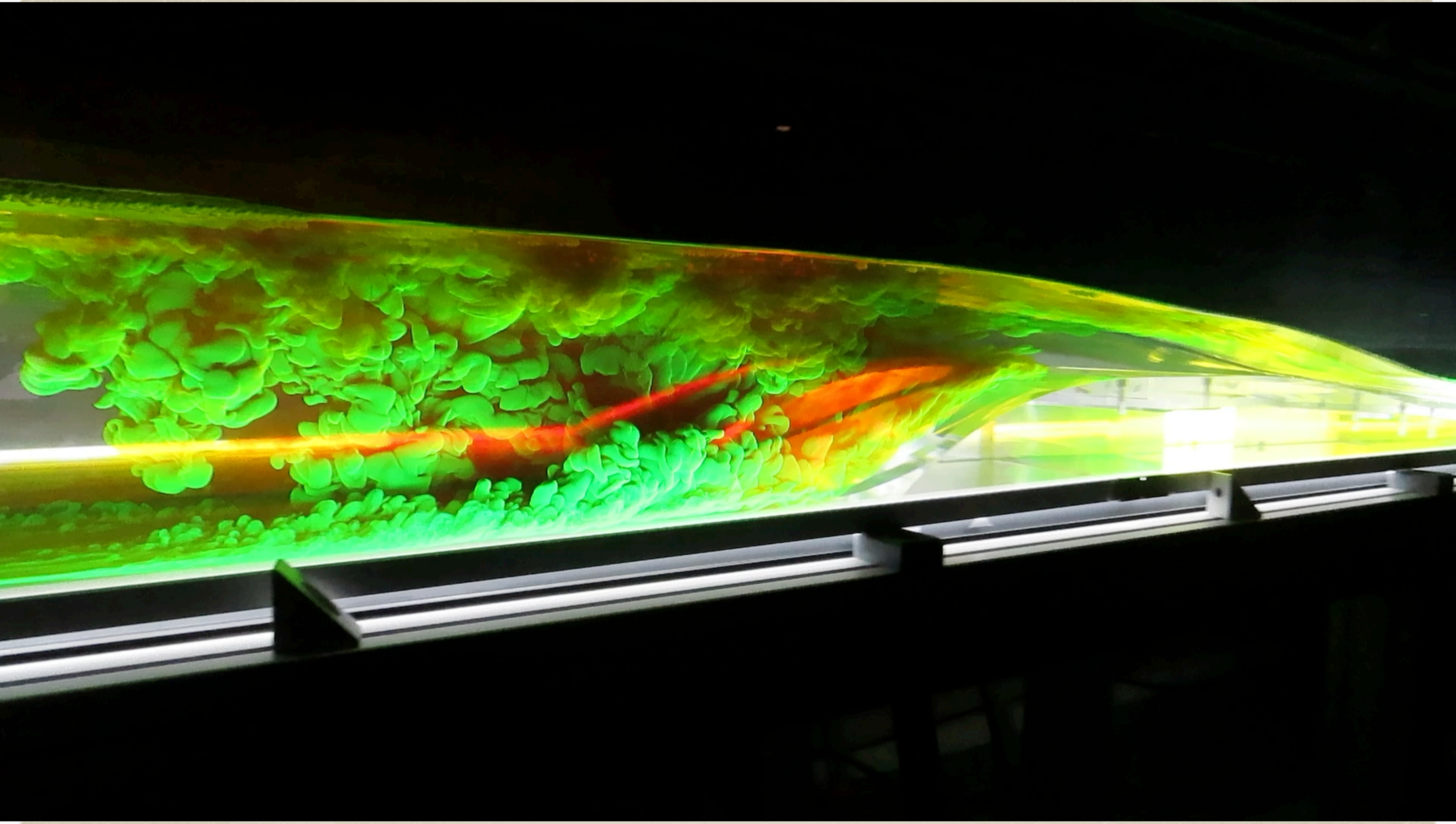
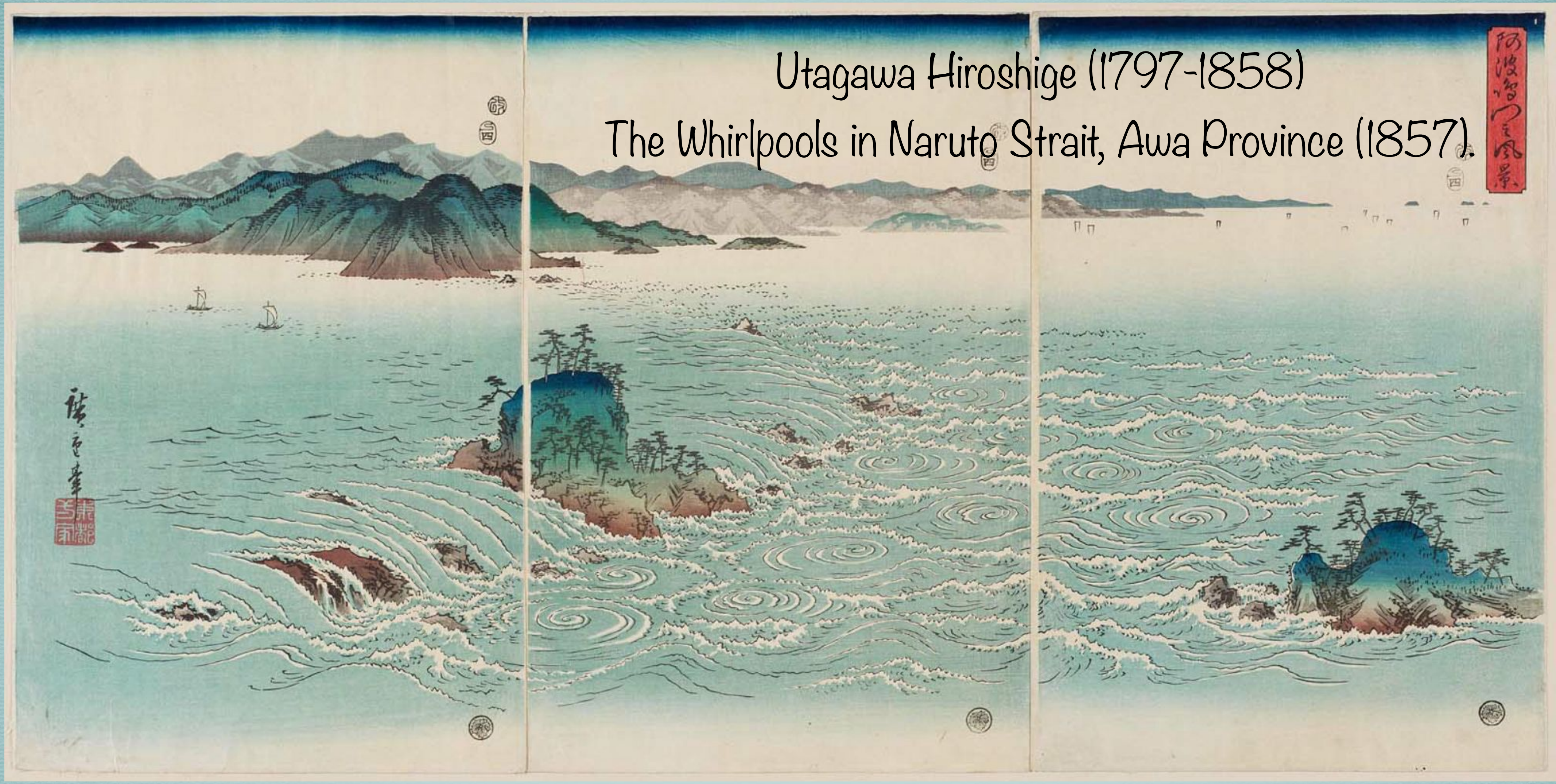


MIMICKING QUANTUM FIELD THEORY IN CURVED SPACETIMES WITH CLASSICAL OPEN WATER CHANNEL FLOWS



Utagawa Hiroshige (1797-1858)
The Whirlpools in Naruto Strait, Awa Province (1857).

Germain Rousseaux (CNRS),
Institut Pprime, Futuroscope, France

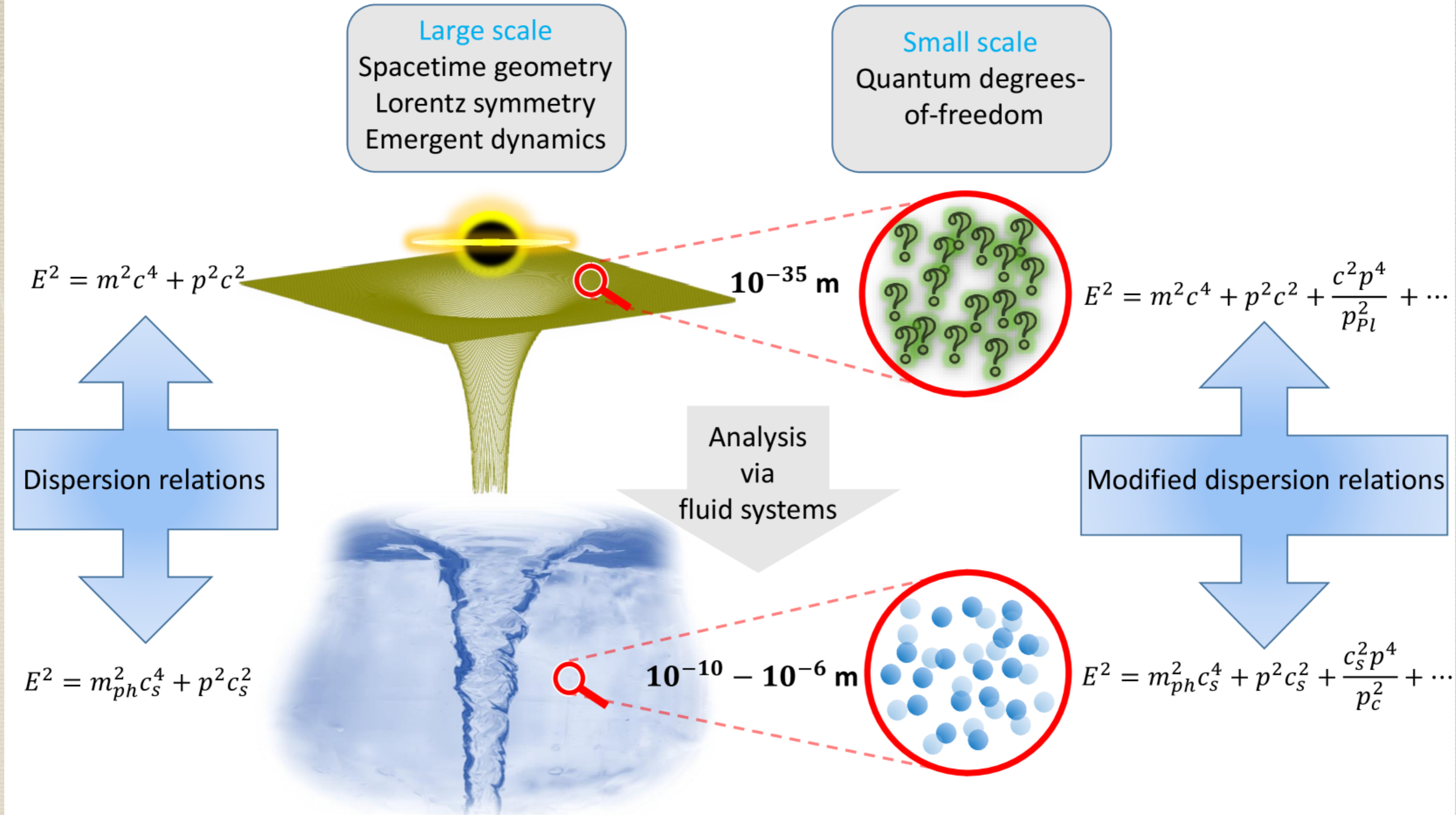
Institut Pprime

cnrs

Avenues of Quantum Field Theory In Curved Spacetime 2025,
22-24/01/2025 Tours (France)


New Scientist
WEEKLY 27 May 2023
CAN NEW ALZHEIMER'S TREATMENTS FINALLY CURB DEMENTIA?
WHY THINKING ABOUT YOUR FUTURE SELF CAN CHANGE YOUR LIFE TODAY
HOW TO BUILD A BLACK HOLE
Inside the audacious attempts to recreate extreme gravity in the lab
JOIN OUR NEW BOOK CLUB...
And discover great reads with us
PLUS THE ORIGINS OF KISSING / HOUSE MADE FROM DIAPERS / LOST BEES / DO OCTOPUSES HAVE NIGHTMARES? / BAT CAVES

Analogue Simulations of Quantum Gravity with Fluids



In fluids, the effective geometry breaks down at scales where the continuous description in terms of macroscopic variables no longer holds, in close analogy to the breakdown of spacetime geometry due to quantum gravitational effects. This scale is manifest by a breakdown of Lorentz symmetry as evidenced by the above modified phonon dispersion relations (see figure, right). There, m_{ph} is the mass of phonon excitations which is the analogue of the rest mass m, with energy and momentum being denoted by E and p for either system. For $m_{ph} = 0$ and truncating the expansion at the fourth-order in p the above expression reduces to the well-known Bogoliubov dispersion relation in a Bose gas [100, 101] or in a circular hydraulic jump [104, 105]. The critical momentum $p_c = Mc_s$, that here plays the role of the Planck momentum p_{Pl}, depends on the mass M of the particles forming the condensate and is inversely proportional to the coherence length $\xi = \hbar/(Mc_s)$. When $p \ll p_c$, the excitations follow the standard relativistic energy-momentum relation.

S. L. Braunstein, M. Faizal, L. M. Krauss, F. Marino, and N.A. Shah, Analogue simulations of quantum gravity with fluids, Nature Reviews Physics, 5 (10), p. 612-622 (2023).

A Hydraulic Black Hole in the Morbihan Gulf (France)

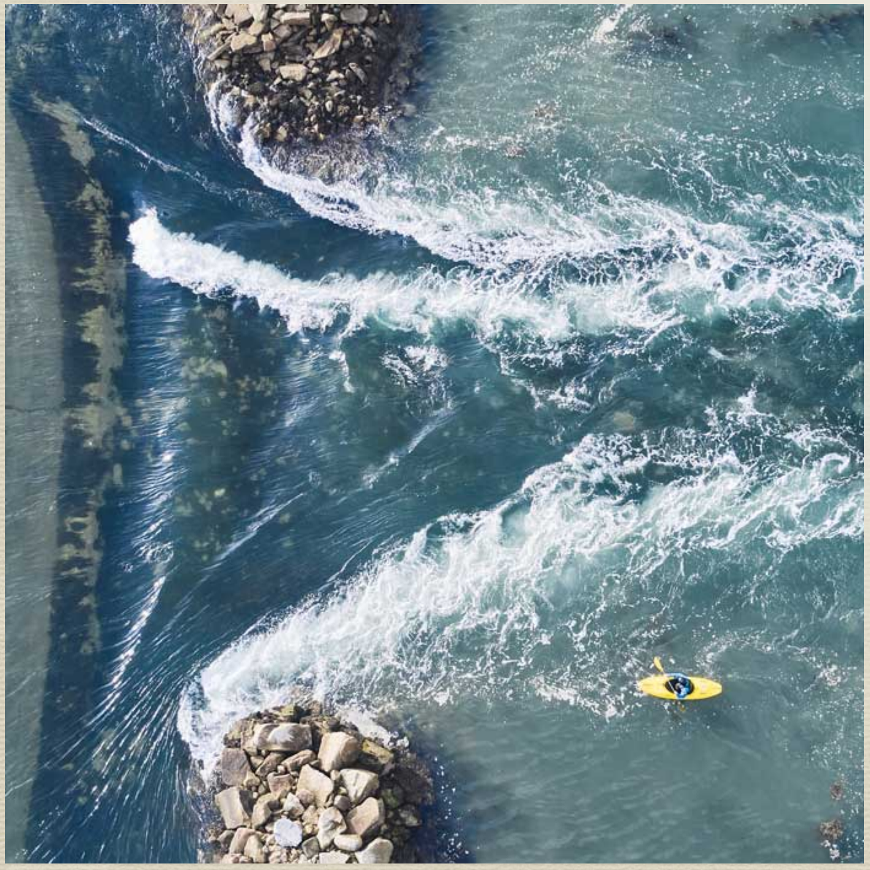
Gois of Berder Island in Britanny, rising tide : « Pascalian » black hole by water depths mismatch.

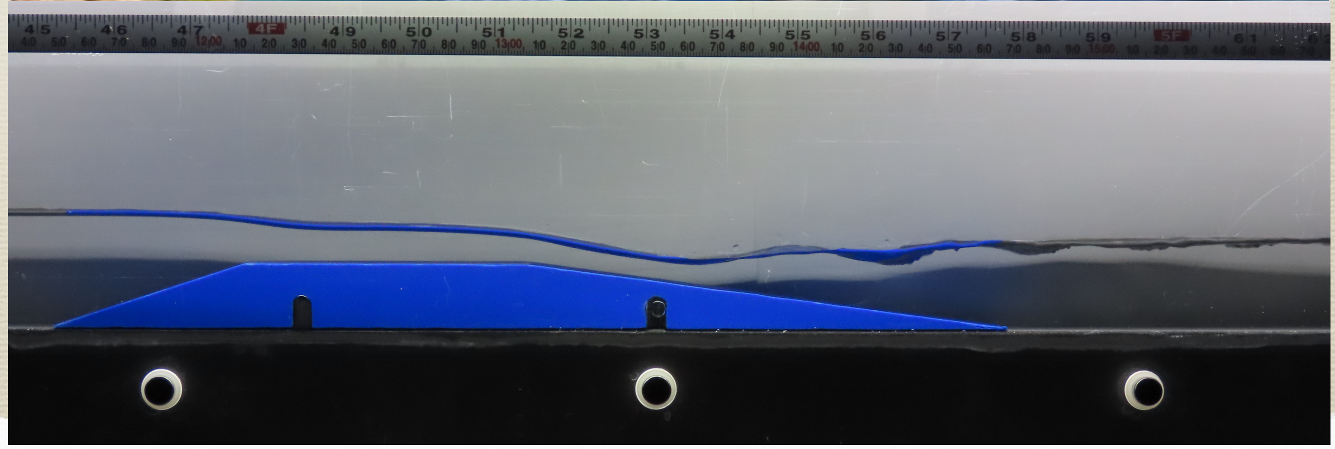
2D Hydraulic Black Hole

2D Hydraulic Black Hole and White Fountain

small tidal range
small upstream
water depth
flood tide

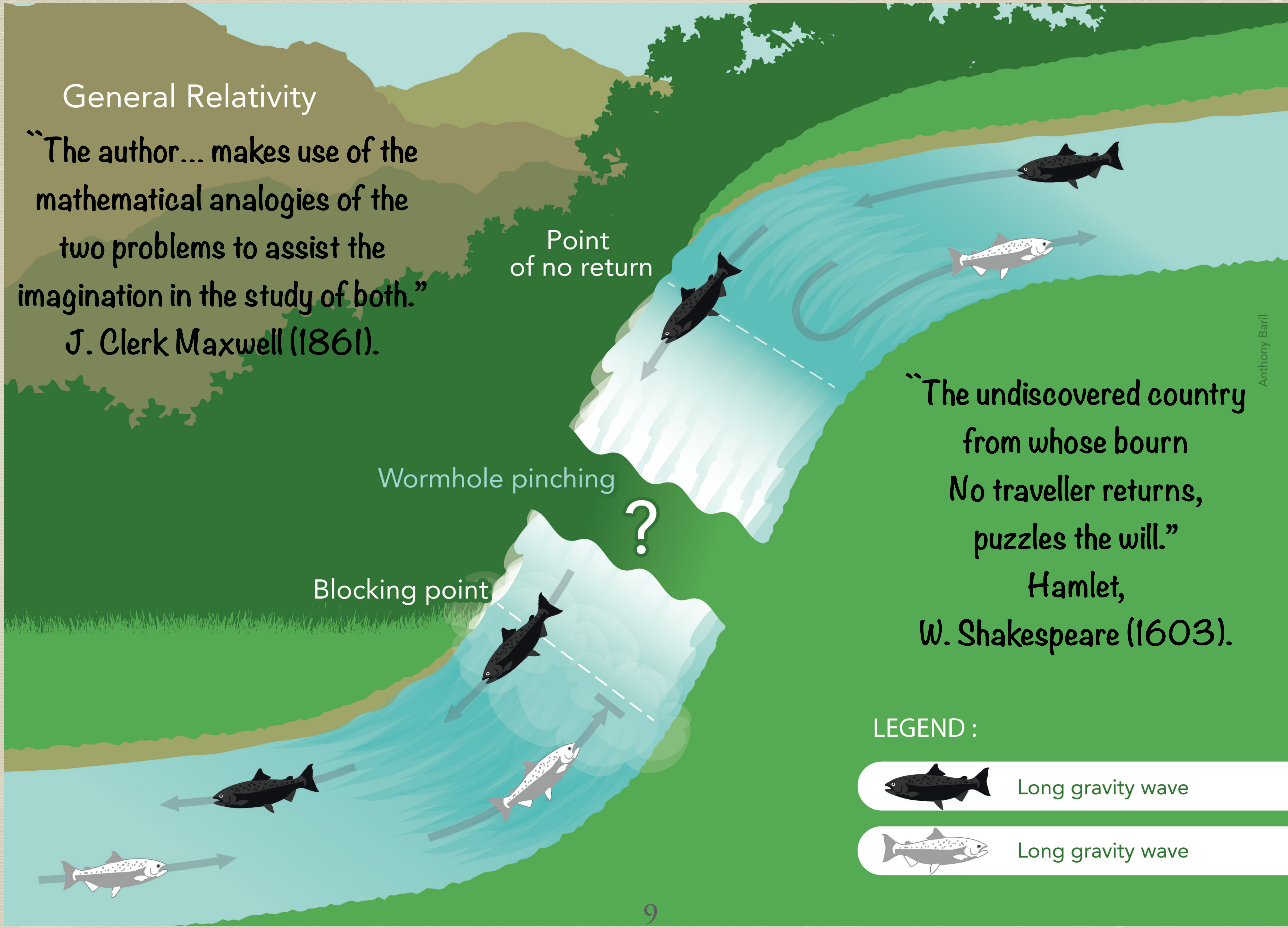
Transcritical accelerating
non-dispersive waterfall followed
by a transcritical decelerating
dispersive undulation





2D Hydraulic White Fountain in the Morbihan Gulf (France)

1D Black Hole, White Fountain and Wormhole in General Relativity



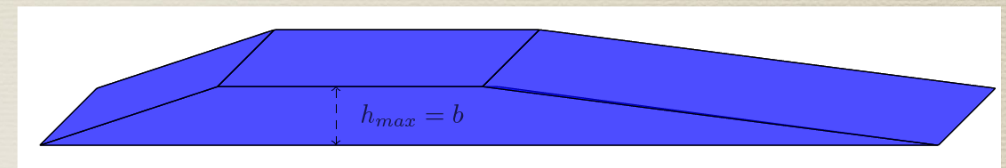
The Mini Open Water Channel

Downstream gate

Flow inlet

Pump

Nice Geometry by Rousseaux et al. (2010)

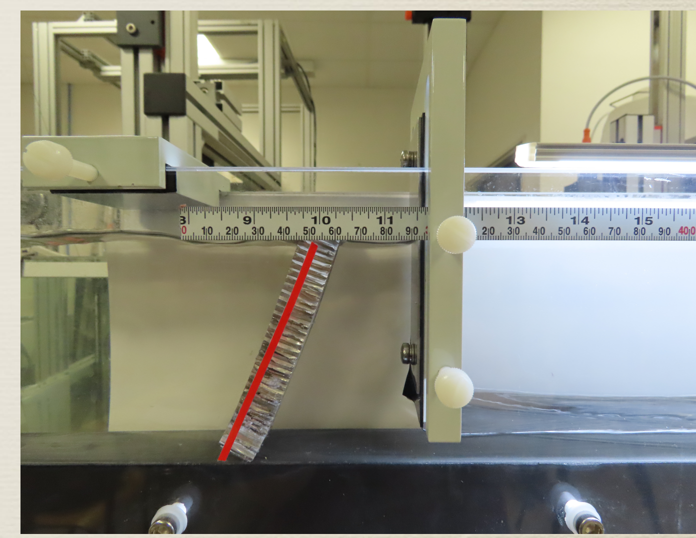


Typical type of asymmetric geometry used:

Length: 32.2 cm

Maximum heigth: 2.1 cm

Sluice gate

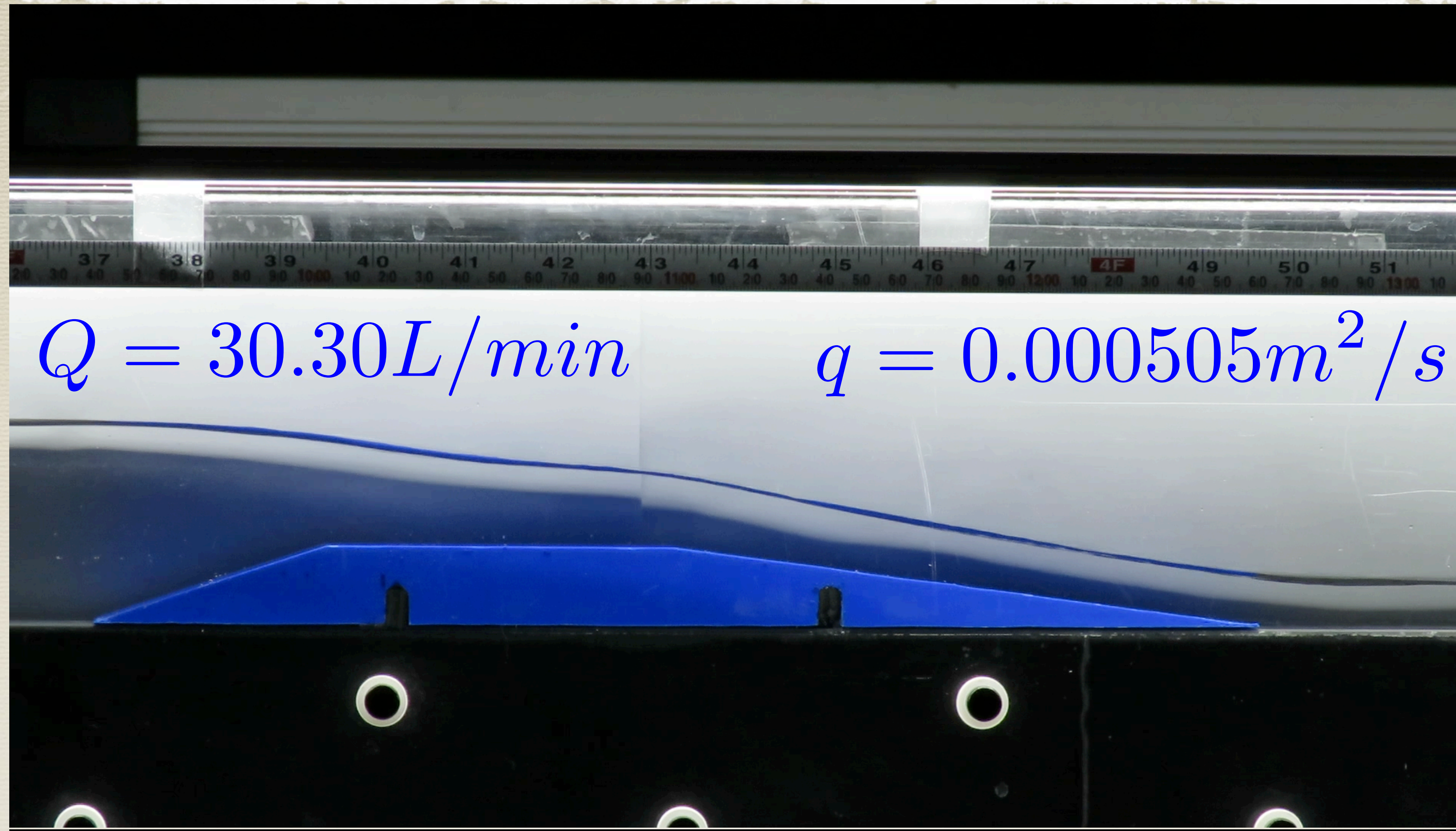


Hypotheses and experimental conditions:

- flow conservation
- $U_{upstream}$=Q/(Wh)
- No downstream condition (door open)
- No initial water level imposed
- Inter-obstacle distance set at 9.2 cm (arbitrary)
- Neglected boundary layer

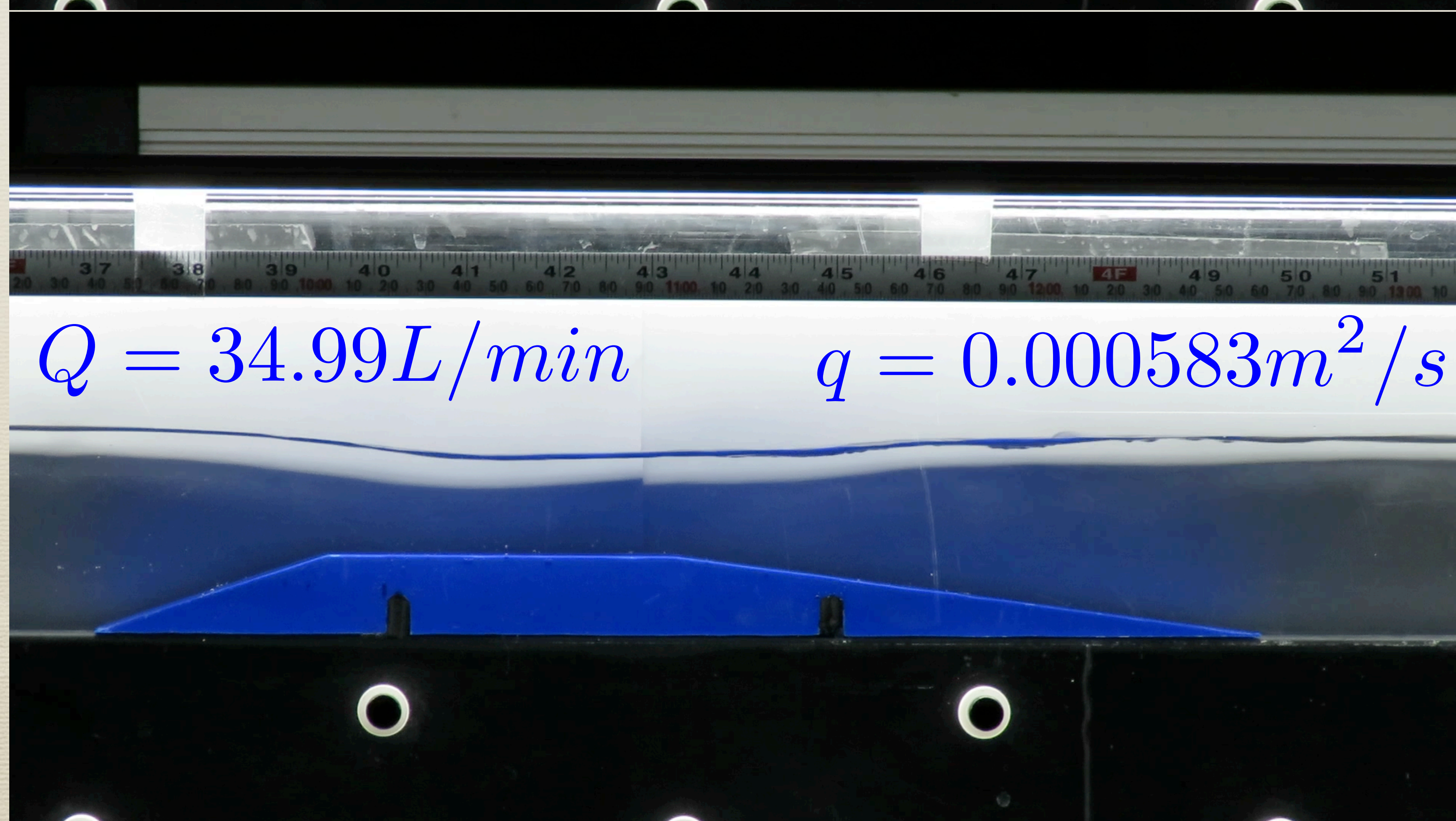
Channel characteristics:

- Length: L=2.5 m
- Wide: W=5.3 cm
- Range of the flow rate: 2 to 38 L/min
- Range of the flow rate: 0.0006 to 0.0115 m^2/s



No downstream gate

Transcritical BH Flow

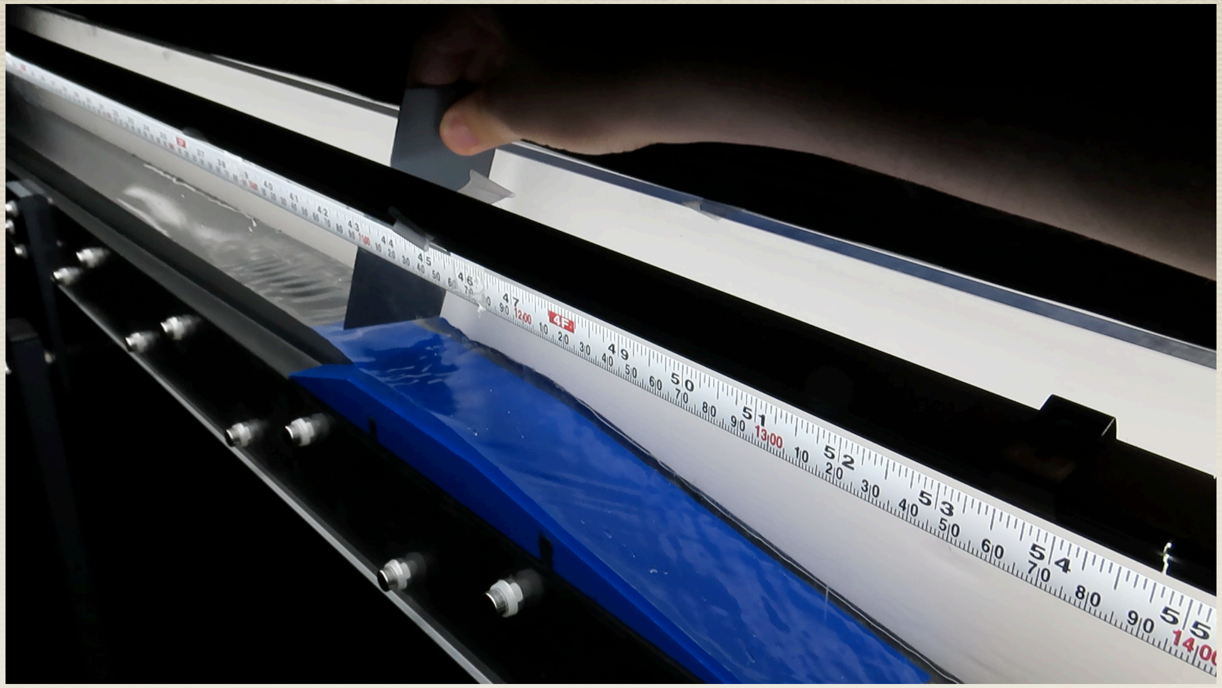


$h_g = 0.0185m$

Subcritical WH Flow

Analogue Black Hole Flows

By courtesy of Eric Lamballais for the lending of the pedagogical open channel

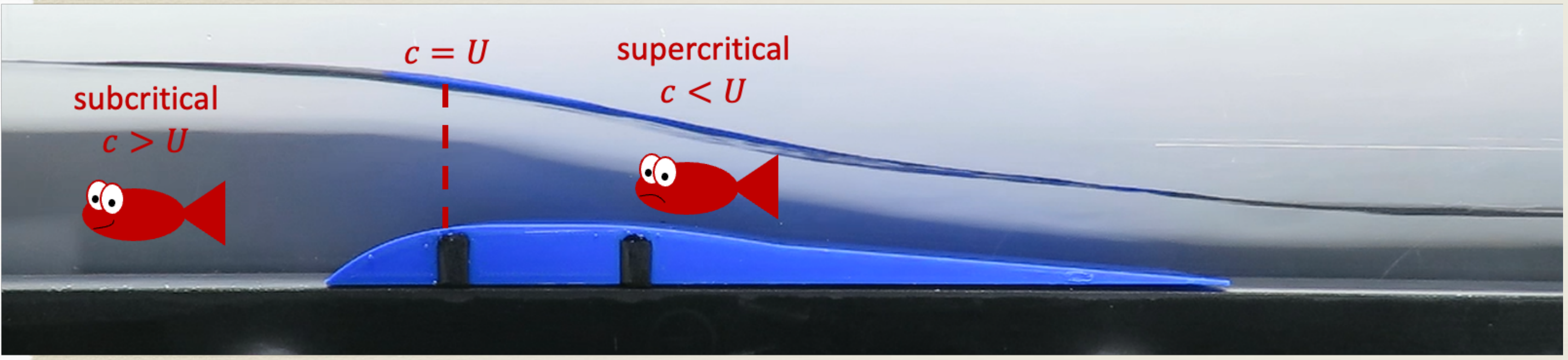


There is NO initial water depth !

