Displacement memory for flyby

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Abstract: Zel'dovich and Polnarev suggested that particles hit by a burst of gravitational waves generated by flyby would merely be displaced. Their prediction is confirmed by fine-tuning the derivative-of-a-Gaussian wave profile proposed by Gibbons and Hawking, or analytically by its approximation by a Pöschl-Teller potential. The study is extended to higher-order derivative profiles as proposed for gravitational collapse.

Based on:

P. M. Zhang and P. A. Horvathy, "Displacement within velocity effect in gravitational wave memory," Annals Phys. **470** (2024), 169784 [arXiv:2405.12928 [gr-qc]].

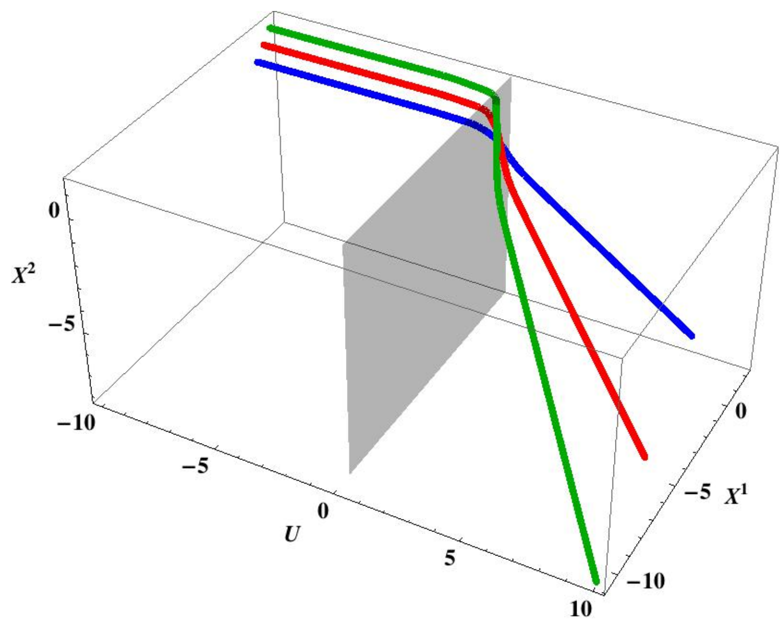
P. M. Zhang, Q. L. Zhao, J. Balog, P. A. Horvathy, "Displacement memory for flyby," Annals Phys. **473** (2025), 169890 [arXiv:2407.10787 [gr-qc]].

P. M. Zhang, Q. L. Zhao, M. Elbistan P. A. Horvathy, "Gravitational wave memory: further examples," [arXiv:2412.02705 [gr-qc]].

Memory effect

A. Velocity VM J. Ehlers and W. Kundt

"Exact solutions of the gravitational field equations," *in Gravitation: An Introduction to Current Research*, edited by L. Witten (Wiley, New York, London, 1962).



Particles hit by GW fly apart with non-zero constant velocity.

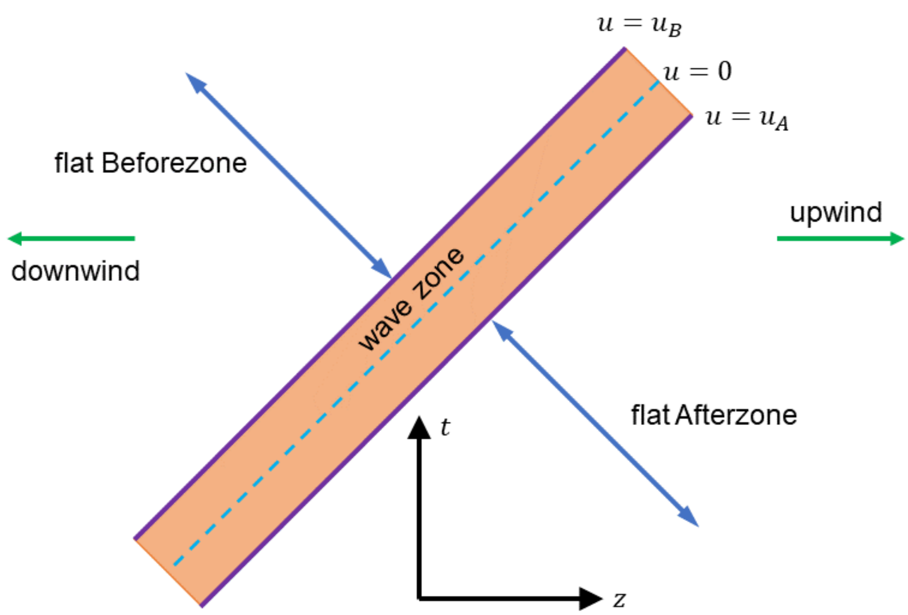
B. Displacement DM Zel'dovich, Polnarev

"Radiation of gravitational waves by a cluster of superdense stars," Astron. Zh. **51**, 30 (1974)

> *. . . [for] two noninteracting bodies (such as satellites). [. . .] the distance should change, and this effect might possibly serve as a nonresonance detector. [. . .] their relative velocity will become vanishingly small as* **flyby** *concludes.*

G. W. Gibbons S. W. Hawking *"Theory of the detection of short bursts of gravitational radiation,"* Phys. Rev. D **4** (1971) 2191.

Sandwich wave: burst of gravitational wave. Spacetime non-flat only in short interval $u_B \leq u \leq u_A$ of retarded time [Wavezone]. Flat both in Beforezone $u < u_B$ that the wave has not reached yet, and in Afterzone $u_A < u$ where has already passed,



$\mathcal{A}(U) \neq 0$ only in "wave zone" $U_B < U < U_A$.

Gibbons - Hawking *flyby* $\sim$ *1st derivative* of Gaussian,

$$\mathcal{A}(U) = \frac{1}{2} \frac{d(e^{-U^2})}{dU}. \tag{1}$$

Geodesics in Brinkmann* coordinates

1. Plane GW in 1 space + 2 lightlike dimensions (toy model).

$$g_{\mu\nu}X^{\mu}X^{\nu} = dX^2 + 2dUdV - \mathcal{A}(U)X^2dU^2 \qquad (2)$$

Sandwich wave: $\mathcal{A}(U) \neq 0$ only in "wave zone" $U_B < U < U_A$.

For non-tachyonic geodesic: Jacobi invariant

$$\mathfrak{m}^2 = g_{\mu\nu}\dot{X}^{\mu}\dot{X}^{\nu} = \text{const} \leq 0\,. \qquad (3)$$

Massive: $\mathfrak{m}^2 < 0$, Lightlike $\mathfrak{m}^2 = 0$.

