Programme of the Conference

New Trends in Geometry, Combinatorics and Mathematical Physics

`https://indico.math.cnrs.fr/event/11259/overview`

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-9:50	Zorich	Thomas	Khesin	Shapiro, A	Felikson
10:00-10:50	Veselov	Tabachnikov	Kirillov	Alekseev	Ustinov
11:00-11:25	Coffee Break				
11:30-12:20	Chapoton	Schwartz	Gurevich	Conley	Shapiro, M
12:30-14:20	Lunch Break				
14:30-16:00	Free time / Time for discussions / Posters				
16:00-16:30	Posters Display				
16:30-17:20	Fomin	Ovenhouse		Millionschikov	
17:30-18:20	Karpenkov	Shemyakova		Williams	
18:30-19:20	Welcoming Apéritif				
19:15-	Dinner				

Titles and Abstracts of the talks

Anton Alexeev
Title: Hamiltonian Virasoro actions on Teichmüller spaces
Abstract: In this talk, we explain that (infinite dimensional) Teichmüller spaces associated to hyperbolic surfaces with absolute boundary carry Hamiltonian actions of the Virasoro algebra. Our study is motivated on the one hand by the work of Saad-Shenker-Stanford on Jackiw-Teitelboim gravity, and on the other hand by the theory of Hamiltonian loop group actions on moduli spaces of flat connections. The talk is based on a joint work with Eckhard Meinrenken, see arXiv:2401.03029.

Frédéric Chapoton
Title: Representations of Posets, product formulas and Fractional Calabi-Yau Categories
Abstract: The notion of fractional Calabi-Yau category, introduced by Kontsevich, appears both in algebraic geometry and representation theory. In algebraic geometry, they can be found as semi-orthogonal pieces in derived categories of Fano varieties, as shown by Kuznetsov. In representation theory, simple examples are derived categories of representations of quivers of ADE type.

Charles Conley
Title: Extensions of tensor modules
Abstract: We will survey some results and questions concerning extensions of tensor modules of Lie algebras of polynomial vector fields on Euclidean spaces. We will also discuss the associated cohomology rings and annihilators. In the case of odd dimension, we will consider the situation for the contact subalgebra.

Anna Felikson
Title: Hyperbolic geometry of friezes
Abstract: We will discuss hyperbolic geometry of friezes from surfaces. This is a joint work with Pavel Tumarkin.

Sergey Fomin
Title: Expressive curves
Abstract: A real plane algebraic curve C is called expressive if its defining polynomial has the smallest number of critical points allowed by the topology of the set of real points of C. We give a necessary and sufficient criterion for expressivity (subject to a mild technical condition), describe several constructions that produce expressive curves, and relate their study to the combinatorics of plabic graphs, their quivers, and links. This is joint work with E. Shustin.

Dimitri Gurevich
Title: Reflection Equation Algebras vs Quantum Groups
Abstract: I plan to compare the roles of Reflection Equation Algebras and Quantum Groups in different problems of Combinatorics and Mathematical Physics. A special attention will be paid to a Quantum version of the Capelli identity.

Oleg Karpenkov
Title: Geometry of multidimensional Farey summation algorithm and frieze patterns
Abstract: In this talk we present a new geometric approach to subtractive continued fraction algorithms in high dimensions. We adapt a version of Farey summation to the geometric techniques proposed by F. Klein in 1895. More specifically we introduce Farey polyhedra and their sails that generalise respectively Klein polyhedra and their sails, and show similar duality properties of the Farey sail integer invariants. The construction of Farey sails is based on the multidimensional generalisation of the Farey tessellation provided by a modification of the continued fraction algorithm introduced by R. W. J. Meester. We classify Farey polyhedra in the combinatorial terms of prismatic diagrams. Prismatic diagrams extend boat polygons introduced by S. Morier-Genoud and V. Ovsienko in the two-dimensional case. As one of the applications of the new theory we get a multidimensional version of Conway-Coxeter frieze patterns. We show that multidimensional frieze patterns satisfy generalised Ptolemy relations. This is joint work with Matty van Son.

Alexandre Kirillov
Title: Groups G_n
Abstract: In this talk I want to speak about the family of finite groups, introduced in our forthcoming joint paper with D.B.Fuchs. For these groups the orbit method gives the simple answers to all questions of representation theory.

Boris Khesin
Title: Helicity and domino tilings
Abstract: We consider domino tilings of $3D$ cubiculated regions. The tilings have two invariants, flux and twist, often integer-valued, which are given in purely combinatorial terms. These invariants allow one to classify the tilings with respect to certain elementary moves, flips and trits. In the talk we present a construction associating a divergence-free vector field to any domino tiling, such that the flux of the tiling can be interpreted as the (relative) rotation class of the field, while the twist of the tiling is the relative helicity of the vector field. This is a joint work with Nicolau Saldanha.