$$Fr = \frac{U}{c} = \frac{U}{\sqrt{gh}}$$

when

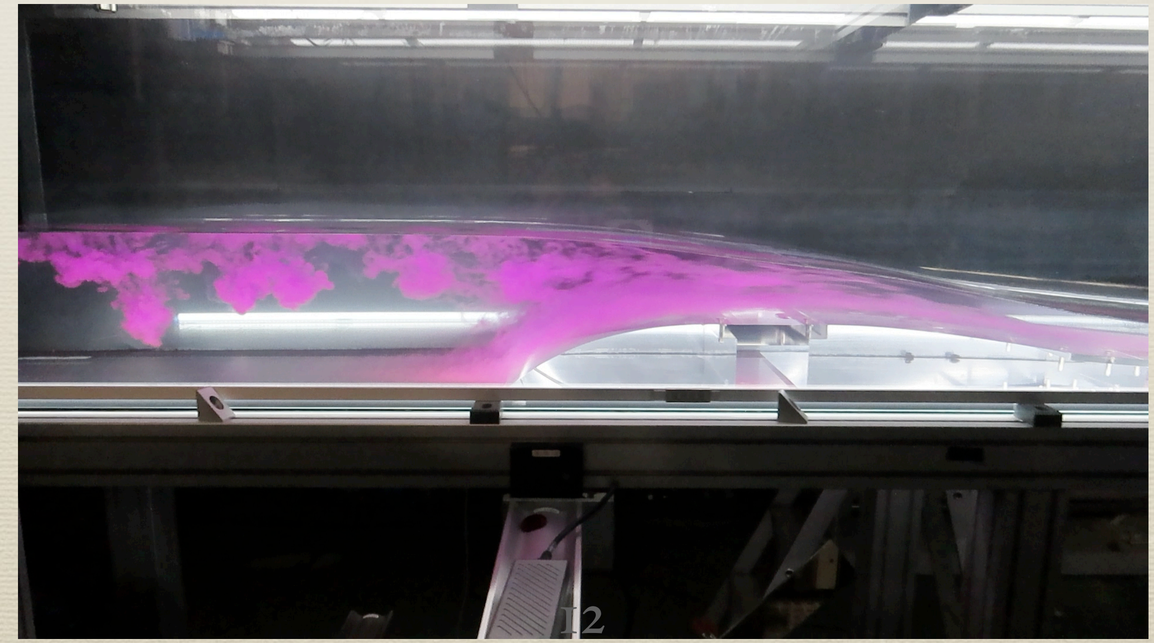
$$kh << 1$$



Small open channel features:

- Length : L=2,5 m
- Width : w=5,4 cm
- Flow rate range in L/min: 2-38
- Flow rate range in m²/s: 0.0006-0.0115

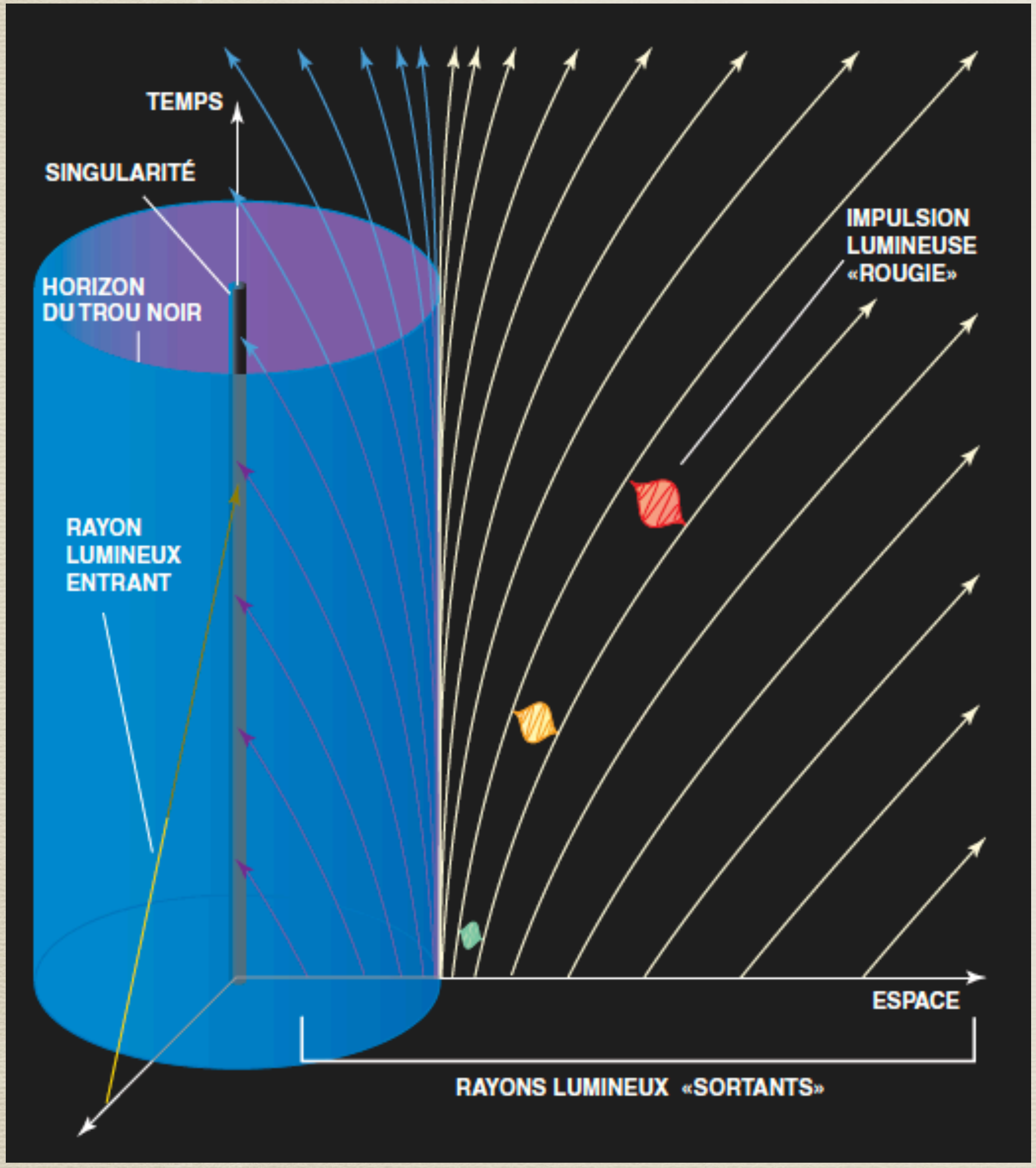
Alexis Bossard,
Ongoing PhD Thesis 2022-2025
(Unpublished)



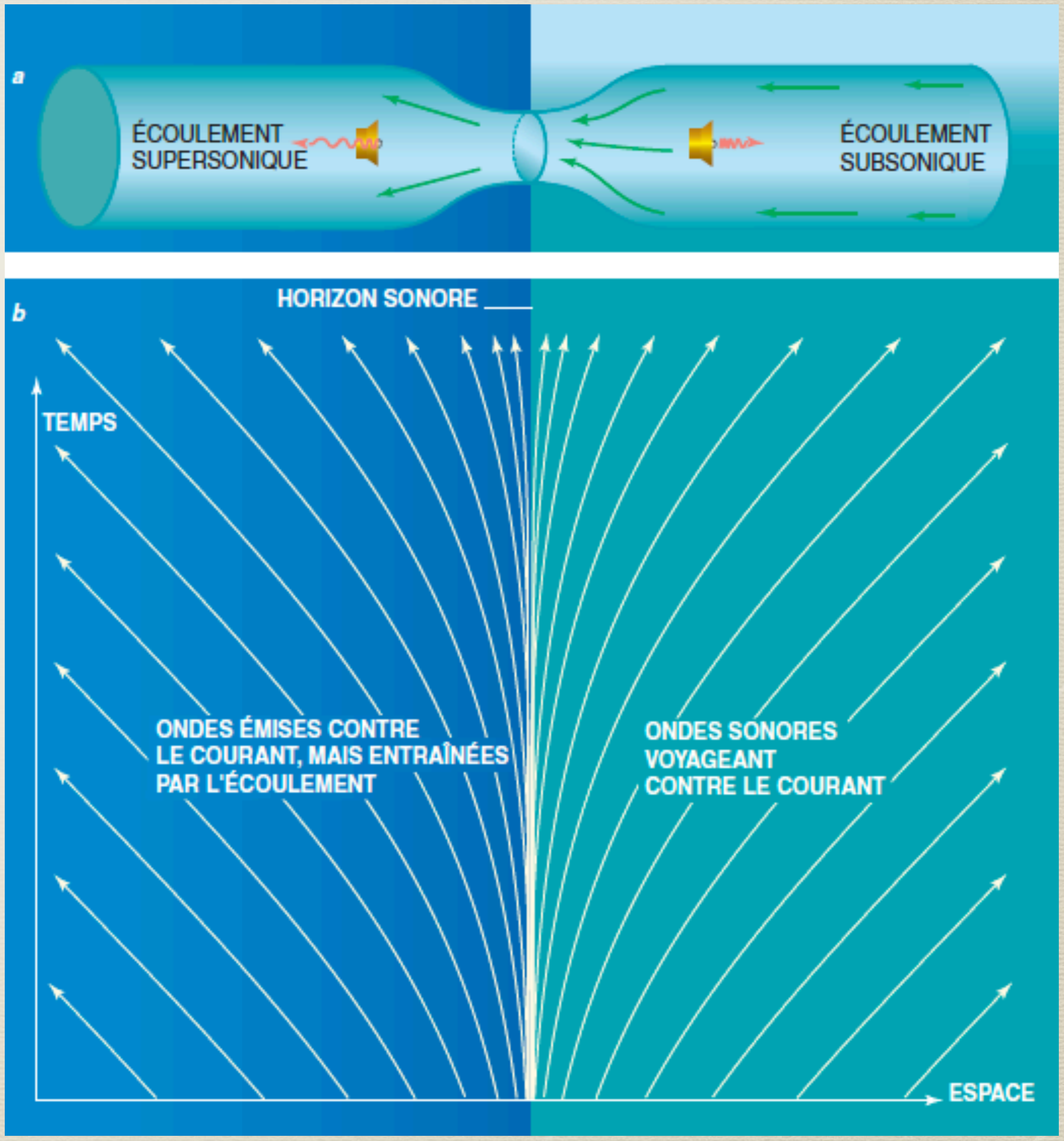
There is an initial water depth !

Space-Time Diagrams of a Black Hole

A Black Hole in Spacetime

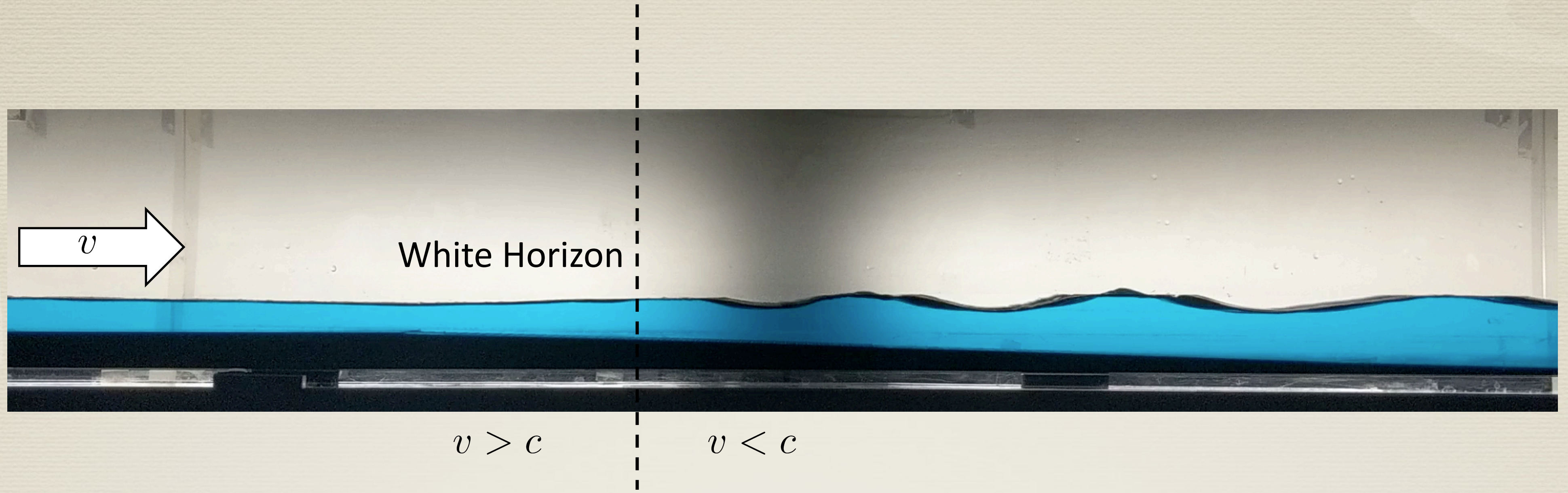


A Dumb Hole in a De Laval Nozzle

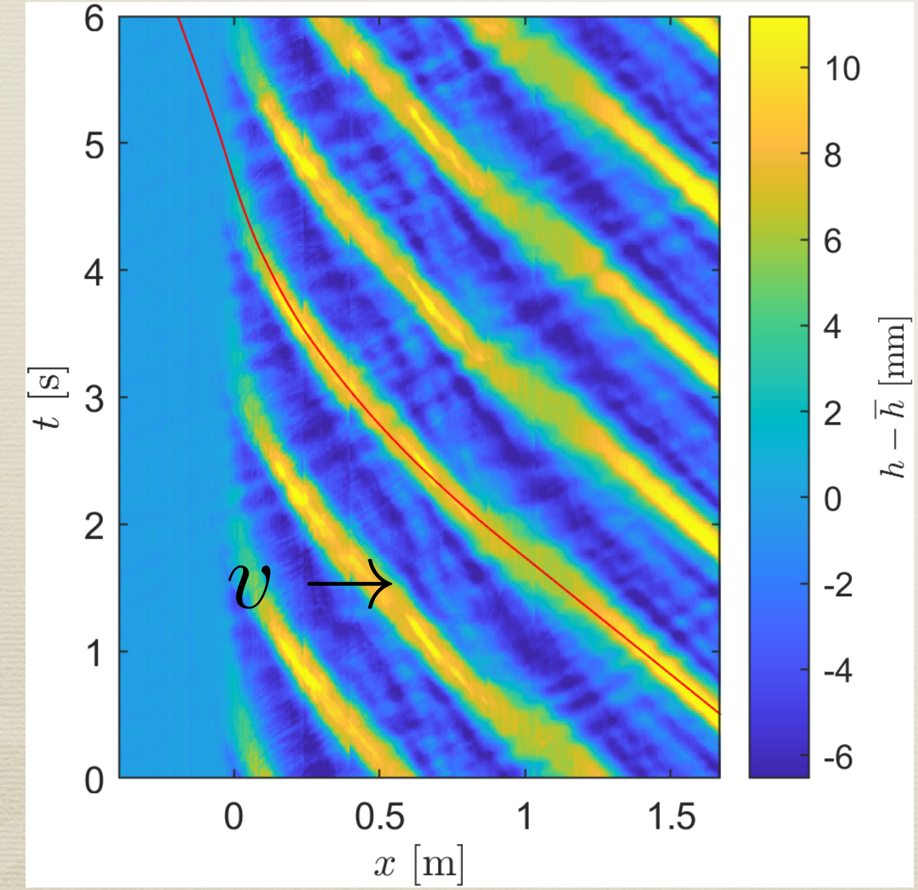


Courtesy Renaud Parentani (Pour La Science, 2002).

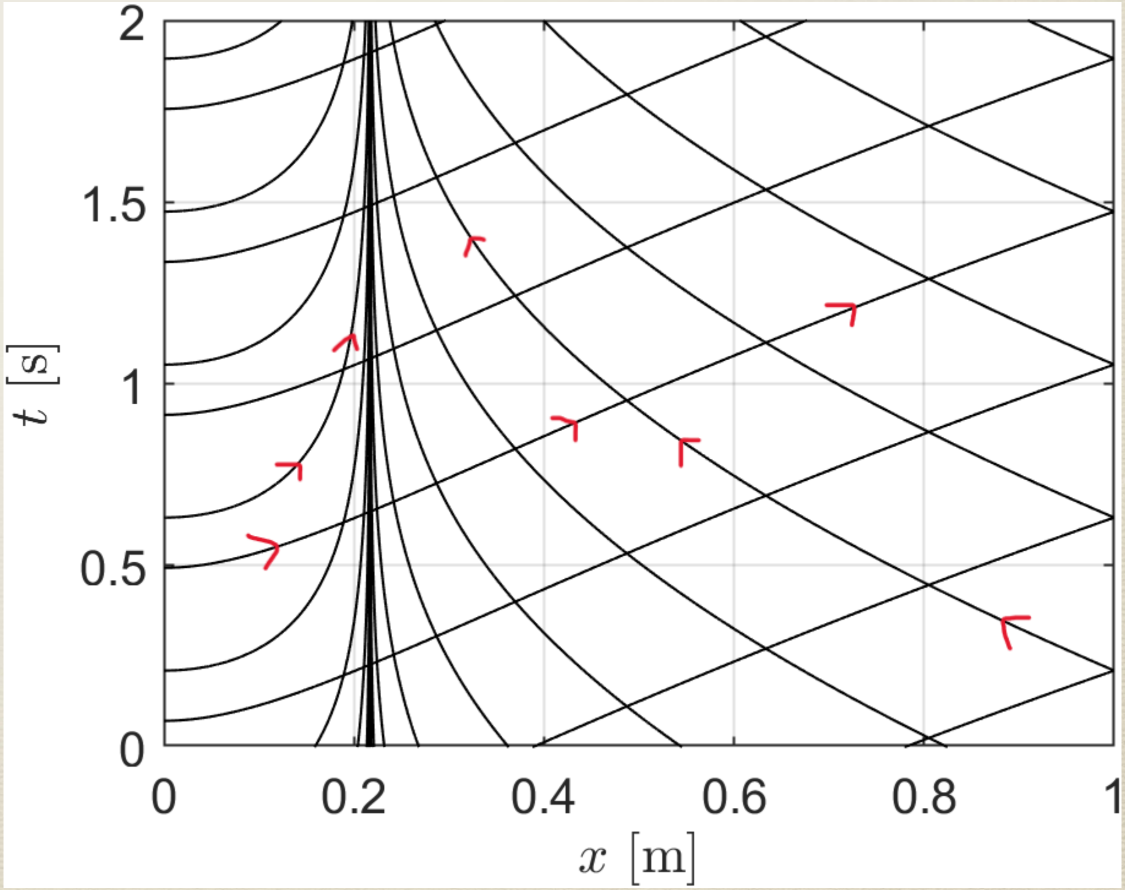
Space-Time Diagrams of a White Fountain=Cataract



Water interface recording

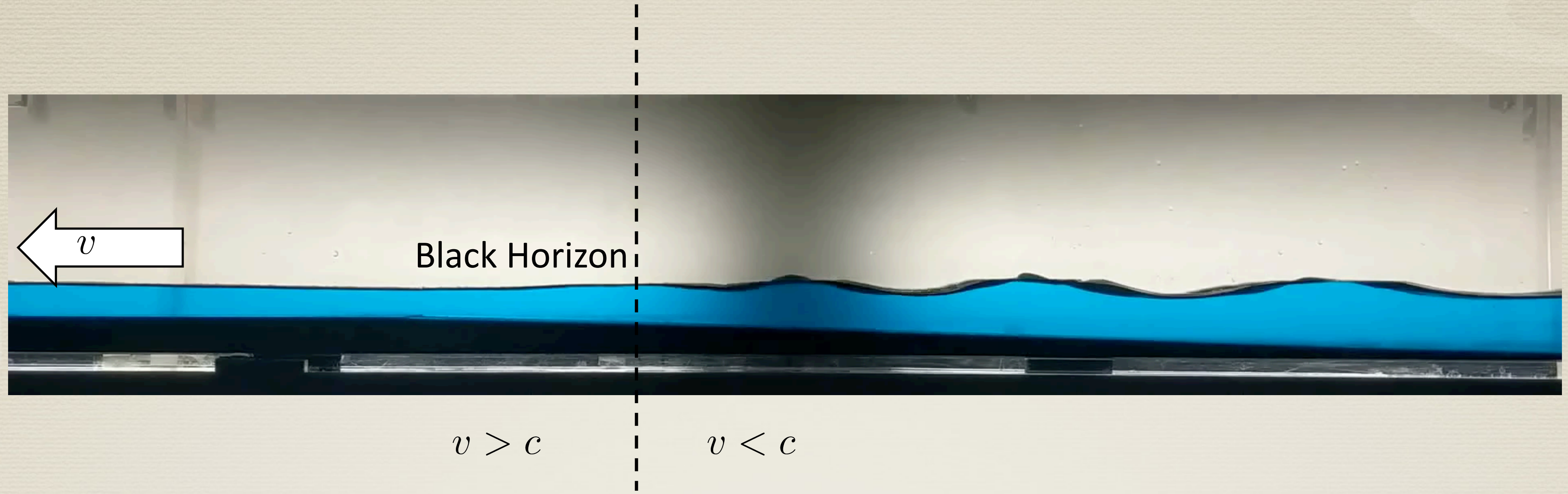


Spacetime diagram

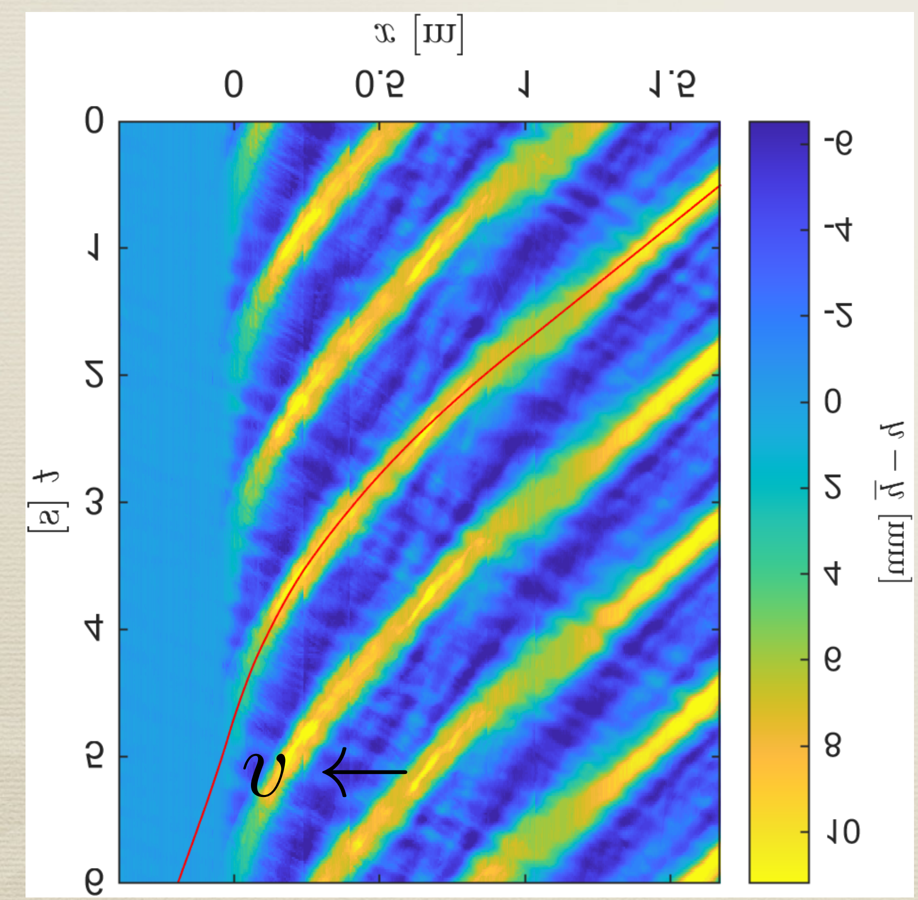


Courtesy Jaro Fransen from his Master Thesis at Eindhoven University of Technology (2024).

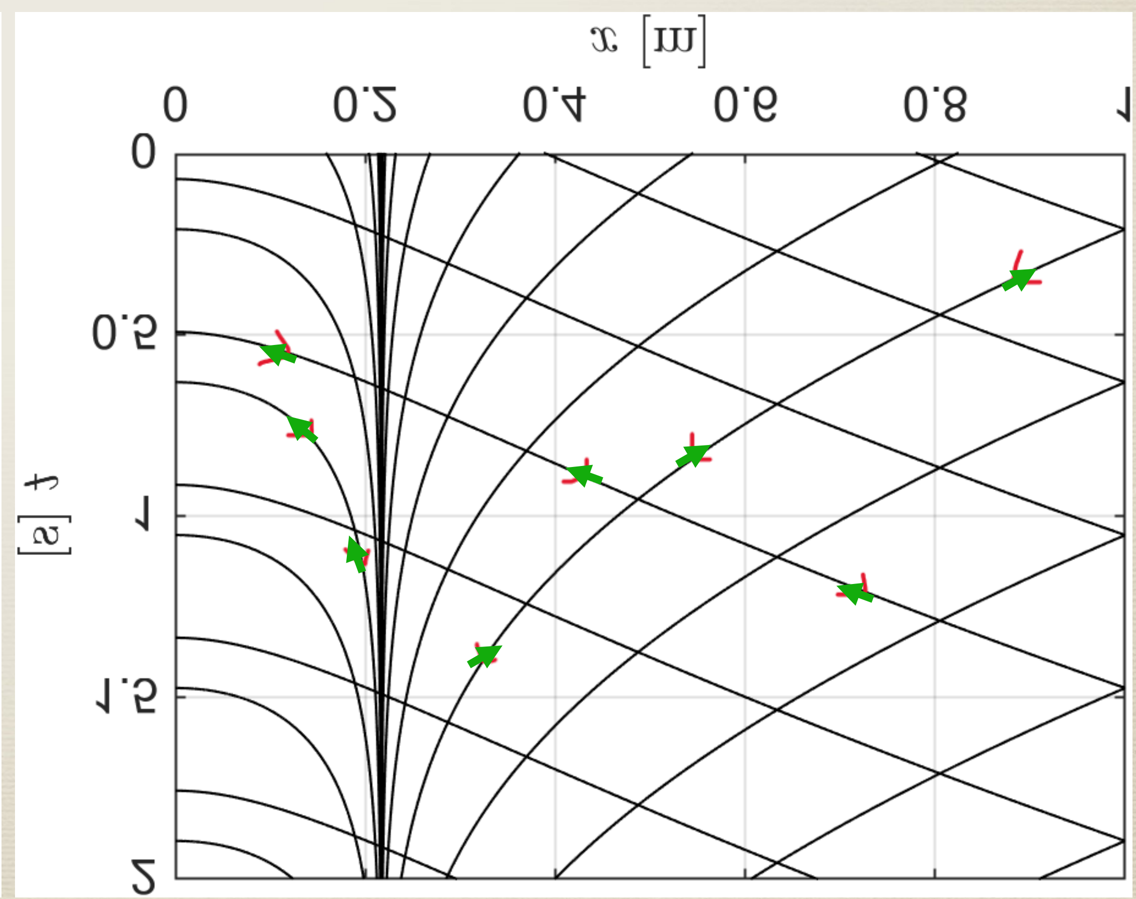
Space-Time Diagrams of a Black Hole=Waterfall



Water interface recording

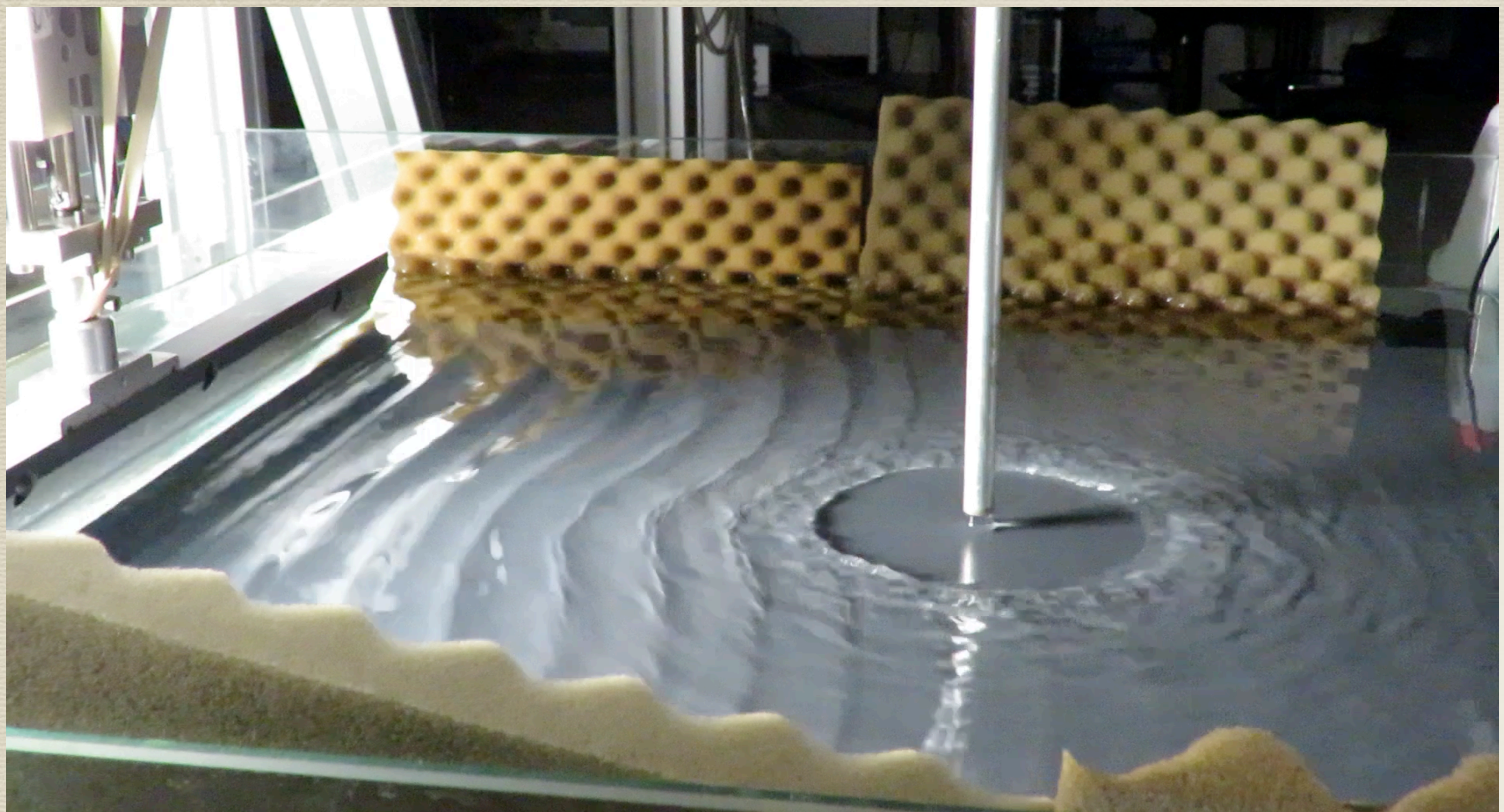


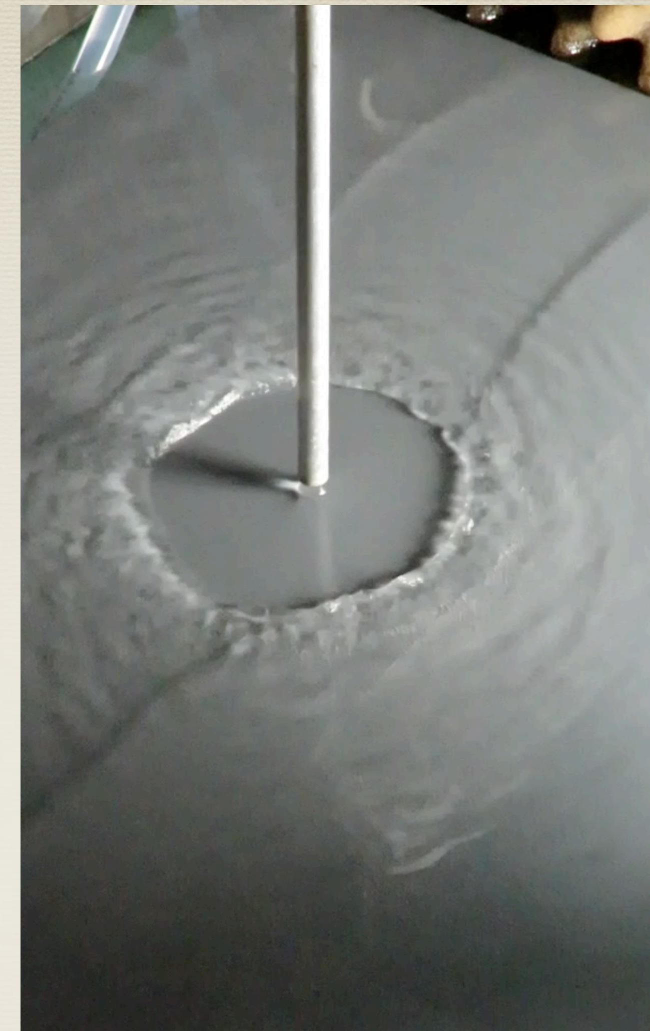
Spacetime diagram

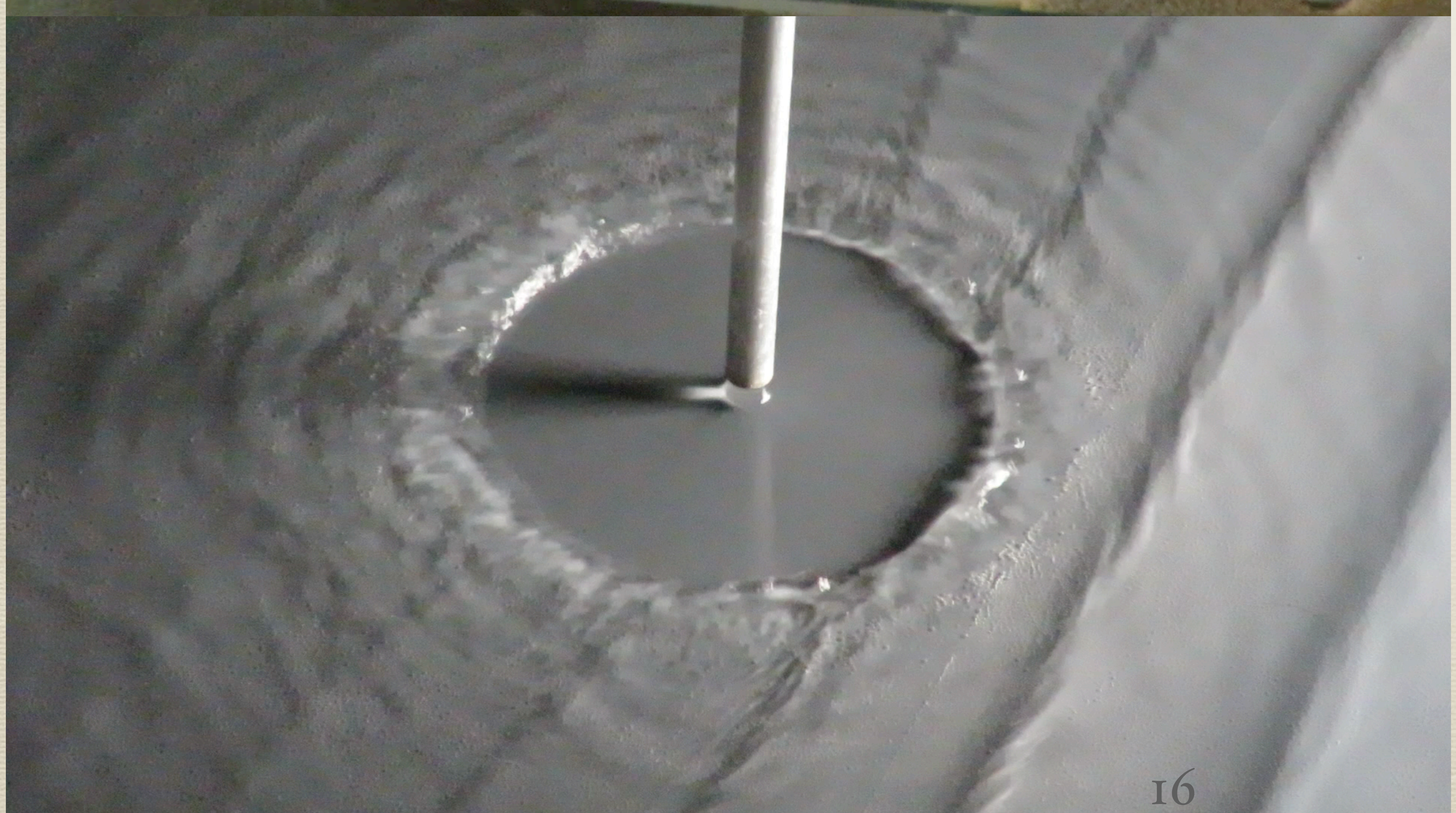


After Jaro Fransen from his Master Thesis at Eindhoven University of Technology (2024).

