tilde{F}_{\tilde{\omega}_0}(\omega) e^{-i\tilde{\psi}(\omega)} \hat{a}_\omega$$

$$\hat{\mathbb{X}}' := \hat{U}^\dagger(\chi)\, \hat{\mathbb{X}}\, \hat{U}(\chi) \equiv \boldsymbol{U}(\chi)\, \hat{\mathbb{X}}.$$

$$\hat{\mathbb{X}} := (\hat{A}_{\omega_0}, \hat{A}_{\tilde{\omega}_0}, \hat{A}_\perp)^{\mathrm{Tp}}$$

$$\boldsymbol{U} \equiv \begin{pmatrix} c_\theta\, c_\phi & -c_\theta\, s_\phi\, c_\psi - s_\theta s_\psi & -c_\theta\, s_\phi\, s_\psi + s_\theta c_\psi \\ s_\phi & c_\phi\, c_\psi & c_\phi\, s_\psi \\ -s_\theta\, c_\phi & s_\theta\, s_\phi\, c_\psi - c_\theta s_\psi & s_\theta\, s_\phi\, s_\psi + c_\theta c_\psi \end{pmatrix}$$

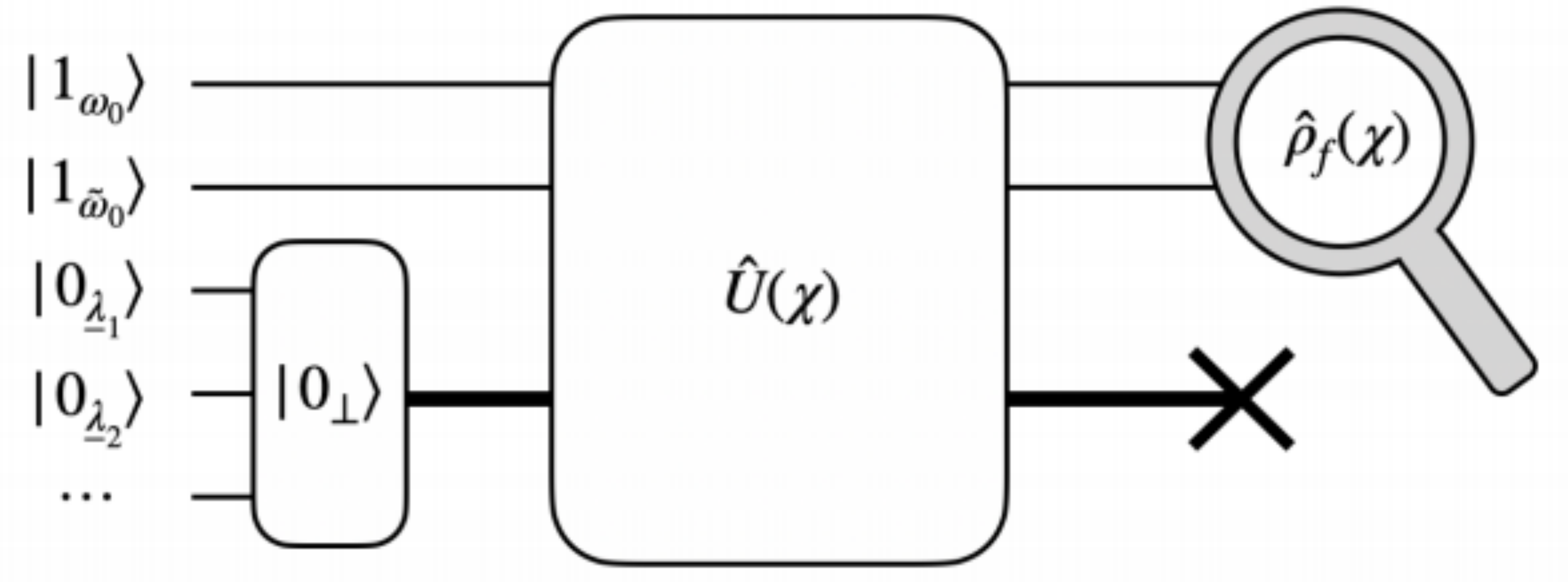
$$\cos\theta\, \cos\phi \equiv |\langle F'_{\omega'_0}, F_{\omega_0}\rangle|,$$
$$\cos\phi\, \cos\psi \equiv |\langle F'_{\tilde{\omega}'_0}, F_{\tilde{\omega}_0}\rangle|,$$
$$\sin\phi \equiv |\langle F'_{\tilde{\omega}'_0}, F_{\omega_0}\rangle|.$$

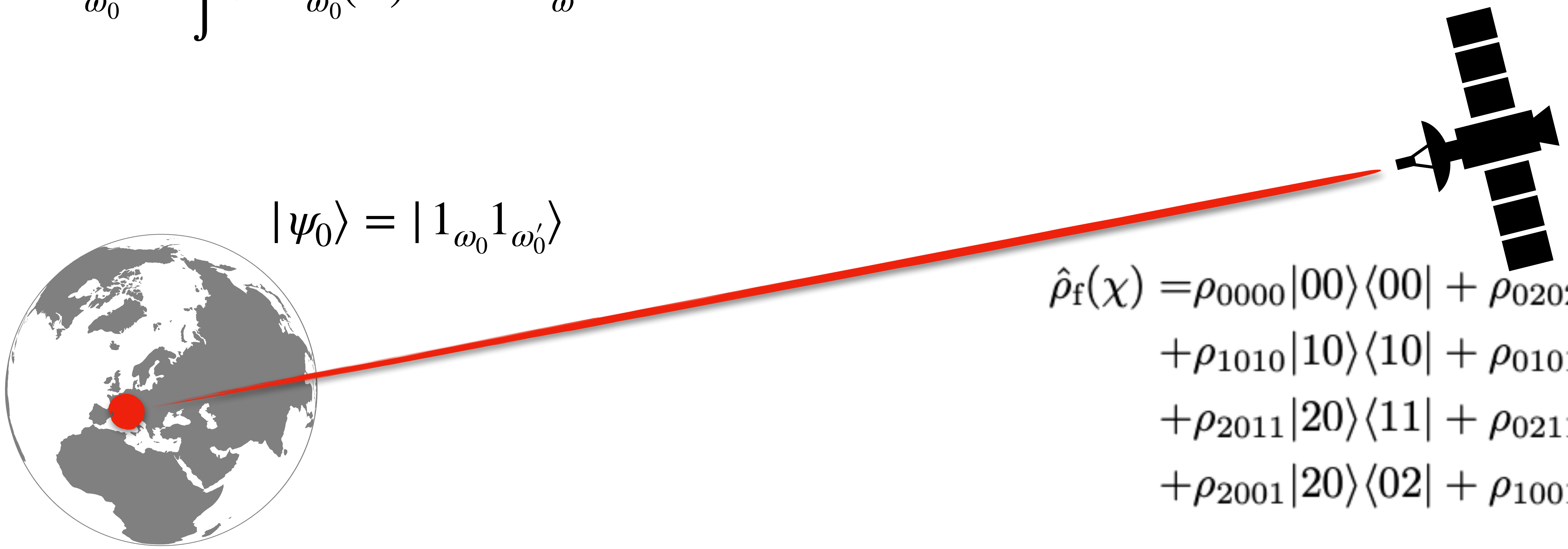
Curved spacetime as a mode mixer

Extended photon

$$\hat{A}_{\omega_0} := \int d\omega F_{\omega_0}(\omega) e^{-i\psi(\omega)} \hat{a}_\omega$$

$$\hat{A}_{\tilde{\omega}_0} := \int d\omega \tilde{F}_{\tilde{\omega}_0}(\omega) e^{-i\tilde{\psi}(\omega)} \hat{a}_\omega$$





$$|\psi_0\rangle = |1_{\omega_0} 1_{\omega_0'}\rangle$$

$$\begin{aligned}\hat{\rho}_{\mathrm{f}}(\chi) =& \rho_{0000}|00\rangle\langle 00| + \rho_{0202}|02\rangle\langle 02| + \rho_{2020}|20\rangle\langle 20| \\ &+ \rho_{1010}|10\rangle\langle 10| + \rho_{0101}|01\rangle\langle 01| + \rho_{1111}|11\rangle\langle 11| \\ &+ \rho_{2011}|20\rangle\langle 11| + \rho_{0211}|02\rangle\langle 11| \\ &+ \rho_{2001}|20\rangle\langle 02| + \rho_{1001}|10\rangle\langle 01| + \mathrm{h.c.}\end{aligned}$$