* M. W. Brinkmann, "Einstein spaces which are mapped conformally on each other," Math. Ann. **94** (1925) 119–145.

Lightlike geodesics $\mathfrak{m}^2 = 0$:

$$\frac{d^2X}{dU^2} + \frac{1}{2}\mathcal{A}X = 0\,, \qquad (4a)$$

$$\frac{d^2V}{dU^2} - \frac{1}{4}\frac{d\mathcal{A}}{dU}(X)^2 - \tfrac{1}{2}\mathcal{A}\frac{d(X^2)}{dU} = 0\,. \qquad (4b)$$

$V(U)$ horizontal lift of $X(U)$

Coordinate X decoupled from V. Projection into transverse space is V-independent.

Conversely, lightlike geo determined by eqn. (4a) with U viewed as Newtonian time *.

*

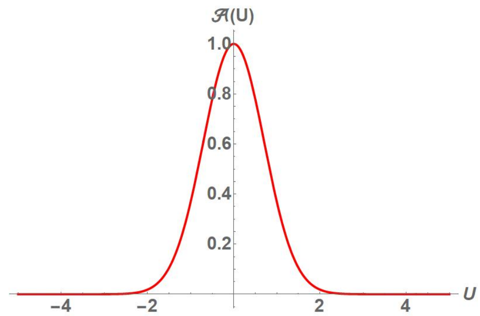
C. Duval, G. Burdet, H. P. Kunzle and M. Perrin, *"Bargmann Structures and Newton-cartan Theory,"* Phys. Rev. D **31** (1985), 1841-1853

C. Duval, G. W. Gibbons and P. Horvathy, *"Celestial mechanics, conformal structures and gravitational waves,"* Phys. Rev. D **43** (1991), 3907-3922 [arXiv:hep-th/0512188 [hep-th]].

L. P. Eisenhart, *"Dynamical trajectories and geodesics"*, Annals. Math. **30** 591-606 (1928).

Gaussian profile

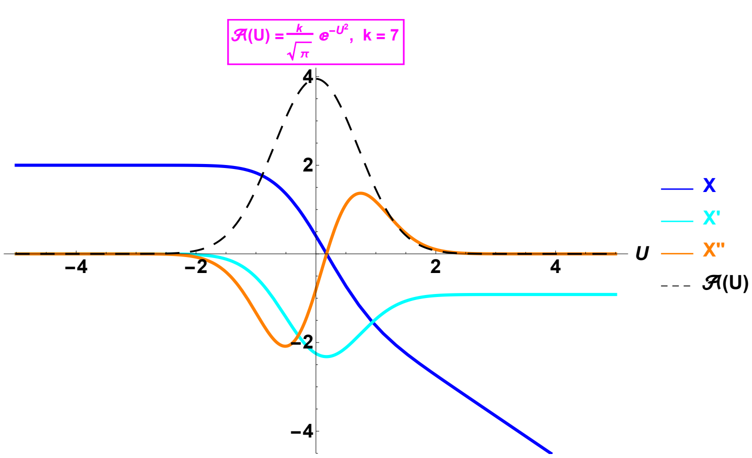
$$\mathcal{A}^{Gauss}(U) = \frac{k}{\sqrt{\pi}} e^{-U^2} \qquad (5)$$

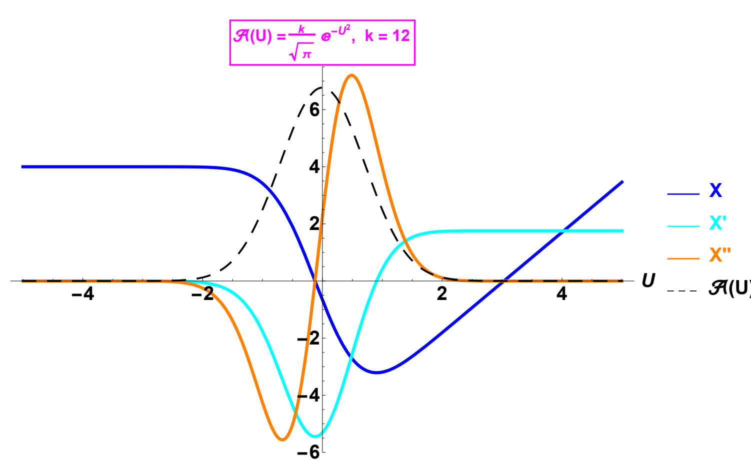


Outside (approximate) Wavezone $U_b < U < U_a$ both *velocity and force* vanish $\Rightarrow$ free motion (Newton).

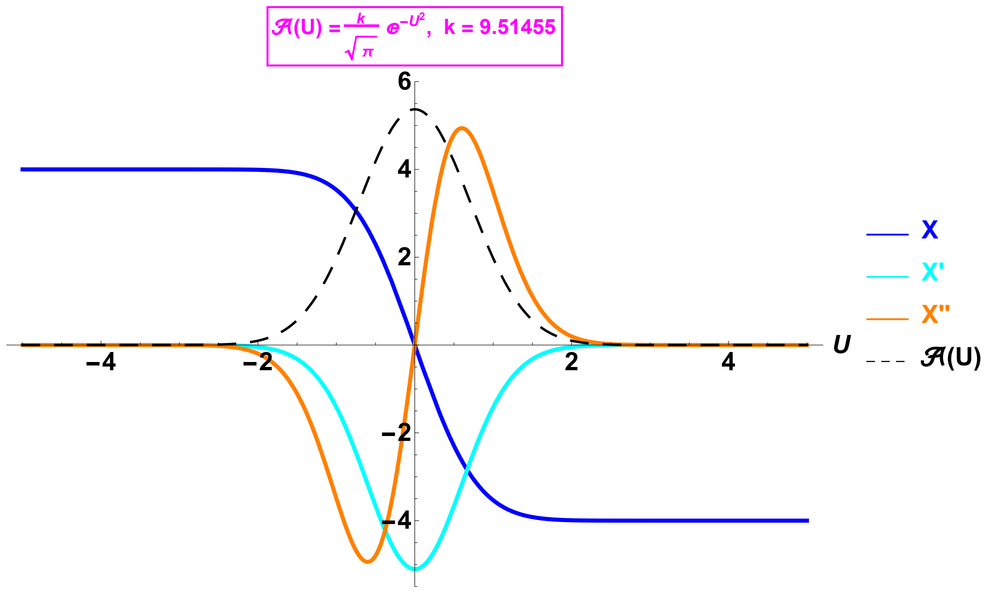
Only numerical solutions.

