Removing spurious degrees of freedom from EFT of gravity

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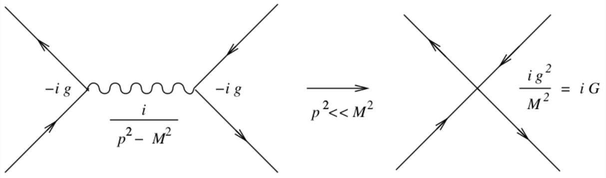
Effective theories

Modern view: every theory of nature is an **effective theory**

Every effective theory has an intrinsic range of applicability
(energy range, velocity range, temperature range, ...)

Central idea: describe particular regime in terms of
degrees of freedom relevant for that regime.

Famous example from particle physics: Fermi's four-interaction



Two interacting oscillators

Two interacting oscillators: “light” X and “heavy” Y:

$$S[X,Y] = \int dt \left[\frac{\dot{X}^2}{2} - \frac{\boldsymbol{\omega}^2}{2} X^2 + \frac{\dot{Y}^2}{2} - \frac{\boldsymbol{\Omega}^2}{2} Y^2 - U(X,Y) \right]$$

Hierarchy: $\boldsymbol{\Omega} \gg \boldsymbol{\omega} \longrightarrow$ easy to excite X, difficult to excite Y.

For small energies $E \ll \hbar\boldsymbol{\Omega}$ system is described well only in terms of the light oscillator degrees of freedom X. The coupling to Y is captured by *effective self-interactions*:

$$\begin{aligned} S_{\text{eff}}[X] = \int dt \Bigg[& \frac{\dot{X}^2}{2} - \frac{\boldsymbol{\omega}^2}{2} X^2 - \frac{\lambda}{4} X^4 \\ & - c_1 \frac{X^6}{\boldsymbol{\Omega}^2} - c_2 \frac{X^2 \dot{X}^2}{\boldsymbol{\Omega}^2} - c_3 \frac{\ddot{X}^2}{\boldsymbol{\Omega}^2} + \mathcal{O}\big(\boldsymbol{\Omega}^{-4}\big) \Bigg] \end{aligned}$$

Integrating out...

Formal procedure: Integrating out degrees of freedom

$\longrightarrow$ effective non-local theory

$\longrightarrow$ local derivative expansion.

$\longrightarrow$ infinite tower of local interactions

Inverse kinetic operators (retarded Green's functions) that appear in the effective nonlocal theory obtained by integrating out are expanded:

$$\frac{1}{\frac{d^2}{dt^2}+\Omega^2+gX^2} \approx \frac{1}{\boldsymbol{\Omega}^2}\left[1-\frac{\frac{d^2}{dt^2}+gX^2}{\boldsymbol{\Omega}^2}+\left(\frac{\frac{d^2}{dt^2}+gX^2}{\boldsymbol{\Omega}^2}\right)^2+\dots\right]$$

full theory ⟶ effective theory ✓

effective theory ⟶ full theory **?**

EFT of gravity

UV gravitational physics not known — consider effective theory

$$\begin{aligned} S[g_{\mu\nu}] = \int d^4x\,\sqrt{-g}\,\Big\{ & -\frac{2\Lambda}{\kappa^2} + \frac{R}{\kappa^2} \\ & + \alpha_1 R^2 + \alpha_2 R^{\mu}{}_{\nu} R^{\nu}{}_{\mu} + \alpha_3 R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\mu\nu} \\ & + \kappa^2 \Big[\beta_1 \nabla^{\mu} R \nabla_{\mu} R + \beta_2 \nabla^{\rho} R^{\mu\nu} \nabla_{\rho} R_{\mu\nu} + \beta_3 R^3 \\ & \qquad + \beta_4 R R^{\mu\nu} R_{\mu\nu} + \beta_5 R^{\mu}{}_{\nu} R^{\nu}{}_{\rho} R^{\rho}{}_{\mu} + \beta_6 R^{\mu}{}_{\rho} R^{\nu}{}_{\sigma} R^{\rho\sigma}{}_{\mu\nu} \\ & \qquad + \beta_7 R R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\mu\nu} + \beta_8 R^{\mu}{}_{\alpha} R^{\alpha\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\mu\nu} \\ & \qquad + \beta_9 R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta} R^{\alpha\beta}{}_{\mu\nu} + \beta_{10} R^{\mu\nu}{}_{\rho\sigma} R^{\rho\alpha}{}_{\mu\beta} R^{\sigma\beta}{}_{\nu\alpha} \Big] \Big\} \end{aligned}$$

Lovelock's theorem: only theory of massless spin-2 graviton is general relativity — our effective theory introduces additional degrees of freedom?