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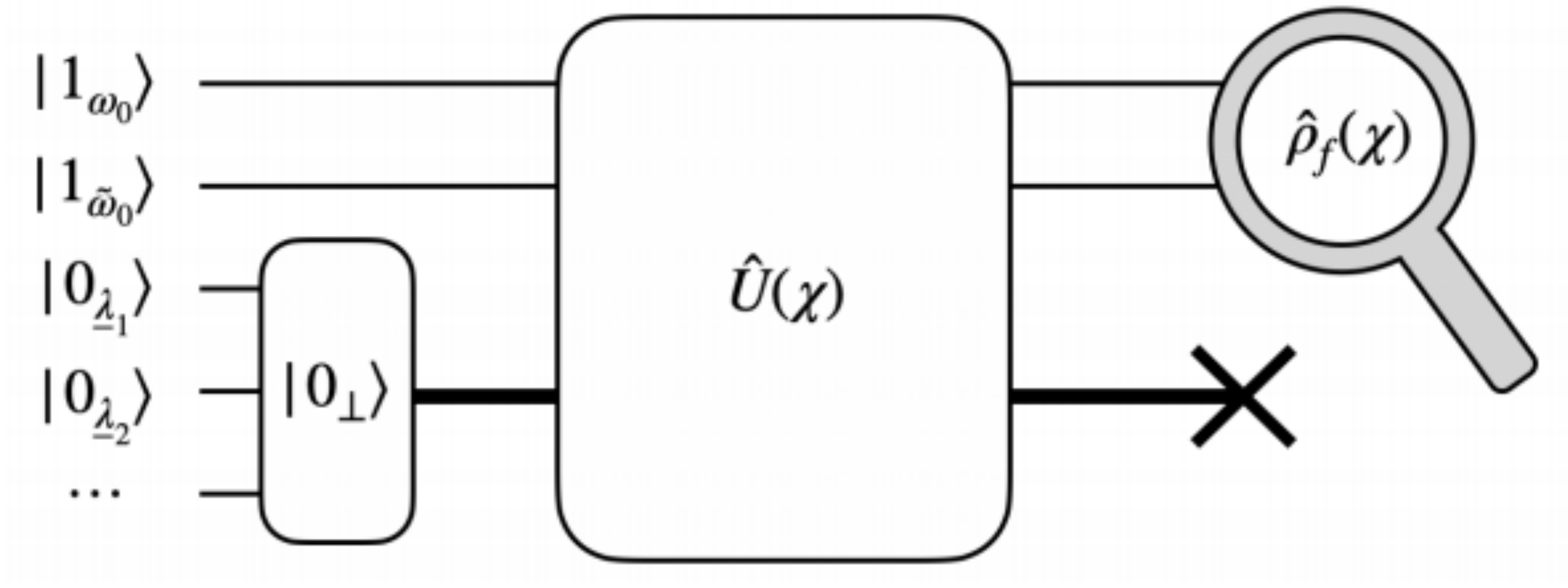
Curved spacetime as a mode mixer

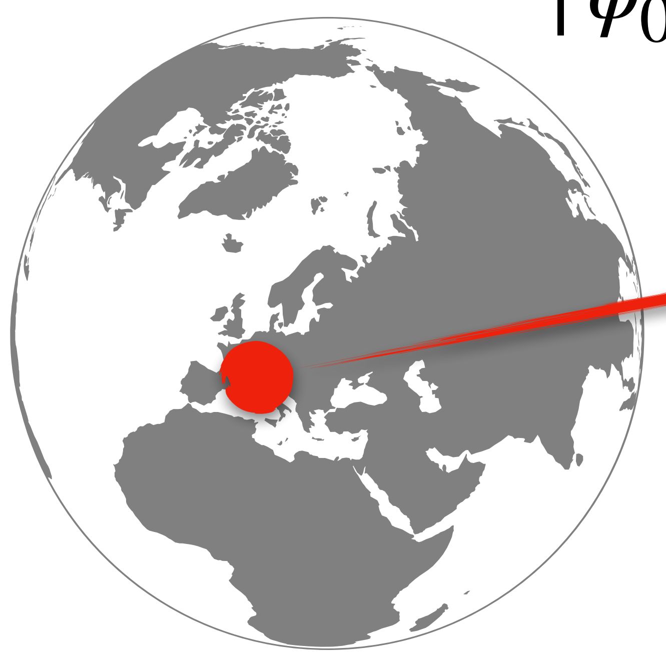
Extended photon

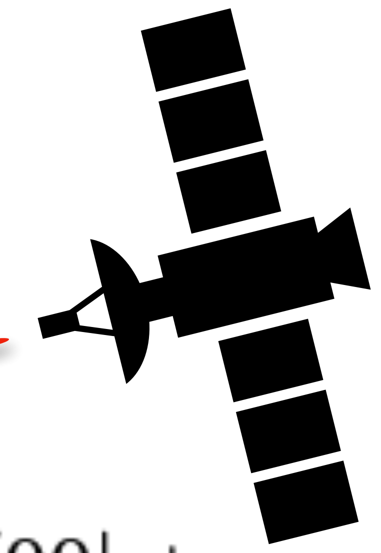
$$\hat{A}_{\omega_0} := \int d\omega F_{\omega_0}(\omega) e^{-i\psi(\omega)} \hat{a}_\omega$$

$$\hat{A}_{\tilde{\omega}_0} := \int d\omega \tilde{F}_{\tilde{\omega}_0}(\omega) e^{-i\tilde{\psi}(\omega)} \hat{a}_\omega$$



$$|\psi_0\rangle = |1_{\omega_0} 1_{\omega_0'}\rangle$$





Engineer all parameters
to selectively have:

$$\begin{aligned}\hat{\rho}_{\mathrm{f}}(\chi) =& \rho_{0000}|00\rangle\langle 00| + \rho_{0202}|02\rangle\langle 02| + \rho_{2020}|20\rangle\langle 20| \\ &+\rho_{1010}|10\rangle\langle 10| + \rho_{0101}|01\rangle\langle 01| + \rho_{1111}\cancel{|11\rangle\langle 11|} \\ &+\rho_{2011}\cancel{|20\rangle\langle 11|} + \rho_{0211}\cancel{|02\rangle\langle 11|} \\ &+\rho_{2001}|20\rangle\langle 02| + \rho_{1001}|10\rangle\langle 01| + \mathrm{h.c.}\end{aligned}$$