• $D = 1$ transverse dim. For randomly chosen parameters: : VM .





Miracle ! Numerical Fine-tuning $\rightsquigarrow$ critical value $k = k_{crit}$ DM !



Half-wave. X : **trajectory**, dX/dU : **velocity**, $\mathbf{d^2X/dU^2}$: **force**.

DM also for higher amplitudes when Wavezone accommodates an integer number of half-waves $\sim$ old quantum mechanics !

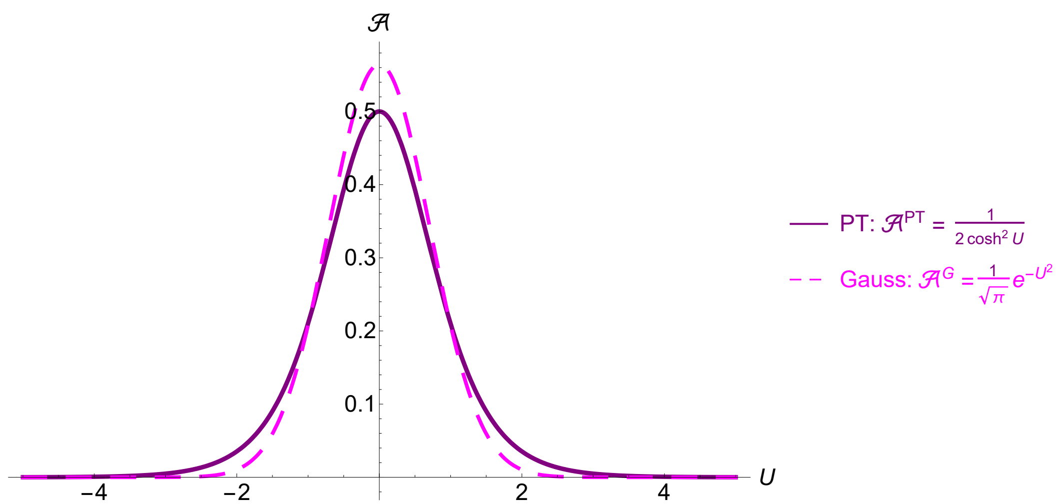
$X \to \psi, U \to x$: DM (4a) $\sim$ zero-energy bound states of time-indept Schrödinger eqn

$$-\frac{d^2\psi}{dx^2} - \mathcal{A}\,\psi = 0\,. \qquad (6)$$

Non-normalizable ground state with $E = 0$ *energy.* (SUSY ?)

Gaussian reminiscent of **Pöschl-Teller** (PT),

$$\mathcal{A}^{PT}(U) = \frac{k}{2\cosh^2 U}\,, \tag{7}$$



Gaussian bell (dashed) is well approximated by Pöschl-Teller potential (7) (solid line).

Putting $k = k_m = 4m(m+1)$ time-indept Schr eqn

$$\boxed{\frac{d^2X}{dU^2} + \frac{m\,(m+1)}{\cosh^2 U}X = 0\,.} \tag{8}$$

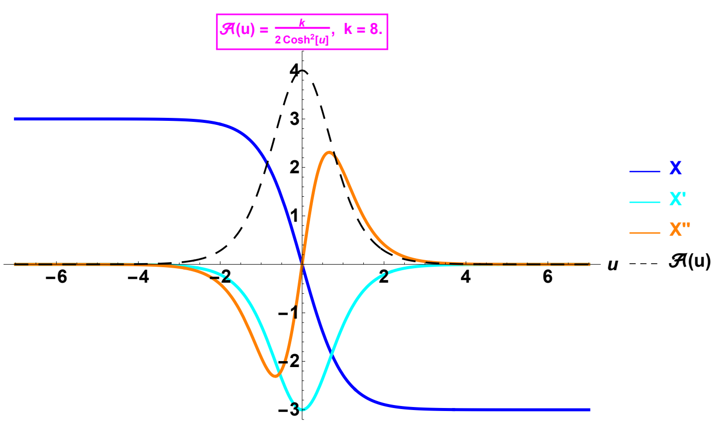
N.B. non-normalizable ground state with $E = 0$ *energy* . . .

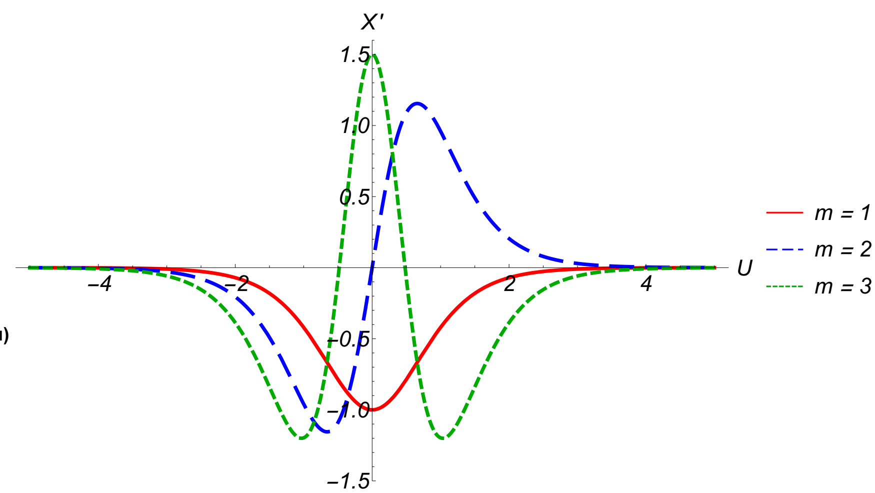
Particle at rest before burst arrives:

$$X(U=-\infty)=X_0,\quad \dot{X}(U=-\infty)=0\,. \qquad (9)$$

DM requires $X(U)\to$ const for $U\to\infty \quad\Rightarrow$ solution propto Legendre polynomial,

$$X_m(U)=P_m(\tanh U),\quad m=1,2,\ldots, \qquad (10)$$





"Vertical" component $V(U)$

$$V(out)=V_0=V(in) \qquad (11)$$

Outside Wave zone, motion purely transverse.