[familiar example: $f(R)$ theories propagate graviton+scalar]

EFT of gravity

UV gravitational physics not known — consider effective theory

$$\begin{aligned}
S[g_{\mu\nu}] = \int d^4x \sqrt{-g}\, \Big\{ &-\frac{2\Lambda}{\kappa^2} + \frac{R}{\kappa^2} \\
&+ \cancel{\alpha_1 R^2} + \cancel{\alpha_2 R^{\mu}{}_{\nu} R^{\nu}{}_{\mu}} + \cancel{\alpha_3 R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\mu\nu}} \\
&+ \kappa^2 \Big[\cancel{\beta_1 \nabla^{\mu} R \nabla_{\mu} R} + \cancel{\beta_2 \nabla^{\rho} R^{\mu\nu} \nabla_{\rho} R_{\mu\nu}} + \cancel{\beta_3 R^3} \\
&\qquad + \cancel{\beta_4 R R^{\mu\nu} R_{\mu\nu}} + \cancel{\beta_5 R^{\mu}{}_{\nu} R^{\nu}{}_{\rho} R^{\rho}{}_{\mu}} + \cancel{\beta_6 R^{\mu}{}_{\rho} R^{\nu}{}_{\sigma} R^{\rho\sigma}{}_{\mu\nu}} \\
&\qquad + \cancel{\beta_7 R R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\mu\nu}} + \cancel{\beta_8 R^{\mu}{}_{\alpha} R^{\alpha\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\mu\nu}} \\
&\qquad + \boxed{\beta_9 R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta} R^{\alpha\beta}{}_{\mu\nu}} + \cancel{\beta_{10} R^{\mu\nu}{}_{\rho\sigma} R^{\rho\alpha}{}_{\mu\beta} R^{\sigma\beta}{}_{\nu\alpha}} \Big] \Big\}
\end{aligned}$$

Pert. shift: $g_{\mu\nu} \longrightarrow g_{\mu\nu} + \kappa^2\big(c_1 R g_{\mu\nu} + c_2 R_{\mu\nu} + c_3 \Lambda g_{\mu\nu}\big) + \ldots$

't Hooft, Veltman, *One loop divergencies in the theory of gravitation*, Ann.Inst.H.Poincare A Phys.Theor. 20 (1974) 69-94

Goroff, Sagnotti, *Quantum gravity at two loops*, Phys.Lett.B 160 (1985) 81-86

van de Ven, *Two loop quantum gravity*, Nucl.Phys.B 378 (1992) 309-366

Minimal higher derivative EFT of gravity

Assume there is some unknown UV physics that is generally covariant.

$$S[g_{\mu\nu}] = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{\kappa^2} + \boldsymbol{\epsilon} \kappa^2 R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta} R^{\alpha\beta}{}_{\mu\nu} \right]$$

Can we remove from this theory the degrees of freedom associated to higher derivatives in the Riemann-cubed term?

$\longrightarrow$ method of perturbative reduction

Glavan, Mukohyama, Zlosnik, *Removing spurious degrees of freedom from EFT of* , *gravity* arXiv:2409.15989 [gr-qc]

Lovelock's theorem: only generally covariant theory of the massless spin-2 graviton is general relativity — <u>what is the reduced theory?</u>

Simple example

Action for harmonic oscillator with higher derivative correction:

$$S[X]=\int d^4x\left[\frac{\dot{X}^2}{2}-\frac{\omega^2X^2}{2}+\frac{\epsilon\ddot{X}^2}{2\omega^2}\right]\quad\Rightarrow\quad\ddot{X}+\omega^2X+\frac{\epsilon}{\omega^2}X^{(4)}=0$$