Hong-Ou-Mandel interference: Purely quantum

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Sensing

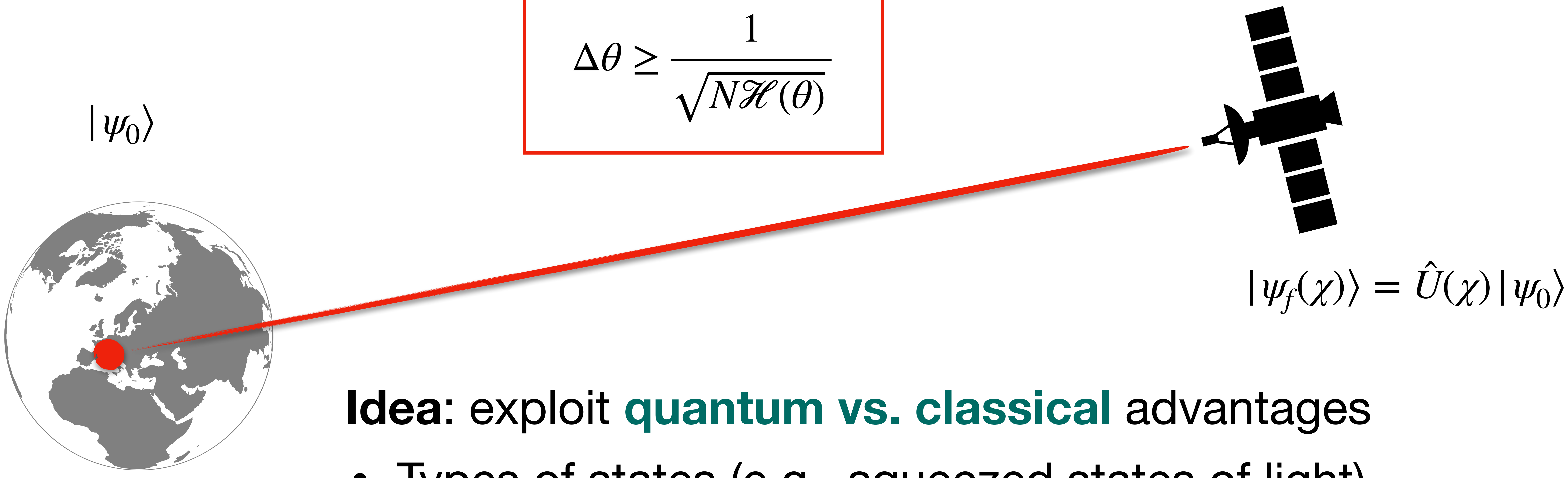
Q: what can we use this effect for?

Curved spacetime as a mode mixer

Quantum metrology

$$|\psi_0\rangle \xrightarrow[\text{(quantum) channel}]{\hat{U}(\chi)} |\psi_f(\theta)\rangle = \hat{U}(\theta)|\psi_0\rangle \xrightarrow[\text{state fidelity}]{\mathscr{F}} \mathscr{F}(|\psi\rangle, |\psi'\rangle) \xrightarrow[\text{Fisher info.}]{\mathscr{H}} \mathscr{H}(\theta) = \lim_{d\theta \to 0} 8\frac{1-\sqrt{\mathscr{F}_{\theta,\theta+d\theta}}}{d\theta^2}$$

$$\Delta\theta \geq \frac{1}{\sqrt{N\mathscr{H}(\theta)}}$$



Idea: exploit **quantum vs. classical** advantages

- Types of states (e.g., squeezed states of light)
- Types of measurements (global quantum)

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Limits to domain of applicability

Q: do all redshifts lead to effective transformations as described above?

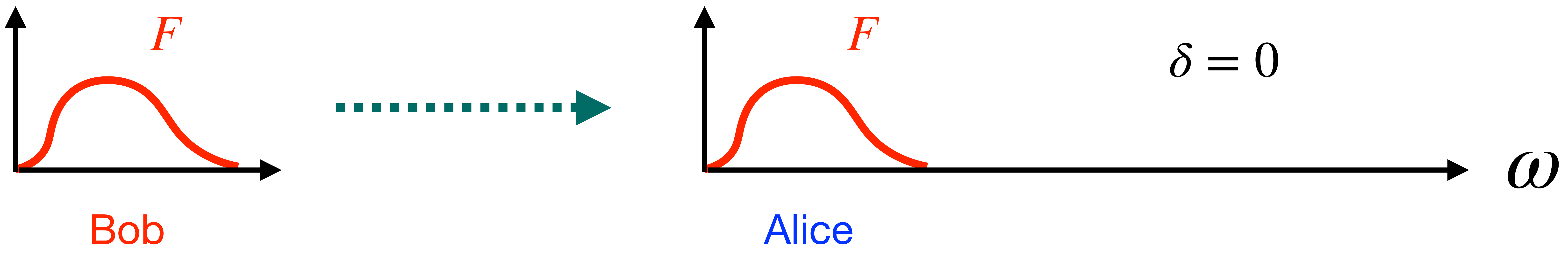
(Credit to Nils Leber (BSc) @ Universität des Saarlandes)

Beyond multimode mixing

One mode $\hat{a}(\theta) = \cos\theta\,\hat{a} + \sin\theta\,\hat{b}$

$\chi = 1 + \delta$

$|\langle F' | F \rangle| = 1$



$$\hat{a}(\theta) = \cos\theta\,\hat{a} + \sin\theta\,\hat{a}_{\perp}$$

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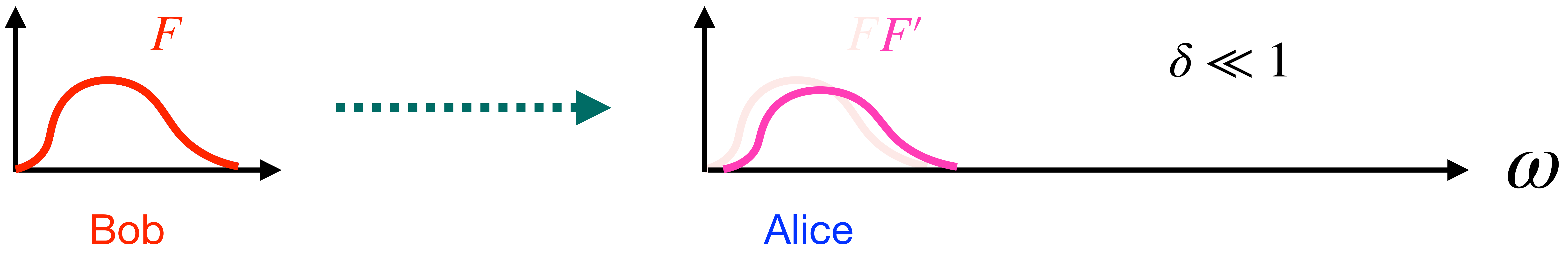
Beyond multimode mixing

One mode

$$\chi = 1 + \delta$$



$$\delta \ll 1$$

$$|\langle F' | F \rangle| = 1 - \delta^2$$

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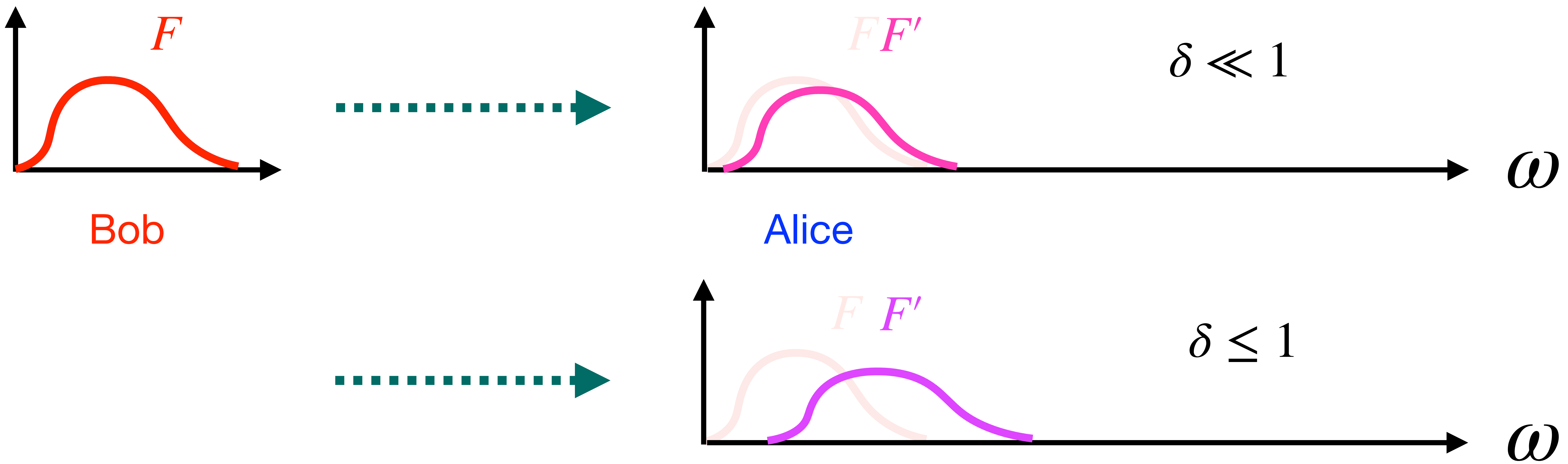
Beyond multimode mixing