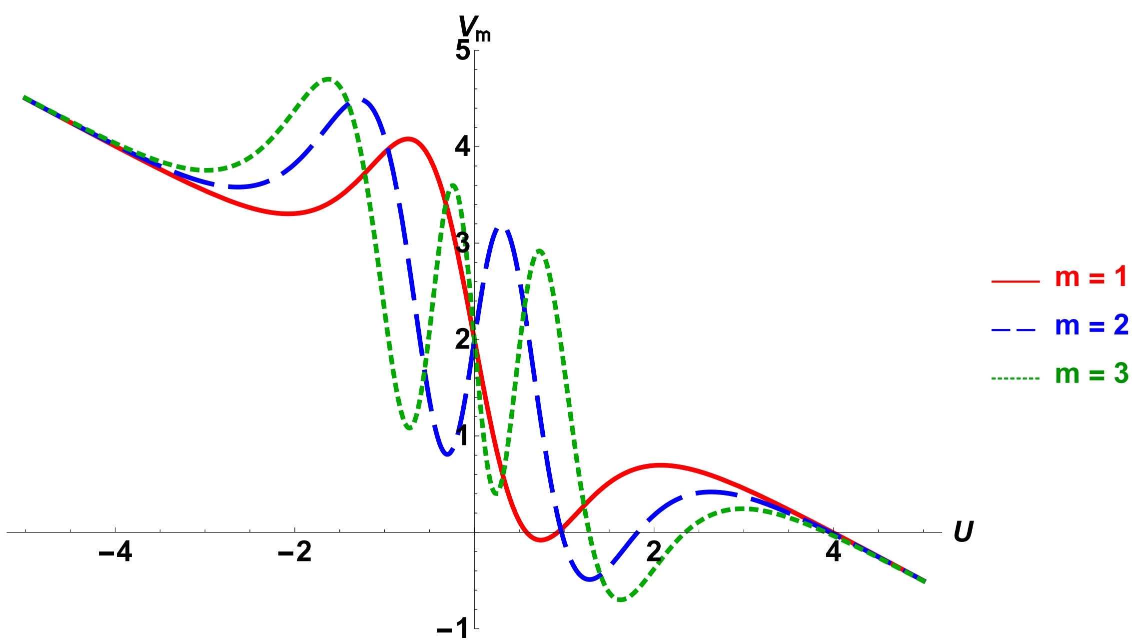
Massive geodesics

Results extend to particles with nonzero relativistic mass, $\mathfrak{m} \neq 0$.

Then* V picks up linear-in-U term,

$$V_{\mathfrak{m}}(U) = V_{null}(U) - \left(\frac{\mathfrak{m}}{2m}\right)^2 U\,, \qquad (12)$$

where $m = p_V$ is conserved quantity generated by Killing vector ∂_V (non-relativistic mass in E-D framework). In units where $\mathfrak{m} = -1$ and $m = 1$, vertical coordinate (12) gets extra term $-\frac{1}{2}U$.

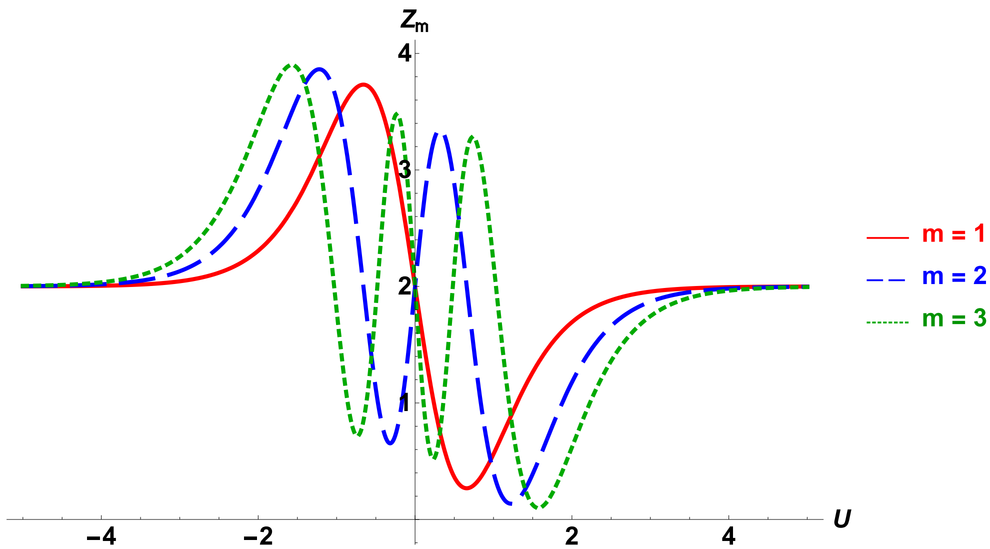


*M. Elbistan et al. Annals Phys. **418** (2020), 168180 [arXiv:2003.07649 [gr-qc]], eqn. # (VI.2).

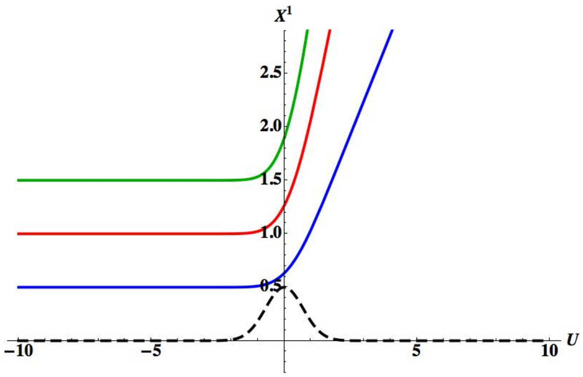
Switching from (lightlike) V to relativistic position coordinate, $Z = V + \frac{1}{2}U$ yields

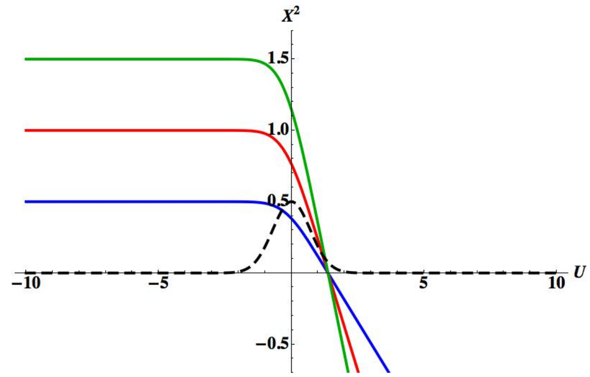
$$Z_{\mathfrak{m}}(U) = V_0 = \text{const} \qquad (13)$$

DM for X coordinates no displacement for Z_m !!



