PBHs abundance : loss of predictability?

Cristiano Germani

Universitat de Barcelona

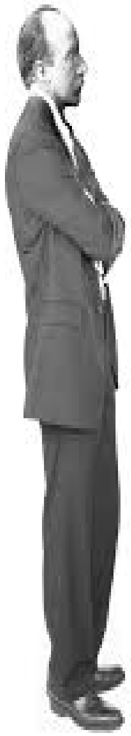
With Aichen Li

Why this is not a black hole?

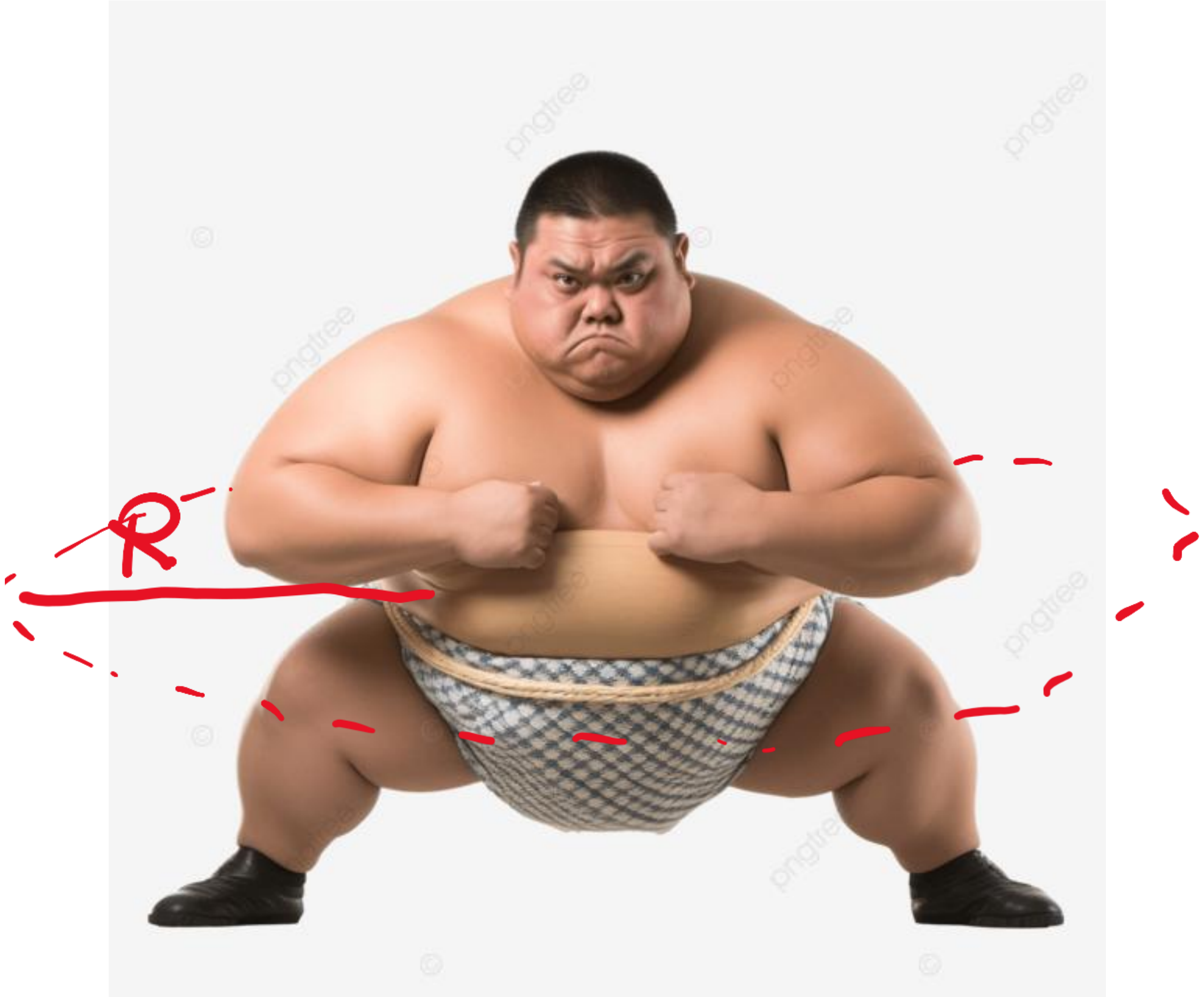
(typically)

$$\boxed{\frac{\delta \rho}{\rho} \ggg 1!}$$

$R \gg 2$  $/$  ^{2}



(Planck)



It is the mass within hoop of radius R_s that matter!

