

Analytic pressure for products of matrices

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Following Landau's approach to phase transitions, they are related to the regularity of some pressure function. We will present some results in this perspective for models defined by products of matrices.

For these models, we will explain how we establish the real-analyticity of the pressure function P under some irreducibility and contractivity assumptions. The proof is based on the fact that for any $q \in \mathbb{R}_+^*$, $P(q)$ is the logarithm of the spectral radius of a bounded endomorphism Γ_q . We further demonstrate that $(\Gamma_q)_q$ forms an analytic family of quasi-compact operators. Leveraging holomorphic functional calculus, we derive the analyticity of the pressure.

First, using Doob's relativisation procedure we construct Markov operators Π_q . Second, we show that for every $q \in \mathbb{R}_+^*$, Π_q is quasi-compact, ensuring the quasi-compactness of Γ_q . Finally we prove that the map $q \mapsto \Gamma_q$ is analytic.

The key unlocking the proof of quasi-compactness lies in a Doeblin-Fortet inequality. The proof of this inequality is inspired by a new approach developed for quantum trajectories in [1]. It is based on the study of a Radon-Nykodim derivative. This new method, leads us to extend some results by Guivarc'h and Le Page in [2]. In particular we do not require that our matrices are invertible or strongly irreducible.

[1] T. Benoist, A. Hautecoeur, C. Pellegrini, « Quantum Trajectories, Spectral Gap, Quasi-compactness & Limit Theorems », <https://arxiv.org/pdf/2402.03879.pdf>

[2] Y. Guivarch, E. Le Page, « Spectral gap properties for linear random walks and Pareto's asymptotics for affine stochastic recursions », Ann. Inst. H. Poincaré Probab. Statist. (2016)

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