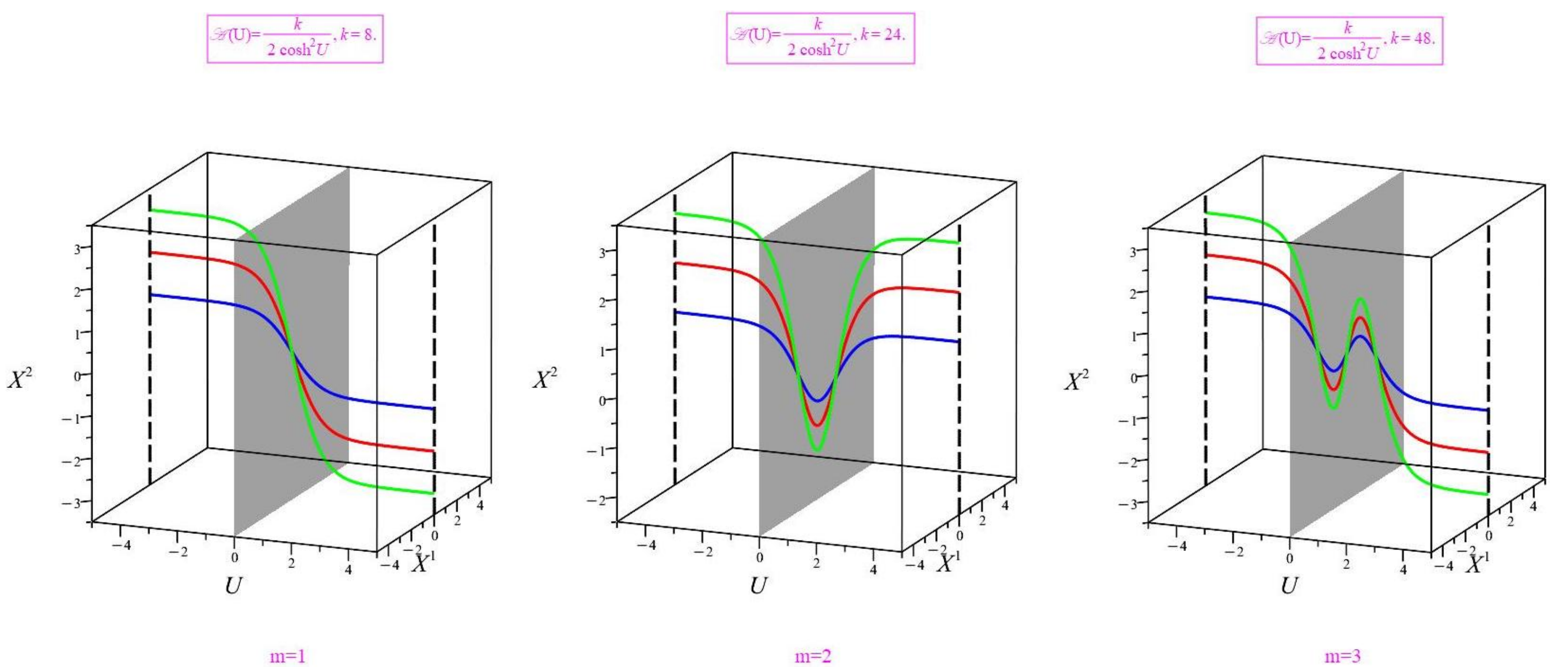
• $D = 2$ transverse dim: potential $\mathcal{A}(X_1^2 \ominus X_2^2)$.





Attractive in X^2 but repulsive in X^1 sector.

DM only for X^2 component: "half DM"

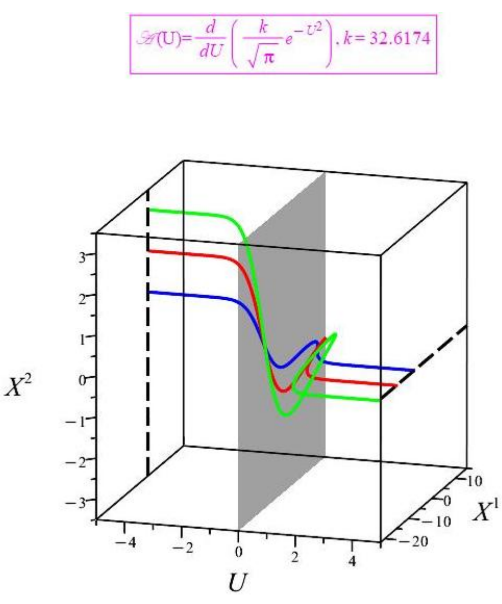


DM for flyby ??

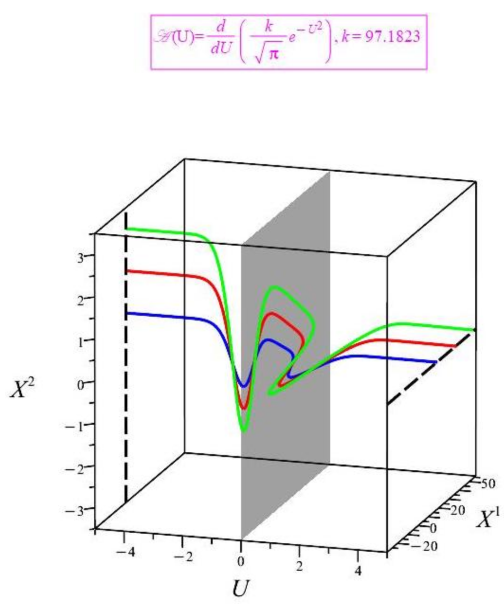
Zel'dovich-Polnarev: pure displacement for **flyby** (give no proof). Gibbons-Hawking: D=2. flyby profile propto **first derivative** of Gaussian,

$$\mathcal{A}(U) = \frac{d}{dU}\left(k\frac{\lambda}{\sqrt{\pi}}e^{-\lambda^2U^2}\right) \qquad (14)$$

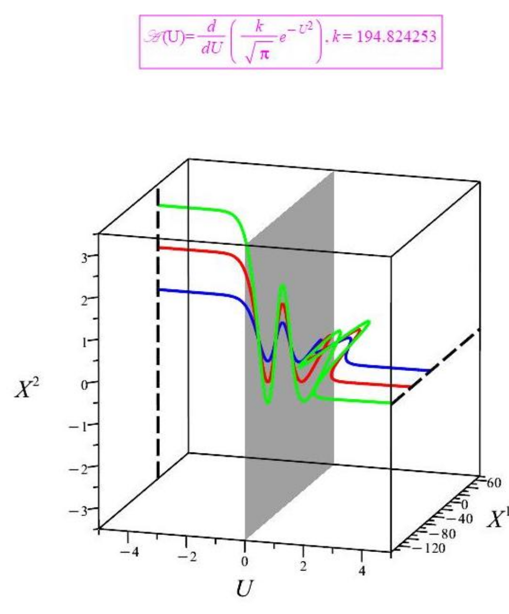
VM . Can become DM ? Miracle ! for (numerically found) specific choices $k = k_m$ DM for both components !