One mode

$$\chi = 1 + \delta$$



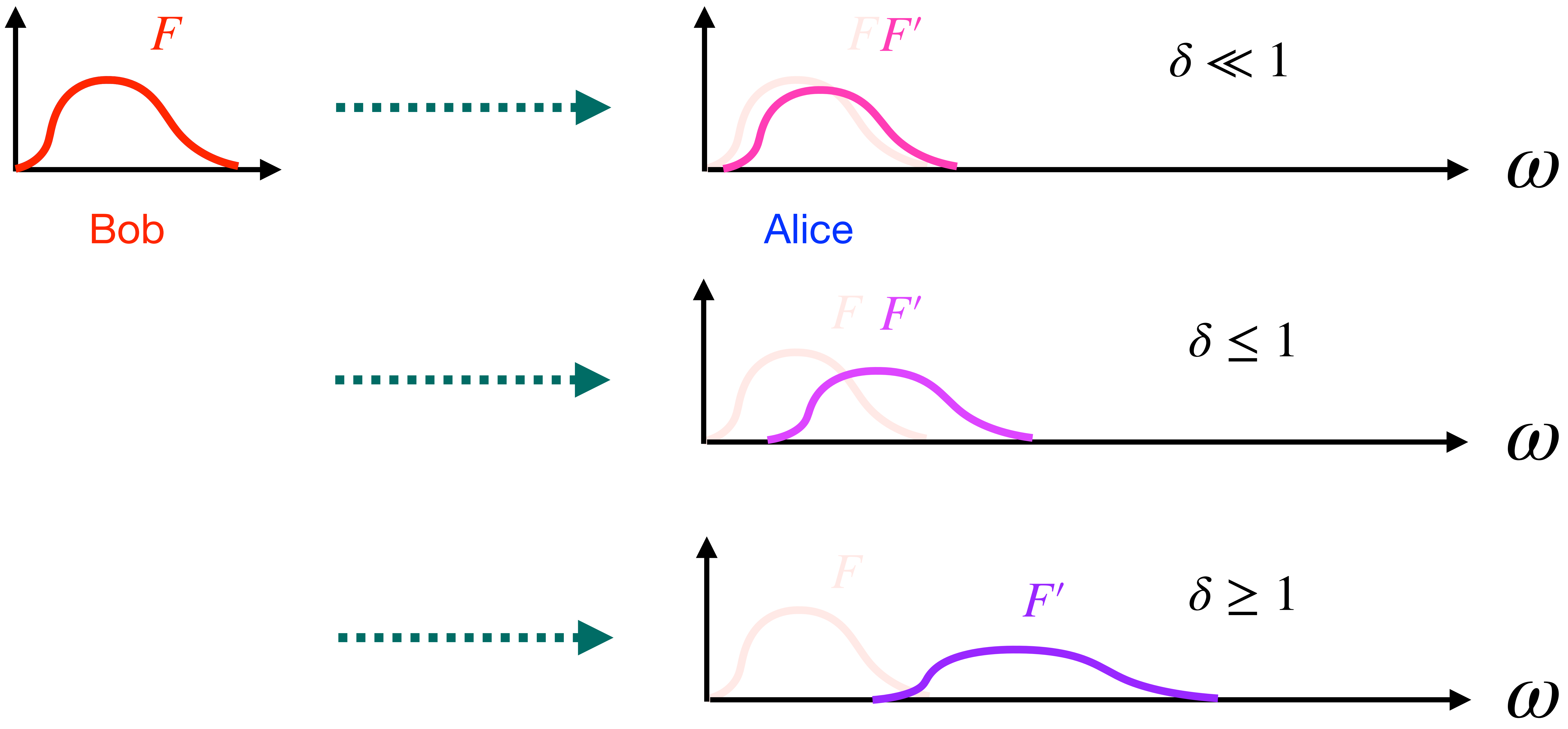
$$|\langle F' | F \rangle| = 1 - \delta^2$$

$$|\langle F' | F \rangle| = \rho < 1$$

Beyond multimode mixing

One mode

$\chi = 1 + \delta$



$|\langle F' | F \rangle| = 1 - \delta^2$

$|\langle F' | F \rangle| = \rho < 1$

$|\langle F' | F \rangle| = \rho \ll 1$

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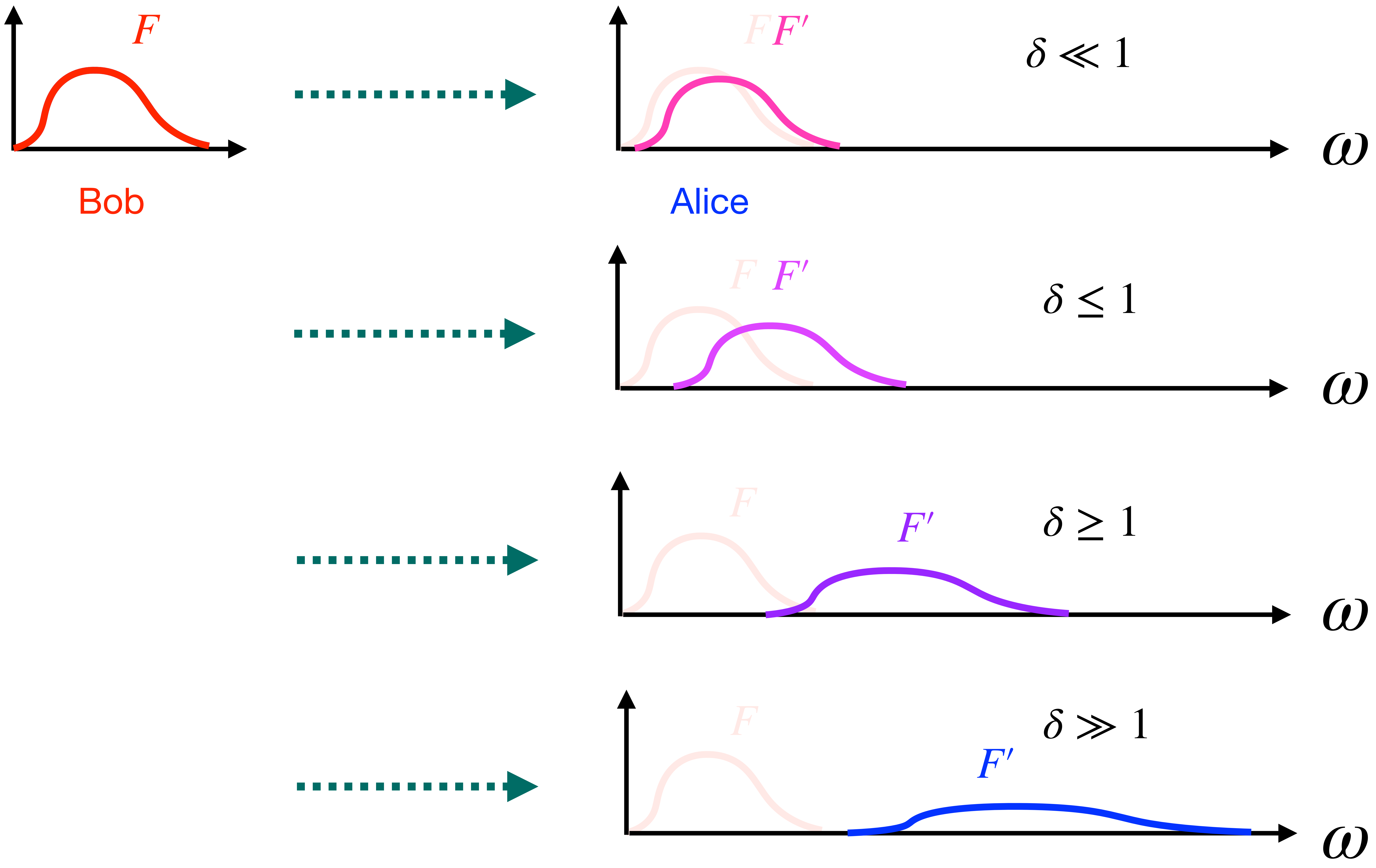
Beyond multimode mixing

