

# Measure estimation on a manifold explored by a symmetric diffusion process

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We explore a compact connected manifold  $\mathcal{M}$  with a diffusion  $(X_t)_{t \geq 0}$  admitting a stationary measure  $\mu$ . Such a process can be obtained as the limit of random walks visiting large sample of points drawn i.i.d. from  $\mu$ . From the observation of a sample path of the diffusion between times 0 and  $T$ , we can approximate the unknown probability measure  $\mu$  by the occupation measure of  $(X_t)_{t \in [0, T]}$ . Smoothing this measure by convolution with a kernel improves the convergence rates, in Wasserstein distance, that were established by Wang and Zhu (2023).

More precisely, we give theorems for the convergence speed in Wasserstein distance for the invariant density estimator  $p_{T,h}$

$p_{T,h}(y) := \frac{1}{T} \int_0^T K_h(X_t, y) dt$ , with  $K_h(x, y) := \eta_h(x)^{-1} K\left(\frac{\rho(x,y)}{h}\right)$  and  $\eta_h(x) = \int_{\mathcal{M}} K\left(\frac{d(x,y)}{h}\right) dy$ , where  $K : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$  is a kernel function.

We also discuss the dependence of the convergence speed on the order of  $K$  and the regularity of  $p$  and  $\mathcal{M}$ .

## References

Wang and Zhu (2023): Limit theorems in Wasserstein distance for empirical measures of diffusion processes on Riemannian manifolds, Ann. Inst. Henri Poincaré, Probab. Stat. 59, 1 (2023), 437–475.

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