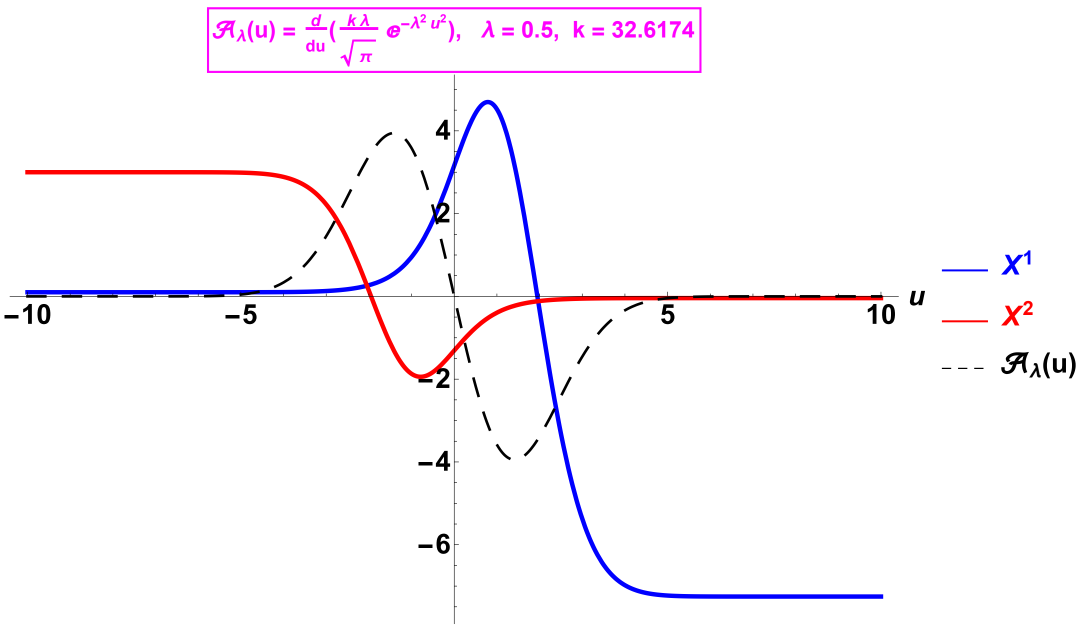
m=1

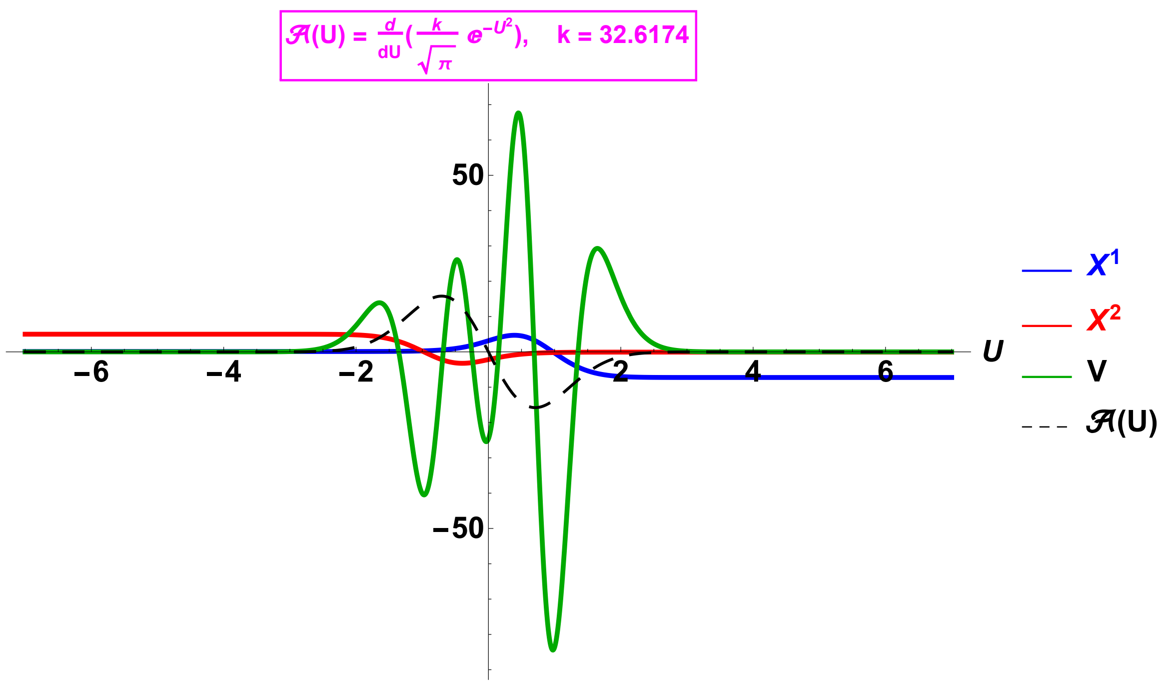


m=2



m=3





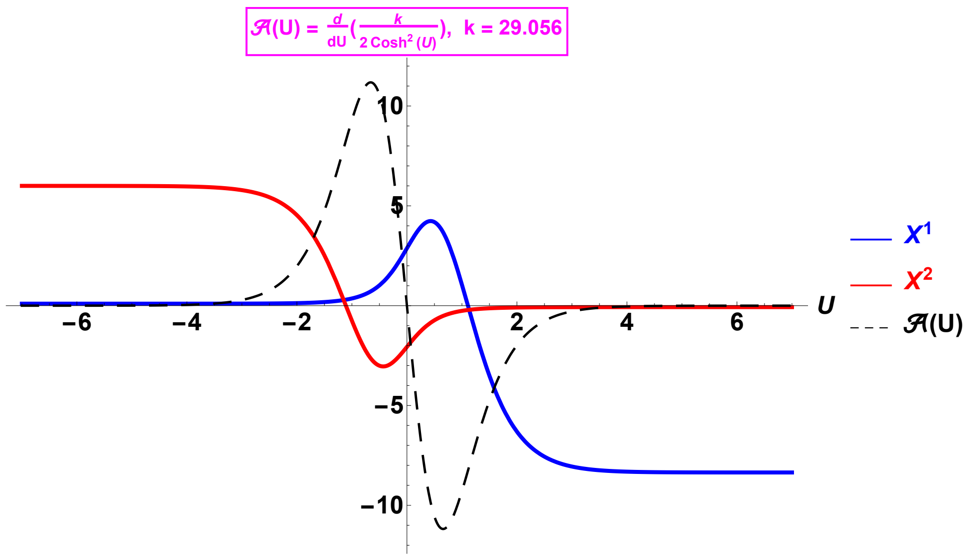
N.B. $X_{in}^{1,2} \neq X_{out}^{1,2}$, but $V_{in} = V_{out}$.

