We consider the equivelence relations arising from $GL(2,\mathbb{Z})$-action, $SL(2,\mathbb{Z})$-action, and continued fractions. Real numbers x and y in the unit interval are said to be continued-fraction-equivalent if there exist nonnegative integers n and m such that $T^n(x) = T^m(y)$ where T is the continued fraction map (Gauss map). It is wellknown that this relation holds if and only if there exists a matrix $A \in GL(2,\mathbb{Z})$ such that $y = Ax$. Now we introduce the notion of α-continued fractions, $0 < \alpha < 1$. Then it is shown that the equivalence relation associated to α-continued fractions for a fixed α is not the same as $GL(2,\mathbb{Z})$-eqivalence if $\alpha < \sqrt{2} - 1$. We discuss the detail of the relation between these equivalence relations.