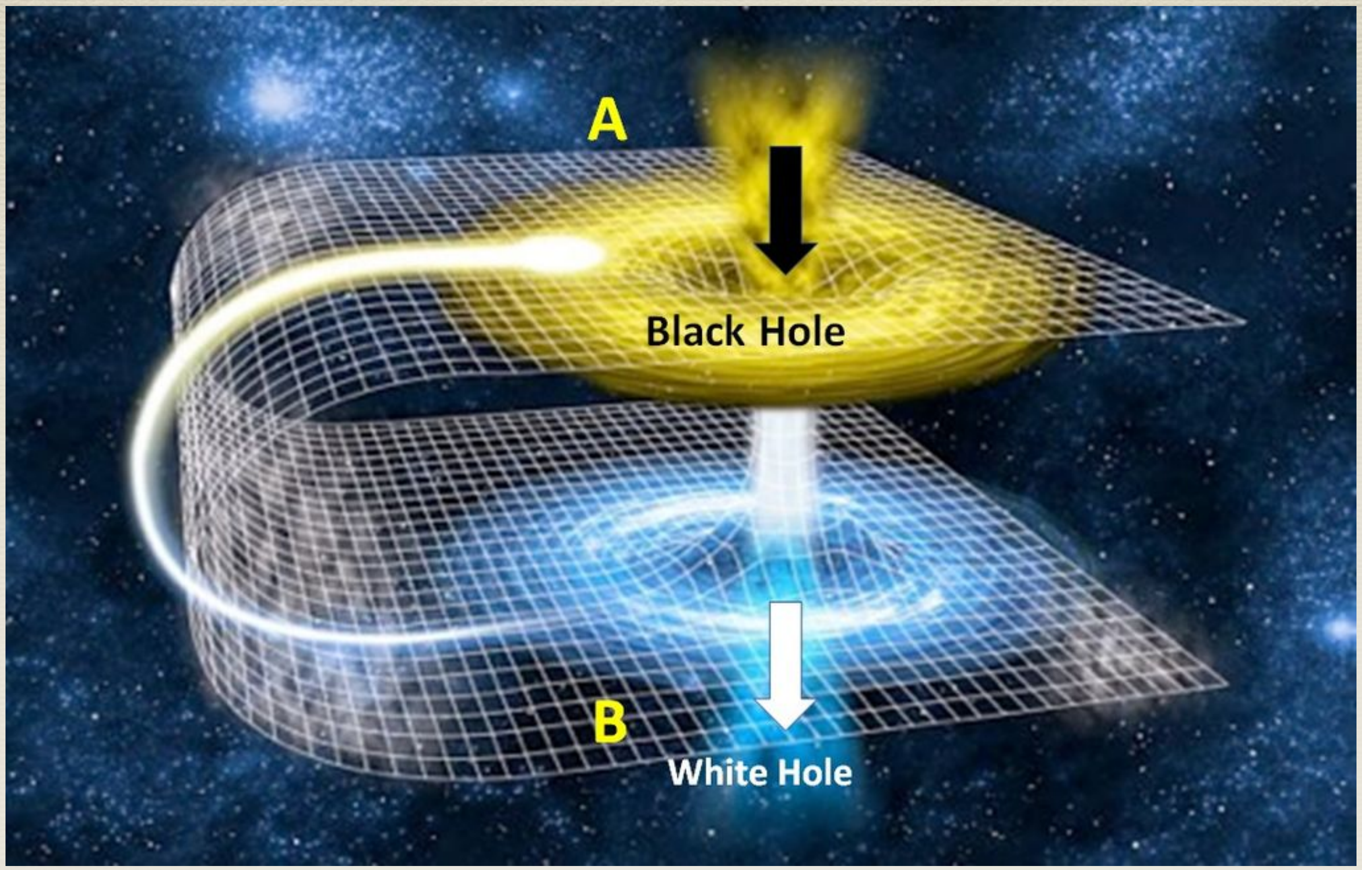
The Circular Jump as a 2D Hydraulic White Fountain

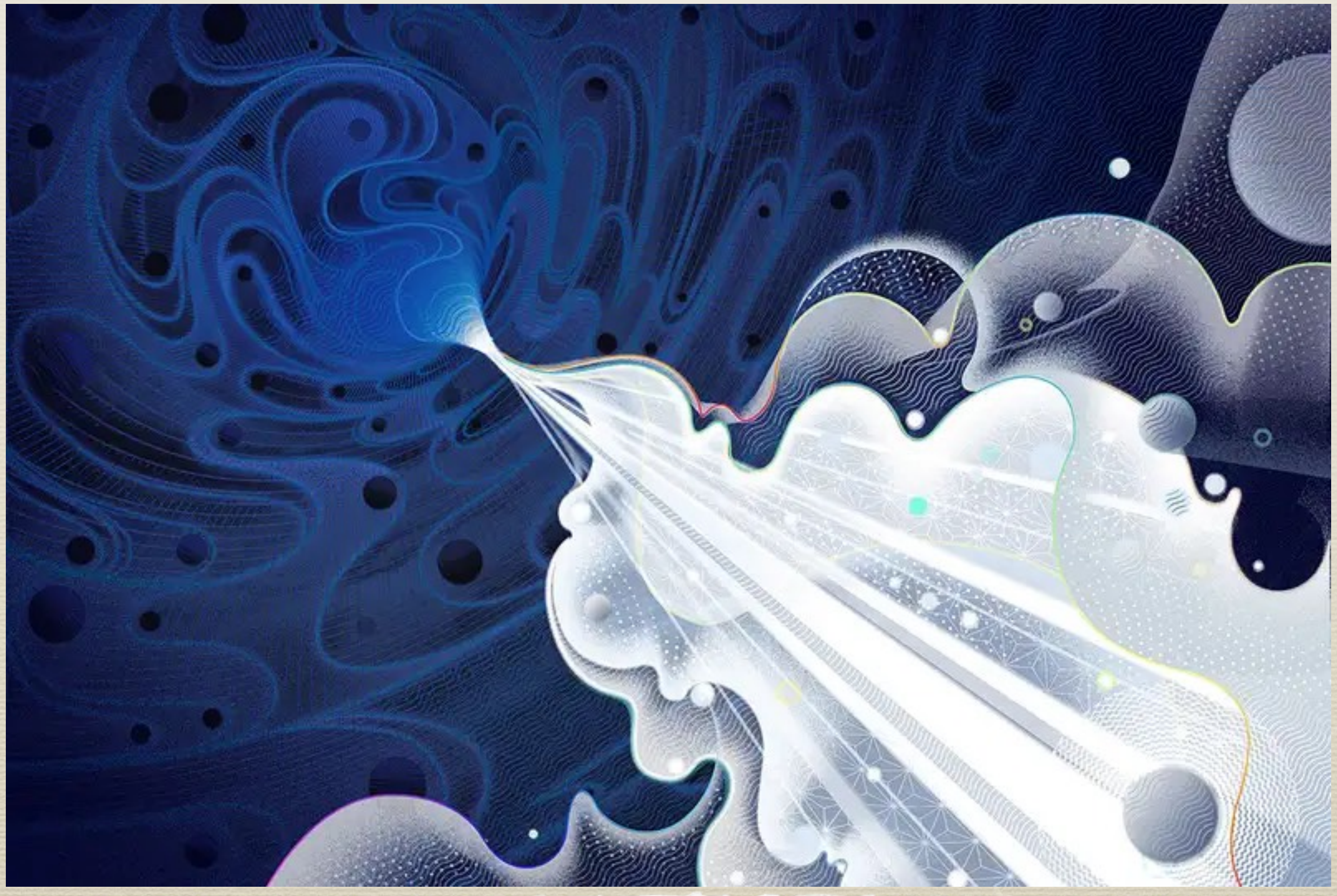




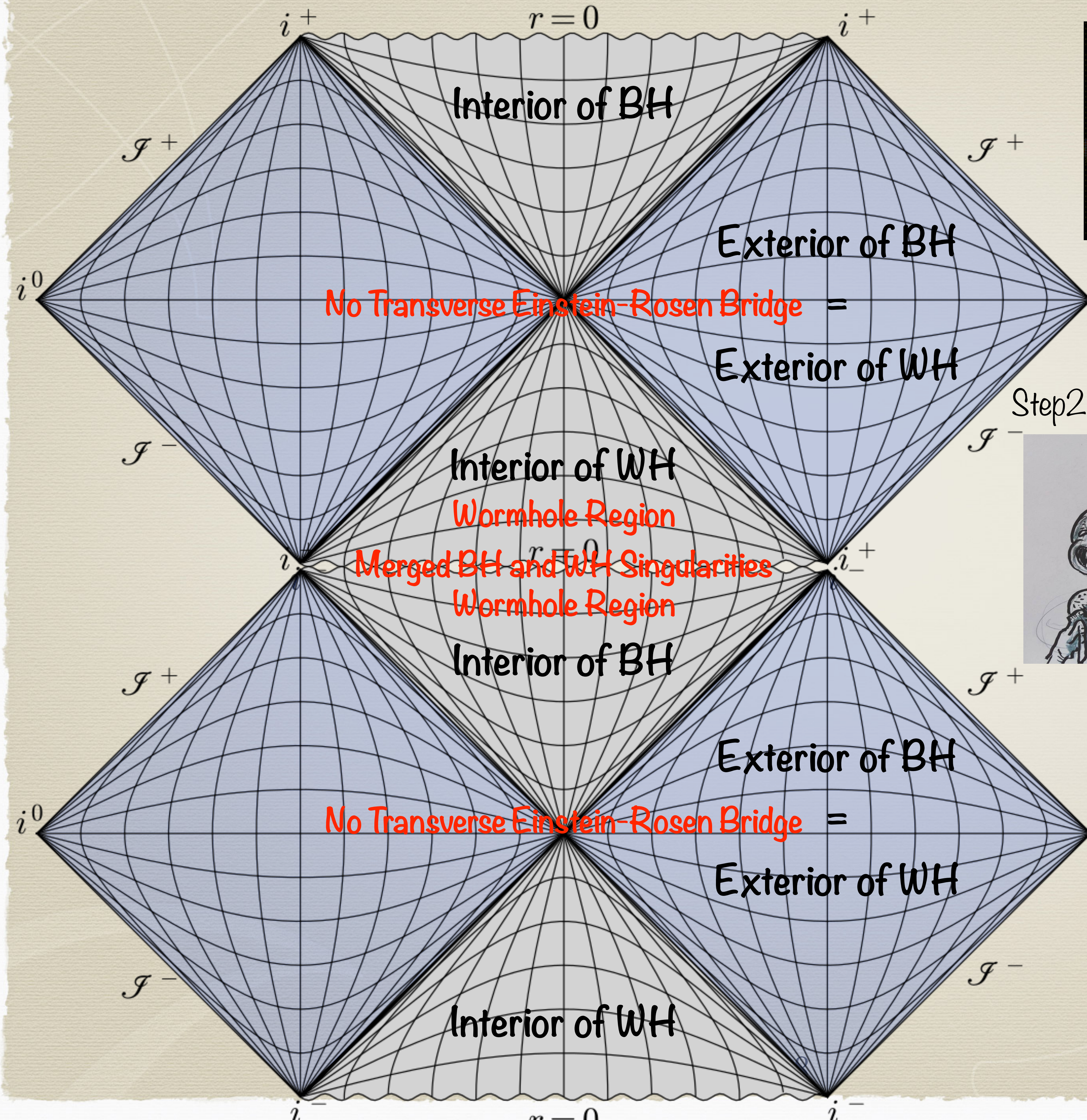


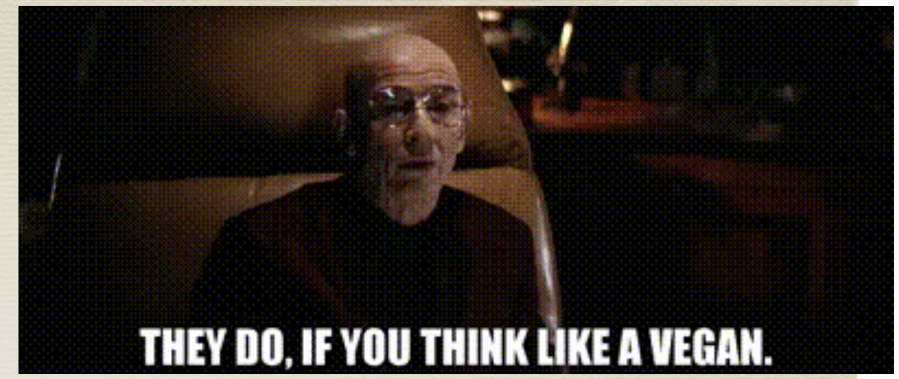
The Right Connexion: merging a pair of NR-BH and WH singularities



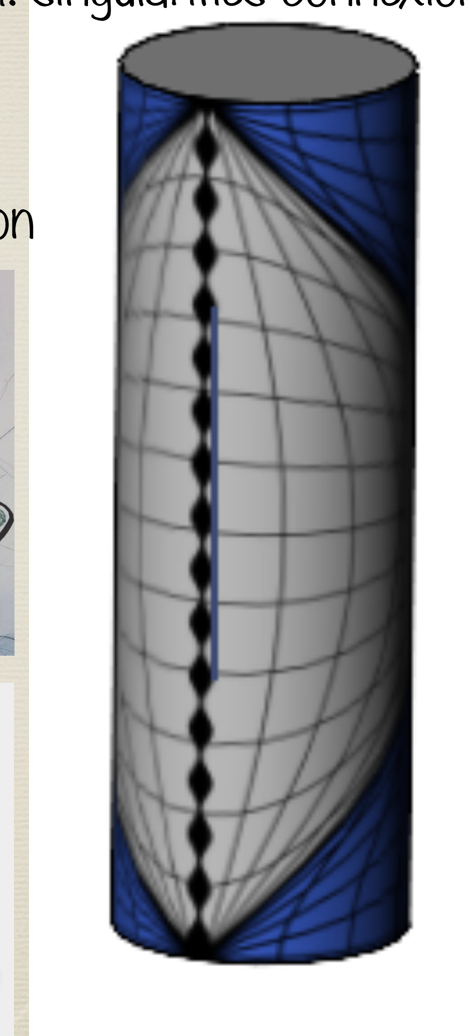


Folding the Penrose Diagram to Connect Past and Future Singularities of a NR-BH/WH pair

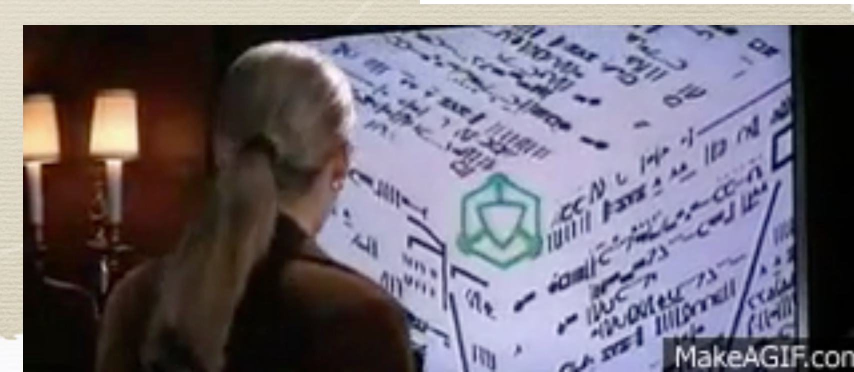




Step 1: singularities connexion

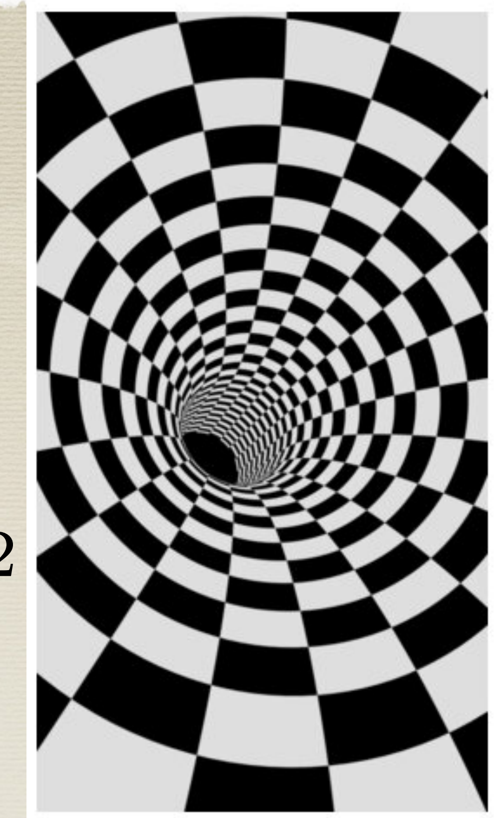


Step2: exteriors connexion



A Bediere with its Waterfall and related Cataract in Bråsvellbreen, Svalbard (Norway)

The Acoustic Metric (Richard White, 1973)



Wave Mechanics
$$\frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right]=0$$

$$dx^\mu=(cdt,d\vec{x})$$

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu=c^2dt^2-(d\vec{x}-Udt)^2$$

$$g_{\mu\nu}=\begin{bmatrix}1-U^2/c^2 & & \mathbf{U}^\intercal/c & \\ & -1 & 0 & 0\\ \mathbf{U}/c & 0 & -1 & 0\\ & 0 & 0 & -1\end{bmatrix}\qquad g_{\mu\alpha}g^{\alpha\nu}=\delta^\nu_\mu=I_4$$

$$g=det(g_{\mu\nu})=-1$$

$$g^{\mu\nu}=\begin{bmatrix}1 & \mathbf{U}^\intercal/c\\ \mathbf{U}/c & \mathbf{U}\otimes\mathbf{U}/c^2-I_3\end{bmatrix}$$

Acoustic ray tracing in moving inhomogeneous fluids

Richard W. White

Naval Undersea Research and Development Center, Pasadena Laboratory, Pasadena, California 91107

(Received 30 September 1971)

Null geodesics of the metric tensor formed from the coefficients of the second-order terms in the partial differential equation for sound are interpreted as the space–time path histories of sound pulses in a geometric ray trace theory for sound propagation in moving inhomogeneous inviscid fluids.

The Journal of the Acoustical Society of America

Geometrical Acoustics
$$k_\mu=(-\frac{\omega}{c},\mathbf{k})\qquad g^{\mu\nu}k_\mu k_\nu=0$$

$$(\omega-\mathbf{U.k})^2=c^2k^2$$

No dispersion

Gravity wave analogues of black holes

Ralf Schützhold* and William G. Unruh†
Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1
(Received 22 May 2002; published 28 August 2002)

It is demonstrated that gravity waves of a flowing fluid in a shallow basin can be used to simulate phenomena around black holes in the laboratory. Since the speed of the gravity waves as well as their high-wave-number dispersion (subluminal vs superluminal) can be adjusted easily by varying the height of the fluid (and its surface tension) this scenario has certain advantages over the sonic and dielectric black hole analogs, for example, although its use in testing quantum effects is dubious. It can be used to investigate the various classical instabilities associated with black (and white) holes experimentally, including positive and negative norm mode mixing at horizons.

« For the Einsteinians, the ds² has a mystical and universal significance, constraining all phenomena to fit themselves in the mold of a sort of space-time form, like water in a vase. »

Paul Painlevé.

$$\left(\frac{\partial}{\partial t}+\boldsymbol{v}_{\mathrm{B}}^{\parallel}\cdot\boldsymbol{\nabla}_{\parallel}\right)^{2}\delta\Phi_{(0)}-g\,h_{\mathrm{B}}\boldsymbol{\nabla}_{\parallel}^{2}\,\delta\Phi_{(0)}=0$$

$$kh << 1$$

$$\Box\,\delta\Phi_{(0)}=\frac{1}{\sqrt{-g}}\partial_{\mu}\left(\sqrt{-g}\;g^{\mu\nu}\partial_{\nu}\,\delta\Phi_{(0)}\right)=0$$

$$\mathrm{g}_{\mathrm{eff}}^{\mu\nu}=\begin{pmatrix}1 & \boldsymbol{v}_{\mathrm{B}}^{\parallel}\\ \boldsymbol{v}_{\mathrm{B}}^{\parallel} & \boldsymbol{v}_{\mathrm{B}}^{\parallel}\otimes\boldsymbol{v}_{\mathrm{B}}^{\parallel}-g\,h_{\mathrm{B}}\mathbf{1}\end{pmatrix}$$

$$c=\sqrt{gh}$$

$$ds^2_{P.-G.}=c^2dt^2-(d\vec{x}-\vec{U}dt)^2$$

$$U(x) <=> V_{Schw}(r)=\sqrt{\frac{2GM}{r}}$$

PHYSICAL REVIEW D **66**, 044019 (2002)

84. To Thomas Barclay

[Berlin,] 14 March 1922

Dear Sir Thomas Barclay–

With reference to your kind inquiry[1] I can inform you that I am coming to Paris on March 27th for ca. 10 days in order to give a few talks at the Collège de France.[2] Considering my imperfect knowledge of the Fren[ch] language, m[y] duties thus already assumed will be a very great effort for me; so it will be scarcely possible for me to follow yet another invitation to speak at the S[ociété] de Physique. However, it will be a great pleasure for me to discuss with [Mr.] P[ainlevé][3] scientif. questions of mutual interest to us. In hope of seeing you again in Paris, I am, with amic. greetings, sincerely yours

182. To Paul Painlevé

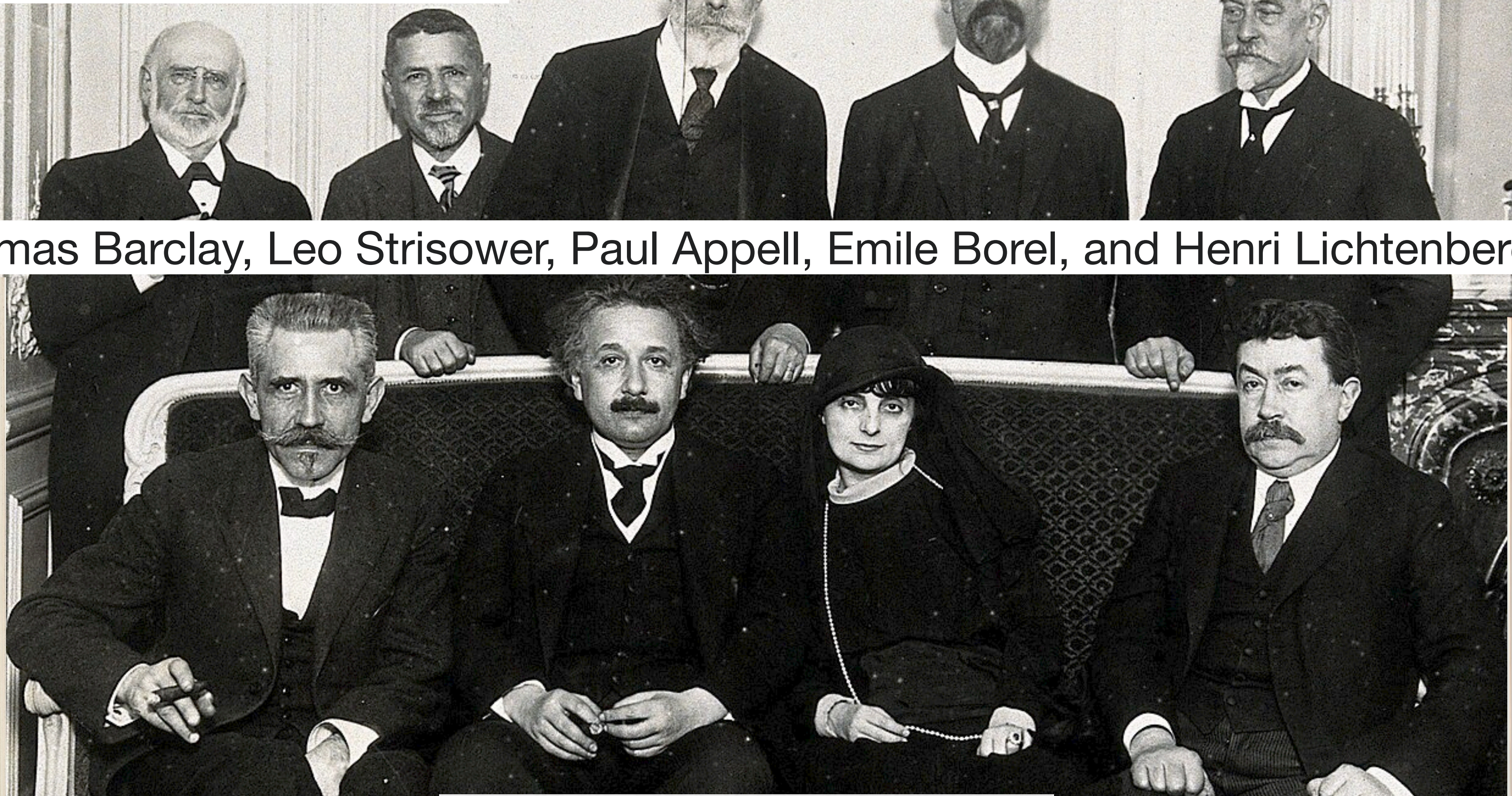
Leyden, 8 May 1922

Dear Mr. Painlevé,

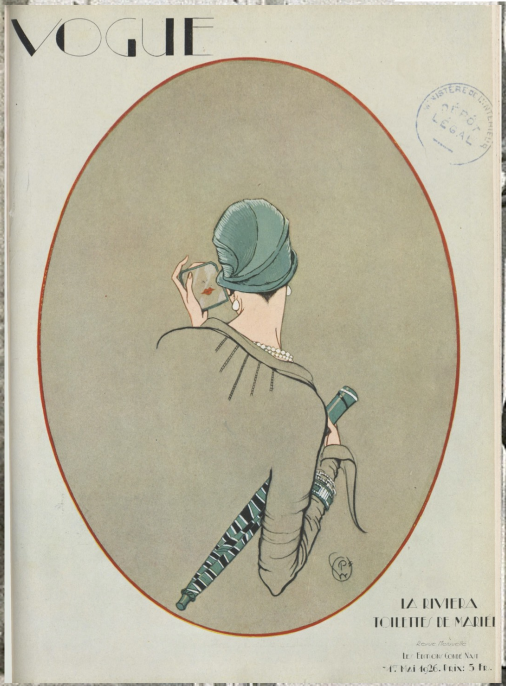
I unfortunately received your letter late; it was forwarded to me.[1] I send you the requested autographs nonetheless in the belief that they have been made use of.[2] The conversations with you were among the most exquisite I experienced in Paris; your intensity and objectivity pleased me very much. All in all, I think back on that sojourn in happy gratitude.

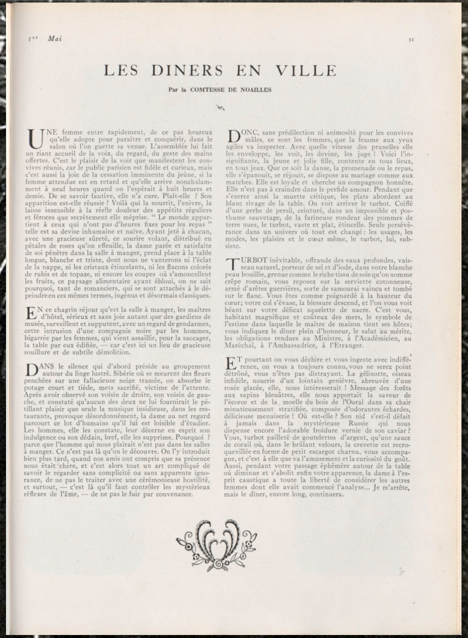
Cordial regards from yours truly,

A. Einstein.



Thomas Barclay, Leo Strisower, Paul Appell, Emile Borel, and Henri Lichtenberger





1er Mai 31

LES DINERS EN VILLE

Par la COMTESSE DE NOAILLES

UNE femme entre rapidement, de ce pas heureux qu'elle adopte pour paraître et conquérir, dans le salon où l'on guette sa venue. L'assemblée lui fait un riant accueil de la voix, du regard, du geste des mains offertes. C'est le plaisir de la voir que manifestent les convives réunis, car le public parisien est fidèle et curieux, mais c'est aussi la joie de la cessation imminente du jeûne, si la femme attendue est en retard et qu'elle arrive nonchalamment à neuf heures quand on l'espérait à huit heures et demie. De se savoir fautive, elle n'a cure. Plaît-elle ? Son apparition est-elle réussie ? Voilà qui la nourrit, l'enivre, la laisse insensible à la réelle douleur des appétits réguliers et féroces que secrètement elle méprise. " Le monde appartient à ceux qui n'ont pas d'heures fixes pour les repas " telle est sa devise inhumaine et naïve. Ayant jeté à chacun, avec une gracieuse sûreté, ce sourire volant, distribué en pétales de roses qu'on effeuille, la dame parée et satisfaite de soi pénètre dans la salle à manger, prend place à la table longue, blanche et triste, dont nous ne vanterons ni l'éclat de la nappe, ni les cristaux étincelants, ni les flacons colorés de rubis et de topaze, ni encore les coupes où s'amoncellent les fruits, ce paysage alimentaire ayant ébloui, on ne sait pourquoi, tant de romanciers, qui se sont attachés à le dépeindre en ces mêmes termes, ingénus et désormais classiques.

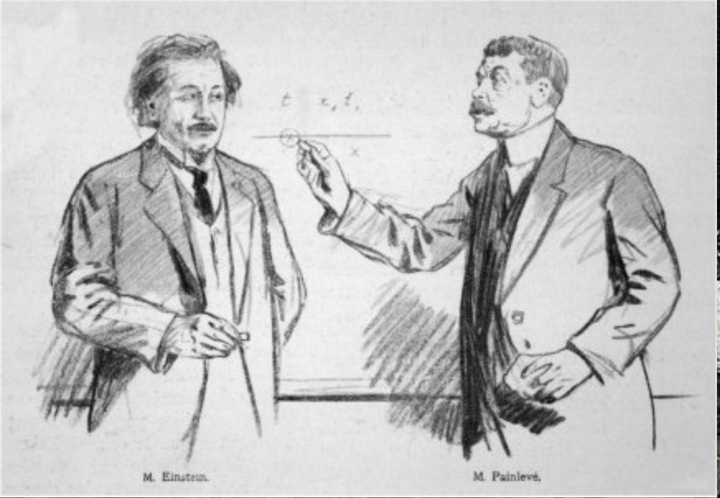
EN ce chagrin séjour qu'est la salle à manger, les maîtres d'hôtel, sérieux et sans joie autant que des gardiens de musée, surveillent et supputent, avec un regard de gendarmes, cette intrusion d'une compagnie noire par les hommes, bigarrée par les femmes, qui vient assaillir, pour la saccager, la table par eux édifiée, — car c'est ici un lieu de gracieuse souillure et de subtile démolition.

DANS le silence qui d'abord préside au groupement autour du linge lustré, Sibérie où se meurent des fleurs penchées sur une fallacieuse neige tramée, on absorbe le potage court et tiède, mets sacrifié, victime de l'attente. Après avoir observé son voisin de droite, son voisin de gauche, et constaté qu'aucun des deux ne lui fournirait le pétillant plaisir que seule la musique insidieuse, dans les restaurants, provoque désordonnément, la dame au net regard parcourt ce lot d'humains qu'il lui est loisible d'étudier. Les hommes, elle les constate, leur décerne en esprit son indulgence ou son dédain, bref, elle les supprime. Pourquoi ? parce que l'homme qui nous plairait n'est pas dans les salles à manger. Ce n'est pas là qu'on le découvre. On l'y introduit bien plus tard, quand nos amis ont compris que sa présence nous était chère, et c'est alors tout un art compliqué de savoir le regarder sans complicité ou sans apparente ignorance, de ne pas le traiter avec une cérémonieuse hostilité, et surtout, — c'est là qu'il faut contrôler les mystérieux réflexes de l'âme, — de ne pas le fuir par convenance.

DONC, sans prédilection ni animosité pour les convives mâles, ce sont les femmes, que la femme aux yeux agiles va inspecter. Avec quelle vitesse des prunelles elle les enveloppe, les voit, les devine, les juge ! Voici l'insignifiante, la jeune et jolie fille, contente en tous lieux, en tous jeux. Que ce soit la danse, la promenade ou le repas, elle s'épanouit, se réjouit, se dispose au mariage comme aux matches. Elle est loyale et cherche un compagnon honnête. Elle n'est pas à craindre dans le perfide amour. Pendant que s'exerce ainsi la muette critique, les plats abordent au blanc rivage de la table. On voit arriver le turbot. Coiffé d'une gerbe de persil, ceinturé, dans un impossible et posthume sauvetage, de la farineuse rondeur des pommes de terre nues, le turbot, vaste et plat, étincelle. Seule persévérance dans un univers où tout est changé : les usages, les modes, les plaisirs et le cœur même, le turbot, lui, subsiste.

TURBOT inévitable, offrande des eaux profondes, vaisseau naturel, porteur de sel et d'iode, dans votre blanche peau bouillie, grenue comme le riche tissu de soie qu'on nomme crêpe romain, vous reposez sur la serviette cotonneuse, armé d'arêtes guerrières, sorte de samouraï vaincu et tombé sur le flanc. Vous êtes comme poignardé à la hauteur du cœur; votre col s'évase, la blessure descend, et l'on vous voit béant sur votre délicat squelette de nacre. C'est vous, habitant magnifique et coûteux des mers, le symbole de l'estime dans laquelle le maître de maison tient ses hôtes; vous indiquez le dîner plein d'honneur, le salut au mérite, les obligations rendues au Ministre, à l'Académicien, au Maréchal, à l'Ambassadrice, à l'Etranger.

ET pourtant on vous déchire et vous ingeste avec indifférence, on vous a toujours connu, vous ne serez point détrôné, vous n'êtes pas distrayant. La gélinotte, oiseau infidèle, nourrie d'un lointain genièvre, abreuvée d'une rosée glacée, elle, nous intéresserait ! Message des forêts aux sapins bleuâtres, elle nous apportait la saveur de l'écorce et de la moelle du bois de l'Oural dans sa chair minutieusement stratifiée, composée d'odorantes échardes, délicieuse menuiserie ! Où est-elle ? Son nid s'est-il défait à jamais dans la mystérieuse Russie qui nous dispense encore l'adorable froidure vernie de son caviar ? Vous, turbot pailleté de goutelettes d'argent, qu'une sauce de corail où, dans le brûlant velours, la crevette est recroquevillée en forme de petit escargot charnu, vous accompagne, et c'est à elle que va l'amusement et la curiosité du goût. Aussi, pendant votre passage éphémère autour de la table où diminue et s'abolit enfin votre apparence, la dame à l'esprit caustique a toute la liberté de considérer les autres femmes dont elle avait commencé l'analyse... Je m'arrête, mais le dîner, encore long, continuera.



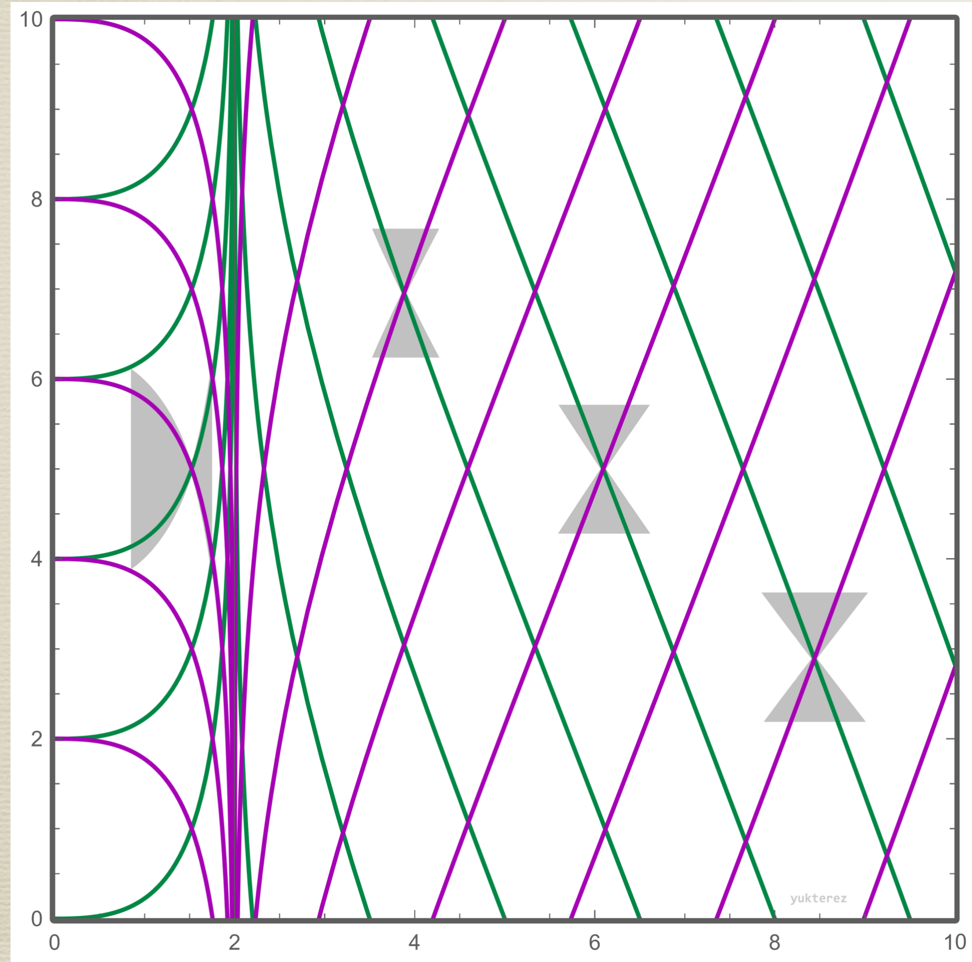
Paul Langevin, Albert Einstein, Anna Comtesse de Noailles (Chroniqueuse pour Vogue), Paul Painlevé (PM)

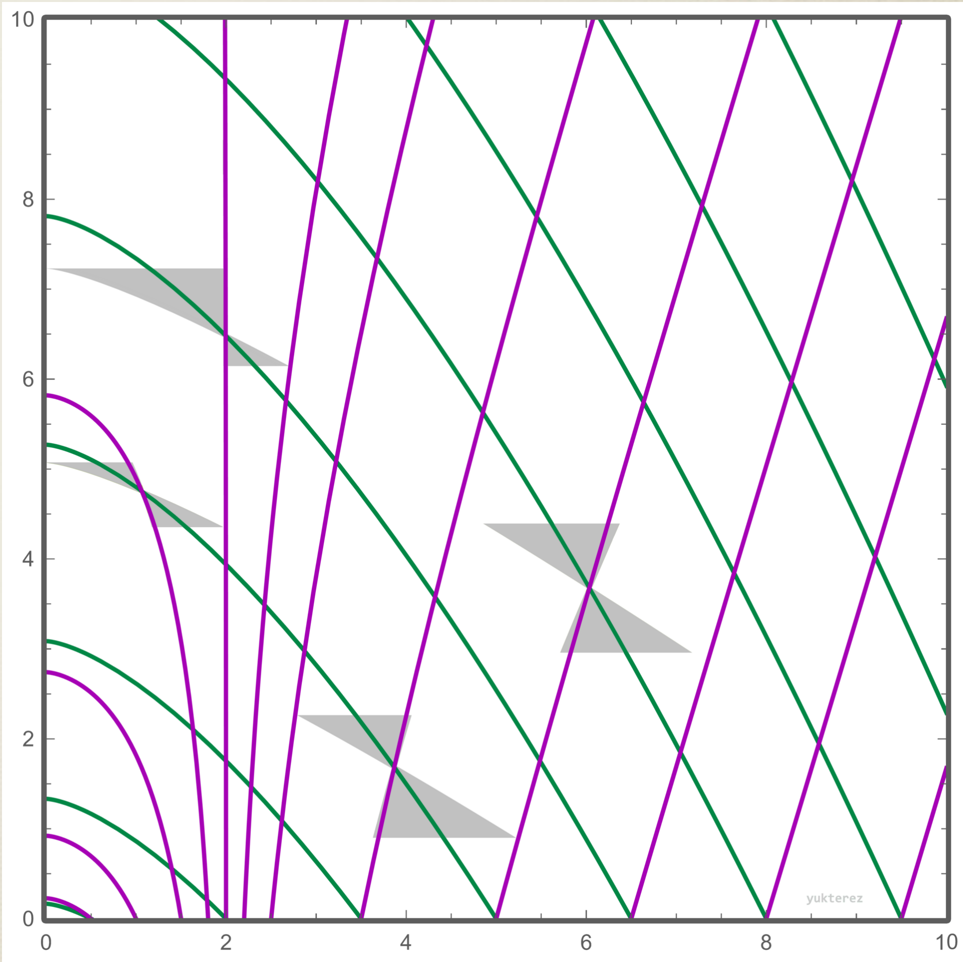
Schwarzschild-Droste versus Painlevé-Gullstrand Space-times

« Painlevé wrote to Einstein to introduce his solution and invited Einstein to Paris for a debate. In Einstein's reply letter (December 7), he apologized for not being in a position to visit soon and explained why he was not pleased with Painlevé's arguments, emphasising that the coordinates themselves have no meaning. Finally, Einstein came to Paris in early April. On the 5th of April 1922, in a debate at the "Collège de France" with Painlevé, Becquerel, Brillouin, Cartan, De Donder, Hadamard, Langevin and Nordmann on "the infinite potentials", Einstein, baffled by the non quadratic cross term in the line element, rejected the Painlevé solution. » J. Fric

« The "trick" of the Painlevé proposal was that he no longer stuck to a full quadratic (static) form but instead, allowed a cross time-space product making the metric form no longer static but stationary and no longer direction symmetric but preferentially oriented. » J. Fric

P-G metric=acoustic metric in AG





Paul Painlevé, La mécanique classique et la théorie de la relativité, C. R. Acad. Sci. (Paris) 173, p. 677-680 (1921).