For $k \neq k_{crit}$ velocity effect.

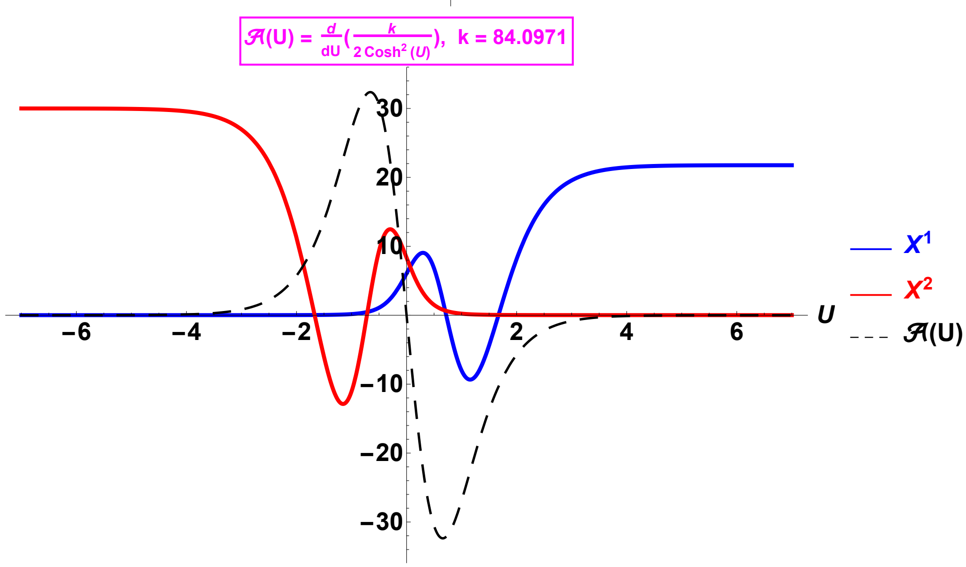
Idem for Pöschl-Teller-flyby profile

$$K_{ij}(U)X^iX^j = \frac{k}{2\cosh^2 U}\Big((X^1)^2 - (X^2)^2\Big). \quad (15)$$

Fine-tuning $\Rightarrow$ DM for *both* components when $k_m = 4m(m+1)$, $m = 1, 2 \ldots$ $\sim$ integer number of half-waves.



$m = 1$



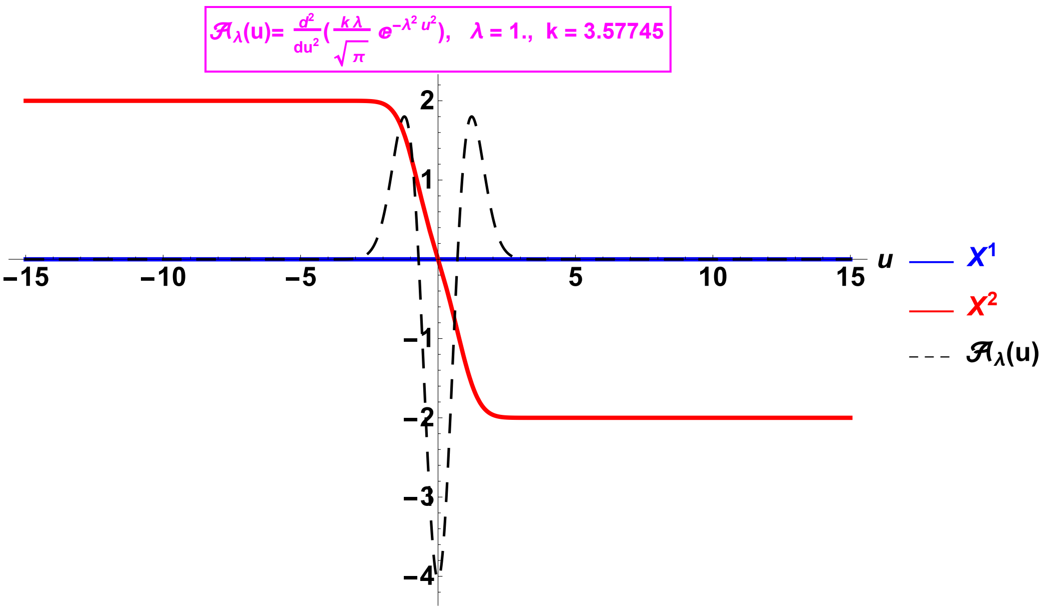
$m = 2$

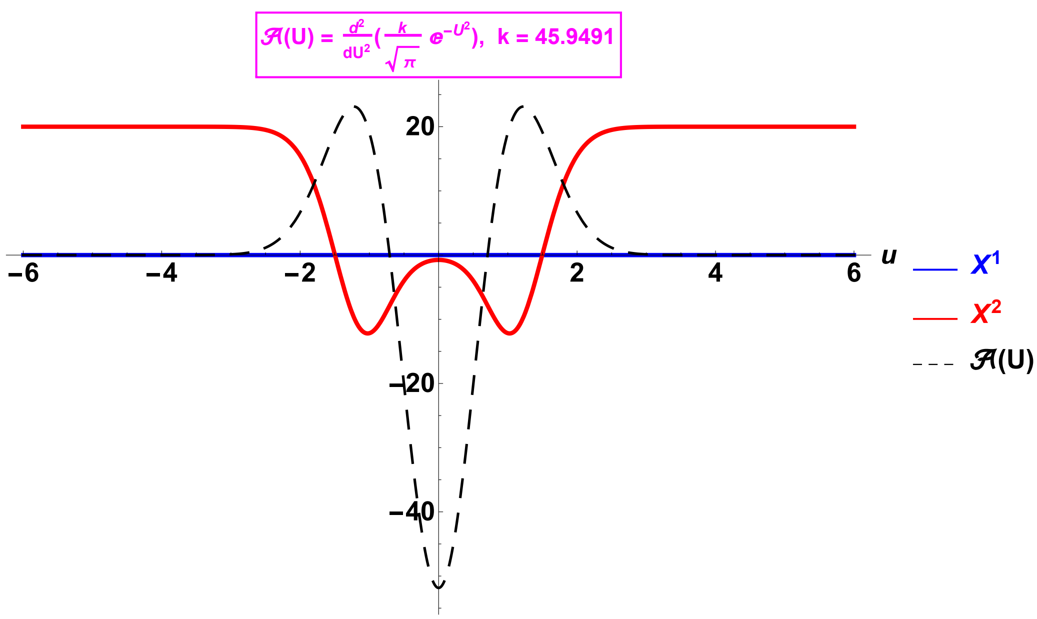
Confirmed analitically $\sim$ confluent Heun fcts.