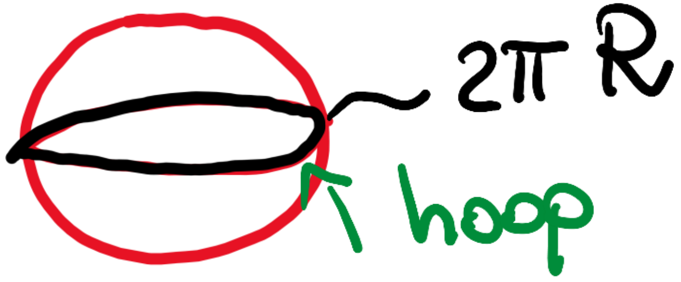
Hoop conjecture

Thorne '75.

What is a hoop?

Suppose we have spherical symmetry

$$ds^2 = \cdots \underbrace{g_{rr}\, dr^2}_{\text{Related to Volume}} + \underbrace{R^2(r,t)}_{\text{Related to hoop and Area}} d\Omega^2$$



If the spacetime is isotropic

R is a scalar

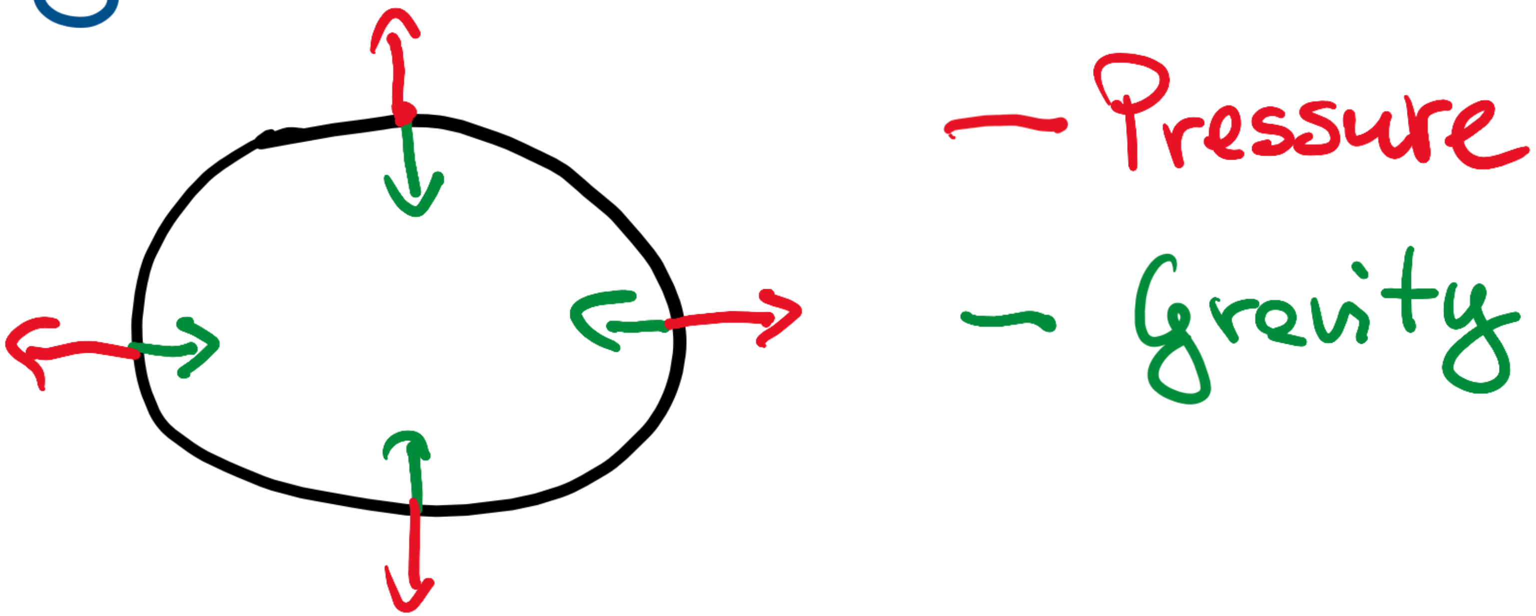
In asymptotically flat space

a BH inexorably form iff

$$\boxed{2\frac{M}{R} \sim o(1)}$$

Question:

How large should initially be the gravitational potential $\left(\frac{2M}{R}\Big|_{t=t_i}\right)$ to eventually lead to BH?



PBH are formed in expanding spece
(FRW)

$\Downarrow$

Universe expension also fights against collapse !

In order to focus on local properties and use the hoop conjecture we could "filter out" the infinitely long modes due to the expansion

Define the compaction function

$$C = 2\left[\frac{M - M_\infty}{R}\right] = 2\,\frac{\delta M}{R}$$

M_∞ is the mass generated by the homogeneous matter

Inflation quantum mechanically generates classically observables perturbations when those cross cosmological horizon

They are:

1) Statistically distributed
2) Averagely small
3) Have "large" wavelenghts

1) Statistically distributed:

There is always a chance that large (collapsing) perturbations are formed

2) Averagely small:

@ formation PBH are rare

Because inflation is isotropic

$\Downarrow$

~ spherical symmetry & isolated perturbations

Important:

Suppose PBHs are formed during radiation era

$$\rho_\circ / \rho_{rad} \sim \left(\frac{1}{a^3}\right) \Big/ \left(\frac{1}{a^4}\right) \sim a$$

PBH relative density grows:

- Initially they are sub-dominant
- @ equality ($\rho_\circ \sim \rho_{rad}$) PBHs are Dark Matter!

3) Classically observable perturbations are at super horizon

This talk: we are going to challenge this....

3) Classically observable perturbations are at super horizon

This talk: we are going to challenge this....

... let us now connect to inflation...

Poisson equation

$\nabla^2\Phi \sim \delta\rho$ and $\Phi \sim e^{2\zeta}$

where $ds^2 = \dots + e^{2\zeta}(dr^2 + r^2 d\Omega^2)$

curvature perturbations

$$C \sim \int \delta\rho \sim \int \nabla^2 e^{2\zeta} = \frac{2}{3}\left[1 - (1 + r\partial_r\zeta)^2\right]$$

This is exact! [Yoo et al '15]

In Fourier space

$$J \sim \int d\vec{k}\, e^{i\vec{k}\cdot\vec{x}}\, J_k(t)$$

$$r^2 J' = \int_{Ball(r)} \nabla^2 J \, dV \equiv g r(\vec{x}_0)\, r$$

Thus:

$$C = g r(\vec{x}_0)\left(1 - \frac{3}{8} g r(\vec{x}_0)\right)$$

If ζ_k is Gaussian

$g_r(\vec{x}_0)$ is Multi-Gaussian

and

all the statistics of PBHs only depends on $\langle \zeta_k \zeta_{k'} \rangle \sim \delta(k-k') P(k)$

P is the power spectrum

To form a BH $g_r(\vec{x}_0) \sim O(1)$

g is Gaussian with

$$\mathrm{PDF}(g) \sim e^{-g^2/2\Sigma_g^2}$$

and $\Sigma_g = \Sigma_g\left[\int \mathcal{P}(k)\right]$

to get enough PBH $\mathcal{P}(K)$ is "large"!

$$\left[\mathcal{P}(x) \sim 10^{-2}\right]$$

CMB requires $P(k) \sim 10^{-9}$ so how do we get to $\sim 10^{-2}$?