Exact solutions:

$$X(t)=e^{i\Omega t}\qquad\Longrightarrow\qquad-\Omega^2+\omega^2+\frac{\epsilon}{\omega^2}\Omega^4=0$$

$$\boldsymbol{\Omega}_0{}^2=\frac{\omega^2}{2\epsilon}\left[1-\sqrt{1+4\epsilon}\right]=\omega^2\left[1+\epsilon+2\epsilon^2+5\epsilon^3+\dots\right]$$

$$\boldsymbol{\Omega}_\epsilon{}^2=\frac{\omega^2}{2\epsilon}\left[1+\sqrt{1+4\epsilon}\right]=\frac{\omega^2}{\epsilon}\left[1-\epsilon-\epsilon^2-2\epsilon^3+\dots\right]$$

$$X(t)=\alpha_+\exp\left(i\boldsymbol{\Omega}_0t\right)+\alpha_-\exp\left(-i\boldsymbol{\Omega}_0t\right)+\beta_+\exp\left(i\boldsymbol{\Omega}_\epsilon t\right)+\beta_-\exp\left(-i\boldsymbol{\Omega}_\epsilon t\right)$$

Simple example

Action for harmonic oscillator with higher derivative correction:

$$S[X] = \int d^4x \left[\frac{\dot{X}^2}{2} - \frac{\omega^2 X^2}{2} + \frac{\boldsymbol{\epsilon}\ddot{X}^2}{2\omega^2}\right] \quad \Rightarrow \quad \ddot{X} + \omega^2 X + \frac{\boldsymbol{\epsilon}}{\omega^2} X^{(4)} = 0$$

Exact solutions:

$$X(t) = e^{i\Omega t} \qquad \Longrightarrow \qquad -\Omega^2 + \omega^2 + \frac{\boldsymbol{\epsilon}}{\omega^2}\Omega^4 = 0$$

$$\boldsymbol{\Omega}_0{}^2 = \frac{\omega^2}{2\boldsymbol{\epsilon}}\left[1 - \sqrt{1 + 4\boldsymbol{\epsilon}}\right] = \omega^2\left[1 + \boldsymbol{\epsilon} + 2\boldsymbol{\epsilon}^2 + 5\boldsymbol{\epsilon}^3 + \dots\right]$$

$$\boldsymbol{\Omega}_{\boldsymbol{\epsilon}}{}^2 = \frac{\omega^2}{2\boldsymbol{\epsilon}}\left[1 + \sqrt{1 + 4\boldsymbol{\epsilon}}\right] = \frac{\omega^2}{\boldsymbol{\epsilon}}\left[1 - \boldsymbol{\epsilon} - \boldsymbol{\epsilon}^2 - 2\boldsymbol{\epsilon}^3 + \dots\right]$$

$$X(t) = \alpha_+ \exp\left(i\boldsymbol{\Omega}_0 t\right) + \alpha_- \exp\left(-i\boldsymbol{\Omega}_0 t\right) + \cancel{\beta_+ \exp\left(i\boldsymbol{\Omega}_{\boldsymbol{\epsilon}} t\right)} + \cancel{\beta_- \exp\left(-i\boldsymbol{\Omega}_{\boldsymbol{\epsilon}} t\right)}$$

Spurious degrees of freedom (often Ostrogradsky ghosts)

Woodard, *Ostrogradsky's theorem on Hamiltonian instability*, Scholarpedia 10 (2015) 8, 32243, arXiv:1506.02210 [hep-th]

Simple example: removing non-analytic dependence 1

Option 1: power series ansatz

Simon, *The Stability of flat space, semiclassical gravity, and higher derivatives,* Phys. Rev. D 43 (1991), 3308-3316

$$X(t) = \sum_{n=0}^{\infty} \epsilon^n X_n(t)$$

⟶ tower of equations:

0th: $\ddot{X}_0 + \omega^2 X_0 = 0 \qquad \Longrightarrow \qquad X_0(t) = \alpha e^{i\omega t} + \beta e^{-i\omega t}$

1st: $\ddot{X}_1 + \omega^2 X_1 = -\frac{X_0^{(4)}}{\omega^2} = -\omega^2 \left[\alpha_+ e^{i\omega t} + \alpha_- e^{-i\omega t}\right]$

$$\Longrightarrow \quad X_1(t) = \frac{e^{i\omega t}}{4}\left[\alpha_- - \alpha_+\left(1 - \boxed{2i\omega t}\right)\right] + \frac{e^{-i\omega t}}{4}\left[\alpha_+ - \alpha_-\left(1 + \boxed{2i\omega t}\right)\right]$$

Secular divergences!

Simple example: removing non-analytic dependence 2

Parker, Simon, *Einstein equation with quantum corrections reduced to second order*, Phys. Rev. D 47 (1993), 1339-1355 [arXiv:gr-qc/9211002 [gr-qc]].

Fröb, Papadopoulos, Roura, Verdaguer, *Nonperturbative semiclassical stability of de Sitter spacetime for small metric deviations,* Phys. Rev. D 87 (2013) no.6, 064019 [arXiv:1301.5261 [gr-qc]].

Option 2: reduce order of equation

$$\ddot{X} + \omega^2 X = -\frac{\epsilon}{\omega^2} X^{(4)}$$

Reduce derivative order by using lower order equation:

$$\longrightarrow \quad (1-\epsilon)\ddot{X} + \omega^2 X - \cancel{\frac{\epsilon^2}{\omega^4} X^{(6)}} = 0$$

$$\longrightarrow \quad \ddot{X} + \omega^2 (1+\epsilon) X + \cancel{\frac{\epsilon^2}{\omega^2} X^{(4)}} - \cancel{\frac{\epsilon^2}{\omega^4} X^{(6)}} = 0$$

$$\longrightarrow \quad \big[1 - \epsilon(1-\lambda)\big]\ddot{X} + \omega^2 (1+\lambda\epsilon) X + \cancel{\lambda \frac{\epsilon^2}{\omega^2} X^{(4)}} - \cancel{\frac{\epsilon^2}{\omega^4} X^{(6)}} = 0$$

Ambiguity in truncation ⟶ energy non-conservation

Method of perturbative constraints 1

Jaen, Llosa, Molina, *A Reduction of order two for infinite order lagrangians,* Phys. Rev. D 34 (1986), 2302

Eliezer, Woodard, *The Problem of Nonlocality in String Theory,* Nucl. Phys. B 325 (1989), 389

Glavan, *Perturbative reduction of derivative order in EFT,* JHEP 02 (2018) 136, arXiv:1710.01562 [hep-th]

Off-shell reduction of derivative order (at the level of action)

— Hamiltonian formulation necessary

Additional ingredient: require analyticity in ϵ

$\longrightarrow$ this removes spurious degrees of freedom

$\longrightarrow$ in practice: forbidden to divide by ϵ

Method of perturbative constraints 2

Starting action:

$$S[X] = \int dt \left[\frac{\dot{X}^2}{2} - U(X) + \frac{\epsilon}{2} \ddot{X}^2 \right]$$

Step 1: promote time derivatives to independent velocity fields:

$$\dot{X} \longrightarrow V\,, \qquad \ddot{X} \longrightarrow \dot{V} \longrightarrow A$$

and introduce Largange multipliers π and ρ for on-shell equivalence

$$\mathcal{S}[X, \pi, V, \rho, A] = \int dt \left[\frac{V^2}{2} - U(X) + \frac{\epsilon}{2} A^2 + \pi\big(\dot{X} - V\big) + \rho\big(\dot{V} - A\big) \right]$$

Method of perturbative constraints 2

$$\mathcal{S}[X,\pi,V,\rho,A] = \int dt\left[\frac{V^2}{2} - U(X) + \frac{\epsilon}{2}A^2 + \pi\big(\dot{X} - V\big) + \rho\big(\dot{V} - A\big)\right]$$

Step 2: Consider equation for auxiliary field:

$$\frac{\delta\mathcal{S}}{\delta A} = \epsilon A - \rho \approx 0$$

Solve for $A = \rho/\epsilon$ and plug it into the extended action to obtain the canonical action (Ostrogradsky's construction)

$$\mathscr{S}[X,\pi,V,\rho] = \int dt\left\{\pi\dot{X} + \rho\dot{V} + \left[-\frac{\rho^2}{2\epsilon} - \pi V + \frac{V^2}{2} - U(X)\right]\right\}$$

Method of perturbative constraints 2

$$\mathcal{S}[X,\pi,V,\rho,A] = \int dt \left[\frac{V^2}{2} - U(X) + \frac{\epsilon}{2}A^2 + \pi\big(\dot{X} - V\big) + \rho\big(\dot{V} - A\big)\right]$$

Step 2: Consider equation for auxiliary field:

$$\frac{\delta\mathcal{S}}{\delta A} = \epsilon A - \rho \approx 0$$

Solve for $A = \rho/\epsilon$ and plug it into the extended action to obtain the canonical action (Ostrogradsky's construction)

$\Longrightarrow$ Appearance of perturbative second-class constraint

$\Longrightarrow$ Dirac-Bergmann algorithm results in reduced action:

$$\mathscr{S}_{\text{red}}[X,\pi] = \int dt \left[\pi\dot{X} - \left(\frac{\pi^2}{2}\left[1 - 2\epsilon U''(X)\right] + U(X) + \frac{\epsilon}{2}\left[U'(X)\right]^2\right)\right]$$

Method of perturbative constraints 3

BUT, it is forbidden to divide by ϵ

$$\mathcal{S}[X,\pi,V,\rho,A] = \int dt\left[\pi\dot{X}+\rho\dot{V}-\left(\pi V-\frac{V^2}{2}+U(X)-\frac{\epsilon}{2}A^2\right)-\rho A\right]$$

$\Longrightarrow$ **appearance of effective constraint**

$$\epsilon A-\rho\approx 0$$

Why constraint? — truncating action to order ϵ means:

$$\epsilon A-\rho\approx 0\ \Big/\times\epsilon \quad\Longrightarrow\quad \boxed{\epsilon\rho\approx 0}$$

$\longrightarrow$ starting point for eliminating degrees of freedom

Conservation of primary constraint generates secondary constraint:

$$\epsilon\dot{\rho}\approx\epsilon(-\pi+V)\approx 0$$

Conservation of secondary constraint determies the auxiliary field:

$$\epsilon(-\dot{\pi}+\dot{V})\approx\epsilon\big[U'(X)+A\big]\approx 0$$

Method of perturbative constraints 4

$$\mathcal{S}[X,\pi,V,\rho,A]=\int dt\left[\pi\dot{X}+\rho\dot{V}-\left(\pi V-\frac{V^2}{2}+U(X)-\frac{\epsilon}{2}A^2\right)-\rho A\right]$$

$$\epsilon\rho\approx 0\,,\qquad \epsilon V\approx\epsilon\pi\,,\qquad \epsilon A\approx-\epsilon U'(X)$$

Equation for A becomes a constraint also at subleading order

$$\frac{\delta\mathcal{S}}{\delta A}=\epsilon A-\rho\approx-\epsilon U'(X)-\rho\approx 0\,.$$

Its conservation generates a secondary constraint:

$$\frac{d}{dt}\left[\epsilon U'(X)+\rho\right]\approx\epsilon U''(X)V-\pi+V\approx-\pi\left[1-\epsilon U''(X)\right]+V\approx 0$$

Its conservation fixes the auxiliary field A

$$\frac{d}{dt}\left(-\pi\left[1-\epsilon U''(X)\right]+V\right)\approx U'(X)\left[1-\epsilon U''(X)\right]+\epsilon U'''(X)\pi^2+A$$

Method of perturbative constraints 5

Reduced canonical action contains only one degree of freedom

$$\mathscr{S}_{\mathrm{red}}[X,\pi] = \int dt \left[\pi \left[(1 + \epsilon U''(X) \right] \dot{X} \right. \\ \left. - \left(\frac{\pi^2}{2} + U(X) + \frac{\epsilon}{2} \left[U'(X) \right]^2 \right) + \mathcal{O}(\epsilon^2) \right]$$

We have effectively solved for *second-class* constraints — appearance of non-canonical symplectic term (Dirac brackets)

Shifting momentum: $\pi \to \pi\left[1 - \epsilon U''(X)\right]$, canonicalizes brackets

$$\mathscr{S}_{\mathrm{red}}[X,\pi] = \int dt \left[\pi \dot{X} - \left(\frac{\pi^2}{2} \left[1 - 2\epsilon U''(X) \right] + U(X) + \frac{\epsilon}{2} \left[U'(X) \right]^2 \right) \right]$$

Method of perturbative constraints 6

Solving for momentum on-shell

$$\pi \approx \left[1 + 2\epsilon U''(X)\right]\dot{X}$$

gives the reduced Lagrangian action

$$S_{\mathrm{red}}[X] = \int dt \left[\frac{1}{2}\left[1 + 2\epsilon U''(X)\right]\dot{X}^2 - U(X) - \frac{\epsilon}{2}\left[U'(X)\right]^2\right]$$

Varying this action gives the equation of motion

$$\left[1 + 2\epsilon U''(X)\right]\ddot{X} + \epsilon U'''\dot{X}^2 + U'(X)\left[1 + \epsilon U''(X)\right] = 0$$

Thus the only equation descending from the action is the one for $\lambda = -1$:

$$\left[1 + (1-\lambda)\epsilon U''(X)\right]\ddot{X} + \epsilon\dot{X}^2 U'''(X) + U'(X)\left[1 - \lambda\epsilon U''(X)\right] = 0$$

EFT of gravity: reduction 1

ADM Hamiltonian formulation of general relativity

$$\mathscr{S}[N, N_i, h_{ij}, \pi^{ij}] = \int d^4x \left[\pi^{ij}\dot{h}_{ij} - N\mathcal{H} - N_i\mathcal{H}^i\right]$$

Hamiltonian constraint & momentum constraint:

$$\mathcal{H} = -\sqrt{h}\left[\frac{\mathscr{Q} - 2\Lambda}{\kappa^2}\right], \qquad \mathcal{H}^i = -\sqrt{h}\left[\frac{2\mathscr{L}^i}{\kappa^2}\right]$$

where

$$\mathscr{K}_{ij} = -\kappa^2\left[\frac{\pi_{ij}}{\sqrt{h}} - \frac{h_{ij}}{2}\frac{\pi}{\sqrt{h}}\right], \qquad \mathscr{K} = \mathscr{K}^i{}_i$$

$$\mathscr{Q}^i{}_j = \mathscr{K}\,\mathscr{K}^i{}_j - \mathscr{K}^i{}_k\mathscr{K}^k{}_j + \mathcal{R}^i{}_j\,, \qquad \mathscr{Q} = \mathscr{Q}^i{}_j$$

$$\mathscr{L}^i{}_{jk} = 2\nabla_{[k}\mathscr{K}_{j]}{}^i\,, \qquad \mathscr{L}_i = \mathscr{L}^j{}_{ji}$$

First-class constraints:

$$\left\{\mathcal{H}(t,\vec{x}), \mathcal{H}(t,\vec{x}\,')\right\} \approx 0\,, \qquad \left\{\mathcal{H}(t,\vec{x}), \mathcal{H}^i(t,\vec{x}\,')\right\} \approx 0\,,$$

$$\left\{\mathcal{H}^i(t,\vec{x}), \mathcal{H}^j(t,\vec{x}\,')\right\} \approx 0$$

EFT of gravity: reduction 2

Reduced canonical action for general relativity + Riemann-cubed

$$\mathscr{S}_{\mathrm{red}}[N, N_i, h_{ij}, \pi^{ij}] = \int d^4x \left[\pi^{ij}\dot{h}_{ij} - N\mathcal{H} - N_i\mathcal{H}^i\right]$$

Hamiltonian constraint & momentum constraint:

$$\mathcal{H} = -\sqrt{h}\Bigg[\frac{\mathscr{Q} - 2\Lambda}{\kappa^2} - 8\alpha\kappa^2\mathscr{Q}^i{}_j\left(2\mathscr{Q}^j{}_k\mathscr{Q}^k{}_i - 3\mathscr{L}^j{}_{kl}\mathscr{L}_i{}^{kl}\right) \\ + 24\alpha\kappa^2\Lambda\left(2\mathscr{Q}^i{}_j\mathscr{Q}^j{}_i - 2\Lambda^2 - \mathscr{L}_k{}^{ij}\mathscr{L}^k{}_{ij}\right)\Bigg],$$

$$\mathcal{H}^i = -\sqrt{h}\left[\frac{2\mathscr{L}^i}{\kappa^2}\right]$$

First-class constraints:

$$\left\{\mathcal{H}(t,\vec{x}), \mathcal{H}(t,\vec{x}\,')\right\} \approx 0 + \mathcal{O}(\alpha^2)\,, \qquad \left\{\mathcal{H}(t,\vec{x}), \mathcal{H}^i(t,\vec{x}\,')\right\} \approx 0\,,$$

$$\left\{\mathcal{H}^i(t,\vec{x}), \mathcal{H}^j(t,\vec{x}\,')\right\} \approx 0\,.$$

EFT of gravity: reduction 3

Reduced theory has the general ADM structure of general relativity, but modified Hamiltonian constraint

$\Longrightarrow$ **Minimally modified gravity**

— theories of the massless spin-2 graviton with a preferred frame

Lin, Mukohyama, *A Class of Minimally Modified Gravity Theories*, JCAP 10 (2017), 033 [arXiv:1708.03757 [gr-qc]]

Mukohyama, Noui, *Minimally Modified Gravity: a Hamiltonian Construction*, JCAP 07 (2019), 049 [arXiv:1905.02000 [gr-qc]]

Type-II class: **not** equivalent to general relativity upon field redefinition.

Conclusions

- Spurious degrees of freedom can be removed from EFT of gravity
- Minimally modified gravity can be seen as an EFT of gravity
- Allows for analyzing effects of UV physics
- Extension to all orders?
- Renormalizability?