One mode

$\chi = 1 + \delta$



$|\langle F' | F \rangle| = 1 - \delta^2$

$|\langle F' | F \rangle| = \rho < 1$

$|\langle F' | F \rangle| = \rho \ll 1$

$|\langle F' | F \rangle| = 0$

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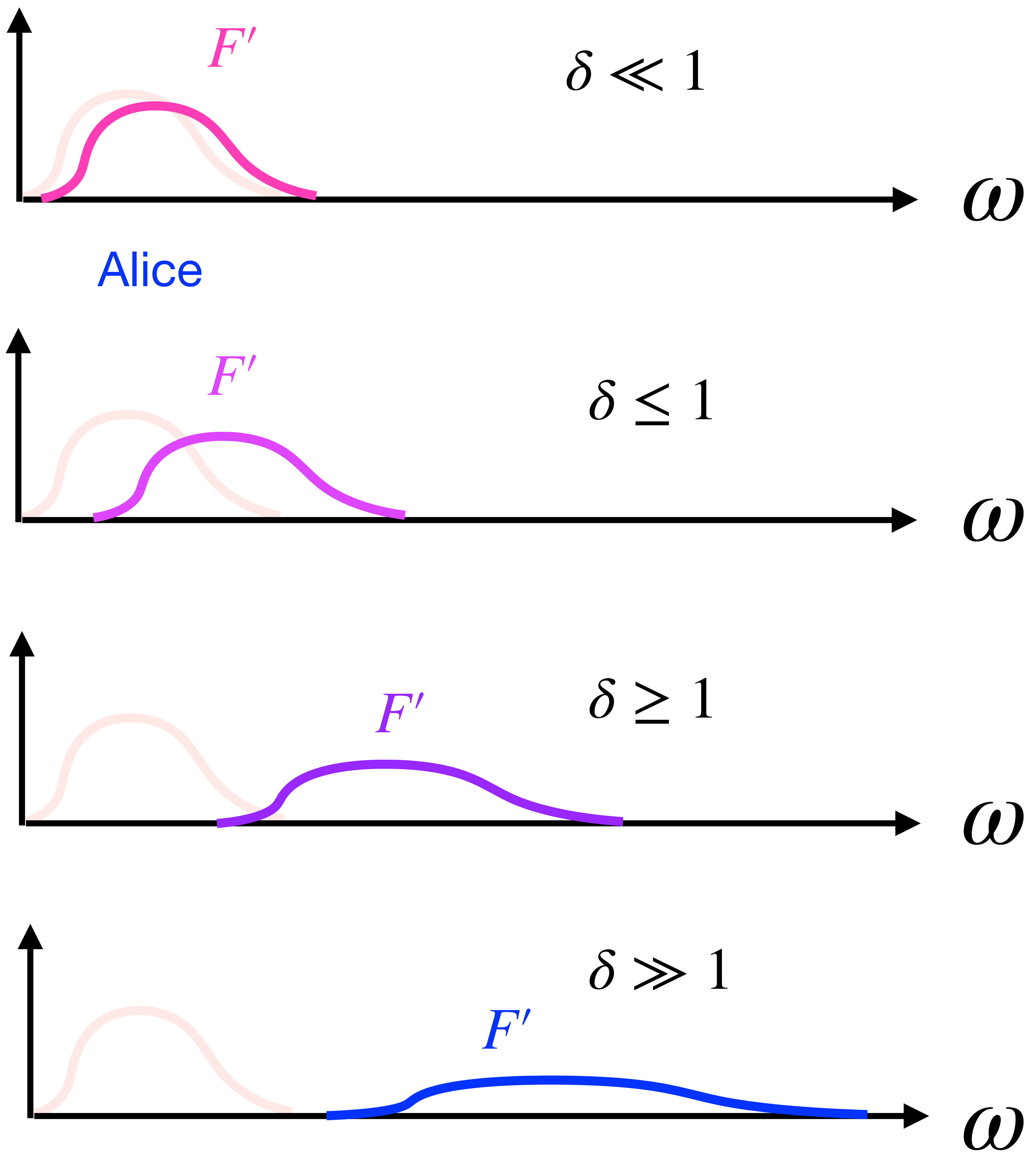
Beyond multimode mixing

One mode



$$\chi = 1 + \delta$$

$$\boldsymbol{U}(\chi) \approx \begin{pmatrix} 1 + ia\delta & ib\delta \\ ib\delta & 1 + ic\delta \end{pmatrix}$$

$$U_{11} \equiv \langle F' | F \rangle$$

$$|U_{12}| \equiv \sqrt{1 - |\langle F' | F \rangle|^2}$$

$$\rho := |\langle F' | F \rangle|$$

$$\theta := arg\langle F' | F \rangle$$

$$U(\chi) \approx \begin{pmatrix} \rho & e^{i\phi}\sqrt{1-\rho^2} \\ -e^{-i\phi}\sqrt{1-\rho^2} & \rho \end{pmatrix}$$

ONE MODE

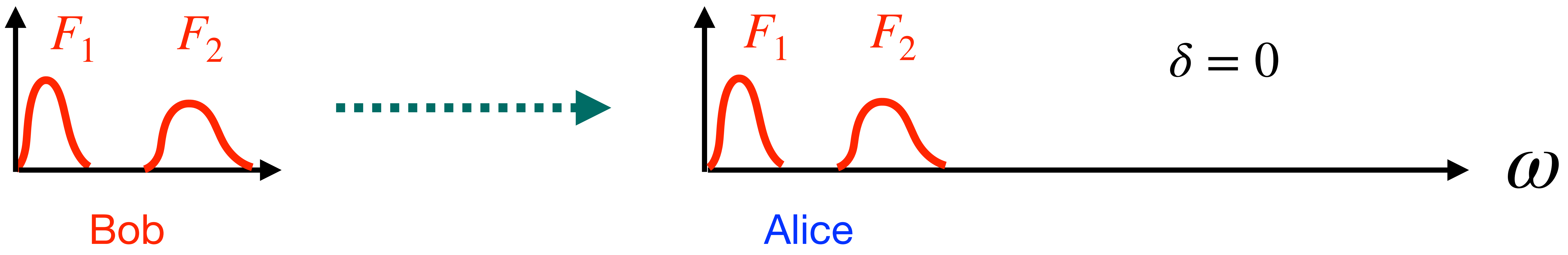
$$\boldsymbol{U}(\chi)\boldsymbol{U}^{\dagger}(\chi) = 1 + \mathcal{O}(\delta^2)$$

$$U(\chi) \approx \begin{pmatrix} 0 & e^{i\phi} \\ -e^{-i\phi} & 0 \end{pmatrix}$$