Allvar Gullstrand, Allgemeine Lösung des statischen Einkörperproblems in der Einsteinschen Gravitationstheorie, Arkiv för Matematik, Astronomi och Fysik, 16 (8), p. 1-15 (1922).

https://astromontgeron.fr/Painleve-article-english.pdf

JOURNAL OF APPLIED PHYSICS VOLUME 35, NUMBER 1 JANUARY 1964

Amplification of Waves due to Quanta with Negative Energy

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(Received 14 March 1963; in final form 29 July 1963)

Wave energy moving faster than the phase velocity of the wave in the medium becomes negative. Such a wave field can equally well be treated as an ensemble of quanta with negative energy. Amplification of waves by mode coupling is represented by pair creation of a negative quantum and a positive quantum. Interaction of an electron stream with a dissipative medium creates the negative quanta.

I. INTRODUCTION

It is sometimes convenient to treat waves as an ensemble of quanta even if the frequency is very low and the quantum effect is not important, since wave motion can be treated quite as well as particle motion with proper energy and momentum. Mode coupling in classical treatments is replaced by creation or annihilation of a pair of quanta in quantum mechanical treatments.

It is well known that the wave energy becomes negative if it is measured in a coordinate system which is moving faster than the phase velocity of the wave relative to the wave-propagating medium.[1–3] If this situation is transferred to quantum mechanics one has, phenomenologically, quanta with negative energy.[4] The behavior of the negative-energy quanta is essential to a kinematics of amplification of waves. Several kinds of amplification phenomena can be understood from kinematics of negative-energy quanta; such as a variety of traveling-wave-tube-type amplifiers,[5] a resistive-wall amplifier,[6] amplification of ultrasound waves in semiconductors,[7] and even masers.

II. ENERGY-MOMENTUM TRANSFORMATION

Although problems about the energy of the wave propagating through a medium moving relative to an observer have been discussed by several authors,[1,2-4] we consider them here again from another point of view. The energy and the momentum of a dynamical system would have no physical meaning apart from a coordinate system in which they are measured. The momentum **G** and the total energy (including the rest energy) E of a dynamical system compose a 4 vector in the Minkovskii space.[8] Suppose a medium at rest with total energy E_0 and momentum zero. In this medium a wave propagating along the x axis is excited through interaction with the other dynamical system, and as a result the total energy and the momentum of the medium increase by ΔE and ΔG_x, respectively. ΔE and ΔG_x are reasonably called the "wave energy" and the "wave momentum." Let this excitation phenomenon be observed by an observer moving with velocity u along the x axis. Then the wave energy $\Delta E'$ and the wave momentum $\Delta G_x'$, which would be observed by the moving observer, are expressed as

$$\Delta E'=(\Delta E-u\Delta G_x)/[1-(u/c)^2]^{\frac{1}{2}}, \tag{2.1}$$

$$\Delta G_x'=[\Delta G_x-(u/c^2)\Delta E]/[1-(u/c)^2]^{\frac{1}{2}}. \tag{2.2}$$

Since the momentum ΔG_x of a wave is related to the wave energy ΔE in general as

$$\Delta G_x=\Delta E/v_{ph}, \tag{2.3}$$

where v_{ph} is the phase velocity of the wave relative to the medium, relations (2.1) and (2.2) become

$$\Delta E'=\{(1-u/v_{ph})/[1-(u/c)^2]^{\frac{1}{2}}\}\Delta E \tag{2.4}$$

$$\Delta G_x'=\{(1-uv_{ph}/c^2)/[1-(u/c)^2]^{\frac{1}{2}}\}\Delta G_x. \tag{2.5}$$

From Eq. (2.4) it follows that if the observer moves faster than the phase velocity of the wave ($u>v_{ph}$) he observes a negative wave energy. This does not mean that absolute negative-energy states exist but that the total energy of a moving medium has decreased after excitation of such a wave. The medium is always needed to obtain a negative-energy wave.

An observer moving faster than the wave velocity observes the phases of the wave in a reversed order as they were emitted by a source oscillator. To him the direction of time evolution is reversed as regards the wave motion. If we take electromagnetic waves propagating through a moving medium, this is equivalent to the electric and magnetic permeabilities, becoming effectively negative.

III. QUANTA WITH NEGATIVE ENERGY

Let us consider propagation of a plane wave along the x axis through a lossless medium. The wave may be of any type: a space-charge wave, an elastic wave, an electromagnetic wave, etc. The generalized wave amplitude a is defined so that the wave energy per unit length becomes $\omega|a|^2$, ω being the angular frequency of the wave. For the sake of generality we assume only that the wave amplitude should satisfy the wave equation

$$(\partial^2a/\partial x^2)-(1/c_0^2)(\partial^2a/\partial t^2)=0, \tag{3.1}$$

if seen in a fixed coordinate system in the medium. Here c_0 is the phase velocity of the wave. If the coordinate system is moving with velocity u relative to the medium the wave equation becomes

$$\frac{\partial^2a}{\partial x^2}+\frac{2u}{c_0^2-u^2}\frac{\partial^2a}{\partial x\partial t}-\frac{1}{c_0^2-u^2}\frac{\partial^2a}{\partial t^2}=0. \tag{3.2}$$

When the wave has a wavenumber k and the spatial part of a is expressed as $\exp(-ikx)$, (3.2) becomes

$$(\partial^2a/\partial t^2)+2iku(\partial a/\partial t)+k^2(c_0^2-u^2)a=0. \tag{3.3}$$

This wave equation gives two modes of propagation, one to the right and the other to the left relative to the medium. For each wave mode the angular frequency ω is related to the wavenumber k as

$$\omega=k(c_0-u) \quad \text{for a right-going wave,} \tag{3.4}$$

$$\omega=-k(c_0+u) \quad \text{for a left-going wave.} \tag{3.5}$$

This wave motion is expressed in canonical form if canonically conjugate variables P and Q are introduced as

$$P=i2^{-\frac{1}{2}}(a-a^*)$$
$$Q=2^{-\frac{1}{2}}(a+a^*). \tag{3.6}$$

Then (3.3) is identical with the canonical equations of motion

$$dP/dt=-\partial H/\partial Q$$
$$dQ/dt=\partial H/\partial P, \tag{3.7}$$

with the Hamiltonian H

$$H=\tfrac{1}{2}\omega(P^2+Q^2). \tag{3.8}$$

When the wave motion is expressed in this form it is quite obvious how the quantum $\hbar$ is introduced. By exact analogy with the ordinary quantum theory we have to consider the canonical variables as noncommutable quantities satisfying

$$PQ-QP=-i\hbar, \tag{3.9}$$

where $\hbar$ is the Planck constant divided by 2π. We are considering boson statistics here. Instead of a we introduce quantum mechanical operators q and q^* as

$$q=(2\hbar)^{-\frac{1}{2}}(Q+iP)$$
$$q^*=(2\hbar)^{-\frac{1}{2}}(Q-iP). \tag{3.10}$$

Then the quantum mechanical Hamiltonian becomes

$$H=\hbar\omega(q^*q+\tfrac{1}{2}). \tag{3.11}$$

The results of this quantization for the Hamiltonian are given by the well-known wave mechanical treatment of a harmonic oscillator.[9] The eigenvalue of the energy of such an oscillator is given by

$$E=(n+\tfrac{1}{2})\hbar\omega, \tag{3.12}$$

where n is the quantum number.

Thus far it is not yet determined whether ω takes a negative or a positive value. On the other hand, ω is related to the wavenumber k as (3.4) and (3.5). As the momentum of a quantum is expressed as $\hbar k$,[10] k should not change its sign when u becomes larger than c_0 because the momentum of a wave transforms as (2.5). Therefore, it follows that ω for a right-going wave is negative for $u>c_0$, and then we have a quantum with negative energy. Jauch and Watson[4] first derived a negative photon through quantization of electromagnetic wave fields in a moving medium. The existence of quanta with negative energy is not peculiar to electromagnetic fields but plasmons, phonons, any surface waves, etc., can take negative energies in this sense.

IV. EMISSION OF QUANTA AND AMPLIFICATION OF WAVES

As an example, we take an electron beam for the moving medium through which space-charge waves propagate. Usually an electron beam in a vacuum is considered an electron plasma with a zero electron temperature. A dispersion equation of a cold plasma is[11]

$$\omega^2=\omega_p^2, \tag{4.1}$$

ω_p being the angular electron–plasma frequency of the plasma. When an electron beam, which is flowing to the right with velocity u, is modulated at angular frequency ω (ω is usually larger than ω_p), two kinds of plasma waves are excited therein. Since the one has a phase velocity $-u(\omega_p/\omega)/(1+\omega_p/\omega)$ and the other $u(\omega_p/\omega)/(1-\omega_p/\omega)$ relative to the cold plasma, the former wave has a negative energy while the latter a positive one if seen from the modulating system. The so-called slow wave is the ensemble of negative plasmons. Suppose there are other waves of any kind with phase velocities $\pm v_{ph}$ propagating through a medium fixed to the modulating system and they are interacting with one another. Here we have four kinds of quanta as shown in Table I.

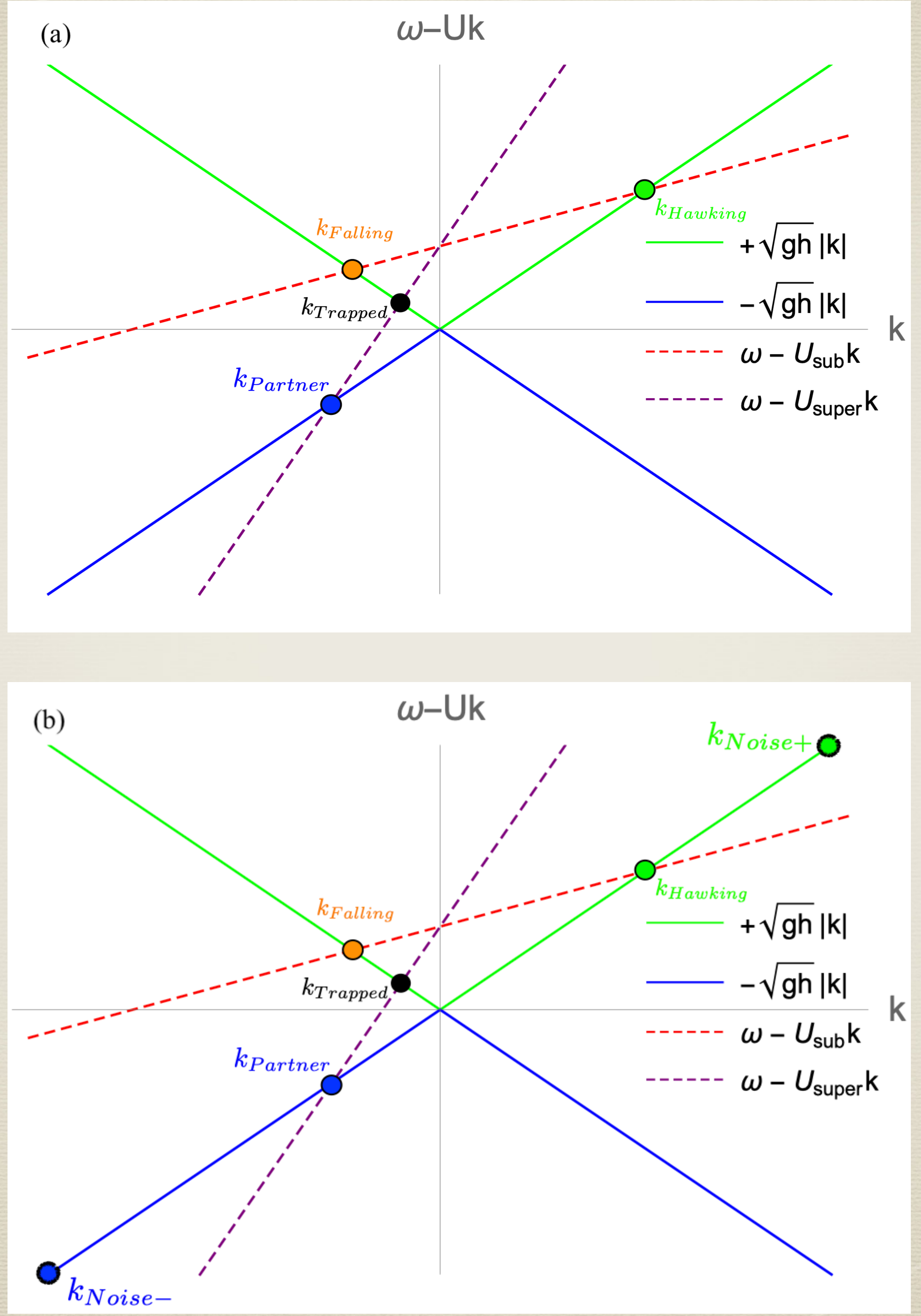
Table I. Four kinds of quanta.

	Plasmons		Quanta in the fixed medium to the observer	
	No. 1 (right going)	No. 2 (left going)	No. 3 (right going)	No. 4 (left going)
Energy	$-\hbar\omega$	$\hbar\omega$	$\hbar\omega$	$\hbar\omega$
Momentum	$-\hbar(\omega+\omega_p)/u$	$\hbar(\omega-\omega_p)/u$	$\hbar\omega/v_{ph}$	$-\hbar\omega/v_{ph}$

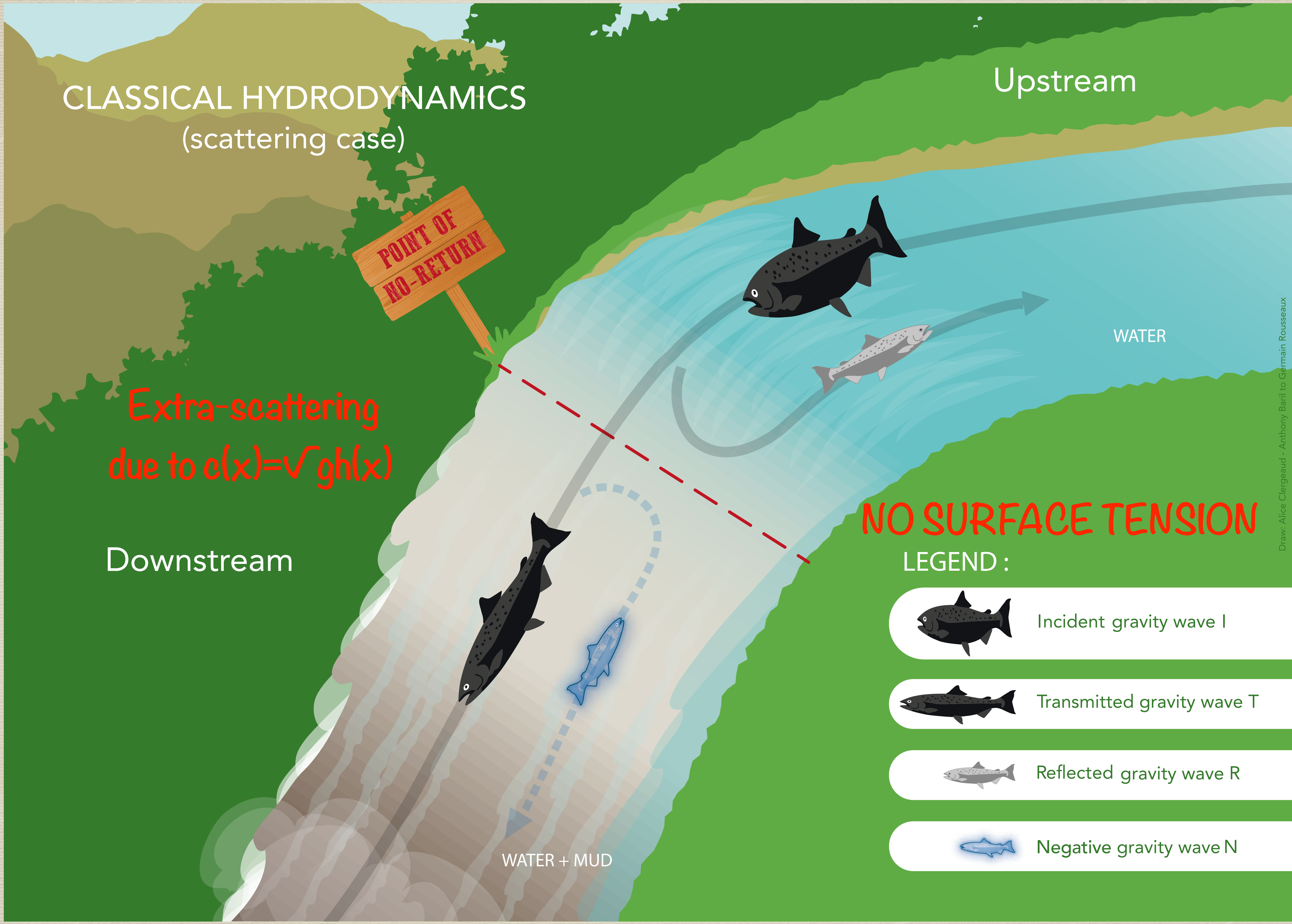
[1] P. A. Sturrock, J. Appl. Phys. **31**, 2052 (1960).
[2] J. R. Pierce, J. Appl. Phys. **32**, 2580 (1961).
[3] G. C. Van Hoven and T. Wessel-Berg, J. Appl. Phys. **34**, 1834 (1963).
[4] J. M. Jauch and K. M. Watson, Phys. Rev. **74**, 950 (1948).
[5] A. H. W. Beck, *Space-Charge Waves and Slow Electromagnetic Waves* (Pergamon Press, Inc., New York, 1958).
[6] C. K. Birdsall, G. R. Brewer, and A. V. Haeff, Proc. IRE **41**, 865 (1953).
[7] A. R. Hutson, J. H. McFee, and D. L. White, Phys. Rev. Letters, **7**, 237 (1961).
[8] W. Pauli, *Theory of Relativity* (Pergamon Press, Inc., New York, 1958).

[9] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1954), 3rd ed.
[10] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949).
[11] L. Tonks and I. Langmuir, Phys. Rev. **33**, 195 (1929).

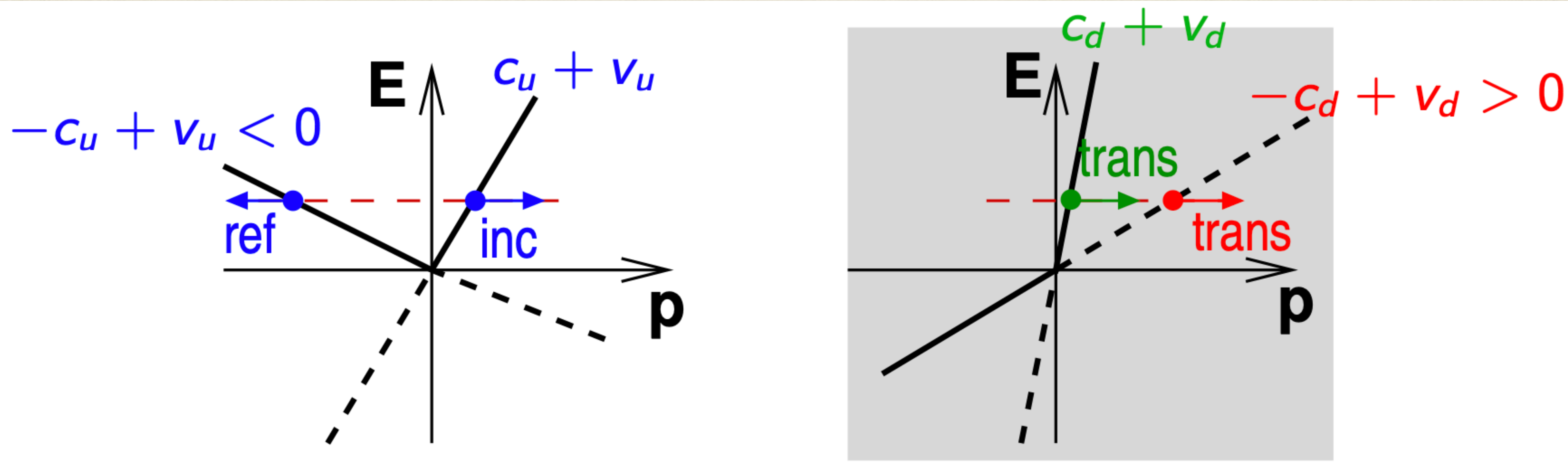
Back-Scattering versus Hawking Process (without dispersion)



Plunge into a Hydraulic Black Hole



Numerical Transmission(s) and Back-Scattering

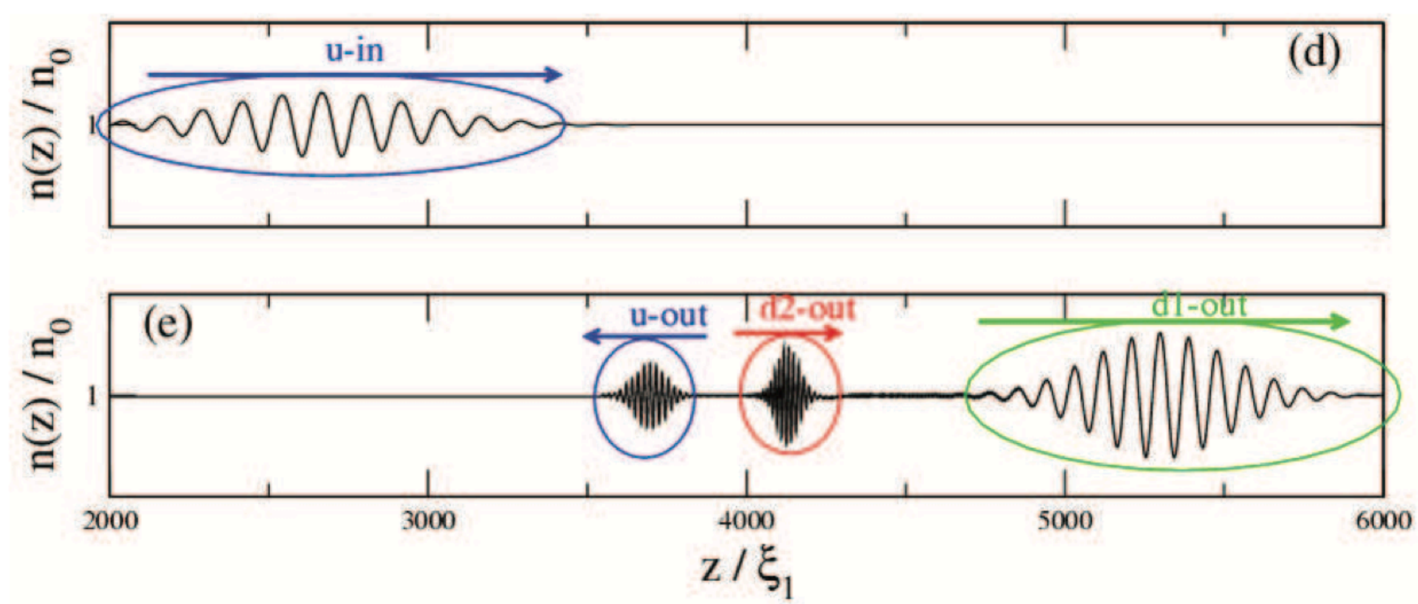


subsonic region *supersonic region*

sound waves
in the lab frame:

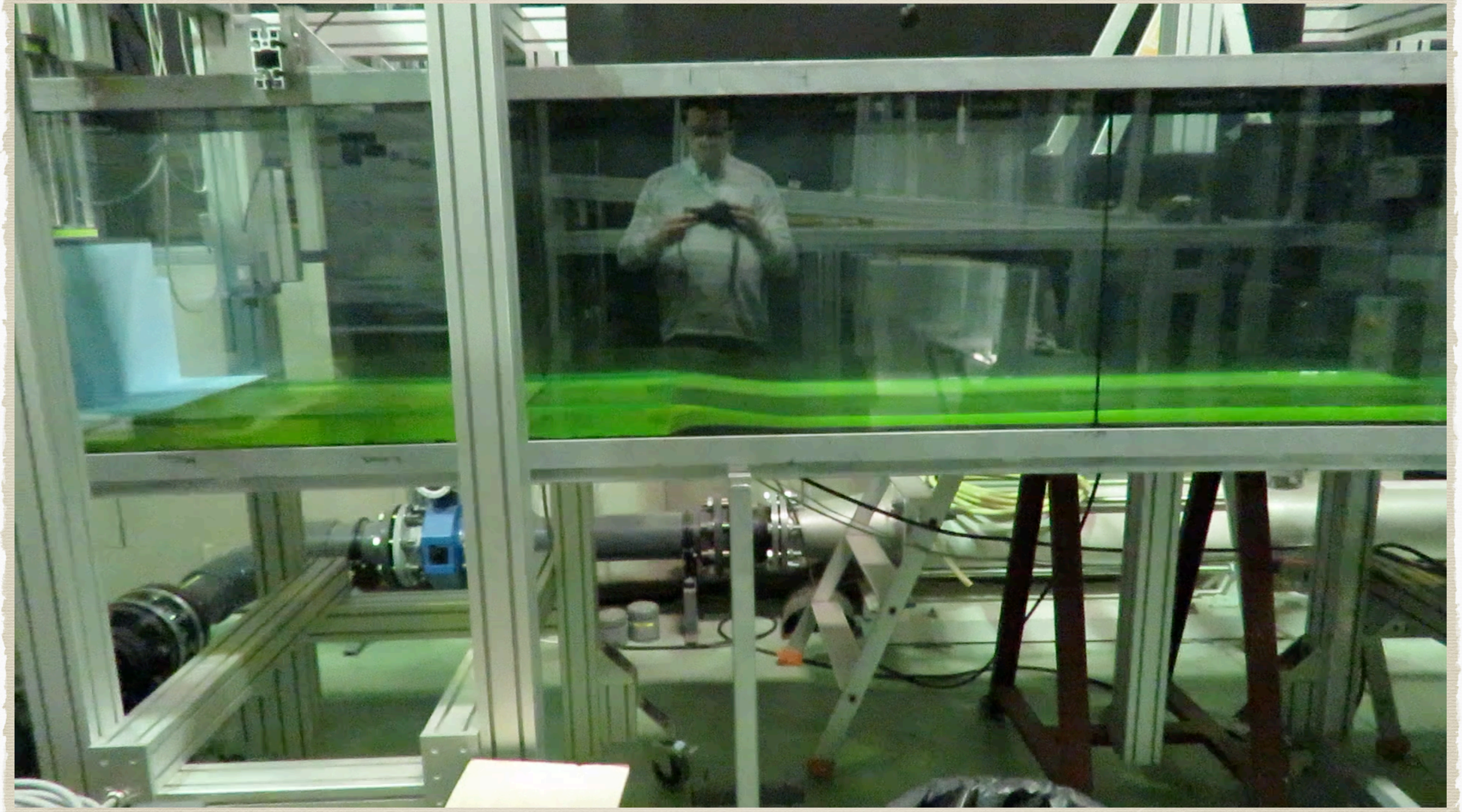
$$E(p) = c\,|p| + v\,p$$

Doppler



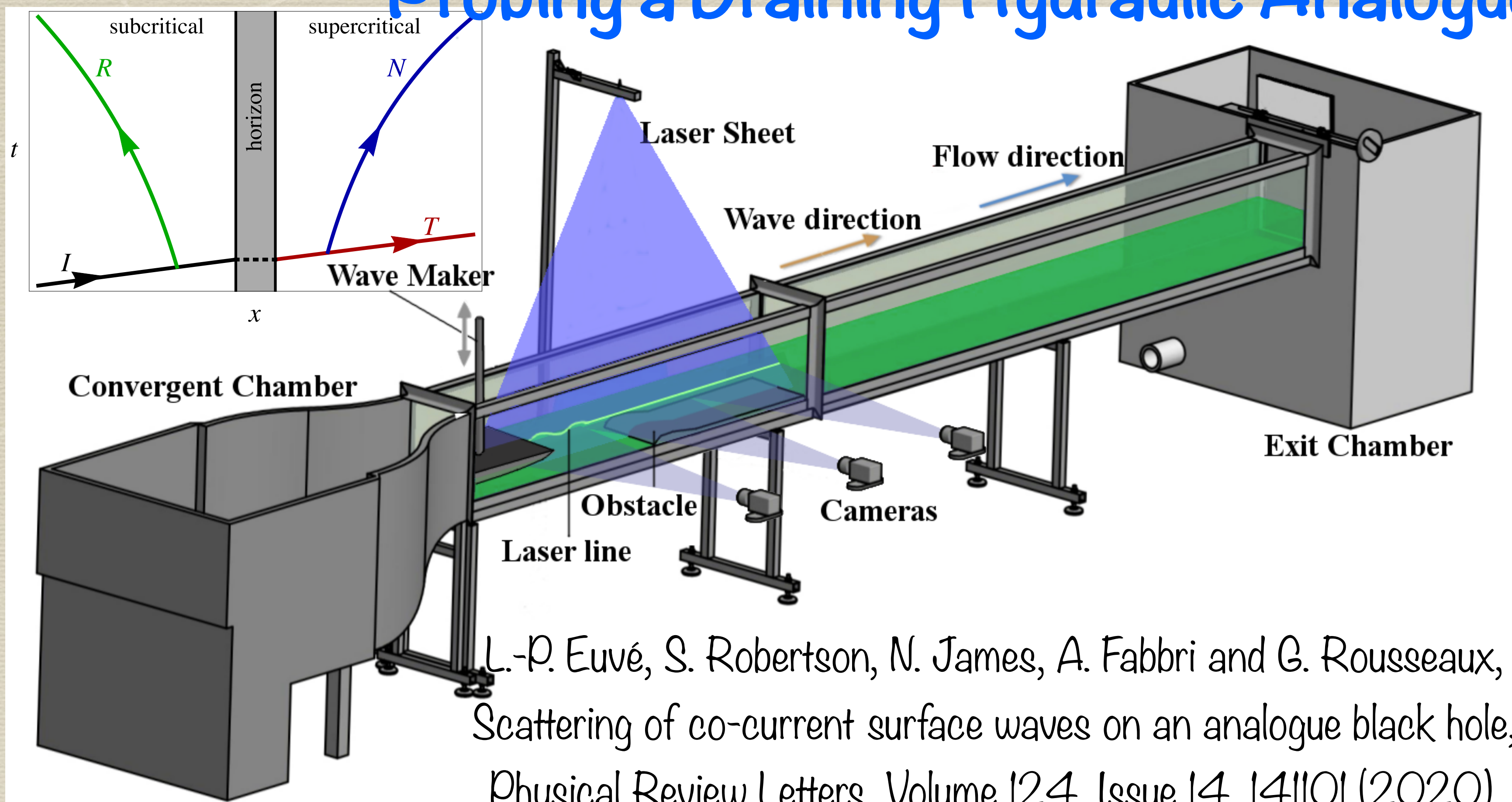
Courtesy Nicolas Pavloff

Sending Sinusoidal Waves into a Hydraulic Black Hole

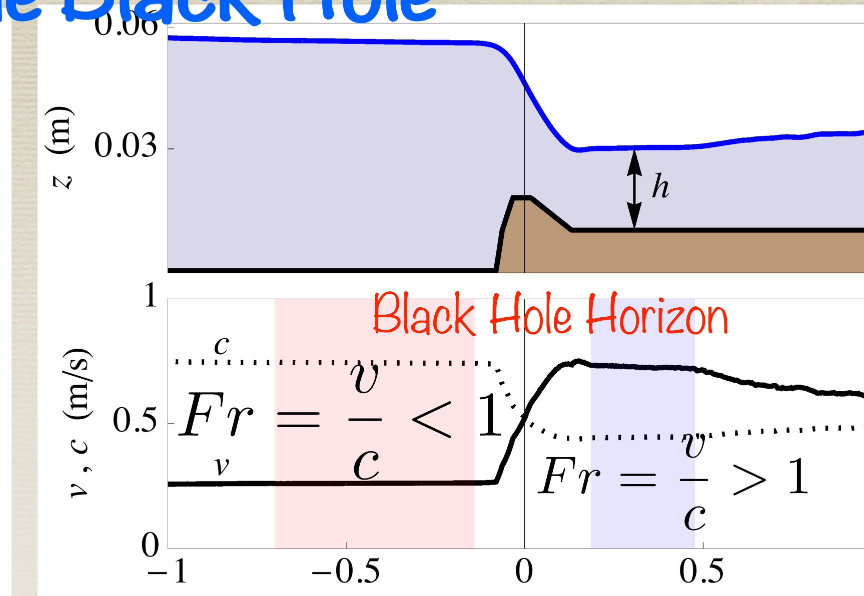


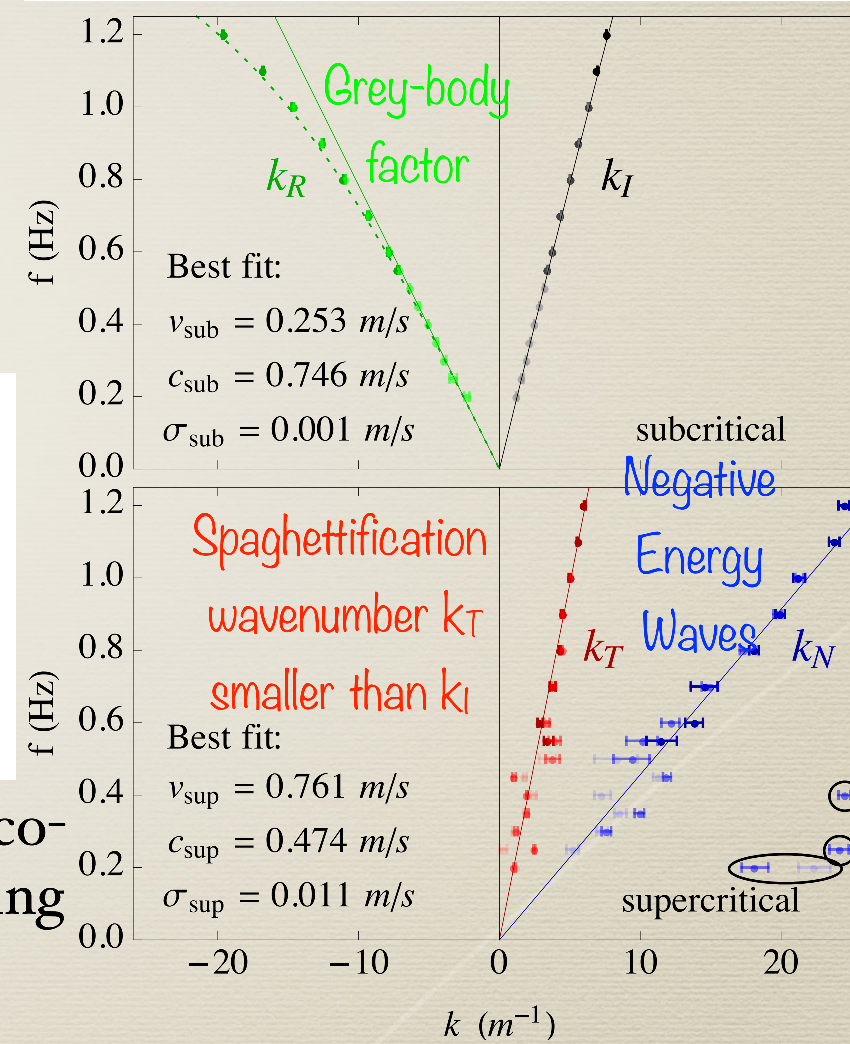
L.-P. Euvé, S. Robertson, N. James, A. Fabbri and G. Rousseaux, Scattering of co-current surface waves on an analogue black hole, Physical Review Letters, Volume 124, Issue 14, 141101 (2020).

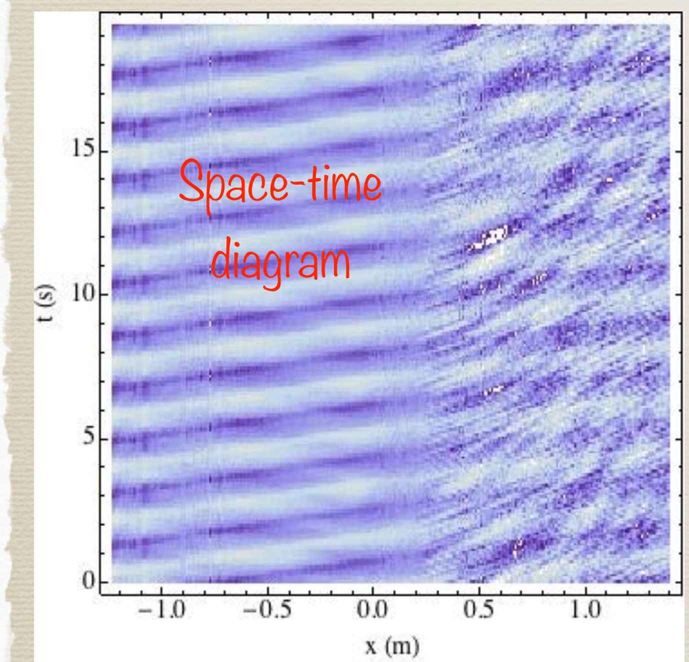
Probing a Draining Hydraulic Analogue Black Hole

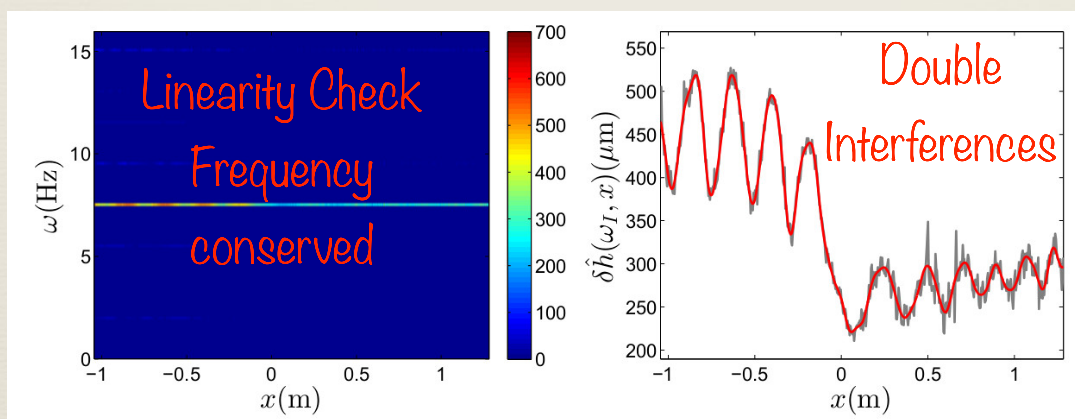


L.-P. Euvé, S. Robertson, N. James, A. Fabbri and G. Rousseaux,
Scattering of co-current surface waves on an analogue black hole,
Physical Review Letters, Volume 124, Issue 14, 141101 (2020).

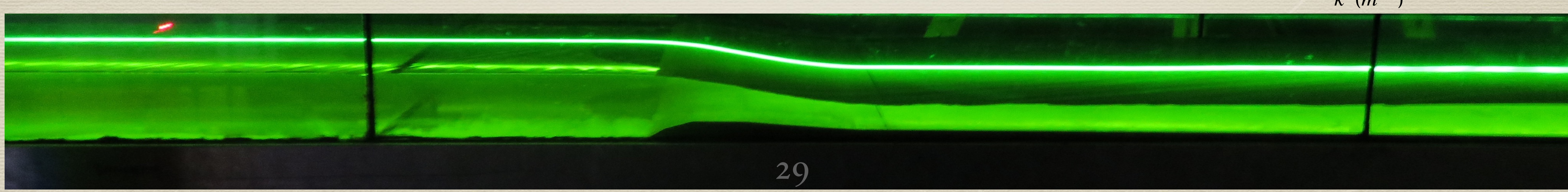








c(x)=√ gh(x) induces an extra scattering of co-propagating modes into counter-propagating modes by an effective potential

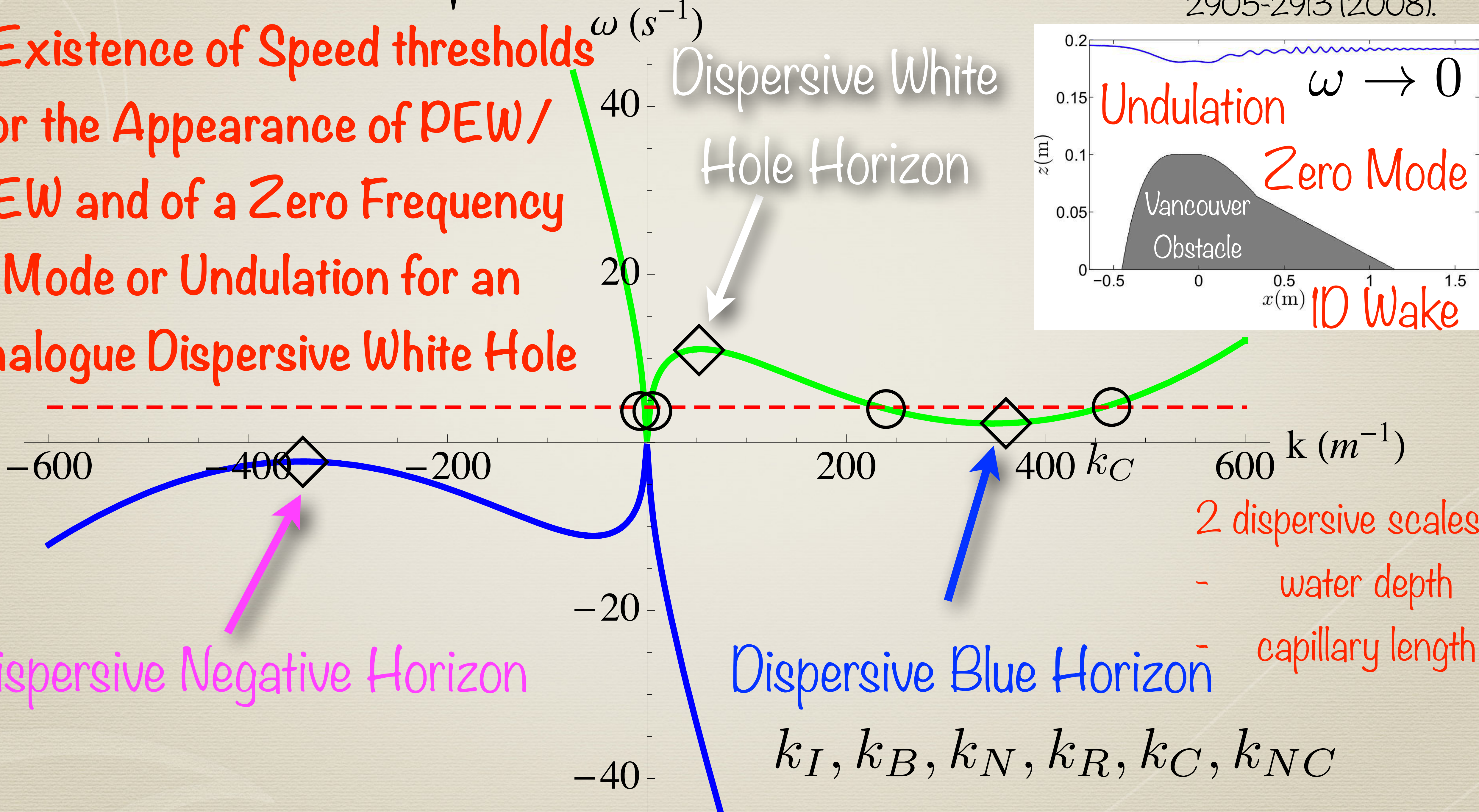


Gravito-capillary Waves and Landau Speed (23cm/s)

$$\omega = \Omega(k) = Uk \pm \sqrt{\left(gk + \frac{\gamma}{\rho}k^3\right)\tanh(kh)}$$

W. G. Unruh, Dumb holes: analogues for black holes, Phil. Trans. R. Soc. A, 366, 2905-2913 (2008).

=> Existence of Speed thresholds for the Appearance of PEW/NEW and of a Zero Frequency Mode or Undulation for an Analogue Dispersive White Hole



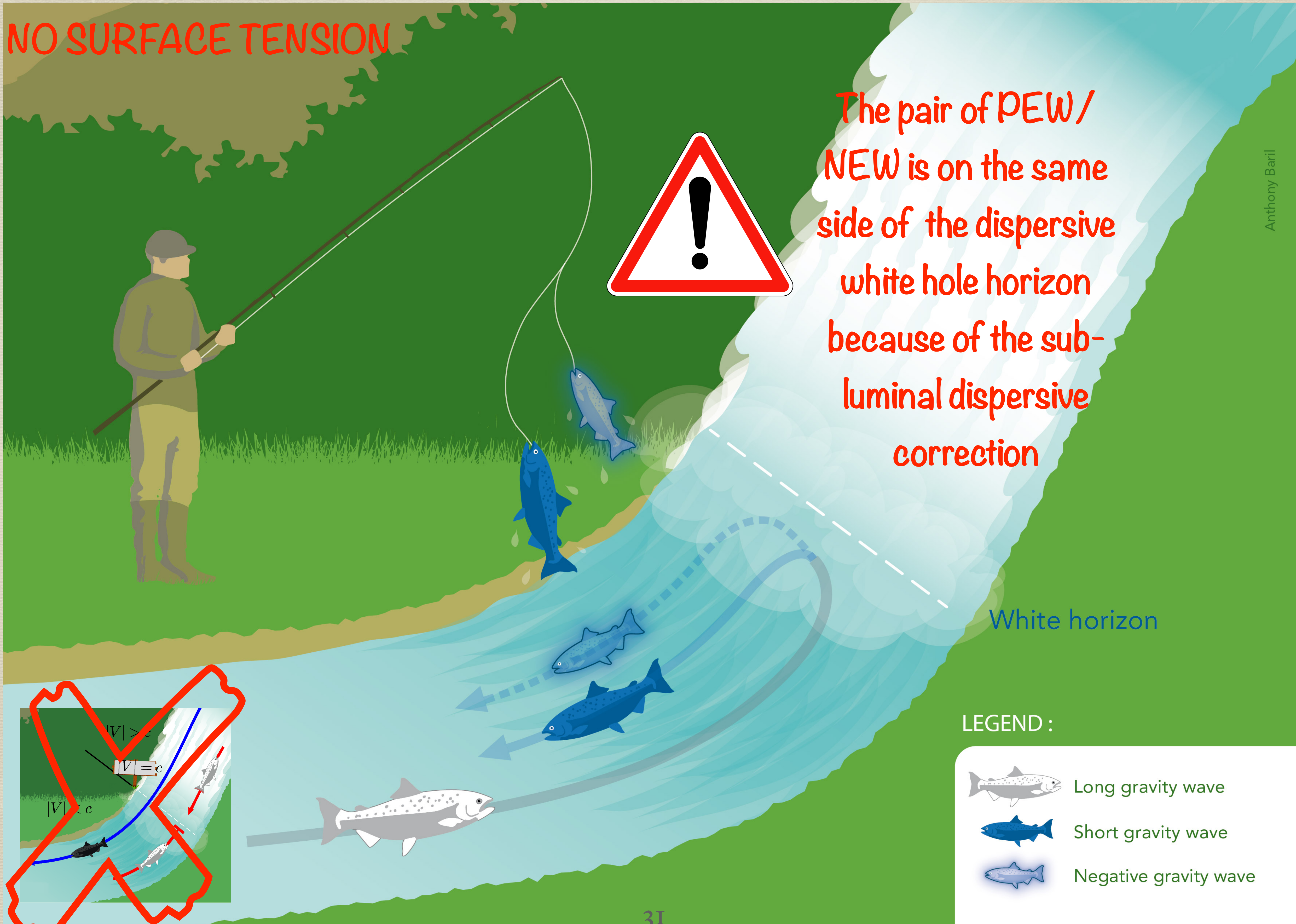
1D Wake

2 dispersive scales:
- water depth
- capillary length

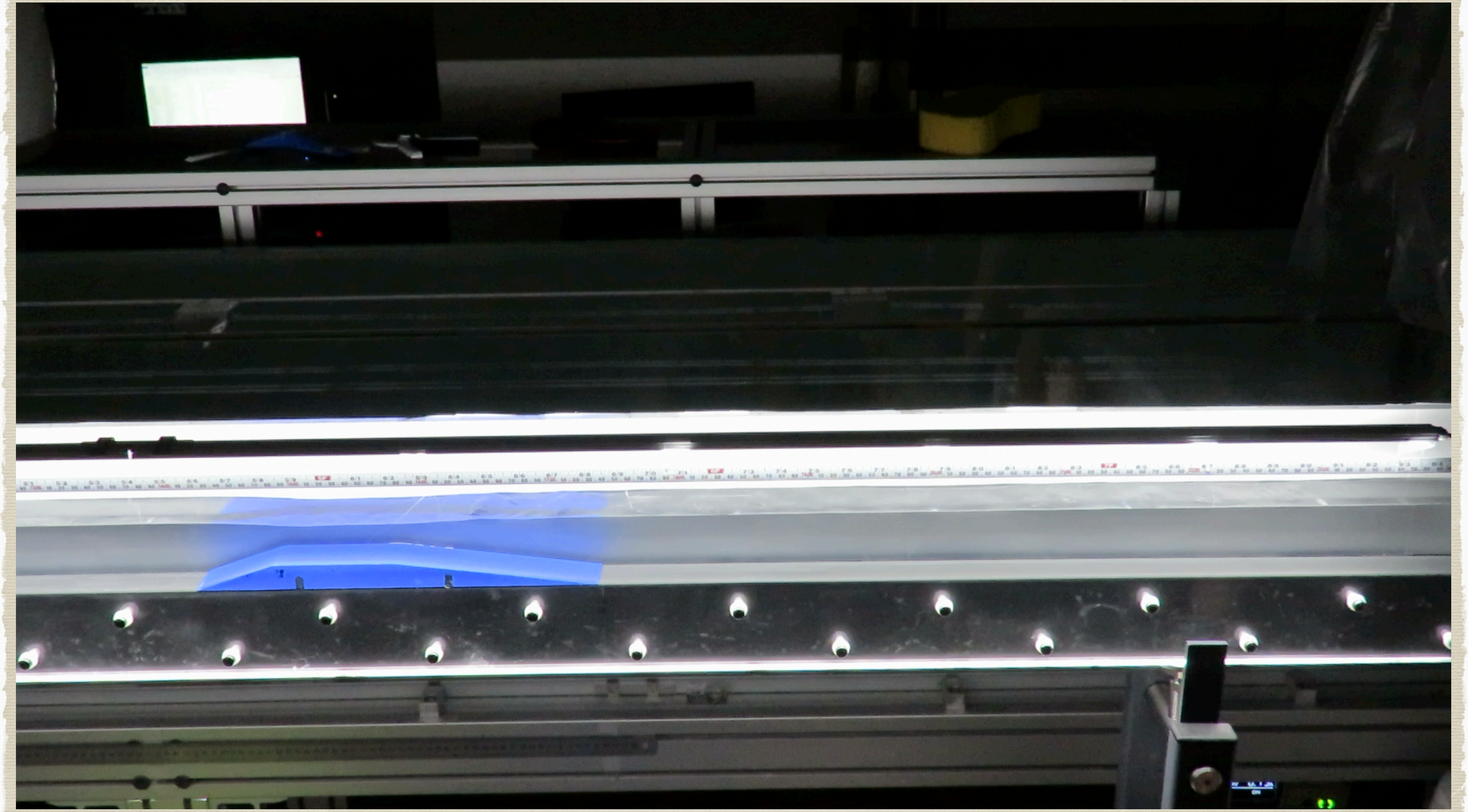
$k_I, k_B, k_N, k_R, k_C, k_{NC}$

Germain Rousseaux, Christian Mathis, Philippe Maïssa, Pierre Coullet, Thomas Philbin and Ulf Leonhardt, Horizon effects with surface waves on moving water, New J. Phys., 12, 095018 (2010).

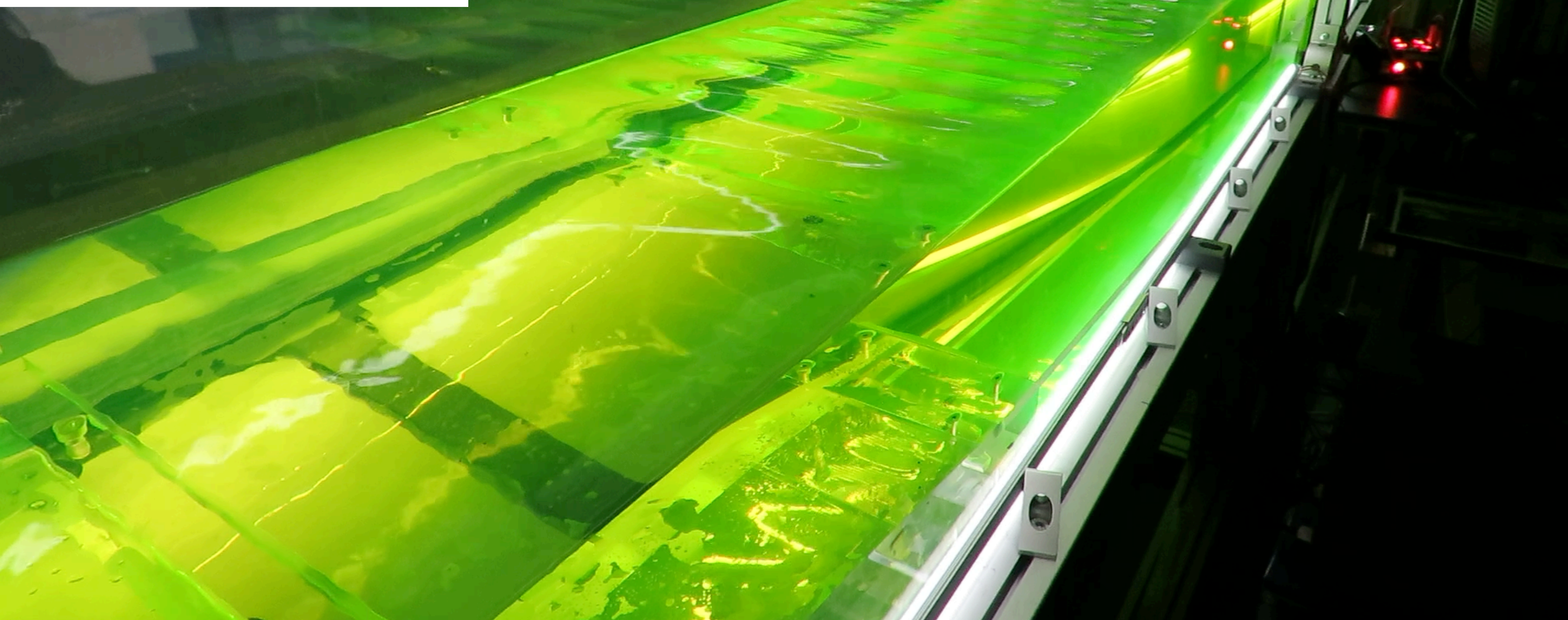
Hawking Radiation of a Dispersive White Hole in Hydrodynamics

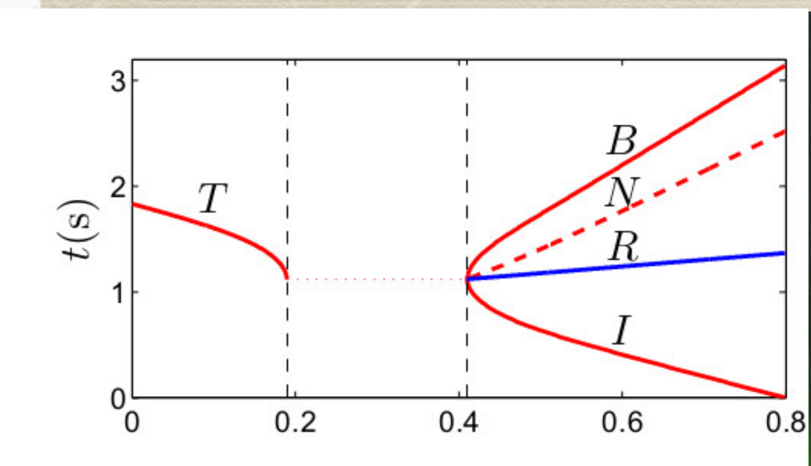


Linear and Non-linear Hawking Radiation

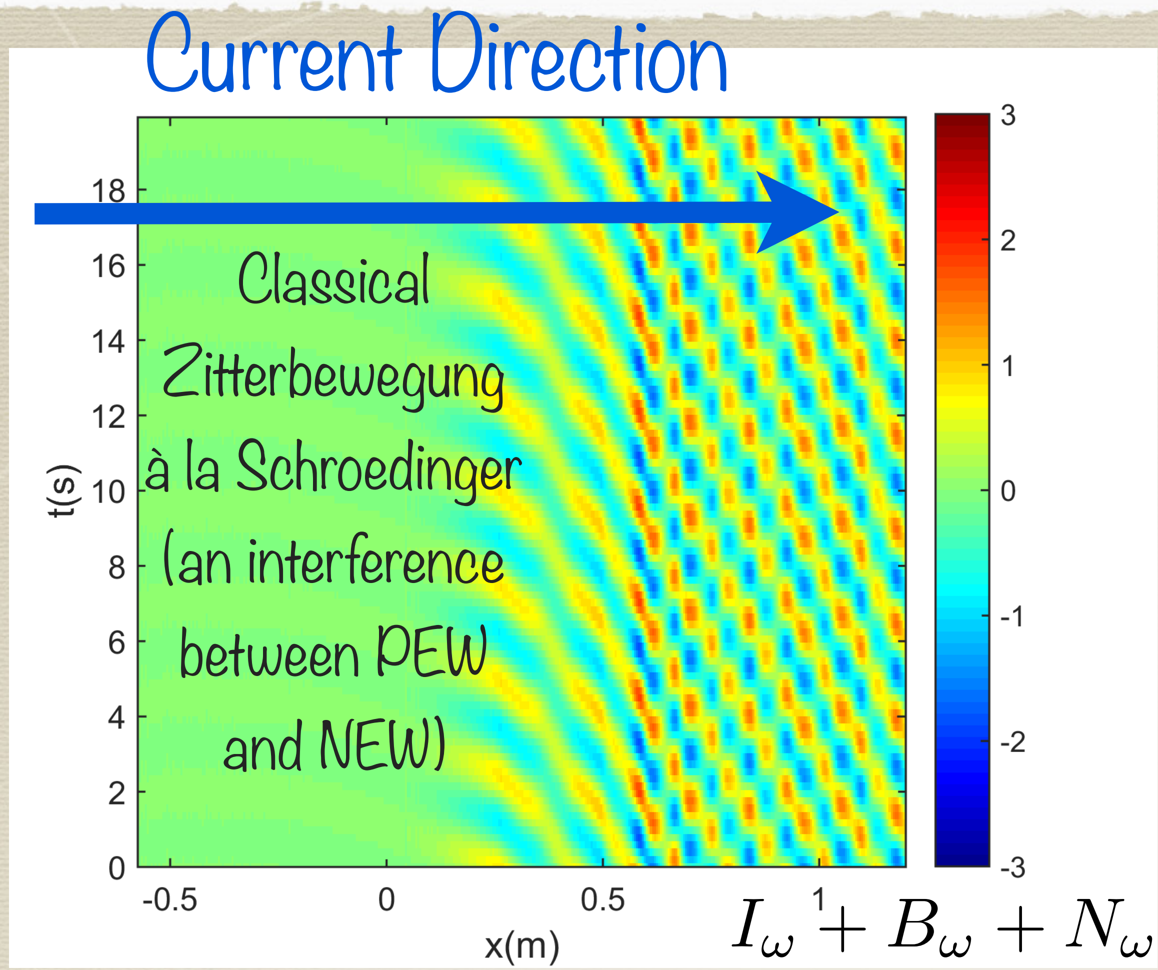


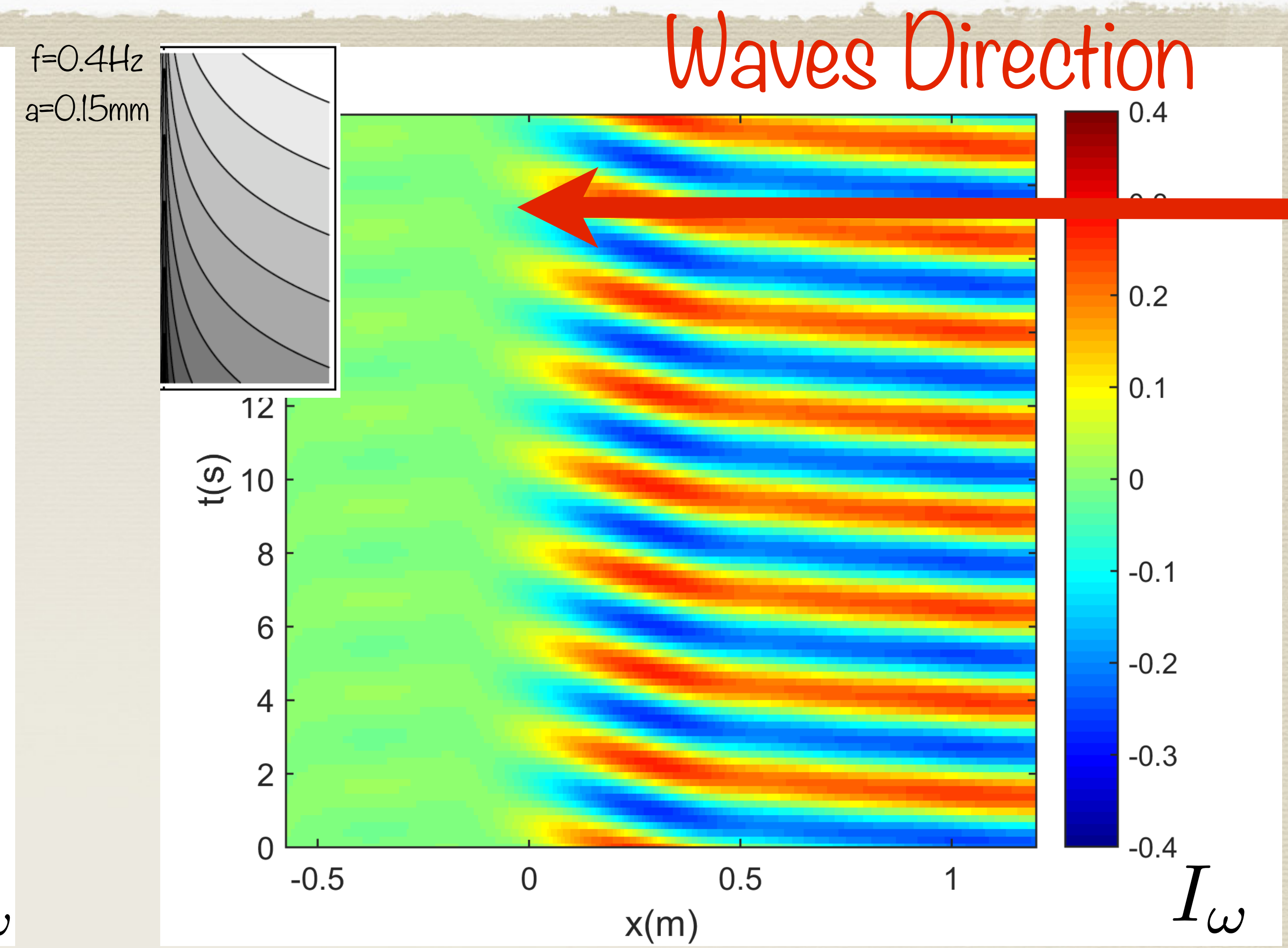
Wave Blocking and Modes Conversion / Large Capillary Dissipation



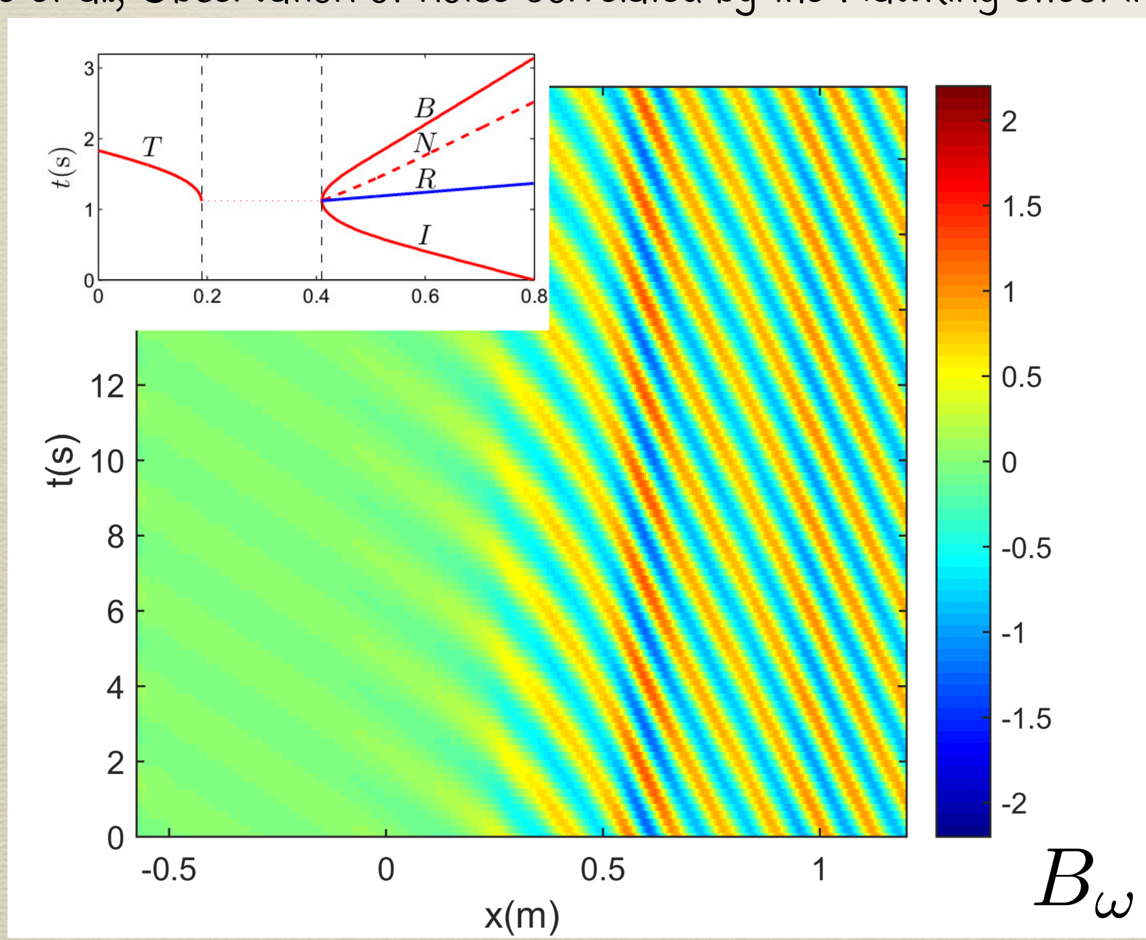


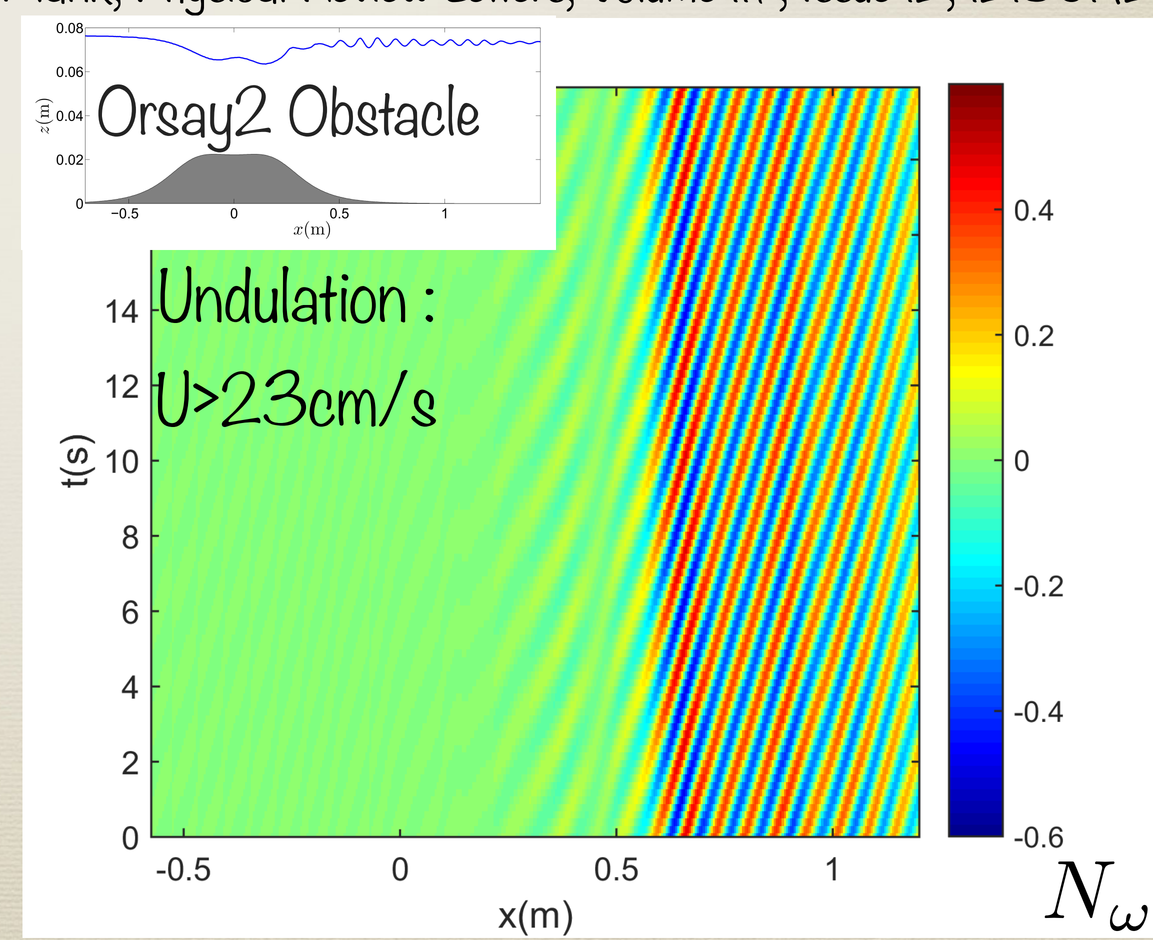
$$V = 20Hz \;\; Q = 16.37l/s \;\; h_d = 23.3cm \;\; T = 0.784s \;\; a_m = 3mm$$

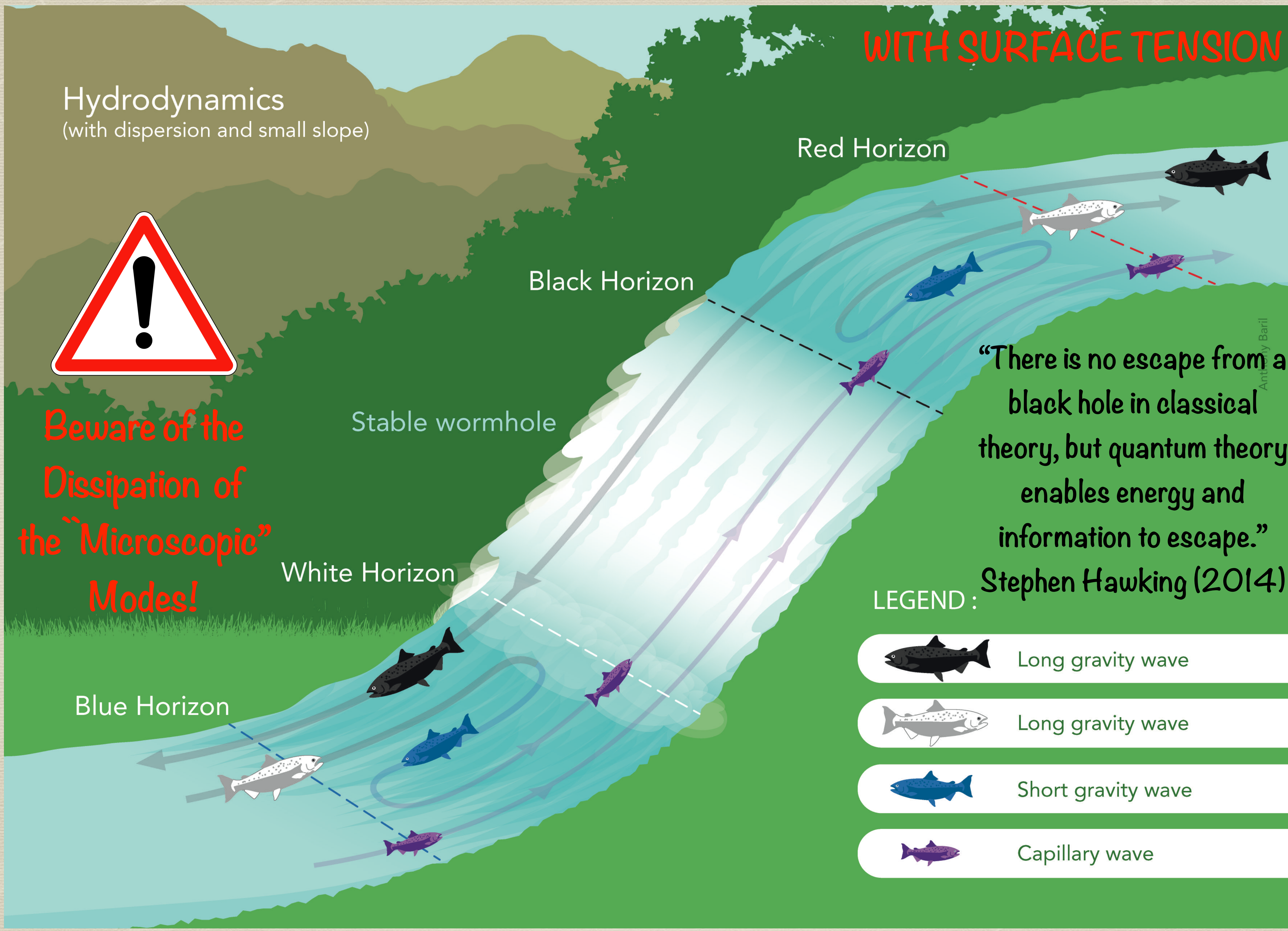




Euvé et al., Observation of noise correlated by the Hawking effect in a water tank, Physical Review Letters, Volume 117, Issue 12, 121301 (2016).

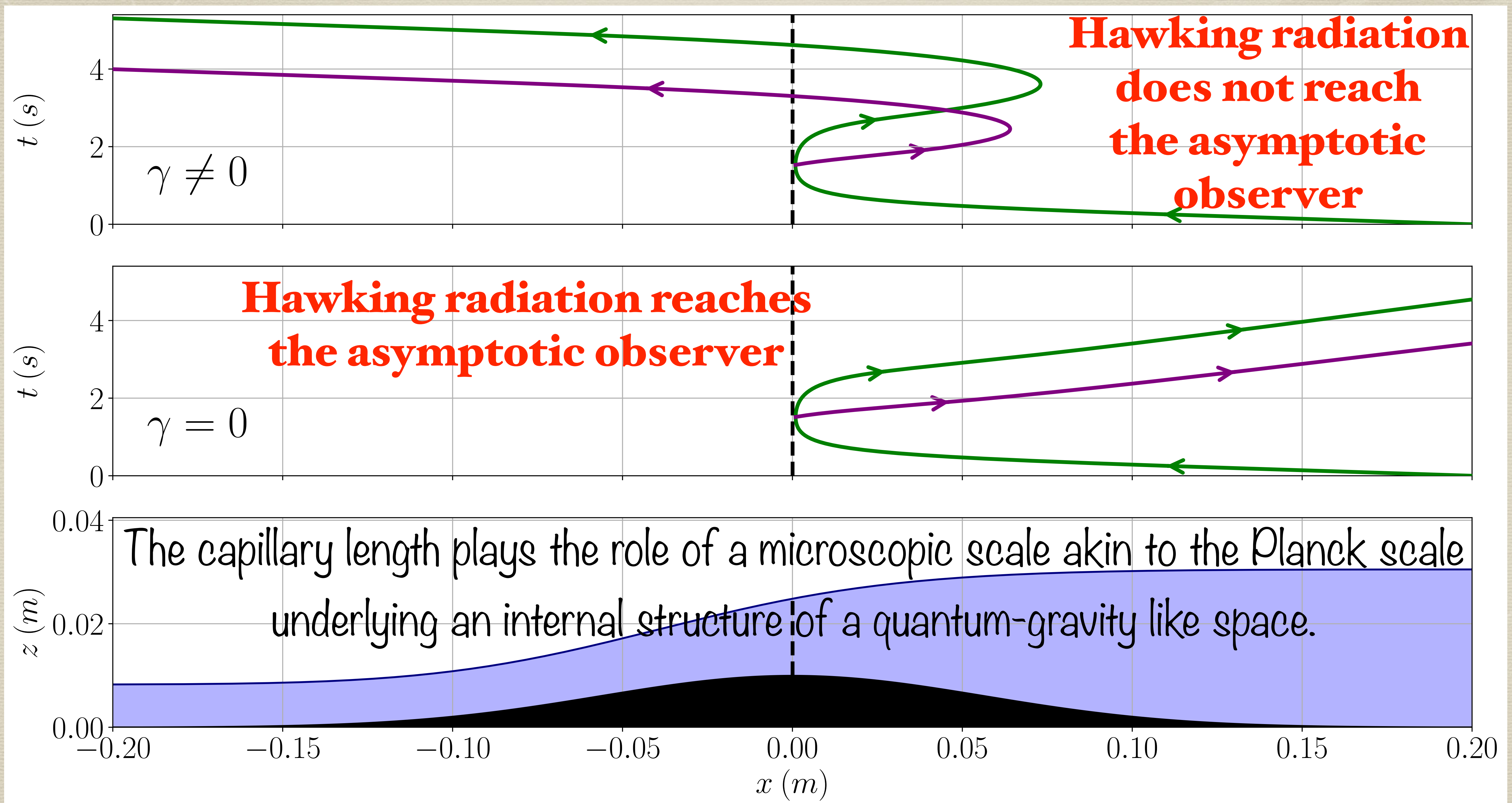






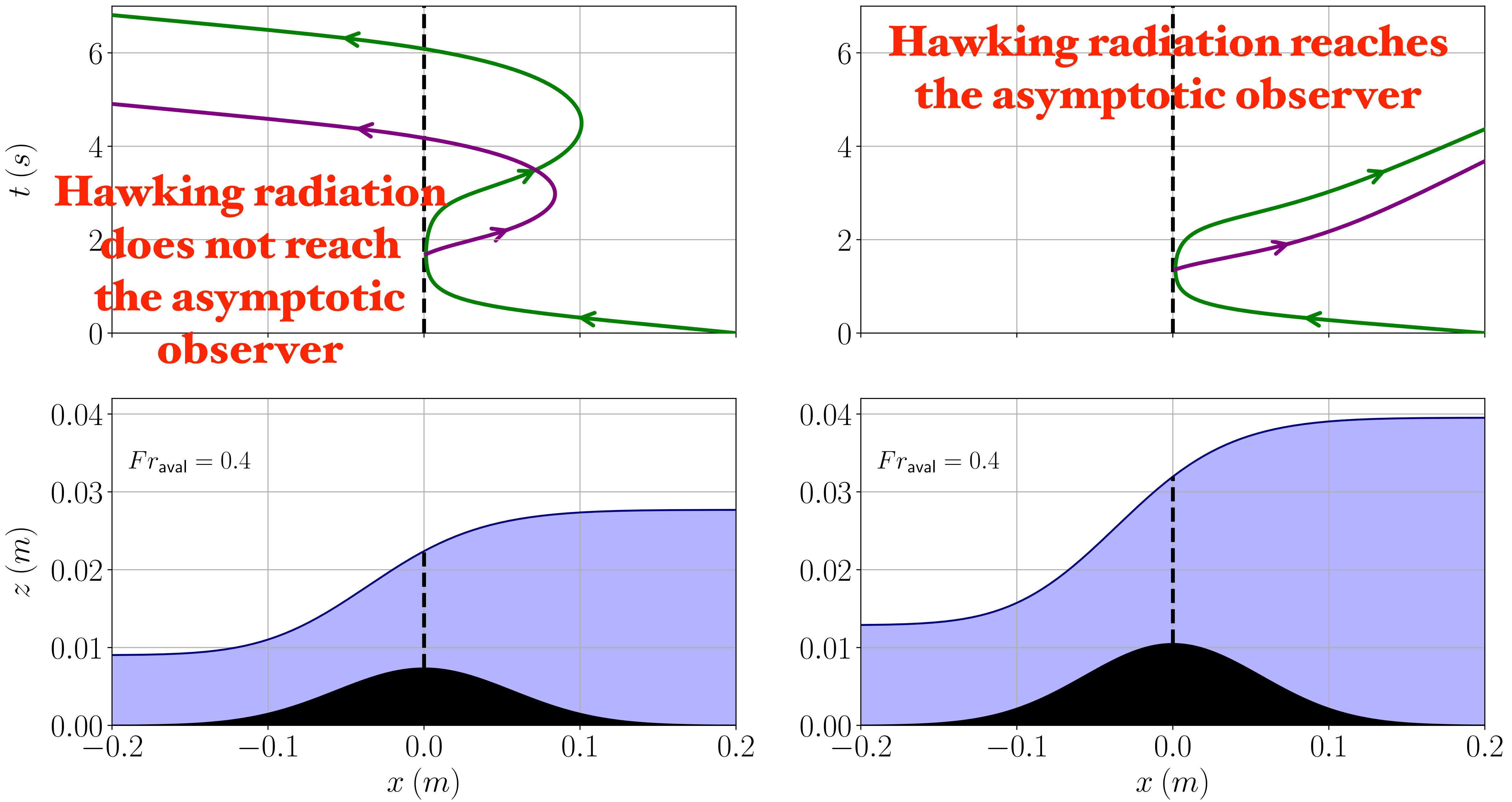
There is no escape from a hydraulic fall in classical theory, but microscopic theory enables energy and information to escape.
There is no entering into a hydraulic cascade in classical theory, but microscopic theory enables energy and information to enter.

Dispersive Effect on Hawking Radiation at a White Fountain Horizon



Alexis Bossard, PhD Thesis 2024 (Unpublished) and
A. Bossard, N. James, V. Jules, J. Fourdrinoy, S. Robertson and G. Rousseaux,
On the art of designing effective space-times with free surface flows in Analogue Gravity,
Comptes Rendus. Physique, 25(G1), p. 457-511 (2024).

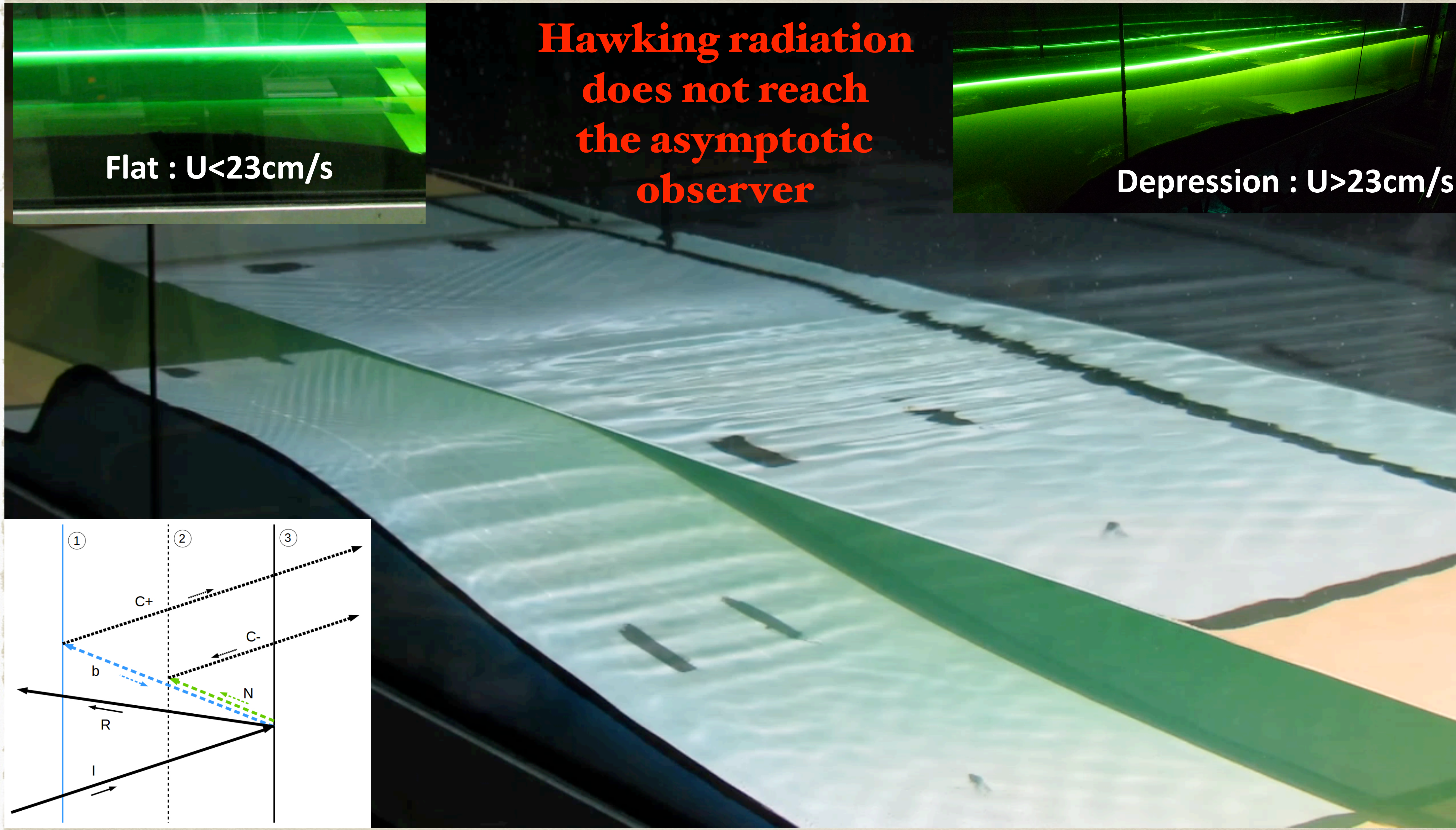
Scale Effect on Hawking Radiation at a White Horizon



The scale of the experiments (here of the obstacle) selects the behavior of the waves which then differs in the presence of surface tension for the same Froude number: (Left) b_{max}=7mm, flow rate per unit width q=0.0058m2/s; (Right) b_{max}=1cm; flow rate per unit width q=0.0098m2/s.

Alexis Bossard, PhD Thesis 2024 (Unpublished)

Experimental Mode Conversions at Dispersive White/Blue Horizon

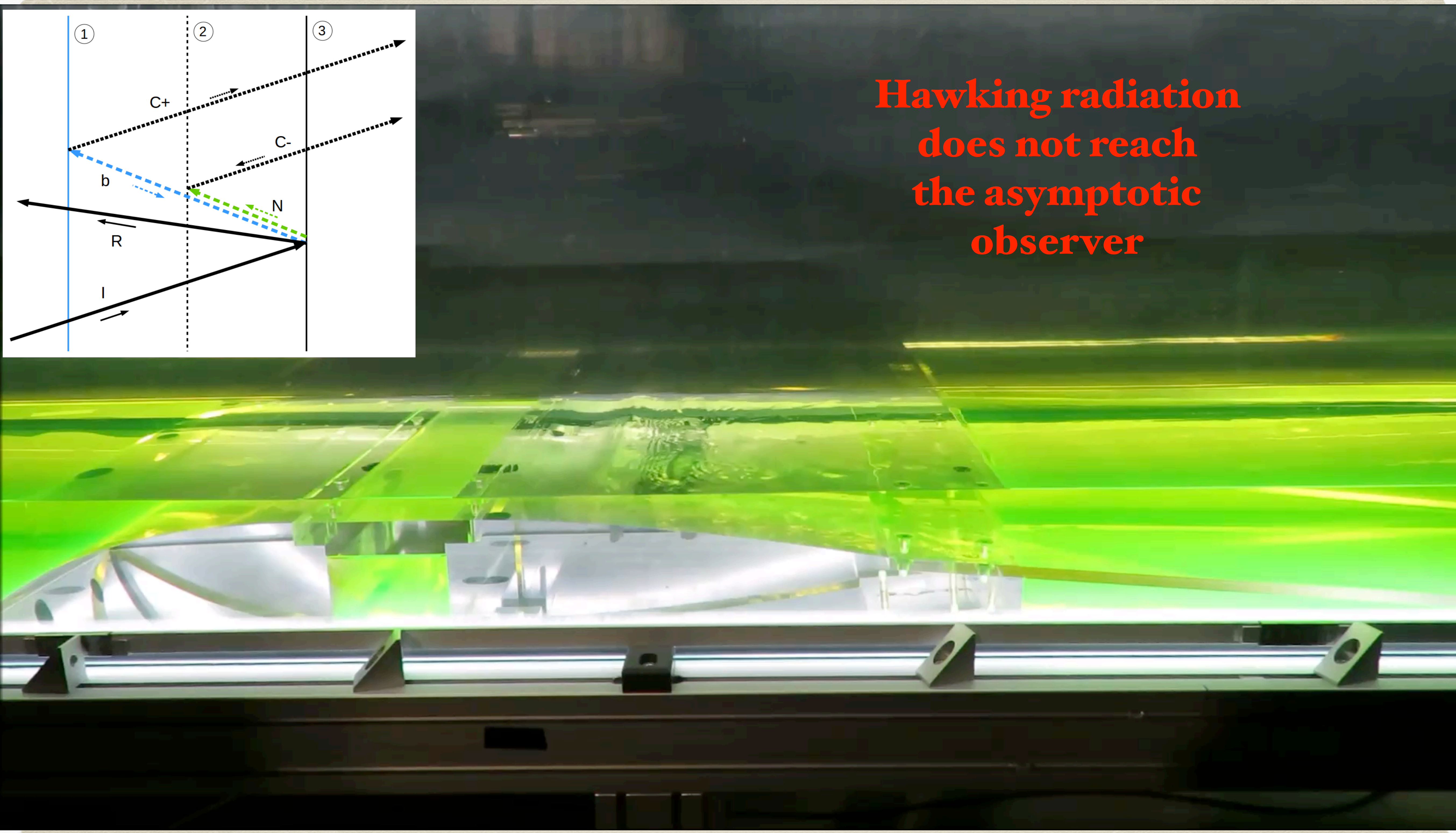


Hawking radiation does not reach the asymptotic observer

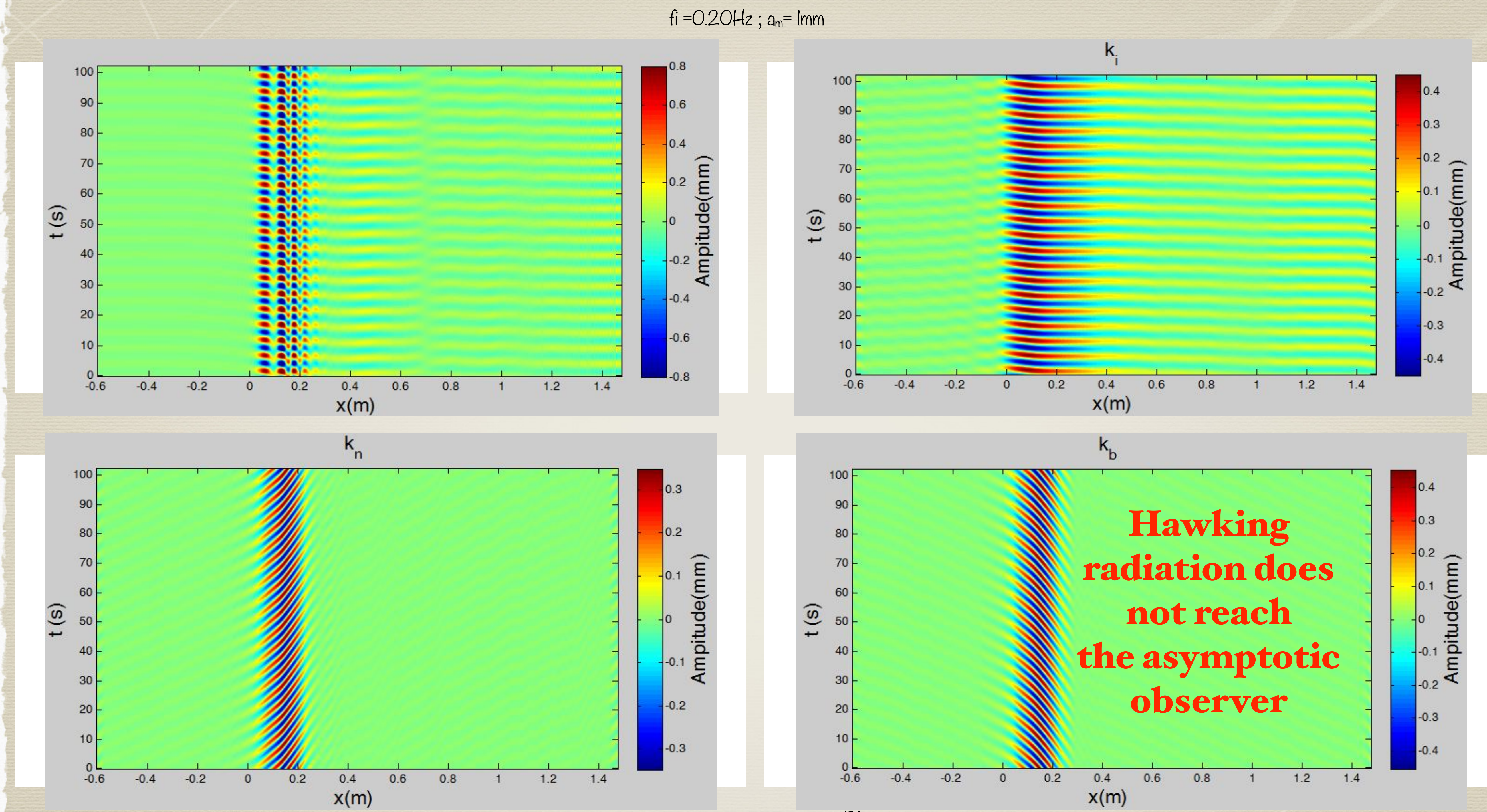
$$Q = 9l/s \quad h_d = 15.7cm \quad T = 0.667s \quad a = 0.2cm$$

Orsay I Obstacle designed by Florent Michel and Renaud Parentani from LPT Orsay.

Killing Hawking Radiation of a White Fountain at a Dispersive Blue Horizon



A Transcritical Flow with the Landau Threshold : Testing the Robustness of Stimulated HR



Regime	Vancouver	New Regime
Fmax (based on hmin)	0.69	1.12
Fmin (based on hmax)	0.16	0.03

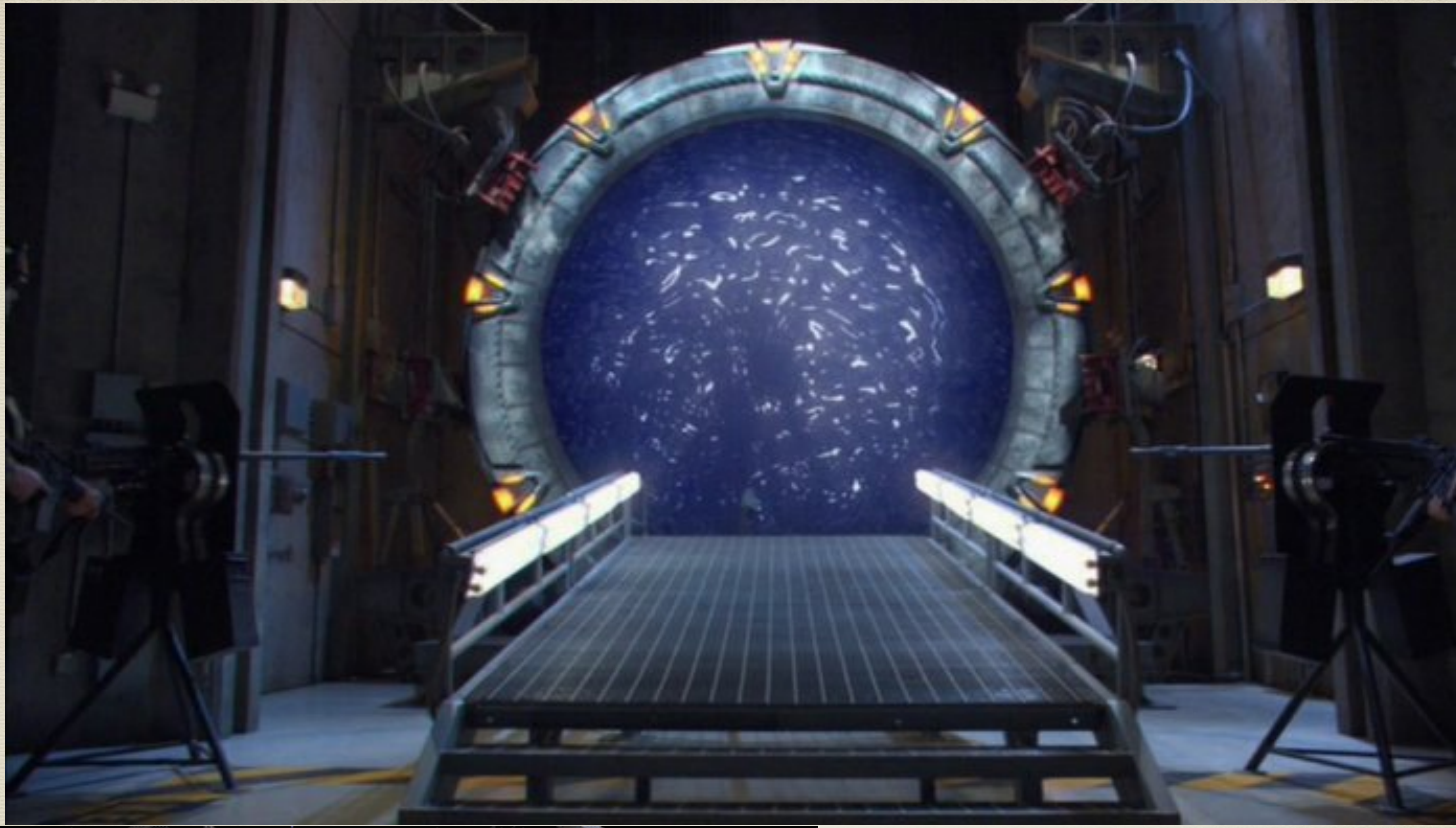
Filters

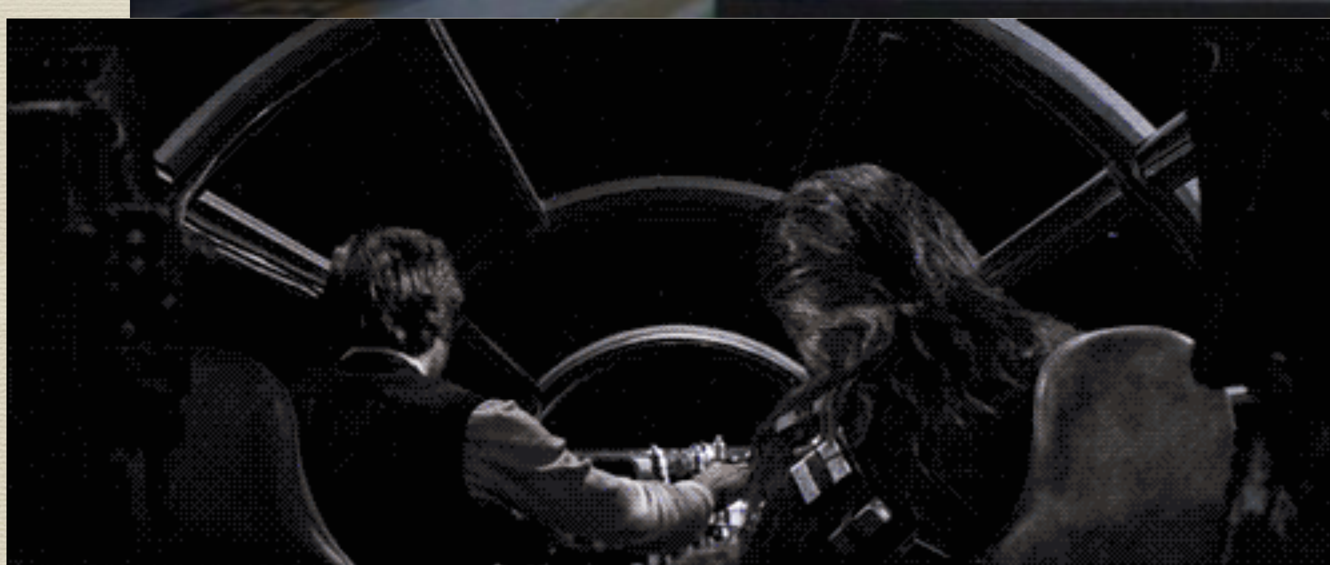
- Temporal : f=0.20Hz
- Blue Peak (alpha) : k=[50 :200] m-1
- Negative Peak (beta) : k=[-200 :-50] m-1
- Incoming Peak (I) : k=[-10 :50] m-1

- Dynamical Height : 12.1cm
- Flow Rate : 1.58l/s (2.1Hz)
- hmin : 1.1cm (Depression=1cm)
- Fmax : 1.12 (Transcritical flow)

Léo-Paul Euvé, Master Thesis 2014 (Unpublished)

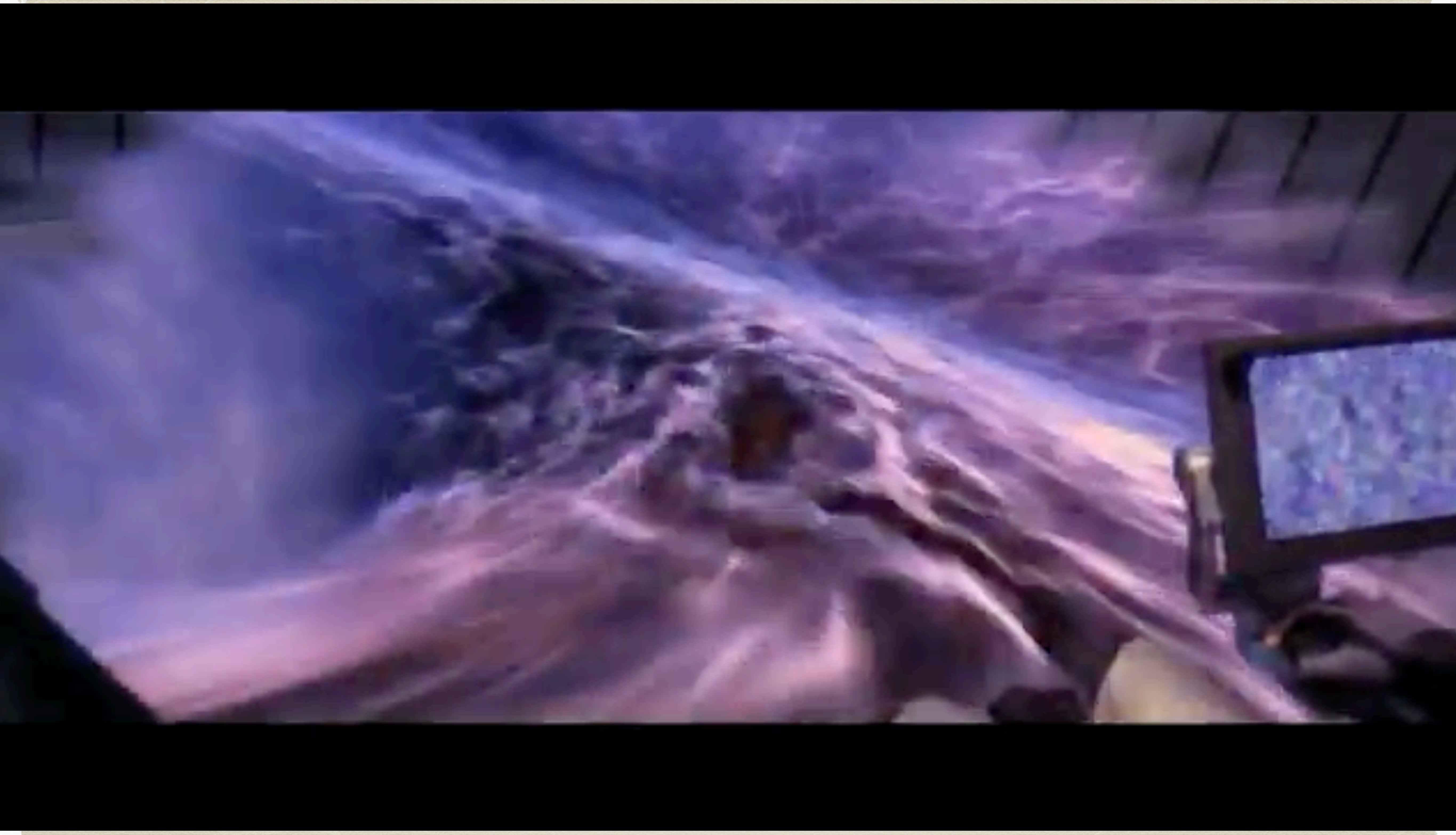
Fluidic Interstellar Travel ?





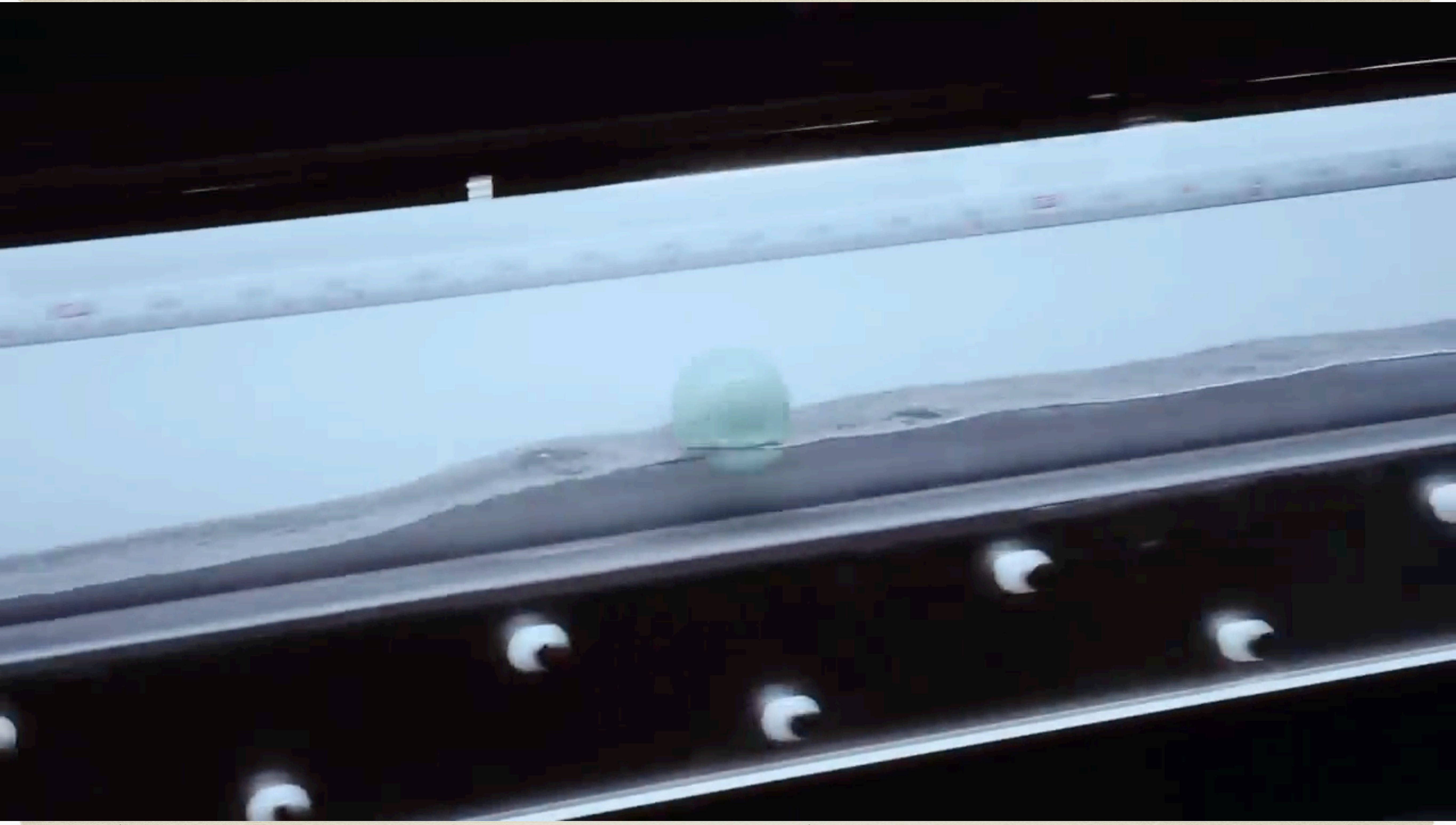
Direct Interstellar Travel in Nature

Direct Interstellar Travel à la Hollywood



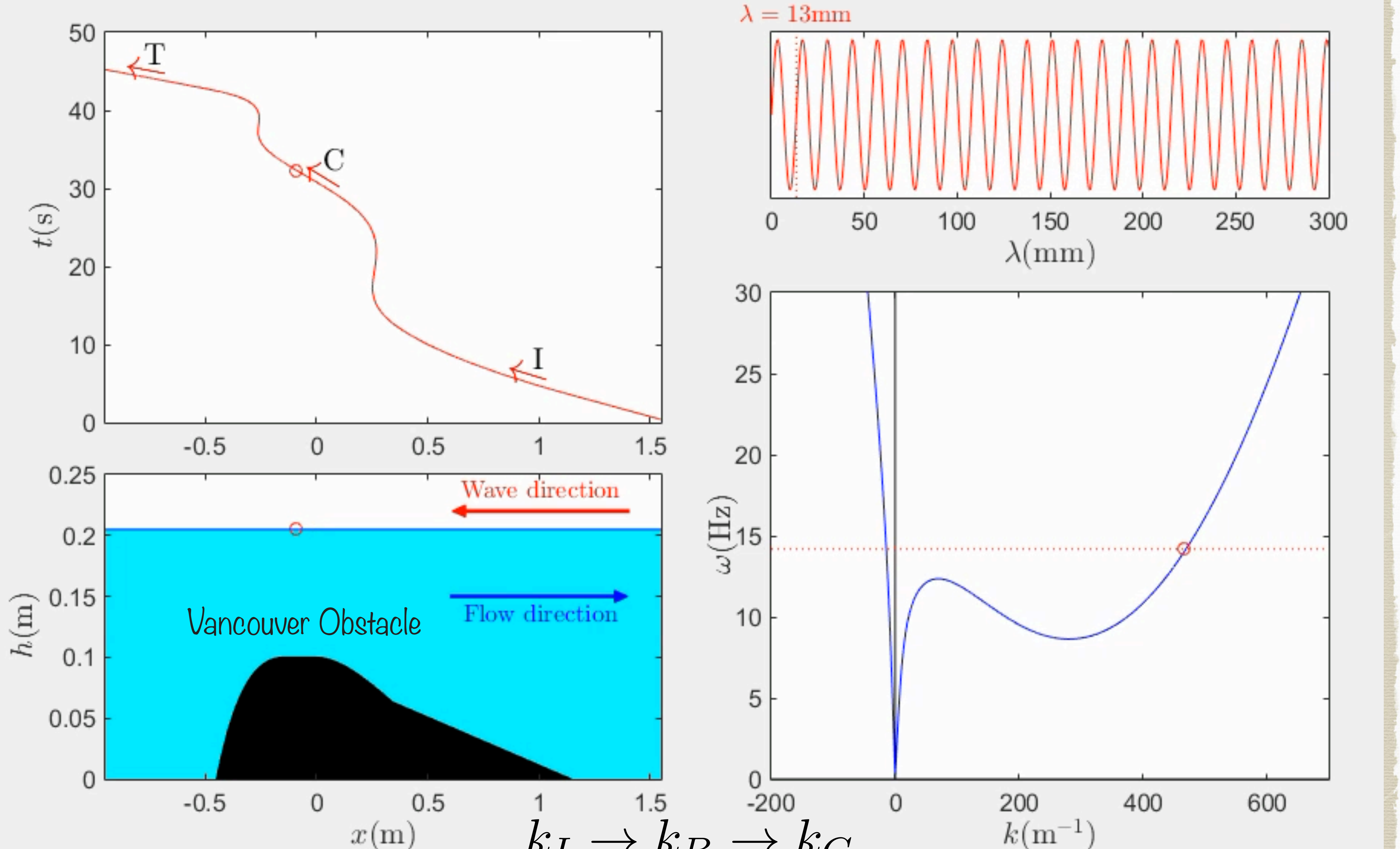
From Robert Zemeckis' Contact Movie (1997), after Carl Sagan's novel Contact (1985).

Direct Interstellar Travel in the Fluidic Lab



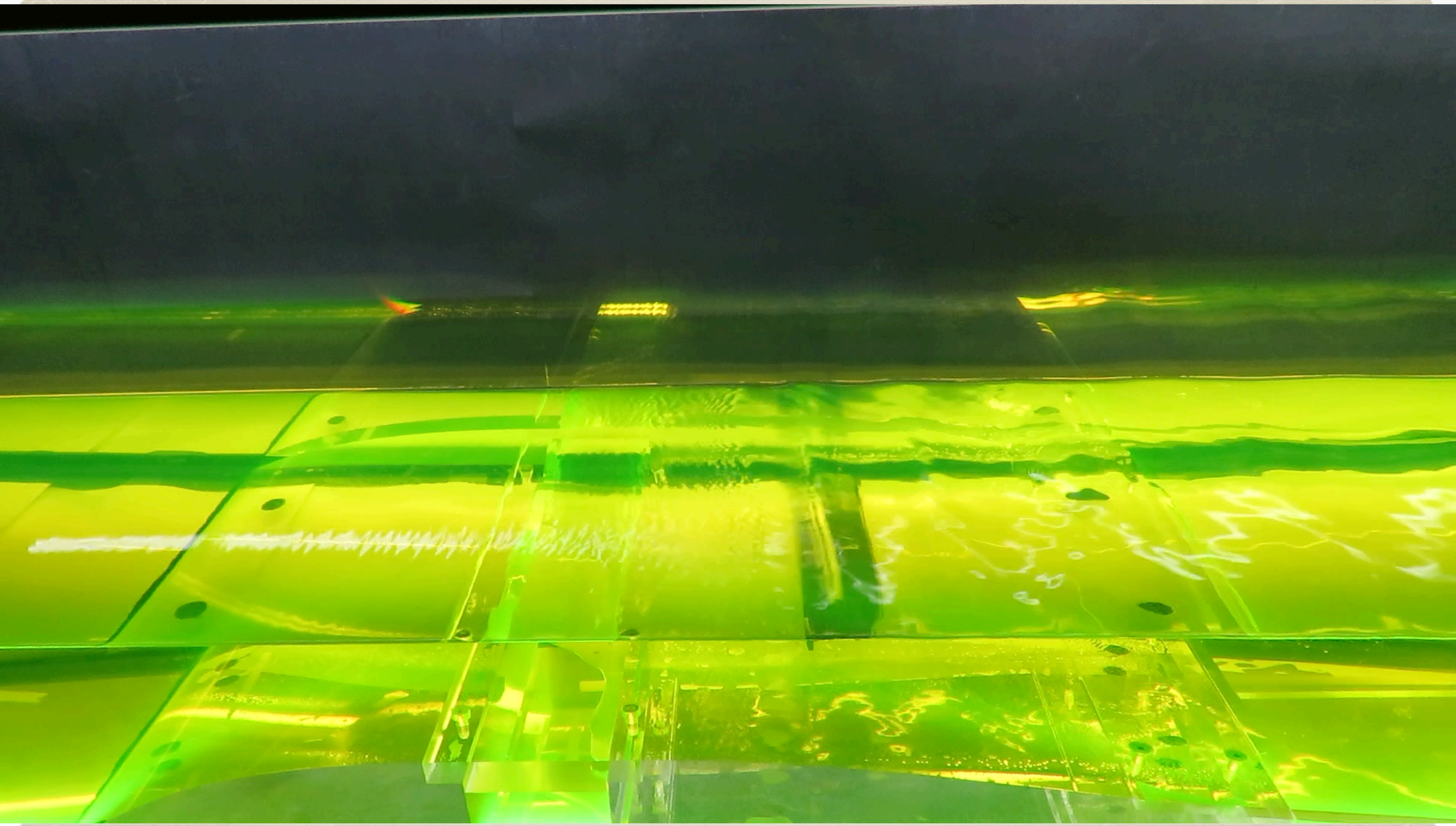
Le blob l'extra média, Video magazine, An edition of the Cité des sciences et de l'industrie and the Palais de la découverte. : Hydraulic black hole. An episode of the series "Lab black holes ». Directed by: Hugo Cayla, in collaboration with Photons Jumeaux. Production: Universcience, 7 Points Productions. Duration: 8min11. March 2024. https://leblob.fr/videos/trou-noir-hydraulique https://leblob.fr/series/trous-noirs-de-labo-0

Transplanckian Physics Ruled by the Capillary Length



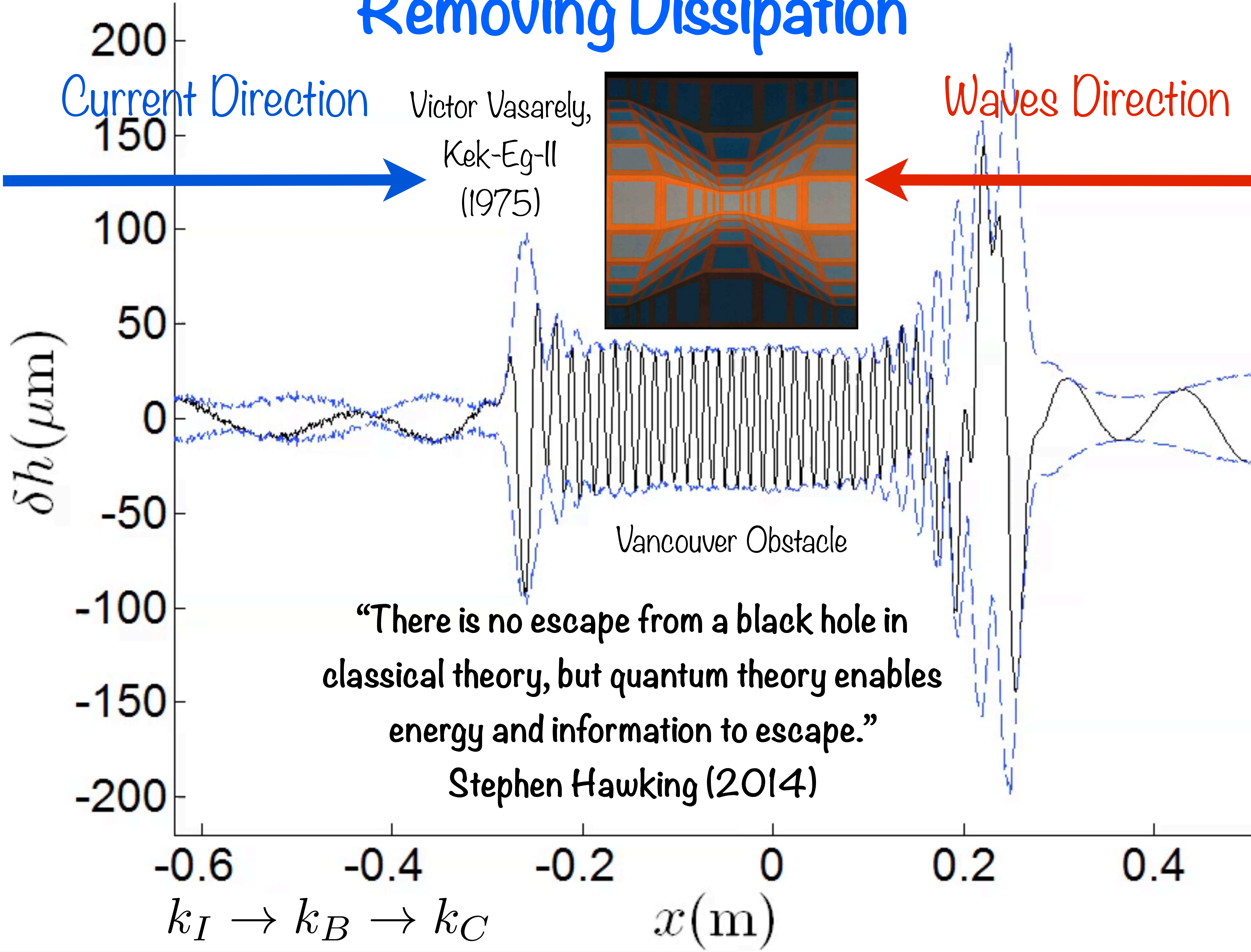
$$k_I \to k_B \to k_C$$

Return Interstellar Travel



$$V = 10Hz \; Q = 8.11 l/s \quad h_d = 20.4cm \; T = 0.444s \; a_m = 4mm$$

Removing Dissipation

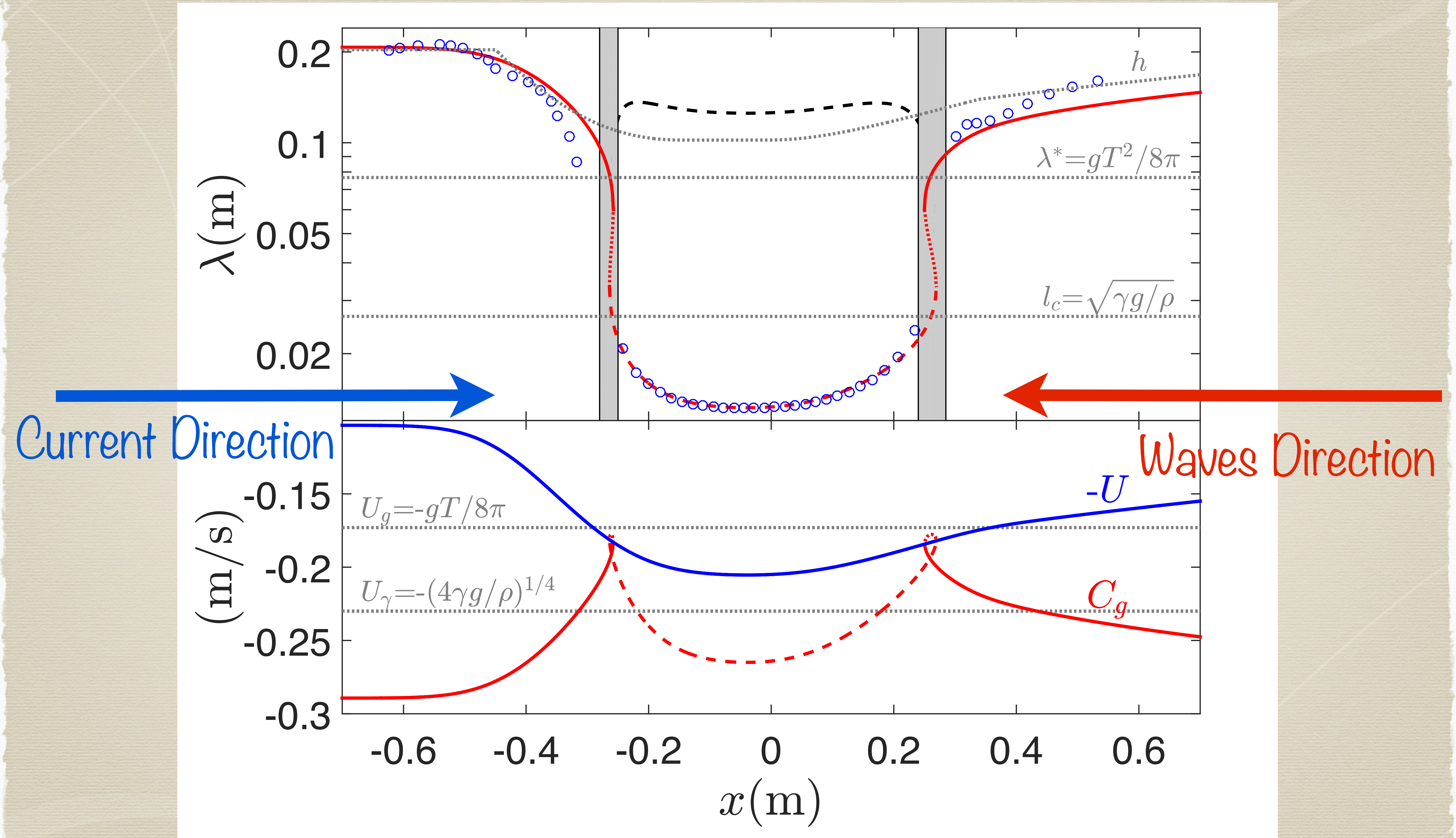


Victor Vasarely, Kek-Eg-II (1975)

"There is no escape from a black hole in classical theory, but quantum theory enables energy and information to escape."
Stephen Hawking (2014)

L.-P. Euvé and G. Rousseaux, Classical analogue of an interstellar travel through a hydrodynamic wormhole, PRD, 96 (6), 064042 (2017).

Blue-Shifting versus Red-Shifting



To what extent is the Hawking radiation dependent on the details of the short distance structure of quantum gravity ?

L.-P. Euvé and G. Rousseaux, Classical analogue of an interstellar travel through a hydrodynamic wormhole, PRD, 96 (6), 064042 (2017).

Avenues of Quantum Field Theory In Curved Spacetime

The dispersive scales (water depth, capillary length, mean free path) in Hydrodynamics play the role of a Planck scale (akin to QG microstructure):

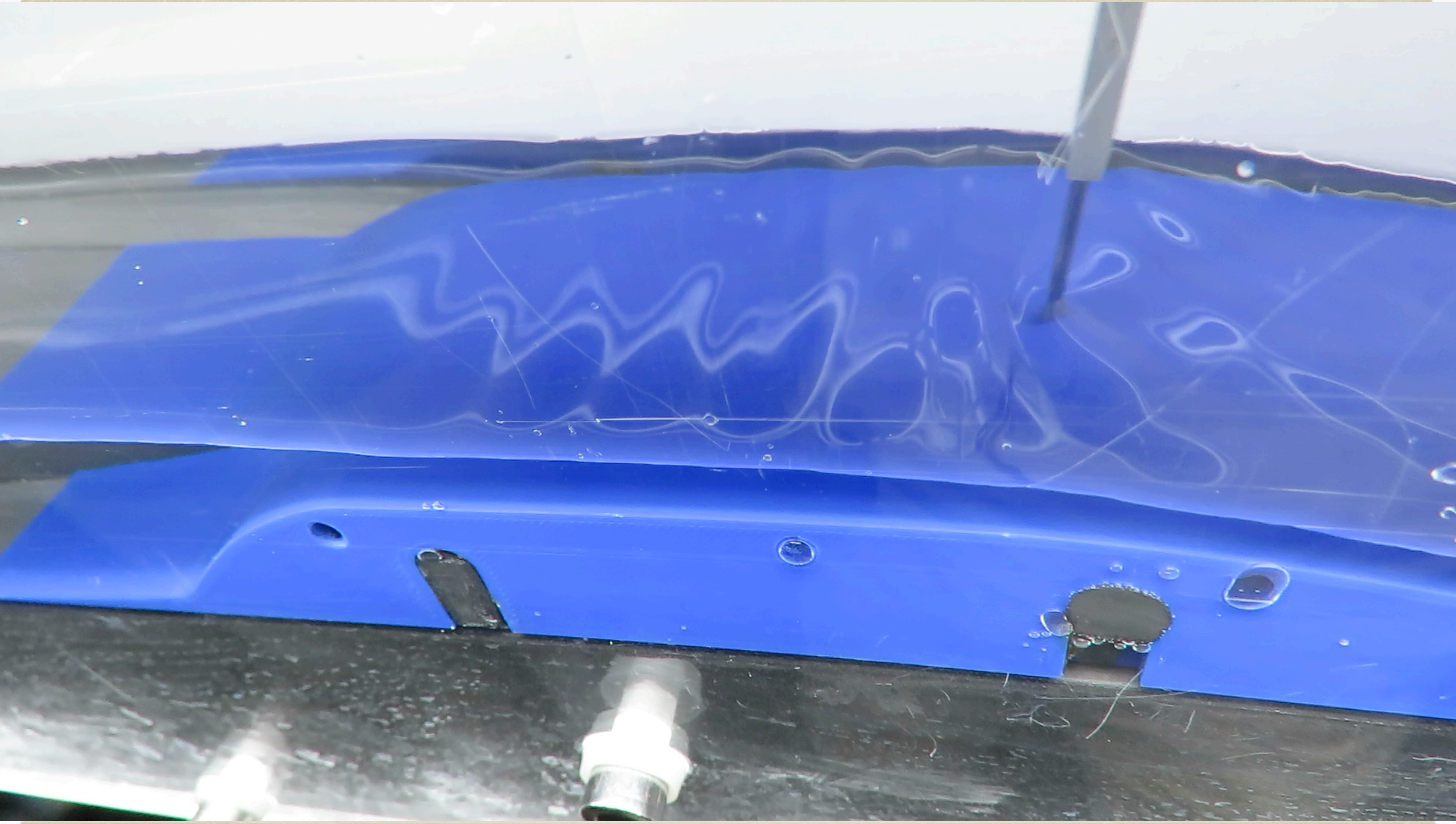
- dispersion (undulation+superluminal behavior) can appear => Lorenz Invariance Breaking
- dissipation (internal degrees of liberty: viscosity, turbulence) can appear
- time-dependence (wave breaking, turbulence, shocks, burst of waves) can appear
- non-linearity (harmonics) can appear
- Hawking radiation may not reach the asymptotic observer (damped by dissipation, localized by harmonics generation, blocked by a dispersive blue group velocity horizon)
- white fountains can be stabilized by dispersion, dissipation and non-linearity
- one can escape from Hydraulic Black Hole/one can enter into Hydraulic White Fountain
- central singularity does not exist and are replaced by classical sometimes time-dependent solutions

``Extraordinary claims require extraordinary evidence” Carl Sagan

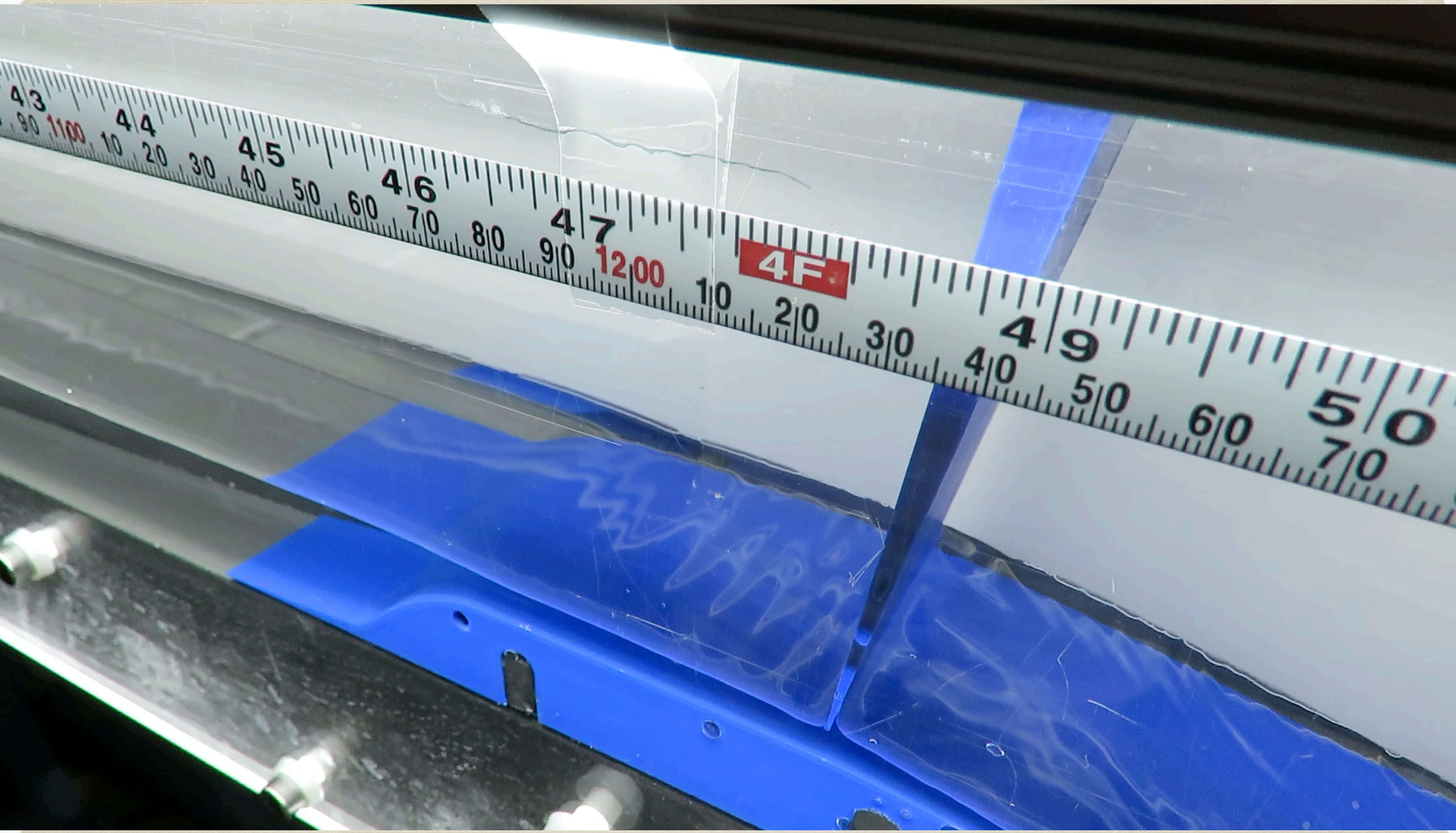
A Hydraulic Black Hole without a Central Singularity

A Hydraulic Black Hole with a (Classical and Time-dependent) Central Singularity

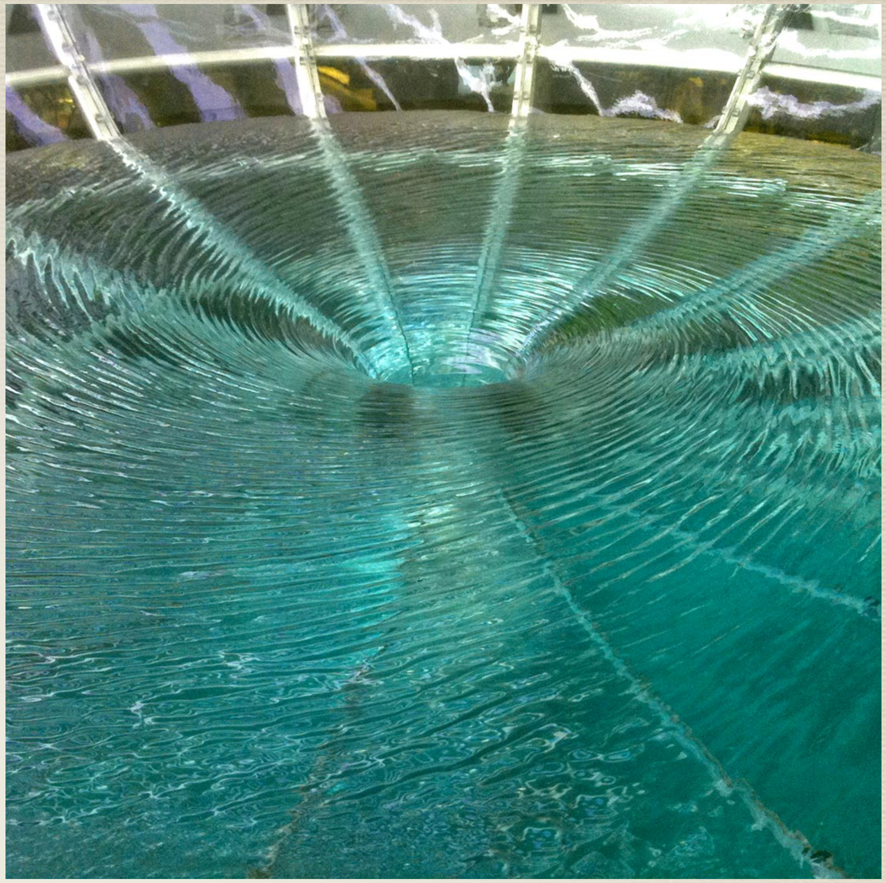
Redshifting from Capillary Waves



Using Turbulence to Modulate in Time the Capillary Undulation



Experimentally Connected Past and Future Singularities of a R-BH



https://nedkahn.com/portfolio/rain-oculus

Velocities in Different Coordinates Systems

The difference between the coordinate systems appears in the first line of Table 2: In Gullstrand–Painlevé coordinates, the coordinate velocity of a freely infalling particle increases smoothly toward the center. Nothing special happens at $r = 2GM$. From a given position, the particle will plunge into the center in a finite time. Even numerically this looks quite Newtonian. In contrast, the velocity with respect to Schwarzschild coordinates approaches zero as the particle approaches $r = 2GM$. Hence, the particle apparently will not be able to go further than $r = 2GM$.

For the Gullstrand–Painlevé metric for incoming light the radial coordinate velocity is always larger in magnitude than -1, at $r = 2GM$ it is -2, for outgoing rays it vanishes at $r = 2GM$ and is negative for $r < 2GM$.

Taking the mere numerical values is misleading. Contemplate for incoming light

$$\frac{\left(\frac{dr}{dt}\right)_{\text{particle}}}{\left(\frac{dr}{dt}\right)_{\text{light}}} = \frac{1}{1 + \sqrt{\frac{r}{2GM}}} \leq 1.$$

So the particle is always slower than light, however it approaches the velocity of light when approaching $r = 0$.

Table 2. Velocities in different coordinate systems.[f]

	Schwarzschild	Gullstrand–Painlevé
Particles		
Coordinate velocity $\frac{dr}{dt}$	$\pm(1 - \frac{2GM}{r})\sqrt{\frac{2GM}{r}}$	$-\sqrt{\frac{2GM}{r}}$
Proper velocity $\frac{dr}{d\tau}$	$\pm\sqrt{\frac{2GM}{r}}$	$\pm\sqrt{\frac{2GM}{r}}$
Light rays		
Coordinate velocity $\frac{dr}{dt}$	$\pm(1 - \frac{2GM}{r})$	$\pm 1 - \sqrt{\frac{2GM}{r}}$

C. Heinicke & F. W. Friedrich, Schwarzschild and Kerr solutions of Einstein's field equation: An introduction, in ``One Hundred Years of General Relativity: From Genesis and Empirical Foundations to Gravitational Waves", Cosmology and Quantum Gravity, p. 109-185 (2017).

Stationary (∂_t=0) But Not Static (t=>-t)

"'With this hypothesis, the Einsteinians propose the, now famous, ds^2 (four variables) whose geodesics define in their theory the motion of a massive point which reads :"'

$$ds^2 = (1 - \frac{a}{r})dt^2 - (\frac{1}{1-\frac{a}{r}})dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{9}$$

"'But this ds^2 is not the only one that meets all the Einsteinian conditions. There is an infinity of others depending on two functions of r, and the choice of the formula (9) between all these formulas is purely arbitrary. These formulas are as simple as formula (9) and involve exactly the same verifications. For instance, we can use instead"':

$$ds^2 = (1 - \frac{a}{r})dt^2 - 2\sqrt{\frac{a}{r}}dr.dt - [dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)] \tag{10}$$

"'where a denotes an arbitrary constant that depends on the mass of the material center O."'

Paul Painlevé [1] part de la métrique de la relativité restreinte :

$$ds^2 = c^2dt^2 - dr^2 - r^2d\omega^2, \tag{1}$$

écrite en coordonnées polaires, où $d\omega^2 = d\theta^2 + sin^2(\theta)d\phi^2$.

Puis il considère la vitesse de chute newtonienne $v(r) = -\sqrt{\frac{2M}{r}}$, où M est la "masse relativiste" du soleil et il met cette vitesse (classique) dans la métrique (1) de Minkowski de la manière suivante :

$$ds^2 = c^2dt^2 - (dr + \sqrt{\frac{2M}{r}}dt)^2 - r^2d\omega^2. \tag{2}$$

Courtesy Jacques Fric and Michel Mizony

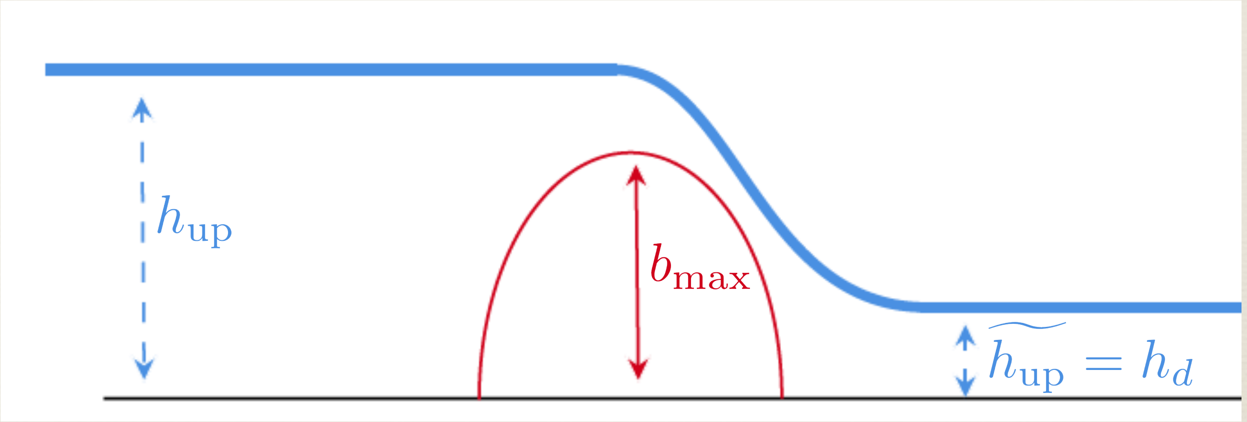
Analogue Gravity Framework for Interfacial Hydrodynamics

$$ds^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = c^2(x)\left[-\left(c^2(x) - U^2(x)\right)\mathrm{d}t^2 \pm 2U(x)\mathrm{d}t\mathrm{d}x + \mathrm{d}x^2\right]$$

$$\begin{cases} c(x) = \sqrt{gh(x)} \\ U(x) = \frac{q}{h(x)} \end{cases}$$

G. Lawrence, S. Weinfurtner, E. Tedford, M. Penrice, W. Unruh, "Waves and currents: Hawking radiation in the hydraulics laboratory?, Chapter 6", Environmental Fluid Mechanics, Memorial Volume in honour of Prof. Gerhard H. Jirka (Taylor and Francis, 2012).

$$dt_{P-G} = dt_{S-D} + \frac{U(r)}{c(r)} \frac{dr}{1 - \left(\frac{U(r)}{c(r)}\right)^2}$$



$$\frac{q^2}{2h(x)^2} + g\left(h(x) + b(x)\right) = \frac{U_{\mathrm{up}}^2}{2} + gh_{\mathrm{up}} \Longrightarrow$$

We can obtain the analytical solutions of the different hydraulic regimes

Non-dispersive description of the flow over an obstacle

Inertia versus Gravity in an Open Water Flow

The flow over a weir (a bottom obstacle) in a water channel is certainly one of the oldest problems in fluid mechanics that dates back to the babylonians suspended gardens and the desert inhabitants that wanted to collect water in oases with irrigation canals featuring spillways to control the water level at constant flow rate of the fluid put into motion by the driving force of gravity.

When comparing inertia to gravity, one usually introduces a dimensionless number known as the **Froude number** which can be seen as the ratio of kinetic to potential gravitational energy or as the ratio between the current speed and the wave speed of long gravity and non-dispersive waves.

$$Fr = \frac{U}{\sqrt{gh}}$$

$$\omega = ck = \sqrt{ghk}$$

$$kh << 1$$

There is an obvious analogy with the Mach number in Aerodynamics

$$Ma = \frac{v}{c}$$

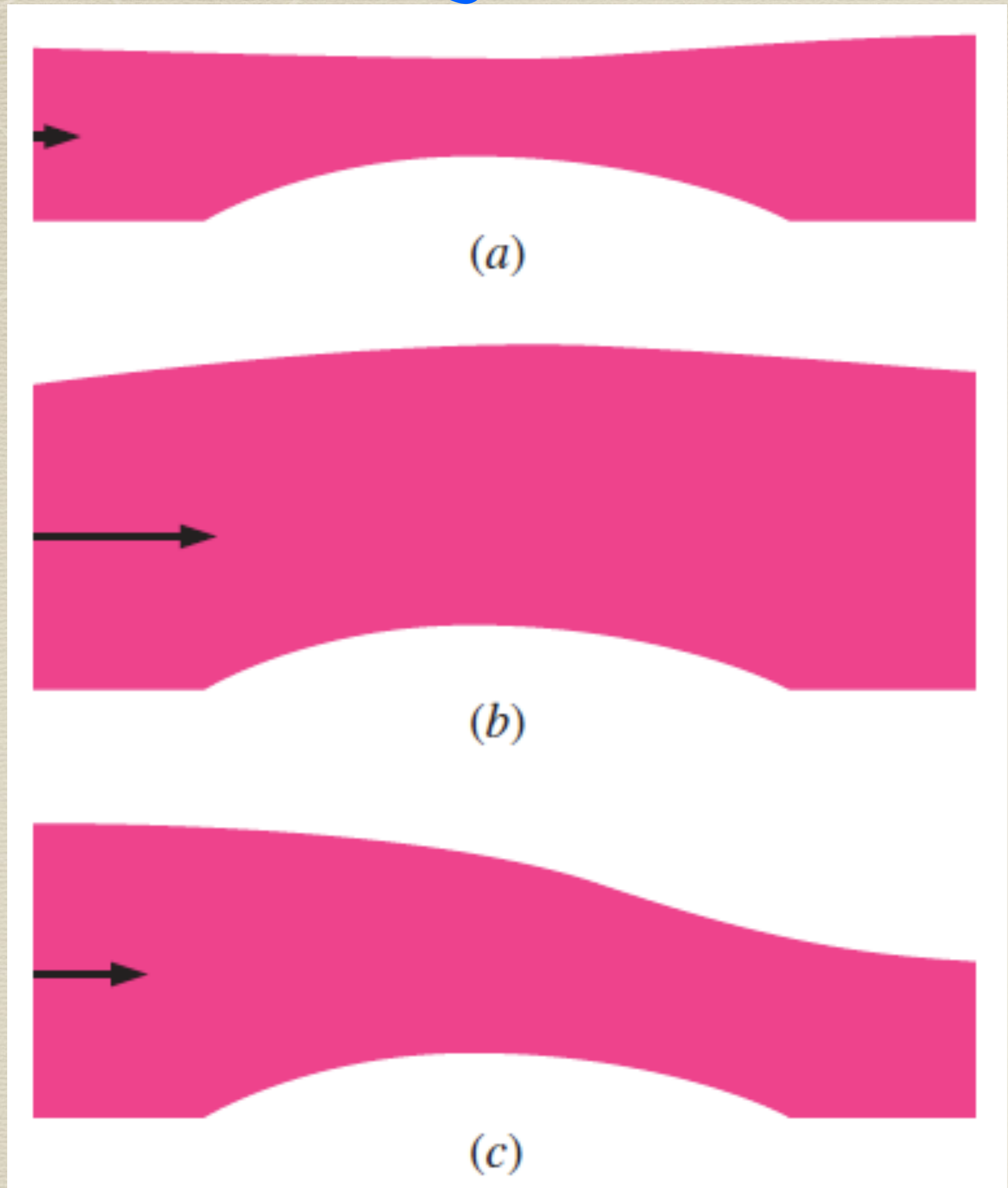
Two regimes appear:

$Fr < 1$ **Subcritical flow**

$Fr > 1$ **Supercritical flow**

separated by a critical point for which the Froude number is unity.

Adding a Bottom Obstacle in an Open Water Flow



In presence of a bottom obstacle, the subcritical regime implies a depression on the top of the obstacle

In presence of a bottom obstacle, the supercritical regime implies a bump on the top of the obstacle

A third regime named transcritical is observed experimentally with a corresponding waterfall

Let us introduce the **obstruction parameter r** which is the ratio between the maximum height of the obstacle b_{max} to the upstream water depth h_{up}.