If perturbation theory is NOT violated $P(k)$ is the variance of the linear ζ (in $\delta\varphi = 0$ gauge)

If we linearize Einstein equations sourced by a scalar field (inflaton φ) we can recast linear perturbations into a gauge invariant [Mukhanov-Sasaki] variable

$$v = a\left[\delta\varphi + \frac{\dot{\varphi}}{H}\zeta\right]$$

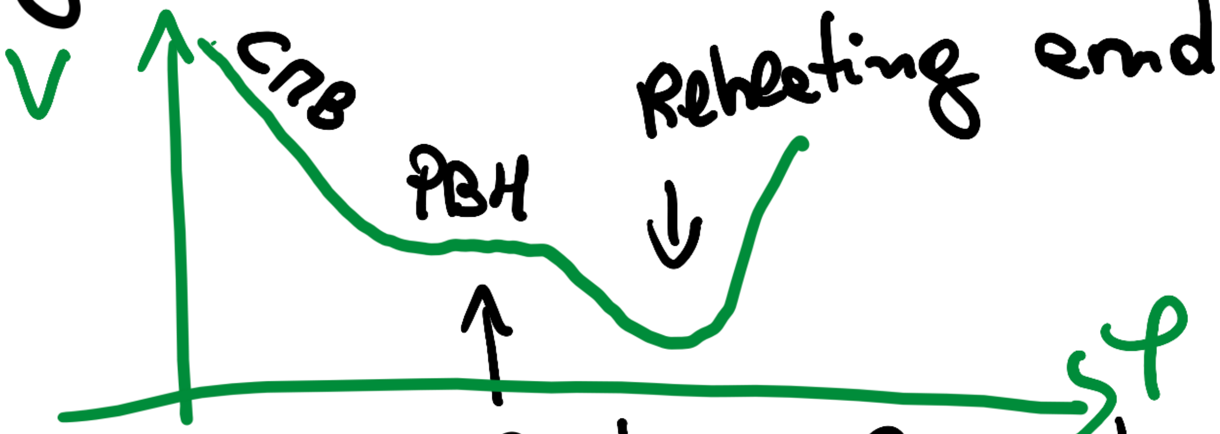
v satisfies an inverted harmonic oscillator

@ super-horizon scales ($k \ll aH$)

$\langle v_k v_{k'} \rangle \propto a^2$ so

$\langle \zeta_k \zeta_{k'} \rangle \big|_{\delta\varphi=0} \propto \frac{H^2}{\dot{\phi}^2} \Rightarrow$ to increase $P(k)$

the inflaton must slow down



Reheating and PBH formation

Inflaton slows down! $\dot{\phi} \sim \frac{1}{a^3}$

Yokoyama - Kristiano challenge:

Is ζ really nearly classical at super-horizon scales?

For a quantum field to behave classically $[\varphi, \dot{\varphi}] \sim 0$

However $[\zeta, \dot{\zeta}]\Big|_{PBH} \sim a^4$

So Yokoyama-Kristiano claimed that $P(k)$ should get large quantum corrections when considering interactions.

The previous discussion is however a bit quick ...

The quantum variable is v by Wronskian condition

$$[v, \dot{v}] \sim i$$

$$v = \delta\varphi + \frac{\dot{\varphi}}{H}\zeta$$

we can consider gauge $\zeta = 0$ (spatially flat) there

$$[\delta\varphi, \delta\dot{\varphi}] \sim \frac{1}{a^2} \;!$$

thus $\delta\varphi$ squeezes and all looks good, $\delta\varphi$ is classical at super-horizon scales

Note that $\delta\varphi\big|_{\zeta=0}$ is strictly speaking the scalar mode!

Nevertheless, in the literature it has been shown that $P(k)$ does get huge contribution from 1-loop in the case of a quick transition from a $\dot{\phi} \sim$ const (CMB case) to $\dot{\phi} \sim \frac{1}{a^3}$ (PBHs)

Here we'll give you the reasons...

Consider an instantaneous transition between $\dot{\phi} \sim 0$ and $\dot{\phi} \sim \frac{1}{a^3}$ at some t_*, at super-horizon scales

$$\delta\ddot{\varphi} + \dots + \delta(t-t_*)\,\delta\varphi = 0$$

Thus around t_* $\delta\dot{\varphi}$ jumps

Sol. $\delta\dot{\varphi} = \underset{\substack{\uparrow \\ \text{cont.}}}{\overline{\delta\dot{\varphi}}} + \Theta(t-t_*)\,\delta\varphi$

Is there an issue for the quantization?