Dmitry Millionshchikov
Title: Positively graded (super) Lie algebras
Abstract: We discuss narrow in the sense of Zelmanov and Shalev positively graded Lie (super)algebras. They appear in different problems of geometry, topology and math physics. The talk will focus on different classification results. One of them concerns two generated positively graded Lie super algebras. In particular we construct two families of positively graded Lie super algebras, the first one contains the positive part R^+ of the Ramond algebra, while the second one contains the positive part NS^+ of the Neveu-Schwarz algebra. This theorem is a super analogue of Benoist's theorem on defining the positive part of the Witt algebra by two generators and two relations.

[1] Millionshchikov D.V., Naturally graded Lie algebras of slow growth, Sb. Math. 2019. V. 210 N 6, P. 862–909

[2] Millionshchikov D.V., Pokrovsky F.I., Ramond and Neveu-Schwarz Algebras and Narrow Lie Superalgebras// Doklady Mathematics. 2024. V. 109, P. 30–32

[3] Shalev A., Zelmanov E.I. Narrow algebras and groups // J. of Math. Sciences. 1999. V. 93, N 6. P. 951–963.

Nicolas Ovenhouse
Title: Higher q-Continued Fractions
Abstract: For certain types of posets, we study the ratios the of their P-partition generating functions, which are generalizations of Morier-Genoud and Ovsienko's q-rational numbers. They satisfy some of the same nice properties, such as a "stabilization phenomenon". When $q = 1$, they give a higher-dimensional continued fraction recurrence which enumerates various combinatorial objects, including P-partitions, lattice paths, and n-dimer covers.

Richard Schwartz
Title: The optimal paper Moebius band
Abstract: Imagine you have a $1 \times L$ strip of paper and you twist it smoothly in space to make a paper Moebius band If L is large this is easy to do and if L is small it is impossible. What is the cutoff? In this talk I will answer this question, proving that you can do it if and only if $L > \sqrt(3)$. B. Halpern and C. Weaver conjectured this answer in 1977.

Alexander Shapiro
Title: Algebraic modular functor
Abstract: It is known from works of Fock and Goncharov, that moduli spaces of local systems provide examples of cluster Poisson varieties. The latter admit natural quantization, defined as corresponding quantum universally Laurent algebras. In my talk I will explain how these universally Laurent algebras behave under cutting and gluing of surfaces.

Michael Shapiro
Title: Cluster superalgebras.
Abstract: We will discuss another generalization of cluster superalgebras. This is a joint work in progress with V.Ovsienko and N.Ovenhouse.

Ekaterina Shemyakova
Title: On differential operators generating higher bracket
Abstract: On supermanifolds, a Poisson structure can be either even, corresponding to a Poisson bivector, or odd, corresponding to an odd Hamiltonian quadratic in momenta. Odd bracket can also be defined by an odd second-order differential operator that squares to zero, known as a "BV-type" operator.

Higher analog, P_∞ or S_∞, is a series of brackets of alternating or odd parities, respectively, that satisfy relations that are higher homotopy analog of the Jacobi identity. These brackets are generated by arbitrary multivector fields or Hamiltonians. However, generating an S_∞-structure by a higher-order differential operator is not straightforward, as this would violate the Leibniz identities. Olga Kravchenko studied these structures, and Ted Voronov addressed the Leibniz identity issue by introducing formal $\hbar$-differential operators. In this talk, we revisit the construction of an $\hbar$-differential operator that generates higher Koszul brackets.

It is well known that a chain map between the de Rham and Poisson complexes on a Poisson manifold at the same time maps the Koszul bracket of differential forms to the Schouten bracket of multivector fields. In the P_∞-case, however, the chain map is also known, but it does not connect the corresponding bracket structures. An L_∞-morphism from the higher Koszul brackets to the Schouten bracket has been constructed recently, using Voronov's thick morphism technique. In this talk, we will show how to lift this morphism to the level of operators.

The talk is partly based on joint work with Yagmur Yilmaz.

Sergei Tabachnikov
Title: 4-point theorems, a nostalgic trip
Abstract: I shall present some recent results involving number 4, some in the spirit of the "Last Geometric Statement of Jacobi" that the conjugate locus of a non-umbilic point on a triaxial ellipsoid has exactly four cusps, and some related to the classic 4-vertex theorem that the curvature of a plane oval has at least four critical points.

Alexander Thomas
Title: Fock bundles and Hitchin components
Abstract: Starting from the Farey triangulation, which is the starting point both for q-rational numbers and for cluster X-coordinates on Teichmüller space, we dive into character varieties with their natural mapping class group symmetry. The classical complex analytic approach via Higgs bundles makes this symmetry invisible. We describe a new approach, using so-called Fock bundles, which allow to parametrize a neighborhood of the Fuchsian locus inside the Hitchin component in a MCG-invariant way. Conjecturally, the approach describes the whole Hitchin component. Joint work with Georgios Kydonakis and Charlie Reid.