Beyond multimode mixing

Two modes

$$\chi = 1 + \delta$$



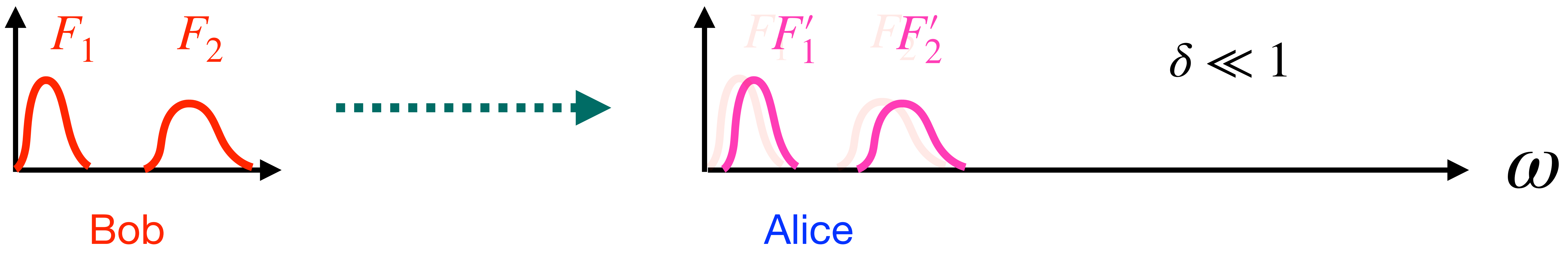
$$\delta = 0$$

$$|\langle F'_n | F_m \rangle| = \delta_{nm}$$

Beyond multimode mixing

Two modes

$$\chi = 1 + \delta$$

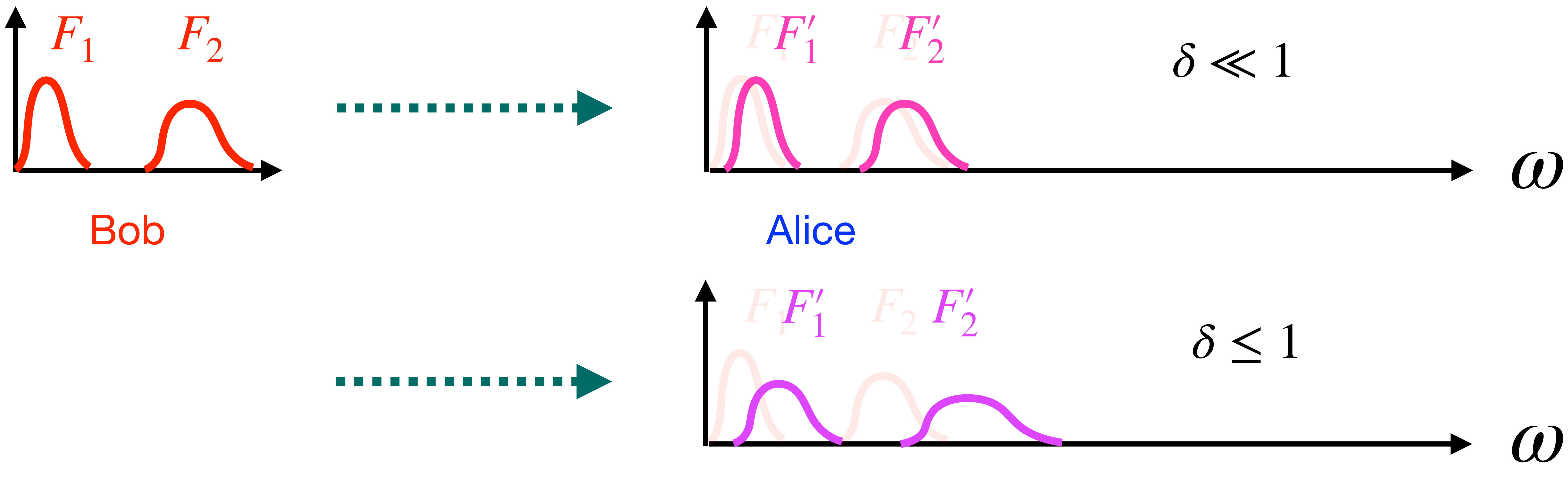


$$\delta \ll 1$$

$$|\langle F'_n | F_m \rangle| = (1 - a_{nm}\delta^2)\delta_{nm}$$

Beyond multimode mixing

Two modes



$$\chi = 1 + \delta$$

$$|\langle F_n' | F_m \rangle| = (1 - a_{nm}\delta^2)\delta_{nm}$$

$$\ldots$$

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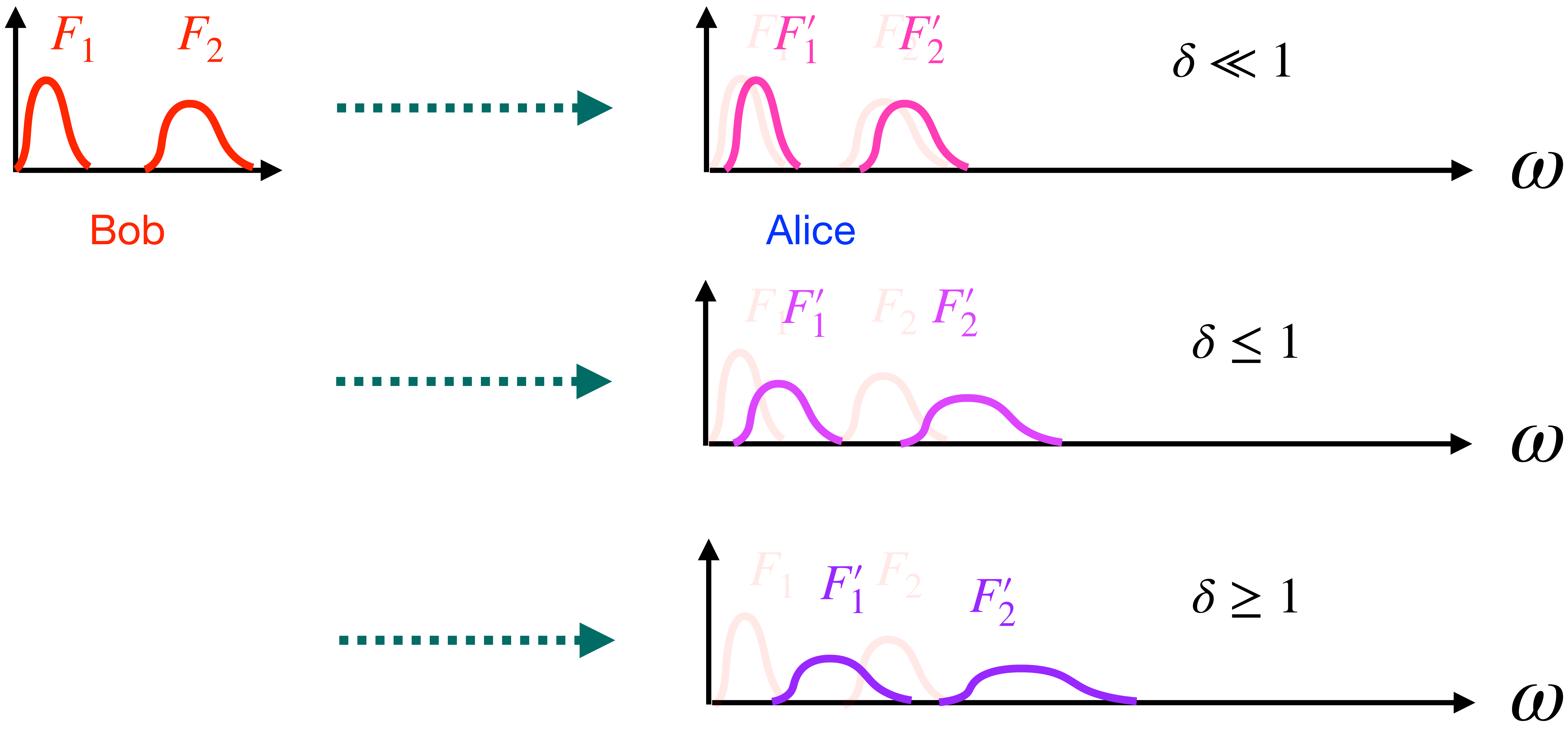
Beyond multimode mixing

Two modes

$$\chi = 1 + \delta$$



$$|\langle F'_n | F_m \rangle| = (1 - a_{nm}\delta^2)\delta_{nm}$$

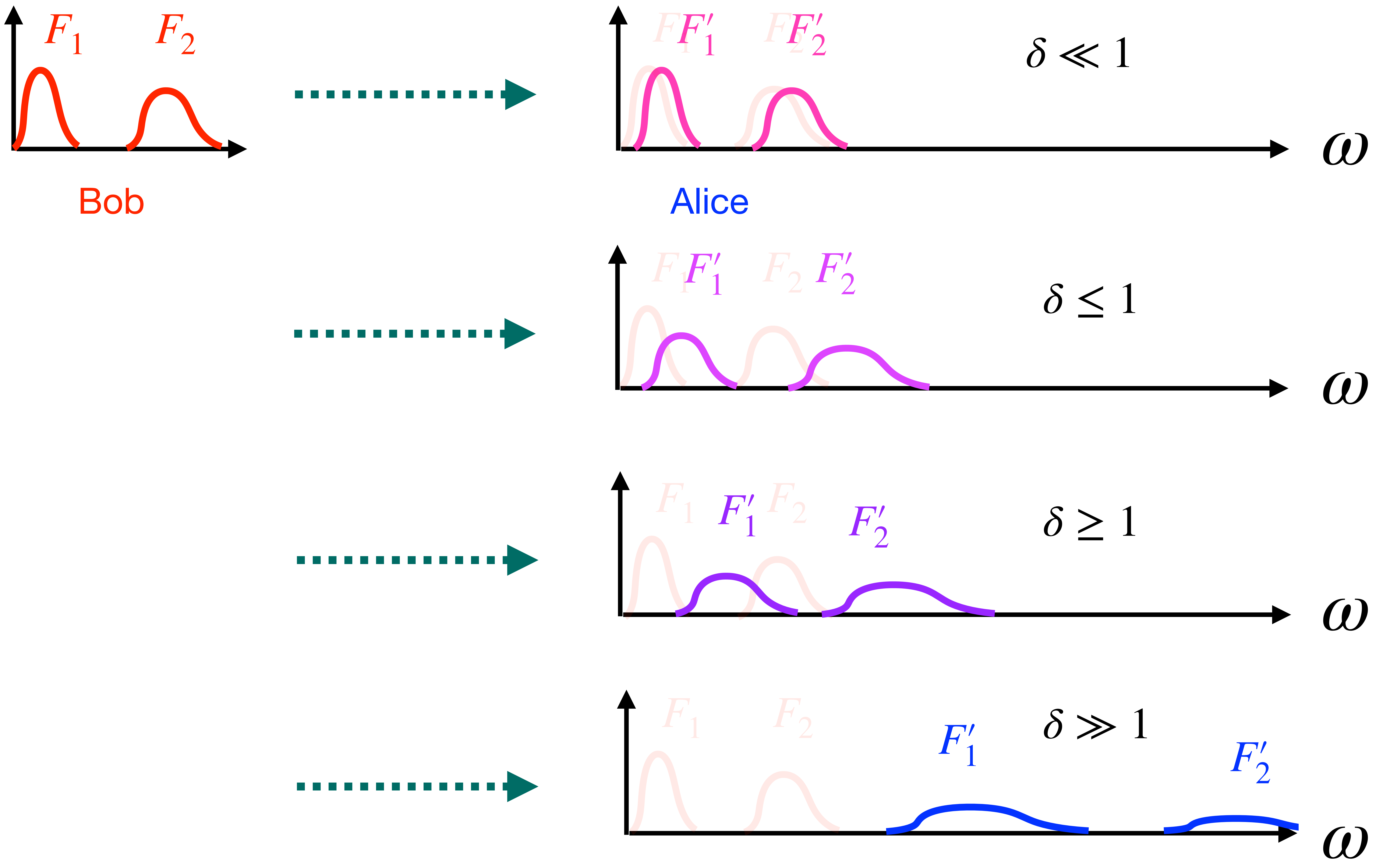
$\dots$

$\dots$

Beyond multimode mixing

Two modes

$$\chi = 1 + \delta$$



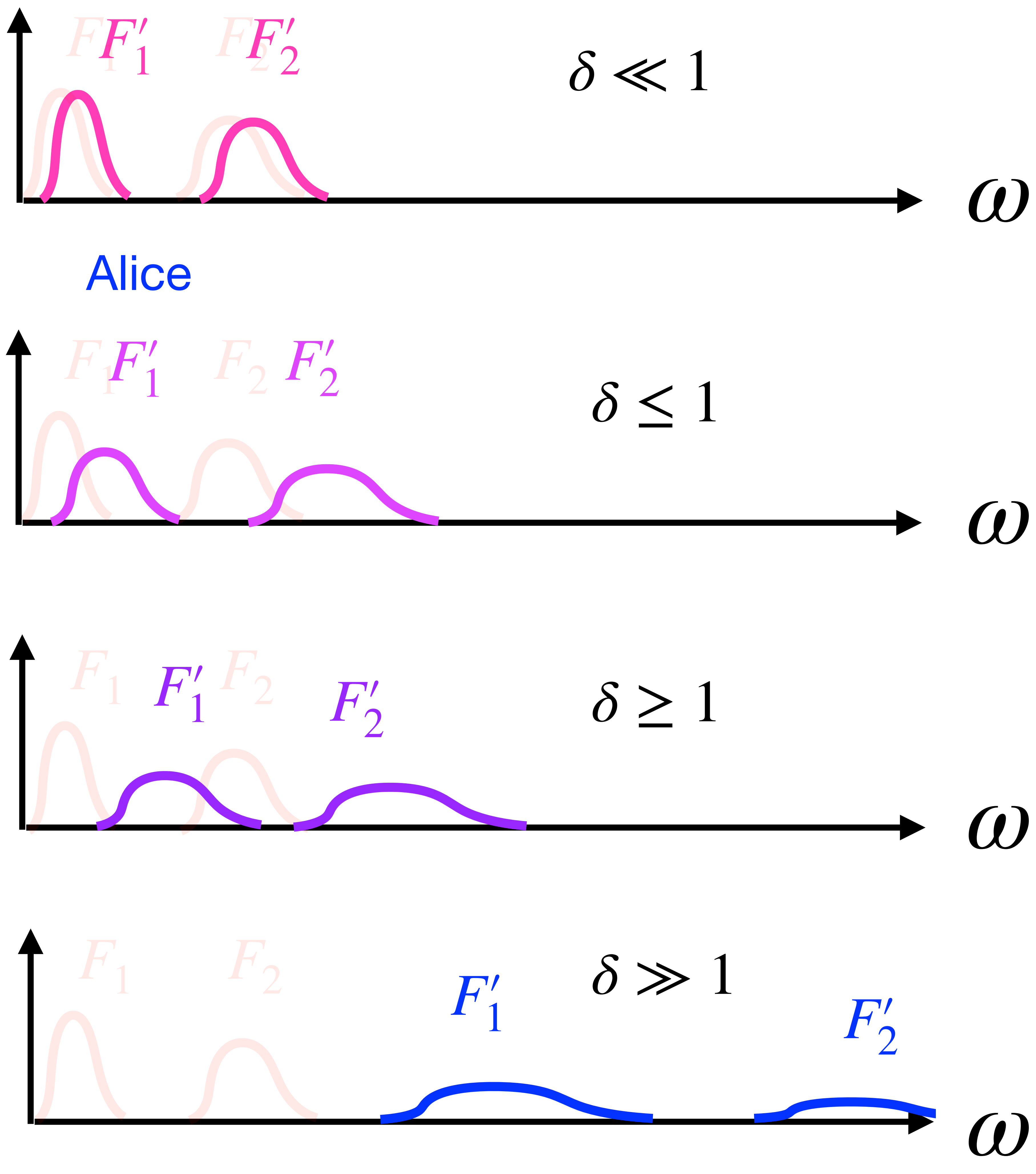
$$|\langle F'_n | F_m \rangle| = (1 - a_{nm}\delta^2)\delta_{nm}$$

...