"Wronskian"

$$\delta\varphi\,\delta\dot{\varphi}^* - \delta\varphi^*\,\delta\dot{\varphi}\Big|_{t\neq t^*} = \delta\varphi\left(\overline{\delta\dot{\varphi}^*} + \delta\varphi^*\theta\right)$$

$$-\delta\varphi^*\,\delta\dot{\varphi} = \cdots + \Big(|\delta\varphi|^2 - \underbrace{|\delta\varphi|^2\Big)}_{=0}\theta$$

So the jump does not change linear quantization!

What happens when we turn on interactions (e.g. cubic)?

For the modes functions:

$$\delta\varphi = \delta\varphi^{(1)} + \delta\varphi^{(2)} \dots$$

$$\delta\ddot{\varphi}^{(2)} + \dots + \#\,\delta(t-t_*)\left[\delta\varphi^{(2)} + \alpha\,\overbrace{\delta\varphi^{(1)}\,\delta\dot{\varphi}^{(1)}}^{\text{source}}\right] = 0$$

$$\Downarrow$$

$$\delta\dot{\varphi}^{(2)} = \dot{\overline{\delta\varphi}}^{(2)} + \#\,\Theta(t-t_*)\left(\delta\varphi^{(2)} + \alpha\,\delta\varphi^{(1)}\delta\dot{\varphi}^{(1)}\right)$$

While $\delta\varphi$ is continous across t_* we have seen that the derivatives are not.

Assume $\delta\varphi^{(2)} \ll \delta\varphi^{(1)}$

$$\delta\dot{\varphi}^{(2)} \sim \overline{\delta\dot{\varphi}^{(2)}} + \# \; \delta\varphi^{(1)} \, \delta\dot{\varphi}^{(1)}$$

So, for the normalized "Wronskian"

$$\frac{\delta\varphi\,\delta\dot{\varphi}^{*} - \delta\varphi^{*}\,\delta\dot{\varphi}}{|\delta\varphi|^2 H} \sim \cdots + \# \frac{\cancel{|\delta\varphi^{(1)}|^2}\,\delta\dot{\varphi}^{(1)}}{\cancel{|\delta\varphi|^2} H} + \mathcal{O}(\delta\dot{\varphi}^{(2)})$$

So if $\delta\dot{\varphi}^{(2)} \sim \# \, \delta\dot{\varphi}^{(1)} \gg \delta\dot{\varphi}^{(2)}$

the Normalized Wronskian grows fast

with the jump of $\delta\dot{\varphi}^{(1)}$!

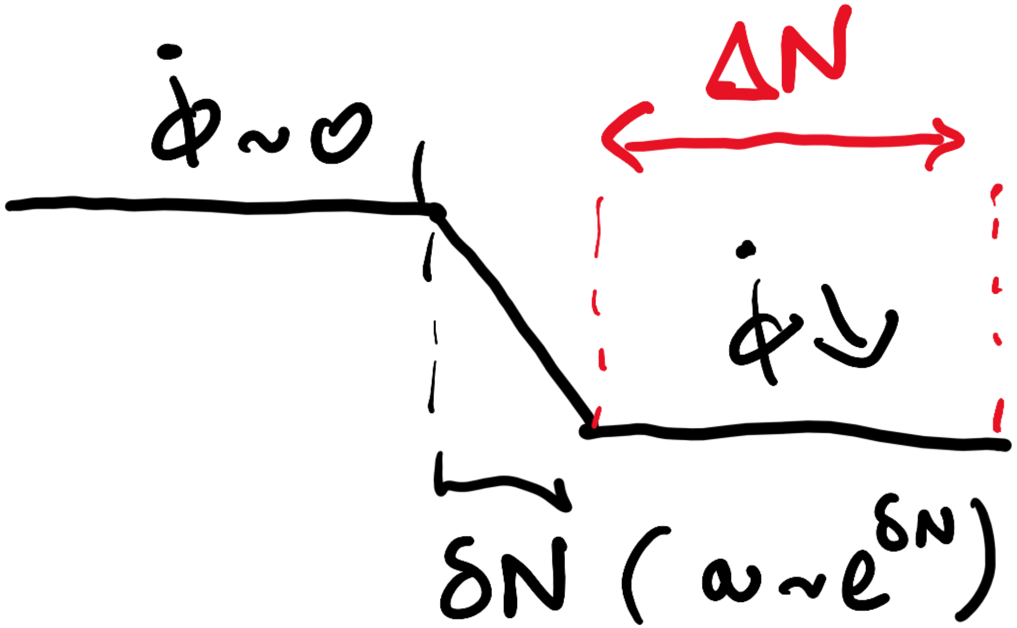
$\Downarrow$

$\boxed{\text{Quantum corrections are large}}$!