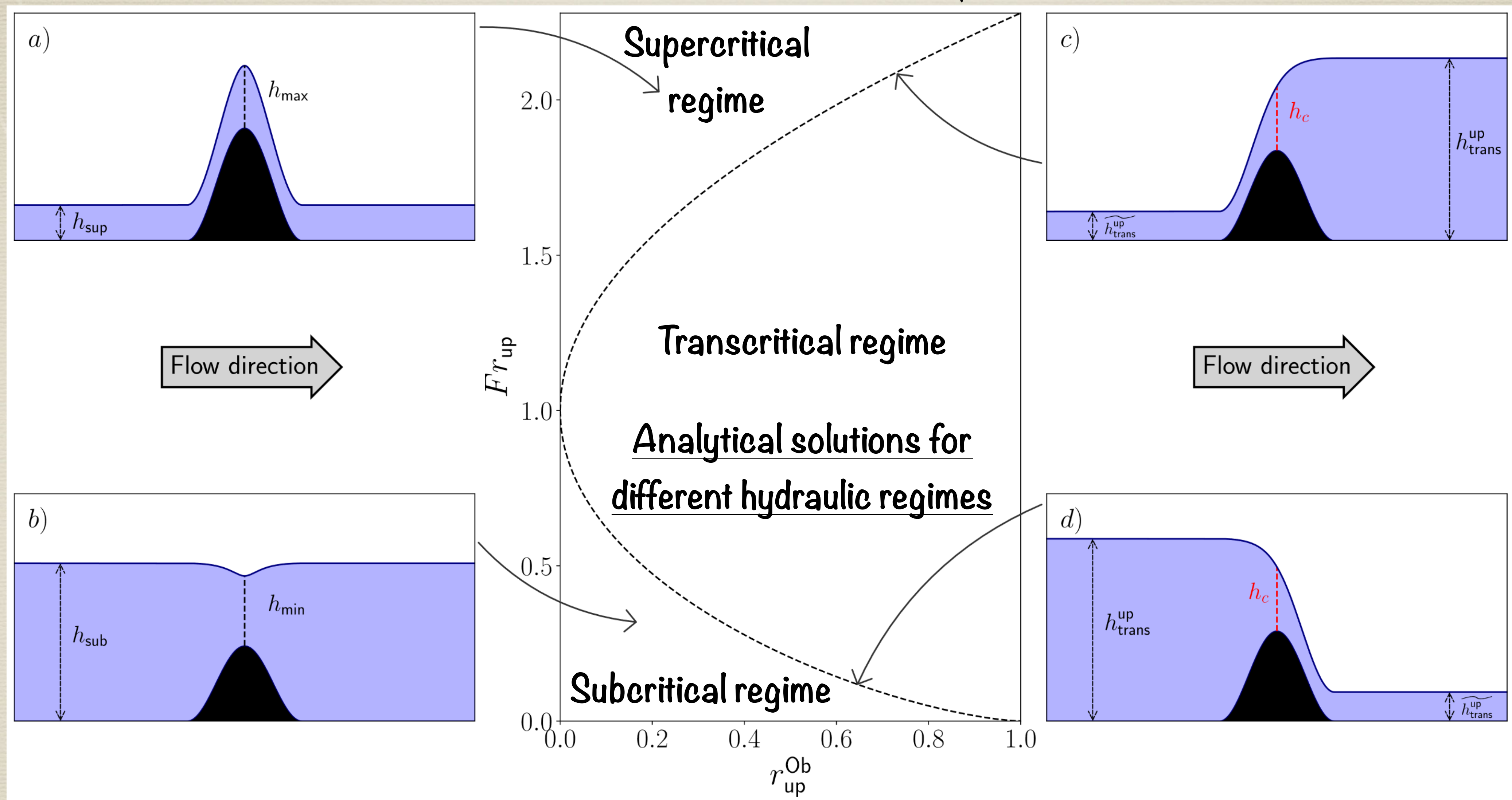
Conservation of the flow rate and Bernoulli equation leads to a **relationship between the upstream Froude number and the obstruction factor** due to Long in 1954.

R. R. Long, Some aspects of the flow of stratified fluids: II. Experiments with a two-fluid system, Tellus 6 (2), p. 97-115 (1954).

The 1954 Long Hydraulic Transcritical Boundaries

$$r_{\mathrm{up}}^{\mathrm{Ob}} = \frac{b_{\max}}{h_{\mathrm{up}}} = 1 + \frac{1}{2} Fr_{\mathrm{up}}^2 - \frac{3}{2} Fr_{\mathrm{up}}^{\frac{2}{3}} \quad \text{with} \quad Fr_{\mathrm{up}} = \frac{U_{\mathrm{up}}}{\sqrt{g h_{\mathrm{up}}}} = \frac{q}{\sqrt{g} h_{\mathrm{up}}^{\frac{3}{2}}}$$

Water depth at the critical transition: $h_c = \sqrt[3]{\frac{q^2}{g}}$

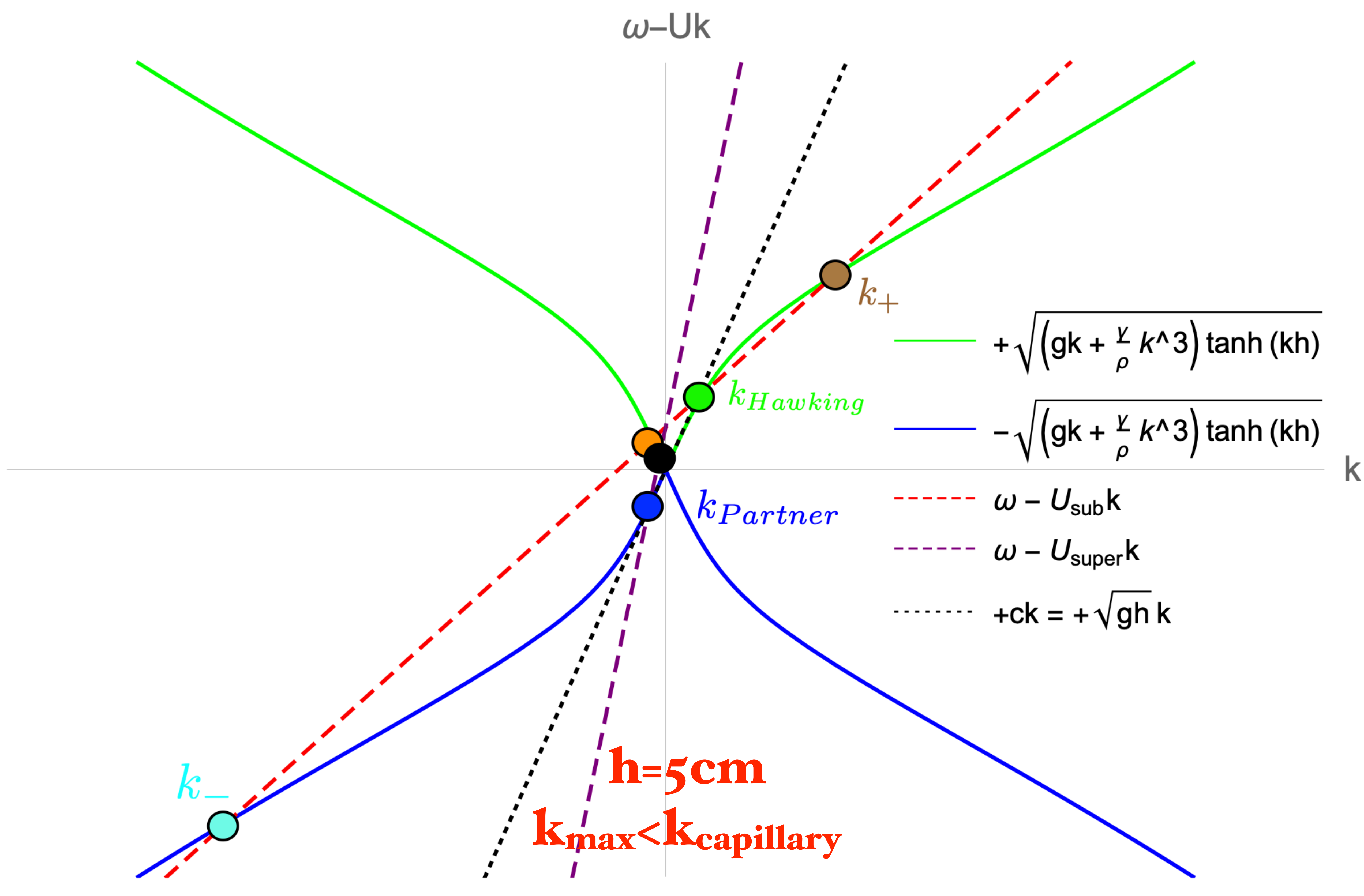


Upstream/downstream condition for a transcritical regime (Bossard et al., 2024)

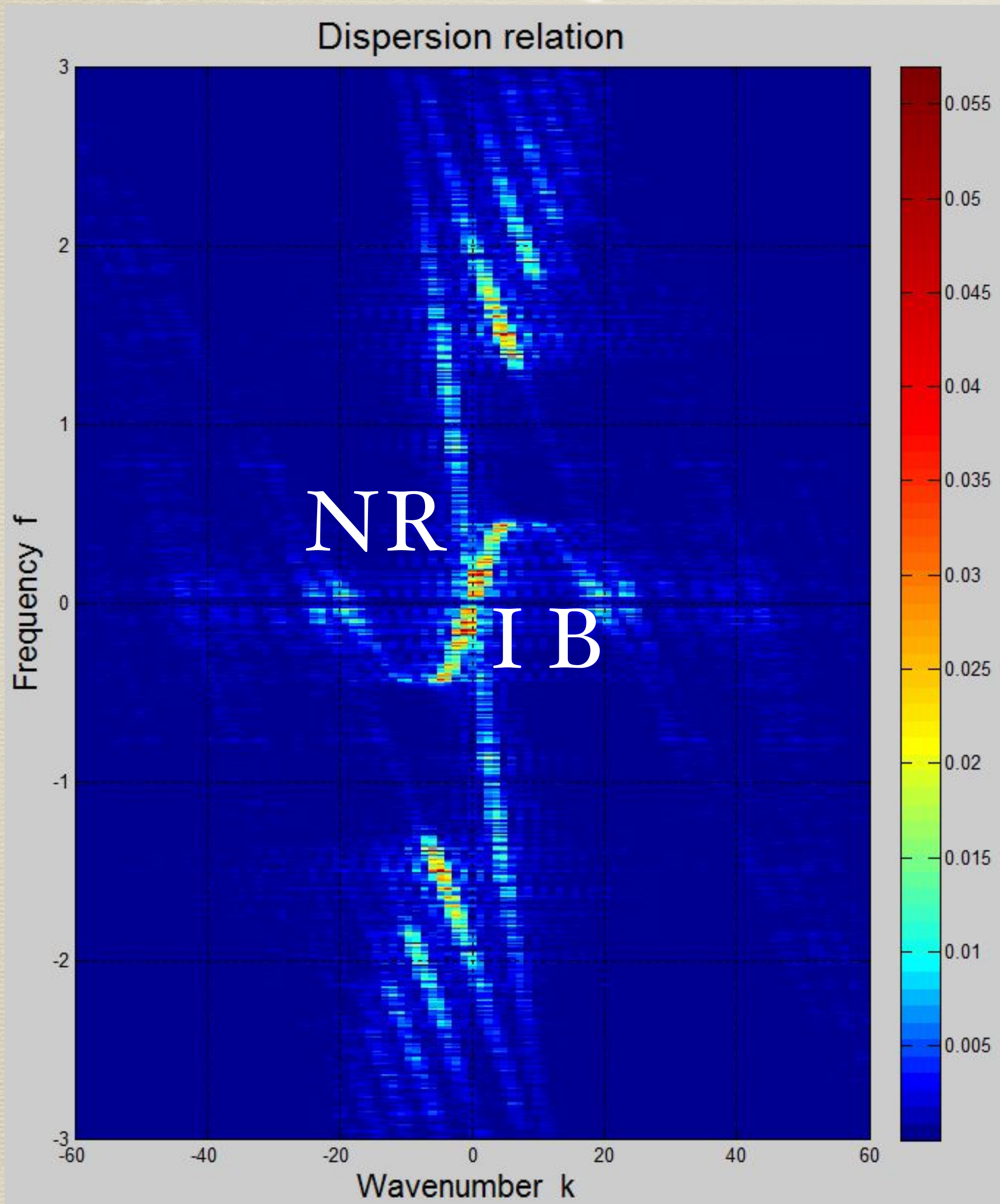
$$h_{\mathrm{trans}}^{\mathrm{up}} = \frac{1}{2}\sqrt[3]{\frac{q^2}{g}} + \frac{b_{\max}}{3} + 2\left(\frac{1}{2}\sqrt[3]{\frac{q^2}{g}} + \frac{b_{\max}}{3}\right)\cos\left(\frac{1}{3}\operatorname{Arccos}\left(1 - \frac{\frac{1}{4}\frac{q^2}{g}}{\left(\frac{1}{2}\sqrt[3]{\frac{q^2}{g}} + \frac{b_{\max}}{3}\right)^3}\right)\right)$$

$$\widetilde{h_{\mathrm{trans}}^{\mathrm{up}}} = h_{\mathrm{trans}}^{\mathrm{up}}\left(\frac{Fr_\infty^2}{4} + \frac{Fr_\infty}{4}\sqrt{Fr_\infty^2 + 8}\right) \qquad Fr_\infty = \frac{U_{\mathrm{up}}}{\sqrt{g h_{\mathrm{trans}}^{\mathrm{up}}}} = \frac{q}{\sqrt{g}\left(h_{\mathrm{trans}}^{\mathrm{up}}\right)^{\frac{3}{2}}}$$

Subluminal Dispersive Corrections (gravito-capillary waves in intermediate waters)

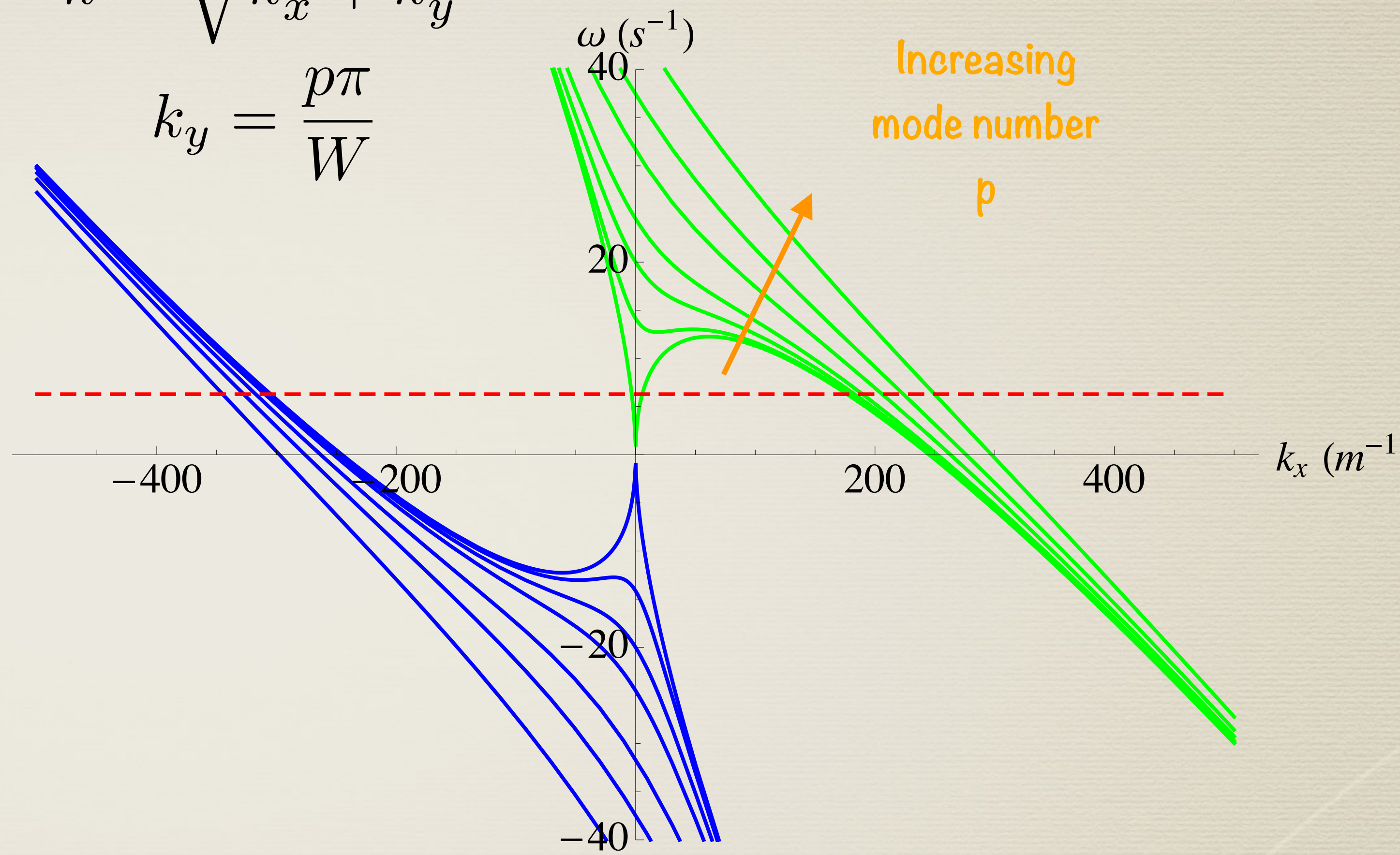


Dispersion Relation of Water Waves in Presence of a Plug-like Uniform Current with Turbulent Fluctuations (including transverse modes but not surface tension effect)



$$k = \sqrt{k_x^2 + k_y^2}$$

$$k_y = \frac{p\pi}{W}$$

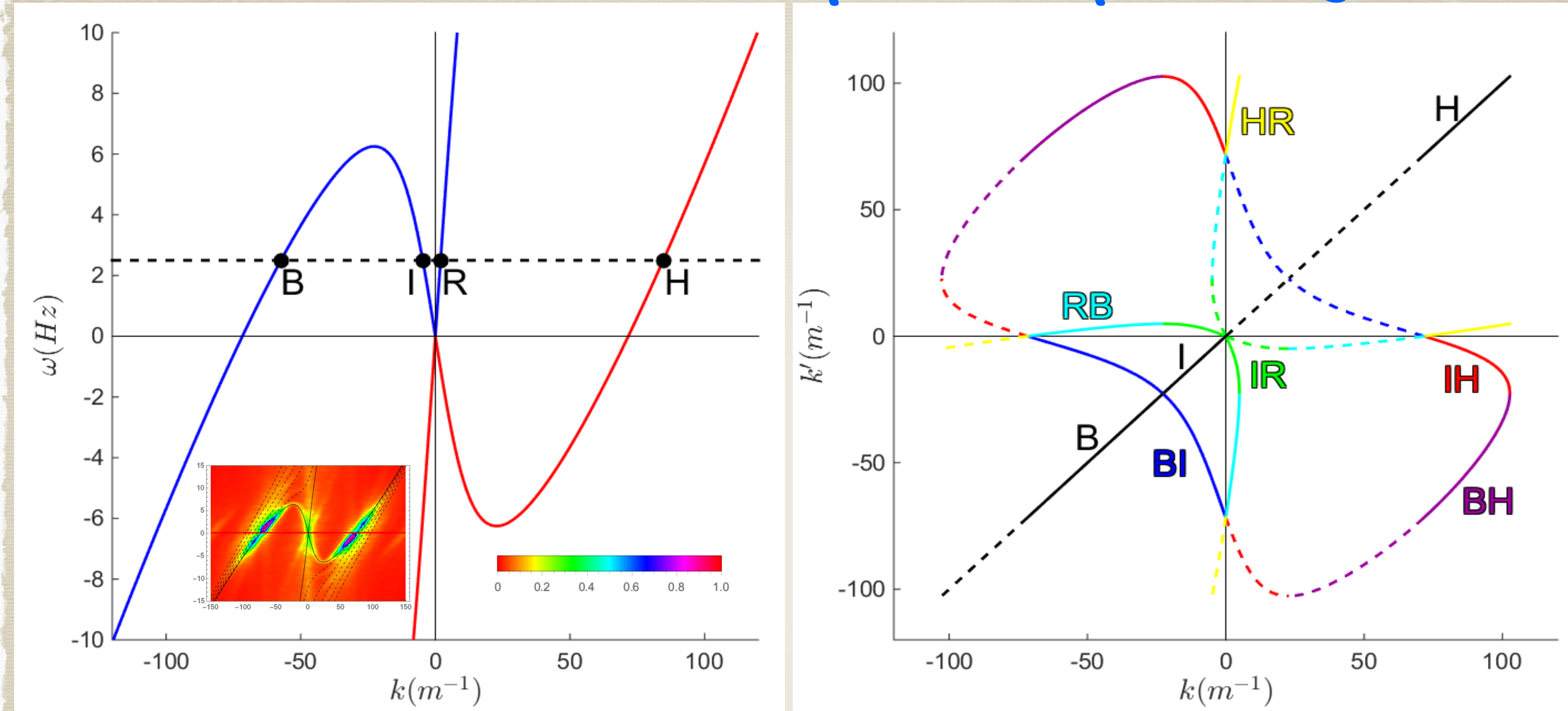


$$sign(v_g^B) = -sign(v_\phi^B)$$

The Blue-Shifted Modes (Hawking radiation when amplified) are such that their Phase Speed is Opposite to their Group Speed

$h_0 = 0.24m \quad q = 0.16m^2.s^{-1}$ Master Thesis of Pierre-Jean Faltot, Poitiers Université (2013)

Theoretical Correlation Map for Deep Gravity Waves



< > : mean over 80 time intervals ; Acquisition duration : 1000 s ; Frequency of acquisition : 5.12 Hz

Power Spectrum $$P(\omega, k) = \frac{<|\delta\tilde{h}(\omega,k)|^2>}{\omega'(k)}$$

Correlation $$G_2(\omega, k, k') = \frac{|<\delta\tilde{h}(\omega,k)\delta\tilde{h}(\omega,k')^*>|}{\sqrt{\omega'(k)\omega'(k')}}$$

Observation of noise correlated by the Hawking effect in a water tank.

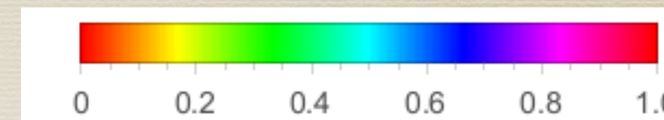
Léo-Paul Euvé, Florent Michel, Renaud Parentani, Thomas Philbin and Germain Rousseaux.

Physical Review Letters, Volume 117, Issue 12, 121301, September 2016.

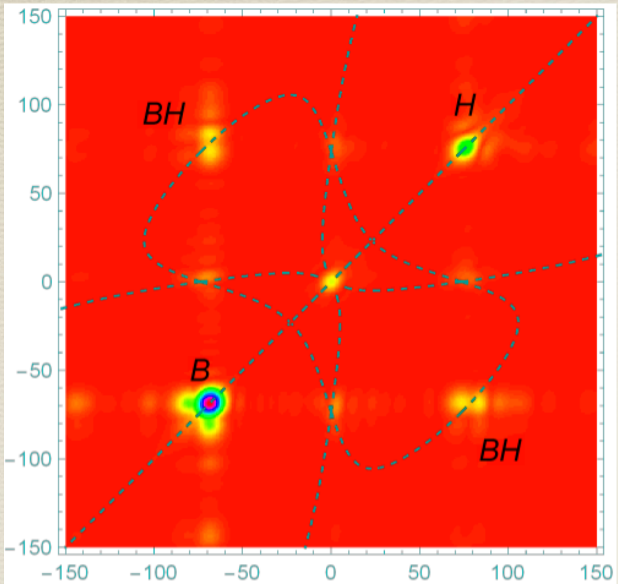
Experimental Correlations

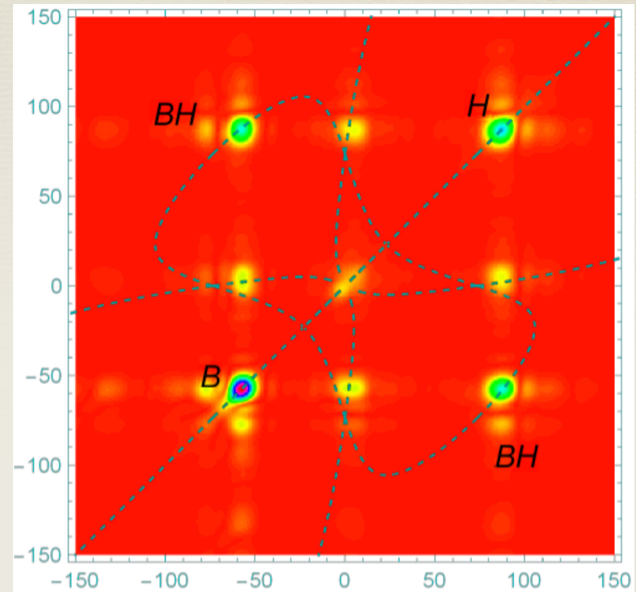
ω = 0,60Hz

ω = 2,50Hz



Noise

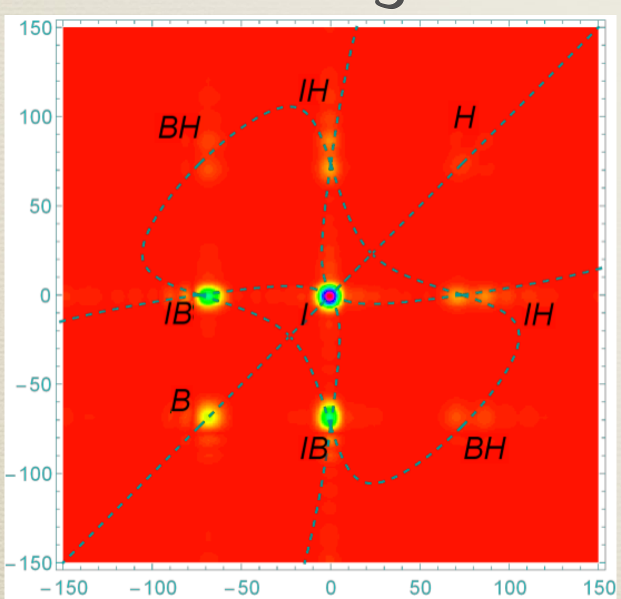


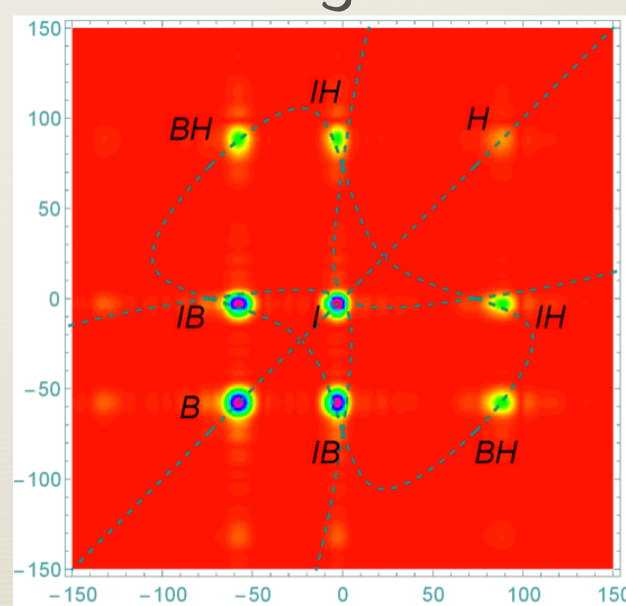


Non-blocking cases

Blocking cases

Wave-maker





<u>Incident Wave:</u>

Amplitude: A~0.2 mm

Wavelength: λ~1 m

Camber: $A/\lambda<3\times10^{-4}$

<u>Converted Modes:</u>

Amplitude: A~1 mm

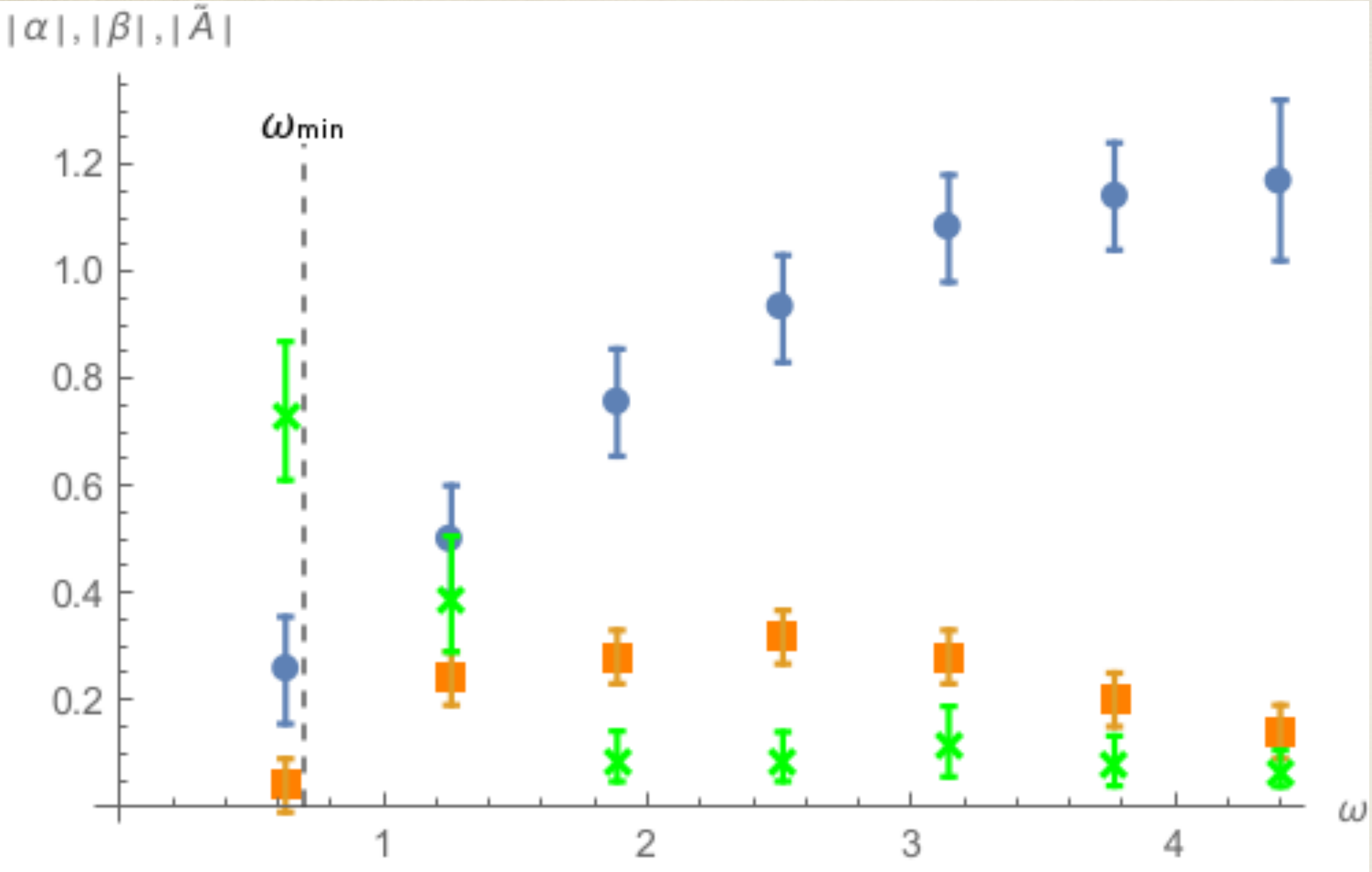
Wavelength: λ~0.1 m

Camber: $A/\lambda<1\times10^{-2}$

=> Spontaneous and Stimulated Mode Conversion With Correlations Between Modes

Scattering Coefficients in the Linear Regime

$$\phi_I = \alpha\phi_B + \beta\phi_H + A\phi_R + \tilde{A}\phi_T$$



$$|\alpha| = \frac{G_2(\omega, k_I, k_B)}{G_2(\omega, k_I, k_I)} \left|\frac{\partial\omega/\partial k_B}{\partial\omega/\partial k_I}\right|^{1/2}$$

$$|\beta| = \frac{G_2(\omega, k_I, k_H)}{G_2(\omega, k_I, k_I)} \left|\frac{\partial\omega/\partial k_H}{\partial\omega/\partial k_I}\right|^{1/2}$$

$|\tilde{A}|$ = upstream measurement

$|A| < 0.2$ theoretical estimation

$$1 = |\alpha|^2 - |\beta|^2 + |A|^2 + \left|\tilde{A}\right|^2$$

Poitiers Regime (Linear and No Landau Threshold) with Orsay 2 obstacle designed by Florent Michel and Renaud Parentani

Euvé et al., Observation of noise correlated by the Hawking effect in a water tank, Physical Review Letters, Volume 117, Issue 12, 121301 (2016).

PHYSICAL REVIEW LETTERS

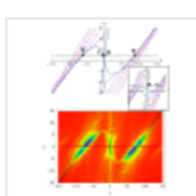
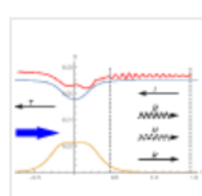
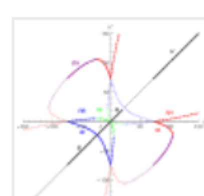
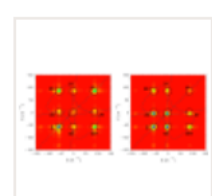
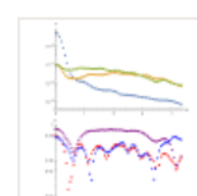
Observation of Noise Correlated by the Hawking Effect in a Water Tank

L.-P. Euvé, F. Michel, R. Parentani, T. G. Philbin, and G. Rousseaux

Phys. Rev. Lett. **117**, 121301 – Published 13 September 2016

ABSTRACT

We measured the power spectrum and two-point correlation function for the randomly fluctuating free surface on the downstream side of a stationary flow with a maximum Froude number $F_{\text{max}} \approx 0.85$ reached above a localized obstacle. On such a flow the scattering of incident long wavelength modes is analogous to that responsible for black hole radiation (the Hawking effect). Our measurements of the noise show a clear correlation between pairs of modes of opposite energies. We also measure the scattering coefficients by applying the same analysis of correlations to waves produced by a wave maker.

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nature physics

ARTICLES

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Observation of quantum Hawking radiation and its entanglement in an analogue black hole

Jeff Steinhauer

We observe spontaneous Hawking radiation, stimulated by quantum vacuum fluctuations, emanating from an analogue black hole in an atomic Bose–Einstein condensate. Correlations are observed between the Hawking particles outside the black hole and the partner particles inside. These correlations indicate an approximately thermal distribution of Hawking radiation. We find that the high-energy pairs are entangled, while the low-energy pairs are not, within the reasonable assumption that excitations with different frequencies are not correlated. The entanglement verifies the quantum nature of the Hawking radiation. The results are consistent with a driven oscillation experiment and a numerical simulation.

Fifty years ago, Bekenstein discovered the field of black hole thermodynamics[1]. This field has vast and deep implications, far beyond the physics of black holes themselves. The most important prediction of the field is that of Hawking radiation[2,3]. By making an approximation to the still-unknown laws of quantum gravity, Hawking predicted that the horizon of the black hole should emit a thermal distribution of particles. Furthermore, each Hawking particle should be entangled with a partner particle falling into the black hole. This presents a puzzle of information loss, and even the unitarity of quantum mechanics falls into question[4–6].

Despite the importance of black hole thermodynamics, there were no experimental results to provide guidance. The problem is that the Hawking radiation emanating from a real black hole should be exceedingly weak. To facilitate observation, Unruh suggested that an analogue black hole can be created in the laboratory, where sound plays the role of light, and the local flow velocity and speed of sound determine the metric of the analogue spacetime[7]. Nevertheless, thermal Hawking radiation had never been observed before this work.

Since the idea of analogue Hawking radiation was presented[7], there has been a vast theoretical investigation of a variety of possible analogue black holes[8–21]. It was predicted that the Hawking radiation could be observed by the density correlations between the Hawking and partner particles[9,10]. The entanglement of the Hawking pairs has also been studied theoretically[22–28]. Recently, we explained that the density correlations could be used to observe the entanglement[24], and we have implemented our technique here.

Over the past several years, we have systematically prepared for the observation of thermal Hawking radiation by studying analogue black hole creation[29], phonon propagation[30], thermal distributions of phonons[31], and self-amplifying Hawking radiation[32]. Our observation of Hawking radiation is performed in a Bose–Einstein condensate, a system in which the quantum vacuum fluctuations are guaranteed by the underlying pointlike atoms composing the condensate. There are experiments in several other systems underway at present with the hopes of observing Hawking radiation[33–37]. Furthermore, stimulated classical mode mixing at a white hole horizon has been observed[38,39].

It has been suggested that the Hawking and partner particles can be observed by studying the two-body correlation function between points on opposite sides of the horizon[9,10,12,40]. The correlation function is given by $G^{(2)}(x,x') = \sqrt{n_{out}n_{in}\xi_{out}\xi_{in}}\langle\delta n(x)\delta n(x')\rangle/n_{out}n_{in}$, where $n(x)$ is the one-dimensional (1D) density of the condensate forming the black hole, and n_{out} and n_{in} are the average densities outside and inside the black hole, respectively. The position x is in units of the shortest length scale of the condensate $\xi \equiv \sqrt{\xi_{out}\xi_{in}}$, where ξ_{out} and ξ_{in} are the healing lengths outside and inside the black hole, respectively, and $\xi_i = \hbar/mc_i$, where c_i is the speed of sound and m is the mass of an atom in the condensate. The strength of the fluctuations are characterized by the prefactor $\sqrt{n_{out}n_{in}\xi_{out}\xi_{in}}$; the lower the number, the larger the signal of Hawking radiation[10]. Figure 1a shows the theoretical $G^{(2)}$ in vacuum, in the hydrodynamic limit of low Hawking temperature in which dispersion can be neglected, in strict analogy with real gravity[9]. Correlations are seen along the line of equal propagation times from the horizon, outside and inside the black hole. These are the correlations between the Hawking and partner particles. Such correlations should also exist in a real black hole, within Hawking's approximation[40].

We find that much insight can be gained by considering the Fourier transform of individual quadrants of $G^{(2)}$ (ref. 24). Most importantly, the Fourier transform of the correlations between points outside and inside the black hole (the quadrant outlined with dotted lines in Fig. 1a) gives the k-space correlation spectrum $\langle\hat{b}_{k_{HR}}\hat{b}_{k_p}\rangle$, where $\hat{b}_{k_{HR}}$ is the annihilation operator for a Hawking particle with wavenumber k_{HR} localized outside the black hole, and $\hat{b}_{k_p}$ is the annihilation operator for a partner particle localized inside the black hole[24]. Specifically,

$$S_0\langle\hat{b}_{k_{HR}}\hat{b}_{k_p}\rangle = \sqrt{\frac{\xi_{out}\xi_{in}}{L_{out}L_{in}}}\int dx\,dx'\,e^{ik_{HR}x}e^{ik_p x'}G^{(2)}(x,x') \qquad (1)$$

where $S_0 \equiv (U_{k_{HR}} + V_{k_{HR}})(U_{k_p} + V_{k_p})$ and U_i and V_i are the Bogoliubov coefficients for the phonons, which are completely determined by $\xi_i k_i$. The length of each region is given by L_i. The coordinates x and x' are integrated over the intervals $[-L_{out}, 0]$ and $[0, L_{in}]$, respectively. If the correlation feature is elongated with a constant cross-section, as in Fig. 1a, then (1) reduces to

$$S_0\langle\hat{b}_{k_{HR}}\hat{b}_{k_p}\rangle = \sqrt{-\tan\theta - \cot\theta}\int dx''\,e^{ikx''}G^{(2)}(x,x') \qquad (2)$$

where x'' is the coordinate perpendicular to the feature in units of ξ, θ is the angle of the correlation feature in the x–x' plane as in Fig. 1a,

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