Alexey Ustinov
Title: On the periodicity of Somos sequences
Abstract: For integer $k \geq 4$ Somos–k sequence is a sequence generated by quadratic recurrence relation of the form

$$s_{n+k}s_n = \sum_{j=1}^{[k/2]} \alpha_j s_{n+k-j}s_{n+j},$$

where α_j are constants and $s_0, \ldots, s_{k-1}$ are initial data. Among them exist a class of sequences with many properties.

They are *finite rank sequences*. The sequence $\{s_n\}_{n=-\infty}^{\infty}$ has a (finite) rank r if maximal rank of two infinite matices

$$(s_{m+n}s_{m-n})\Big|_{m,n=-\infty}^{\infty}, \qquad (s_{m+n+1}s_{m-n})\Big|_{m,n=-\infty}^{\infty}$$

is r. If $r = 2$ then general term of Somos sequence can be expressed in terms of elliptic function. One can consider a general finite rank sequence as a sequence admitting more complicated addition theorem.

Presumably the following properties are more or less equivalent: finitness of the rank, Laurent phenomenon, periodicity (mod N), solvability in theta-functions. The talk will be mostly devoted to periodicity (mod N) of general finite rank sequences.

Alexander Veselov
Title: Quantum Kronecker fractions
Abstract: Few years ago, Sophie Morier-Genoud and Valentin Ovsienko introduced an interesting quantization of the real numbers as certain power series in quantization parameter. It is known now that the quantum golden ratio has the minimal radius of convergence among the quantum real numbers.

I will present some results about quantum rational numbers having the maximal radius of convergence (equal to 1), which we call Kronecker fractions. We discovered several infinite series of such fractions and found all of them with the denominator less than 1000. The talk is based on the ongoing joint work with Sam Evans and Brian Winn.

Harold Williams
Title: Homological combinatorics of Lagrangian coamoebae
Abstract: The amoeba and coamoeba of a subvariety $Z \subset (\mathbb{C}^\times)^n$ are its images under the projections to $\mathbb{R}^n$ and T^n, respectively. In this talk we discuss joint work with Chris Kuo studying the coamoebae of Lagrangian submanifolds of $(\mathbb{C}^\times)^n$, specifically how the combinatorics of their degenerations encodes the homological algebra of mirror coherent sheaves. Concretely, we associate to a free resolution $F^\bullet$ of a coherent sheaf on $(\mathbb{C}^\times)^n$ a tropical Lagrangian coamoeba $T(F^\bullet)$, a certain simplicial complex in T^n. We show that the discrete information in $F^\bullet$ can be recovered from $T(F^\bullet)$ in common situations, and that in general there is a constructible sheaf supported on $T(F^\bullet)$ which is mirror to the coherent sheaf in the relevant sense. The resulting interplay between coherent sheaves on $(\mathbb{C}^\times)^n$ and simplicial complexes in T^n provides a higher-dimensional generalization of the spectral theory of dimer models in T^2, as well as a symplectic counterpart to the theory of brane brick models.

Anton Zorich
Title: Random square-tiled surfaces and random multicurves in large genus (after joint works with V. Delecroix, E. Goujard and P. Zograf)
Abstract: Moduli spaces of Riemann surfaces and related moduli spaces of quadratic differentials are parameterized by a genus g of the surface. Considering all associated hyperbolic (respectively flat) metrics at once, one observes more and more sophisticated diversity of geometric properties when the genus grows. However, most metrics, on the contrary, progressively share certain rules. Here the notion of "most of" has explicit quantitative meaning, for example, in terms of the Weil-Petersson measure. I will present some of these recently discovered large genus universality phenomena.

I will use a count of metric ribbon graphs (after Kontsevich and Norbury) to express Masur-Veech volumes of moduli space of quadratic differentials through Witten-Kontsevich correlators. Then I will present Mirzakhani's count of simple closed geodesics on hyperbolic surfaces. We will proceed with description of random geodesic multicurves and of random square-tiled surfaces in large genus. I will conclude with a beautiful universal asymptotic formula for the Witten-Kontsevich correlators predicted by Delecroix, Goujard, Zograf and myself and recently proved by Amol Aggarwal.

Posters information

Posters will be displayed starting from Monday 4pm and till the end of the week.
Take some time to have a look at them during the coffee breaks and free time, and to talk with their authors.

Amanda Burcroff
Title: Cluster Scattering Diagrams and Generalized Positivity

Perrine Jouteur
Title: Burau representation of B_4 and q-deformed rational projective plane.

Justin Lasker
Title: The First and Second Derivatives of the q-Rationals.

Ritesh Kumar Pandey
Title: Classification of Irreducible Harish-Chandra Modules for Map-Extended Witt Algebras: