Symmetry Breaking in Accelerated Frames: Can it Actually be Restored?

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@ Avenues of Quantum Field Theory In Curved Spacetime

Based on:

”Symmetry restoration and uniformly accelerated observers in Minkowski spacetime”, J. High Energ. Phys. 2024, 218

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Problem Statement

Can a spontaneously broken symmetry be restored under the effect of Unruh temperature?

Tension in literature:

- Symmetry Phase Preservation;
- Symmetry Phase Restoration;
- Symmetry Phase 'Anti-Restoration'.

Challenged physical concepts:

- Invariance of Scalars under General Coordinates Transformation;
- Thermodinamical Nature of Unruh Temperature.

Symmetry Breaking

Real, scalar, ϕ^4 field theory with spontaneous symmetry breaking:

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4, \qquad \text{with} \quad m^2 < 0.$$

Broken Symmetry: $\mathbb{Z}_2$

Quantum corrections, one-loop effective action
background field method $\phi \to \Phi + \tilde{\phi}$:

$$\Gamma[\Phi] = S[\Phi] - i\mathcal{N} \ln \mathrm{Det} \left[\sqrt{-g} \left(\Box + m^2 + \frac{\lambda}{2} \Phi^2 \right) \right]$$

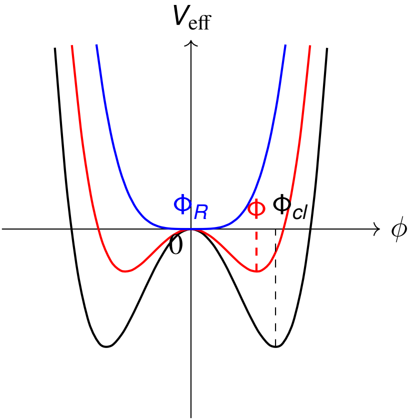
$$\Downarrow \qquad \text{Equation of Motion}$$

$$-\partial_\mu \left(\sqrt{-g} \partial^\mu \Phi \right) - \Phi \left(\frac{\lambda}{6} \Phi^2 + m^2 + \frac{1}{2} \lambda \langle \tilde{\phi}^2 \rangle \right) = 0$$

Effective Potential: Symmetry Breaking and Restoration

Flat Spacetime

Minima: $$\Phi = \pm\sqrt{-6\frac{m^2}{\lambda} - 3\langle\phi^2\rangle_{\mathrm{ren}}}$$



Classical Potential:
$\langle\phi^2\rangle_{\mathrm{ren}} = 0$
$\Phi_{cl} = -\sqrt{6m^2/\lambda}$

Effective Potential:
$0 < \langle\phi^2\rangle_{\mathrm{ren}} < -2m^2/\lambda$

Effective (Restored) Potential:
$\langle\phi^2\rangle_{\mathrm{ren}} = -2m^2/\lambda$
$\Phi_R = 0$

Unruh Effect

- Vacua relation - Unitarity loss:

$$|0\rangle_M = \sqrt{1 - e^{-2\pi\frac{\Omega}{a}}}\,\exp\left(e^{-\pi\frac{\Omega}{a}} b_k^{(R)\dagger} b_{\tilde{k}}^{(L)\dagger}\right)|0\rangle_R$$

- Thermal spectrum - Different particle content:

$$_M\langle 0|\, b_k^{(R)\dagger} b_k^{(R)}\, |0\rangle_M = \frac{1}{e^{\beta_U \Omega} - 1}, \qquad \text{with} \quad \beta_U = \frac{2\pi}{a}$$

- Thermal spectrum - Observables expectation values:

$$_M\langle 0|\, \phi(x)\phi(x')\, |0\rangle_M \quad \Longrightarrow \quad \mathrm{Tr}\left[e^{-\beta_U H^R}\phi(x)\phi(x')\right] / \mathrm{Tr}\left[e^{-\beta_U H^R}\right]$$

State of the Art

- Symmetry Phase Preservation:
  [D.N. Page, Phys.Rev.D 25, (1982)]
  [W.G. Unruh and Weiss, Phys.Rev.D29, (1984)]
  [C. Hill, Phys.Lett.B 155 (1985)]

- Symmetry Phase Restoration:
  [T. Ohsaku, Phys.Lett.B 599 (2004)]
  [D. Ebert and V.C. Zhukovsky, Phys.Lett.B 645 (2007)]
  [P. Castorina and M. Finocchiaro, J.Mod.Phys. 3 (2012)]
  [A. Casado-Turrión and A. Dobado, Phys.Rev.D 99 (2019)]

- Symmetry Phase 'Anti-restoration':
  [P. Candelas and D.J. Raine, J.Math.Phys. 17 (1976)]
  [S. Benic and K. Fukushima, arXiv:1503.05790 (2015)]
  [M. Chernodub, (poster session)]

Symmetry Phase Preservation - Unruh-Weiss Contribution

Thermalization Theorem for Interactive Fields:

$$ {}_M\langle 0 | \left(\phi\left(x_1\right), \cdots, \phi\left(x_n\right)\right)_t |0\rangle_M = \frac{\mathrm{Tr}\left[e^{-\beta_U H^R}\left(\phi\left(x_1\right), \cdots, \phi\left(x_n\right)\right)_\tau\right]}{\mathrm{Tr}\left[e^{-\beta_U H^R}\right]} $$

Φ and $\langle \phi^2 \rangle$ in a thermal bath of Rindler quanta are equal to their zero-temperature Minkowski counterparts!

On the renormalization issue:

”Finally, we note that the results of this paper should not be affected by renormalization of the theory. [...]
Thus any Feynman graph regulated using dimensional regularization in the Rindler system at temperature $T = a/2\pi$ will be equal to the corresponding regulated $T = 0$ graph in Minkowski space. Since all counterterms are Lorentz invariant, the renormalized graphs should be equal as well.”

Symmetry Phase Preservation - Hill and Page Contribution

"[...] will a stationary observer (Schwarzschild) around a black hole of sufficiently small mass witness symmetry restoration? Simple heuristic arguments suggest that the answer must be no. In the ground state we have the order parameter, $\phi_{\text{ren}} = \langle 0| \phi |0\rangle$ which is a scalar and must transform into itself in the Rindler coordinate system"

For Einstein Metrics, $R_{\mu\nu} = \Lambda g_{\mu\nu}$,

$$\langle \phi^2 \rangle_{\text{ren}} = \frac{1}{12}\left(T_{loc}^2 - T_{acc}^2\right) - \frac{1}{48\pi^2}\Lambda$$

"...At $T = 0$ the Minkowski vacuum has $\langle \phi^2 \rangle_{\text{ren}} = 0$, but in Rindler coordinates a static observer is accelerating and sees thermal radiation with $T_{loc} = T_{acc}$, so again [equation above] gives exactly the right answer."

Symmetry Phase Preseravation - Covariant Point Splitting

Christensen counter-terms:

$$G_{\rm CT}\left(x,x'\right)=i\left(\frac{1}{8\pi^2\sigma}+\frac{m^2+\left(\xi-\frac{1}{6}\right)}{8\pi^2}\left[\gamma_E+\frac{1}{2}\ln\left(\frac{m^2\sigma}{2}\right)\right]-\frac{m^2}{16\pi^2}+\frac{1}{96\pi^2}R_{\alpha\beta}\frac{\sigma^\alpha\sigma^\beta}{\sigma}\right)$$

$\Downarrow$ Minkowski space

$$G_{CT}\left(x,x'\right)=i\left(\frac{1}{8\pi^2\sigma}+\frac{m^2}{8\pi^2}\left[\gamma_E+\frac{1}{2}\ln\left(\frac{m^2\sigma}{2}\right)\right]-\frac{m^2}{16\pi^2}\right)$$

Minkowski propagator UV-structure:

$$G_M\left(x,x'\right)=i\frac{m}{4\pi^2\sqrt{2\sigma}}K_1\left(m\sqrt{2\sigma}\right)$$

$\Downarrow$ $\sigma\ll 1$

$$G_{CT}\left(x,x'\right)=i\left(\frac{1}{8\pi^2\sigma}+\frac{m^2}{8\pi^2}\left[\gamma_E+\frac{1}{2}\ln\left(\frac{m^2\sigma}{2}\right)\right]-\frac{m^2}{16\pi^2}\right)+O\left(\sigma\right)$$

Symmetry Phase Preservation - Result

$$\langle\phi^2\rangle_{\text{ren}} = \text{Tr}\left[e^{-\beta_U H^R}\phi^2\right]/\text{Tr}\left[e^{-\beta_U H^R}\right] - {}_M\langle 0|\,\phi^2\,|0\rangle_M$$
$$= {}_M\langle 0|\,\phi^2\,|0\rangle_M - {}_M\langle 0|\,\phi^2\,|0\rangle_M = 0$$

Symmetry can NOT be Restored!

Frame Independent Renormalization Scheme

Symmetry Phase Restoration Perspective

Symmetry Restoration Conditions:

$$\Phi = \pm\sqrt{-6\frac{m^2}{\lambda} - 3\langle\phi^2\rangle} = 0 \quad \Longleftrightarrow \quad \left.\frac{\partial V_{\text{eff}}}{\partial \Phi^2}\right|_{\Phi=0} = \frac{m^2}{2} + \frac{\lambda}{4}\langle\phi^2\rangle = 0$$

Propagators ($x \to x'$ limit):

$${}_M\langle 0|\,\phi(x)\,\phi(x')\,|0\rangle_M = \frac{\text{Tr}\left[e^{-\beta_U H^R}\phi(x)\,\phi(x')\right]}{\text{Tr}\left[e^{-\beta_U H^R}\right]}$$

Acceleration ($\frac{m}{a} \to 0$ limit):	Temperature ($m \to 0$ limit):
$\langle\phi^2\rangle = \frac{1}{4\pi^2}\int_0^\infty d\Omega\,\Omega\left(1 + \frac{2}{e^{\frac{2\pi\Omega}{a}} - 1}\right)$	$\frac{\partial V^{(1)}_{T=0}}{\partial\Phi^2} = \frac{\lambda}{16\pi^2}\int d\omega\,\omega$ $\frac{\partial V^{(1)}_{T}}{\partial\Phi^2} = \frac{\lambda}{16\pi^2}\int d\omega\,\frac{2\omega}{e^{\beta\omega} - 1}$ (cf. [Dolan, Jackiw (1974)])

Symmetry Phase Restoration Perspective

Renormalized vacuum polarization:

$$\langle \phi^2 \rangle_{\text{ren}} = \frac{1}{4\pi^2} \int_0^\infty d\Omega \left(\cancel{\Omega} + \frac{2\Omega}{e^{\frac{2\pi\Omega}{a}} - 1} \right) = \frac{a^2}{48\pi^2}$$

Critical Unruh temperature/acceleration:

$$T_c^2 = \frac{a_c^2}{4\pi^2} = -\frac{24m^2}{\lambda}$$

Symmetry Phase Restoration Perspective - Result

$$ {}_R\langle 0|\, \phi^2 \,|0\rangle_R = {}_M\langle 0|\, \phi^2 \,|0\rangle_M - \frac{a^2}{48\pi^2} $$

Minkowski vacuum is not hot, Rindler vacuum is 'depressed' by temperature $-T_U$.

$$ {}_M\langle 0|\, \phi^2 \,|0\rangle_M - {}_R\langle 0|\, \phi^2 \,|0\rangle_R = \frac{a^2}{48\pi^2} $$

Symmetry can be Restored!

Frame Dependent Renormalization Scheme

Symmetry Phase 'Anti-Restoration' - Candelas, Fukushima, Chernodub Contribution

$$\langle \phi^2 \rangle_{\text{ren}} = {}_R\langle 0 | \, \phi^2 \, | 0 \rangle_R - {}_M\langle 0 | \, \phi^2 \, | 0 \rangle_M = -\frac{a^2}{48\pi^2}$$

Symmetry is resistant to restoration!

Frame Independent Renormalization Scheme

$$\langle \phi^2 \rangle_{\text{ren}} = \frac{1}{12} \left(T^2 - T_{acc}^2 \right) \qquad \text{with } T_{min} = 0$$

It require a higher temperature to restore the broken symmetry.

Conclusions

Result	Phys. quan.	Renorm. term	$\langle\phi^2\rangle_{\text{ren}}$	Ren. scheme
Covariant	${}_M\langle 0\| \phi^2 \|0\rangle_M$	${}_M\langle 0\| \phi^2 \|0\rangle_M$	0	Frame ind.
Restor.	${}_M\langle 0\| \phi^2 \|0\rangle_M$	${}_R\langle 0\| \phi^2 \|0\rangle_R$	$a^2/(48\pi^2)$	Frame dep.
Anti-Rest.	${}_R\langle 0\| \phi^2 \|0\rangle_R$	${}_M\langle 0\| \phi^2 \|0\rangle_M$	$-a^2/(48\pi^2)$	Frame ind.

- *Covariant result*: aligns with curved spacetimes approach. Unruh temperature is not able to restore a broken symmetry and can not be regarded as 'regular' temperature.
- *Restoration result*: Unruh temperature is in every respect a 'regular' temperature. Need to adapt the gravitational, curved spacetime case for accelerated observers.
- *Anti-restoration result*: Unruh temperature behaves like a negative temperature. Need to adapt the gravitational, curved spacetime case for accelerated observers.

Thank you
for your attention

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