ZZBH AoP **473** (2025), 169890 [arXiv:2407.10787 [gr-qc]]

Higher order derivatives in 2 tr dim

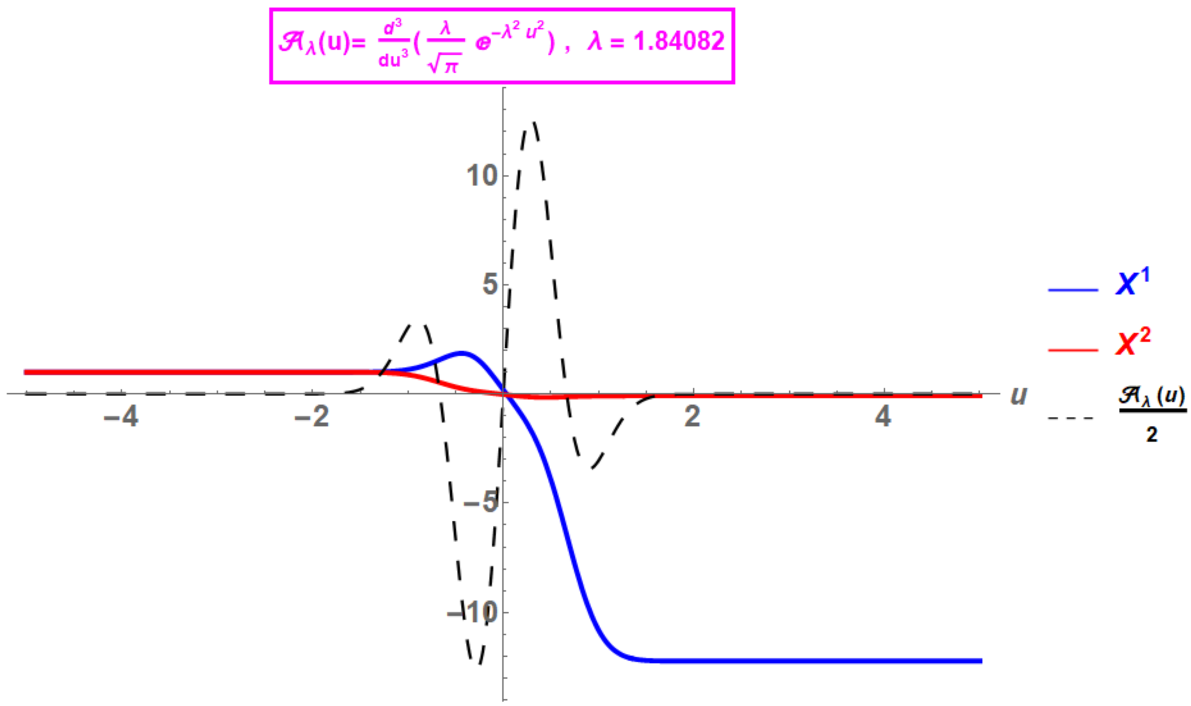
- $d = 2$ Braginsky - Thorne

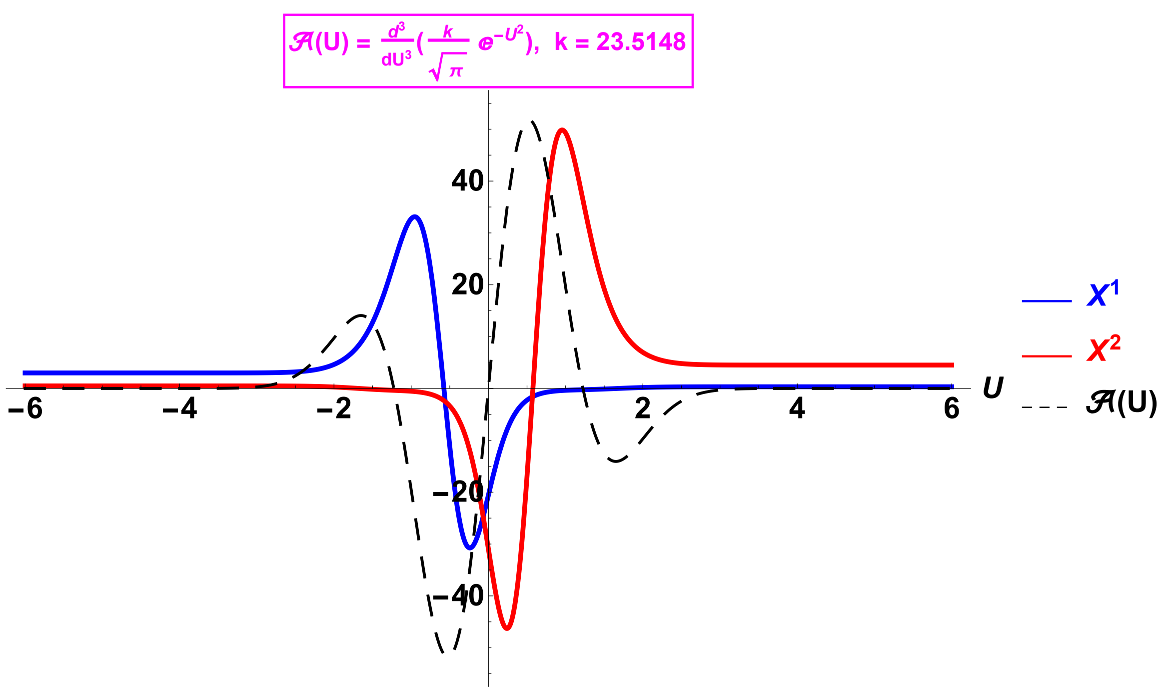




$\frac{1}{2}$ DM for **even** order $d = 2n$

- $d = 3$ gravitational collapse (Gibbons-Hawking)





DM for **both** components for $d = 2n + 1$ odd

CONCLUSION

Particles at rest hit by a burst of GWs fly generically apart, moving freely along straight lines: get VM .

For exceptional ("quantized") values of wave parameters which correspond to integer # of half-wave trajectories in wave zone DM is possible, confirming prediction of Zel'dovich-Polnarev.

Shklovsky and Zel'dovich (1977).