But also classical perturbation theory

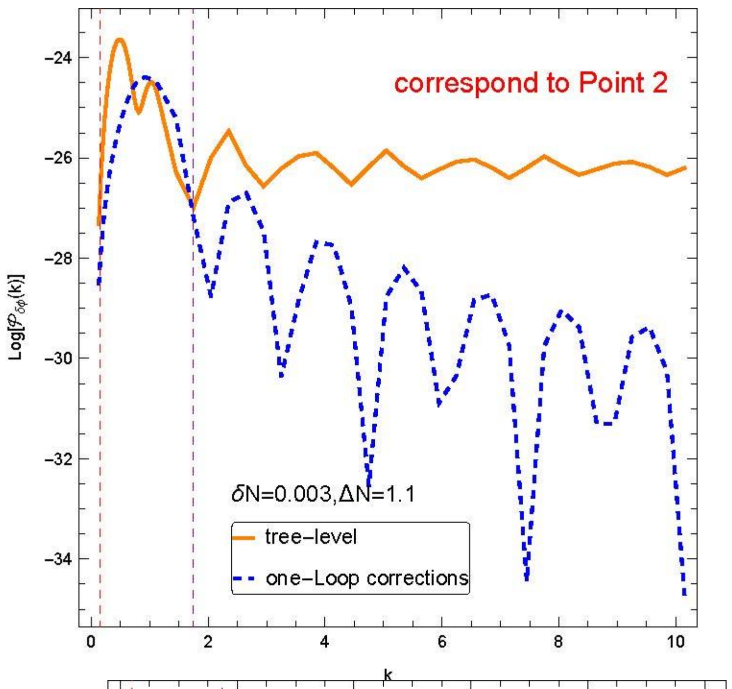
is invalid!

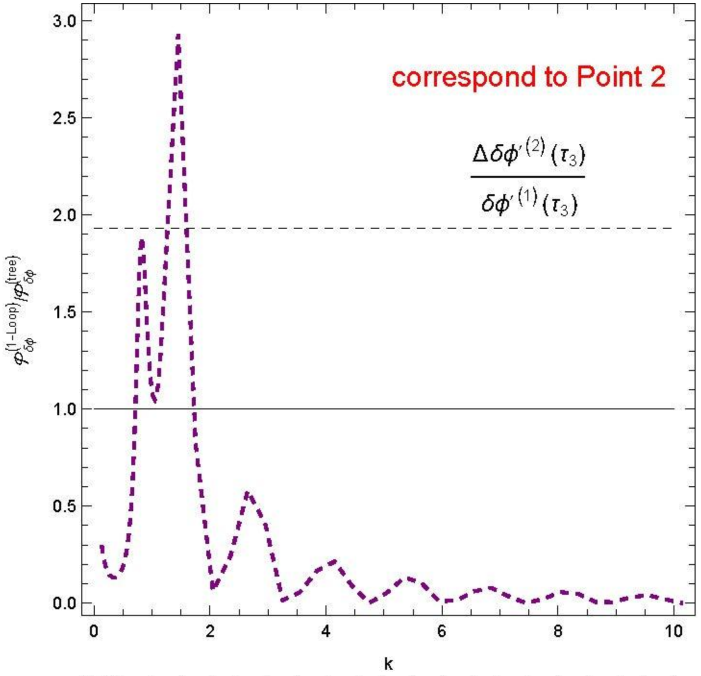
Suppose



We found
$$\frac{\delta\dot\phi^{(2)}}{\delta\dot\phi^{(1)}} \sim \frac{1}{\delta N} \sqrt{\underset{\substack{\uparrow \\ \text{CMB}}}{P_0}} \; e^{6\Delta N}$$

Let us then calculate loop corrections.....