...

$$|\langle F'_n | F_m \rangle| = 0$$

Beyond multimode mixing



Two modes

$$\chi = 1 + \delta$$

$$U(\chi) \approx \begin{pmatrix} 1 + ia_{11}\delta & 0 & ia_{13}\delta \\ 0 & 1 + ia_{22}\delta & ia_{23}\delta \\ -ia_{13}\delta & -ia_{23}\delta & 1 + ia_{33}\delta \end{pmatrix}$$

$$U_{nm} \equiv \langle F'_n | F_m \rangle \quad n = 1,2$$

$$|U_{n3}| \equiv \sqrt{1 - |U_{n1}|^2 - |U_{n2}|^2}$$

$$|U_{3n}| \equiv \sqrt{1 - |U_{1n}|^2 - |U_{2n}|^2}$$

$\ldots$

TWO MODES

$\ldots$

$$U(\chi) \approx \begin{pmatrix} 0 & 0 & e^{i\theta_{13}} \\ 0 & 0 & e^{-i\theta_{23}} \\ e^{-i\theta_{31}} & e^{-i\theta_{32}} & ? \end{pmatrix}$$

$$U(\chi)U^\dagger(\chi) = 1 + \mathcal{O}(\delta^2)$$

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Beyond multimode mixing

Conclusion

One mode

$$U(\chi) = \begin{pmatrix} \cos\theta(\chi) & e^{i\phi}\sin\theta(\chi) \\ -e^{-i\phi}\sin\theta(\chi) & \cos\theta(\chi) \end{pmatrix}$$

✓ For small redshifts

✓ For all redshifts

Two or more modes

$$U(\chi) \approx 1 + iU^{(1)}\delta$$

✓ For small redshifts

✗ For all redshifts

Limits to domain of applicability

Q: do all redshifts lead to effective transformations as described above?

Limits to domain of applicability

Q: do all redshifts lead to effective transformations as described above?

A: No

Limits to domain of applicability

Q: do all redshifts lead to effective transformations as described above?

A: No

Solutions: not clear what process is occurring for very large redshifts

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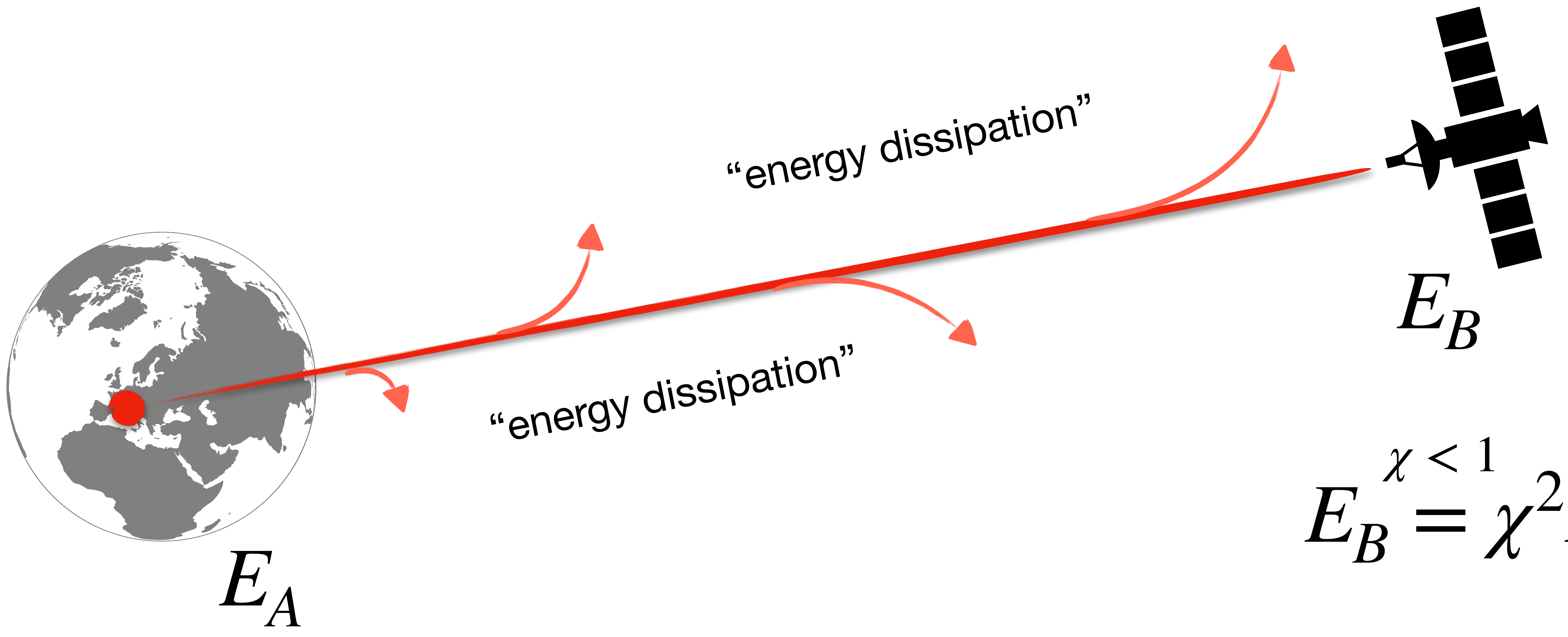
Limits to domain of applicability

Considerations



$$E_B \overset{\chi<1}{=} \chi^2 E_A$$

Q1: where does the energy go (redshift) or come from (blueshift)?

Q2: for large energy loss, which transformation beyond mode-mixing applies?

Q3: do we need to consider the (quantum) dynamics of gravity?

Conclusions and Outlook

We studied the **action of gravitational redshift on modes of light.**

In particular we:

- **Computed** the effects of **deformation** of wave packets
- **Determined** the **rigid shift** of wave packets (“redshift”)
- **Obtained** **genuine deformation** of wave packets
- **Modelled** the transformation as **mode-mixer** for realistic photons
- **Predicted** **quantum interference** due to propagation
- **Encountered** **limitation** to domain of validity

Future ambitions:

- **Strengthen** the **theoretical understanding**
- **Propose** for **tests with cubesats/nanosatellites**
- **Develop** applications for **sensing**

Merci

- **Introduction to gravitational redshift of quantum photons propagating in curved spacetime**
  J. Phys.: Conf. Ser. 2531, 012016 (2023)
- **Gravitational redshift induces quantum interference**
  Ann. Phys. 535, 2200468 (2022)
- **Spacetime effects on wavepackets of coherent light**
  Phys. Rev. D 104, 085015 (2021)
- **Quantum-metrology estimation of spacetime parameters of the Earth outperforming classical precision**
  PRA 99, 032350 (2019)
- **Quantum communications and quantum metrology in the spacetime of a rotating planet**
  EPJ Q Techn. 4:7 (2017)
- **Quantum estimation of the Schwarzschild space-time parameters of the Earth**
  PRD 90, 124001 (2014)
- **Spacetime effects on satellite-based quantum communications**
  PRD 90, 045041 (2014)

Non prævalebunt

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