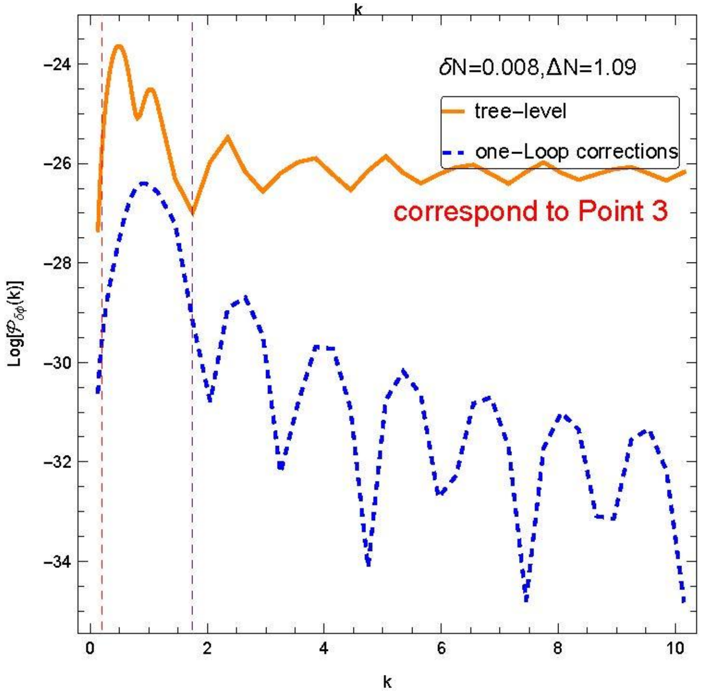


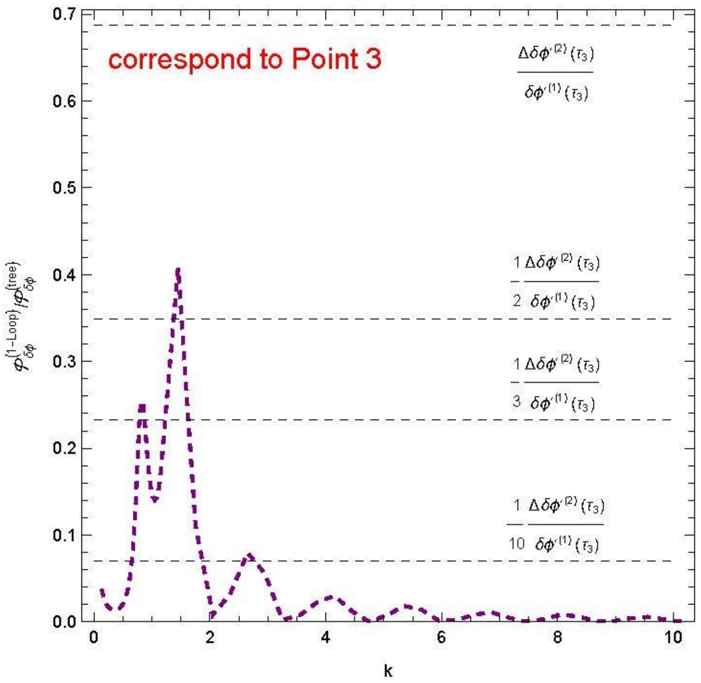


Perturbation theory broken!

$\Downarrow$

P(k) makes no sense!





Perturbation theory not broken

$P(k)^{tree}$ is stable!

Conclusions:

- To have enought PBHs for DM we need to exponentially grow the power spectrum from its CMB value

- If the transition from the CMB scales to the exponential growing is too fast perturbation theory (classial and therefore quantum) is broken.

Sub-conclusion:

Inflationary scenarios never sharply change from attractor (CMB) to non-attractor (PBHs) regimes
$\Rightarrow$ Potentials are smooth and classical perturbation theory is NEVER broken
$\Rightarrow$ PBH abundance is predictable

Morality:

Be careful to draw conclusions from over-simplified (academic) cases.

Nevertheless :

Enjoy playing with your toys !

Thanks for listening